Multi-constrains logistic growth model: An application to tourism carrying capacity

ABSTRACT. This original research article is intended to develop logistic growth models addressing multiple constrains, as J-shape growth have no constrains, S-shape growth model have only one constrain i.e availability of food, but in businesses and economic growth models there are multiple constrains, this article aims to ...nd mathematical model of growth with multiple constrains.

1. Introduction

Growth of population of species, any business, economy of a country etc. has multiple constrains. There are some factor which favors the growth and some others restraints the growth. Like Curie-Weiss model of paramagnetic substances if growth succeeds then it further align some factors to it's favour, some remain una¤ected and there are some which restricts or diminishes the growth. How the growth aligns factors favors to boost further growth is called susceptibility χ,

$$\mathsf{M} = \chi H_{effective},$$

here M is magnetization, and H is the magnetizing field, in paradigm of Curie-Weiss

$$H_{effective} = H_{applied} + \alpha M.$$

Where α, is multiplier called coupling constant by which magnetization enhances magnetizing ...eld, unlike the Curie-Weiss model of paramagnetism in growth of business a particular factor may favour to some threshold value thereafter it opposes growth, as growth due to favouring factor it reaches to Pareto optimal ([2]) point further increasing it will cause decline of growth, whereas in Curie-Weiss model it attains saturation. Here we may call the threshold value after which a growth favouring factor starts opposing growth as carrying capacity, in this article multi-constrains model is discussed with respect to tourism business and it's growth, tourism as a products is carved out of integrating various services viz. travel, accommodation, ...tness, cultural, adventurism, health, eco-tourism, pilgrimage sites etc. Through this model stages of growth of tourism and all its integrands could be e¢ ciently and e¤ectively managed and risk could be minimized, and welfare of hosts could be assured. It analyses the saturation point i.e. the maximum growth

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This paper is in ...nal form and no version of it will be submitted for publication elsewhere.

of tourism until physical carrying capacity is being achieved. This model also aims to predict that tourism gain could be realized and sustained through monetary feedback mechanism responsive to coupling effects, and sustainable development of natural environment.

Carrying Capacity" is defined by the World Tourism Organization as "The maximum number of people that may visit a tourist destination at the same time, without causing destruction of the physical, economic, sociocultural environment and an unacceptable decrease in the quality of visitors' satisfaction". Whereas Middleton and Hawkins Chamberlain (1997) define it as "the level of human activity an area can accommodate without the area deteriorating, the resident community being adversely affected or the quality of visitors experience declining"([**1**]) what both these definitions pick up on is that the carrying capacity is the point at which a destination or attraction drifts from Pareto efficient equilibrium ([**2**]) as a result of the number of visitors. Number of tourists is restricted to a maximum of 15000 visitors per day by Govt. of Uttarakhand restricts, to visit to Badrinath shrine to sustain fragile ecosystem and ensure safety of tourists in case of cloud bursts ([**4**]), locals of Joshimath municipal council protested to halt tourism activity to zero defying restriction of the govt. as land slide and subduction continued around the highway passing though Joshimath([**5**]), at the same time general secretary of Uttarakhand Char Dham Teerth Purohit Mahapanchayat, the body representing the shrine town's priests vehemently pursued to lift the restriction on the number of visitors in order to have sustainable living out of their investments in tourism products ([**6**], [**7**]) .Government of Peru limits the number of hikers on the Inca Trail to 500 per day, because geologists advised to limit it based on physical carrying capacity of the Peru trail ([**3**]).

The term "carrying capacity" are uncertain, with sources variously stating that it was originally used "in the context of international shipping" in the 1840s,([**8**], [**9**]) or that it was first used during 19th-century laboratory experiments with micro-organisms([**10**]). In the terms of Mathematics the specific reason why a population stops growing is known as a limiting or regulating factor ([**11**])

The difference between the birth rate and the death rate is the natural increase. If the population of a given organism is below the carrying capacity of a given environment, this environment could support a positive natural increase; should it find itself above that threshold the population typically decreases([**12**]). Thus, the carrying capacity is the maximum number of individuals of a species that an environment can support in long run.([**13**])

Population size decreases above carrying capacity due to a range of factors depending on the species concerned, but can include insufficient space, food supply, or sunlight. The carrying capacity of an environment varies for different species. In the standard ecological algebra as illustrated in the simplified Verhulst model([**14**]) of population dynamics, carrying capacity is represented by the constant K

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{1.1}$$

where, N is the population size, r is the intrinsic rate of natural increase
K is the carrying capacity of the local environment, and
dN/dt, the derivative of $N$with respect to time t, is the rate of change in population with time. Equation (1.1) is the one of the simplest model, representing a

ecology's carrying capacity for a population of a particular species, in case of tourist destination's carrying capacity must be figured out not only for long run ([**13**]) but also for short run. And a destination's carrying capacity must account all the dimensions viz., social carrying capacity ([**18**]) economic carrying capacity ([**17**]), physical carrying capacity ([**16**]), biophysical carrying capacity ([**19**]). Equation (1.1) species is consumer while ecology is feeder, but in case of tourist destination, all the stake holders are producer of goods and services related to tourism and are also consumer thereof, except the visitors which are merely consumer. Therefore in order to determine carrying capacity of a tourist destination in short and long run we must find the Pareto efficient equilibrium for all stake holder keeping in mind physical carrying capacity of natural environment of the specific destination.

Literature Review:

Equation (1.1) is the one of the simplest model, representing a ecology's carrying capacity for a population of a particular species, in case of tourist destination's carrying capacity must be figured out not only for long run ([**13**]) but also for short run. And a destination's carrying capacity must account all the dimensions viz. physical carrying capacity ([**16**]), economic carrying capacity ([**17**]), social carrying capacity ([**18**]), biophysical carrying capacity ([**19**]). Equation (1.1) species is consumer while ecology is provider without getting anything from the species, but in case of tourist destination, all the stake holders are producer of goods and services related to tourism and are also consumer thereof, except the visitors which are merely consumer. Therefore in order to determine carrying capacity of a tourist destination in short and long run we must find the Pareto efficient equilibrium for all stake holder keeping in mind physical carrying capacity of natural environment of the specific destination. Various researchers have worked on assessing carrying Capacity of a destination could be find in ([**21**], [**22**], [**23**], [**24**], [**25**], [**26**], [**27**], [**28**], [**29**], [**30**], [**31**], [**32**], [**33**], [**34**], [**35**]) Research work carried in these referenced articles is basically empirical Research or review works based on secondary data for example no of visitors to Badrinath shrine from 1992-2024 is plotted in plot no-1. But none have put forward any theory to assess carrying capacity w.r.t. Pareto optimally of tourism.

Research work carried in ([**21**] *to* [**35**]) focuses on single components like availability of hotels, climatic conditions, local resistance, etc.

Now to find interlinking between various factors affecting tourism in totality, for example in ([**21**]) study developed a novel methodology comprising five interlinked steps for assessing carrying capacity in tourism destinations. The methodology was also pilot-tested in four representative case studies: one Slovenian/Italian cross-border destination and three Slovenian destinations. It's application by Bled, Divāca, Brēzice, and Nova Gorica – Gorizia, it was possible to demonstrate the viability of the methodology for destinations that are very different in terms of territorial context and degree of tourism development, and therefore also in terms of carrying capacity.

Findings and Discussion:

Data Collection and Analysis: Secondary data available ([**20**]), for occupancy, availability of hotels, no of visitors, seasonal variation in no of visitors etc. Theoretical interlinking of various parameters affecting tourism is inevitable to understand

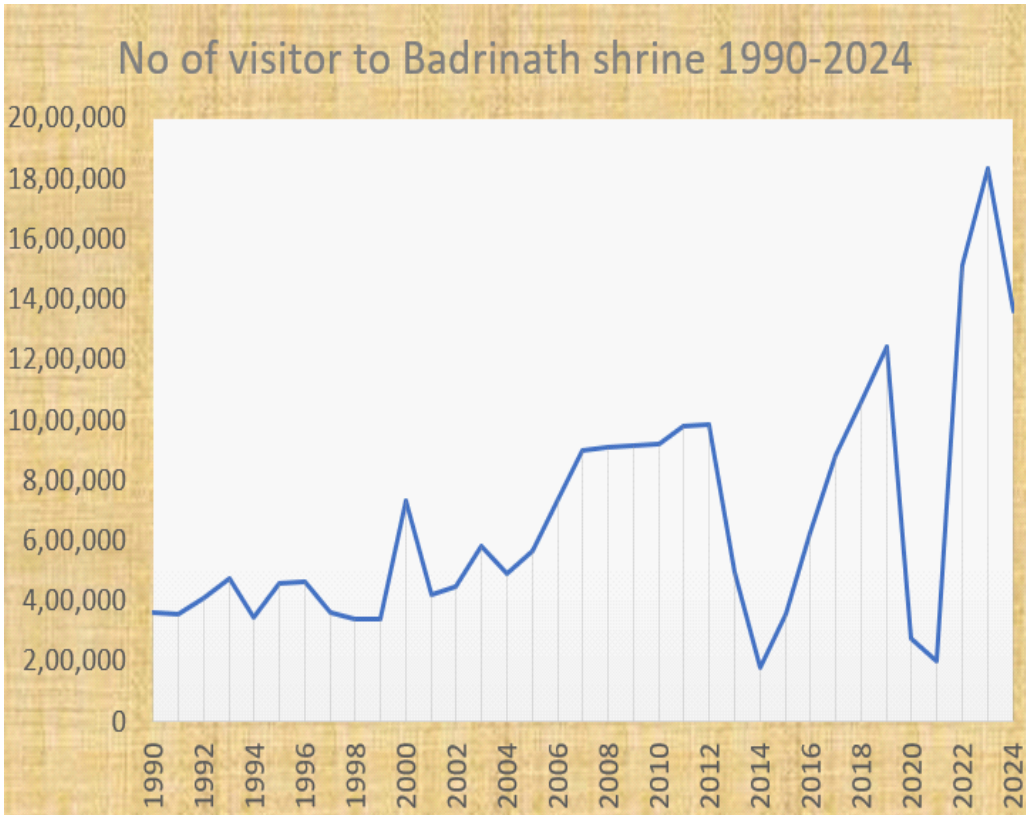

FIGURE 1. .

the exact course of planning and ensuring safety and security of visitors and residents of the tourist place. For logistic variable N

$$dN/dt = rN(1 - N/P)(1 - N/E)(1 - N/S)(1 - N/B) \tag{1.2}$$

Where, in this case N is the no of visitors at a tourist destination on time t, dN/dt is the rate of change in no of visitors at time t, P is physical carrying capacity , E is Economic carrying capacity, S is social carrying capacity, B is Biophysical carrying capacity, r is intrinsic constant for a tourist place it is measure of how susceptible/ hospitable the hosts are for visitors. Mathematical model for rate of change in no of visitors yields no of visitors at

time t is the solution of the equation given below:

$$\frac{dN}{N(1 - N/P)(1 - N/E)(1 - N/S)(1 - N/B)} = rdt$$

$$\frac{PESB}{N\,(P - N)\,(E - N)\,(S - N)\,(B - N)}dN = rdt$$

$$\left(\frac{C_1}{N} + \frac{C_2}{P - N} + \frac{C_3}{E - N} + \frac{C_4}{S - N} + \frac{C_5}{B - N}\right)dN = rdt$$

gives

$$C_1 = 1,\ C_2 = \frac{ESB}{(E-P)(S-P)(B-P)},$$
$$C_3 = \frac{PSB}{(P-E)(S-E)(B-E)}, C_4 = \frac{PEB}{(E-S)(P-S)(B-S)}$$
$$C_5 = \frac{PES}{(E-B)(P-B)(S-B)}.$$

Therefore solution of equation (1.2) is given by

$$\log N - \frac{ESB\log(P-N)}{(E-P)(S-P)(B-P)} - \frac{PSB\log(E-N)}{(P-E)(S-E)(B-E)}$$
$$-\frac{PEB\log(S-N)}{(E-S)(P-S)(B-S)} - \frac{PES\log(B-N)}{(E-B)(P-B)(S-B)} = rt + c, \tag{1.3}$$

where c is the constant of integration.

For a given set of $P, S, E, B, N(t)$ could be determined numerically as obvious from table no(**??**) growth of $N(t)$ is a stochastic growth, however trend could be predict if we know the solution of equation (1.3)

COROLLARY 1. *Consider two constrains model, where only constrains are physical carrying capacityP and economic carrying capacity E, then logistic growth is geven by*

$$\frac{N^{(E-P)}(E-N)^P}{(P-N)^E} = Ae^{rt(E-P)},$$

where N, is the logistic variable, t is the time, r is intrinsic parameter.

PROOF. Since only constrains areP, E,equation (1.2) could be written as

$$dN/dt = rN(1-N/P)(1-N/E). \tag{1.4}$$

Solve above equation to get

$$\int \frac{PEdN}{N(P-N)(E-N)} = \int rdt$$
$$(E-P)\log N - E\log(P-N) + P\log(E-N) = rt(E-P) + c,$$

since N, E, P all are functions of time let at $t = 0$, denote their values at N_0, E_0, P_0,then

$$c = \log N_0 - \frac{E_0}{E_0 - P_0}\log(P_0 - N_0) - \frac{P_0}{P_0 - E_0}\log(E_0 - N_0). \tag{1.5}$$

This gives

$$\frac{N^{(E-P)}(E-N)^P}{(P-N)^E} = Ae^{rt(E-P)}, \tag{1.6}$$

where $A = e^c$, assuming E, P nearly stable and take their integer values (for approximate trend) then equation (1.6) becomes a polynomial equation in variable N, of degree $E + P$ as given below

$$Ae^{rt(E-P)}(P-N)^E N^P - N^E(E-N)^P = 0 \tag{1.7}$$

Since $E + P$ could be very large so equation (1.7) could not be solved in general as polynomial equation of degree five or more have no formulae for their roots see abel impossibility theorem ([**37**],[**38**]) □

Limitations Theoretical solution of equation (1.3) is not possible as coefficients C_1, C_2, C_3, C_4 and C_5 need not be integers, even if all these coefficients are integers such that after taking exponential of both side of equation (1.3) we get polynomial equation of degree five or more it could not be solved in general as predicted by abel impossibility theorem ([**37**], [**38**]). Moreovere P, E, S and B are also time dependent therefore a smooth solution of equation (1.2) is turns stochastic i.e. i depends on probability of change in P, E, S and B, a subtle glimpse could be seen due to change in physical carrying capacity of Badrinath shrine could be seen in figure-1.

REMARK 1. *Nonetheless numerical solutions of equations of the form (1.3) and (1.7) always exists, however based on these equations one could approach non-linear regression model for such multiconstrained logisitc growth.*

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