Original Research Article

On SR-fuzzy transportation problem models and their solution algorithm

Abstract

Businesses have been focusing on the need to increase profit by reducing operational costs. In operations research, the transportation problem is essentially a logistics puzzle focused on moving goods from various suppliers to their final destinations as efficiently as possible. By balancing availability constraints with customer requirement, this model identifies the specific shipping routes that will either drive down total costs or boost the bottom line. In this paper, we explore a transportation model where availability, requirement, and shipping costs are all treated as square root (SR)-fuzzy numbers. The majority of existing studies regarding fuzzy transportation issues revolve around intuitionistic, Pythagorean, and Fermatean fuzzy numbers, leaving other fuzzy frameworks less explored. However, the current study is the first to consider the fuzzy parameter as square root fuzzy numbers. A transportation problem is formulated along with the corresponding algorithms that are applicable to SR-fuzzy data. A key contribution of this work is the formulation of a unique score function designed to the specific characteristics of SR-fuzzy sets. We integrate this score function with standard SR-fuzzy arithmetic to build a method for solving transportation problems in SR-fuzzy environment.

Keywords: Square root fuzzy numbers (SR-FNs), Transportation Problems, Score Function, Intuitionistic Fuzzy Numbers, Pythagorean Fuzzy Numbers.
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1. INTRODUCTION

In recent years, strong competition in many industries has pushed companies to improve their pricing to stay competitive. They are spending more on research and development to find cheaper ways to make products and deliver them to customers. The transportation problem (TP) is one of the most useful linear programming (LP) models for solving these kinds of issues, especially when there are lots of goods and production sites. TP helps us to choose the best production locations to lower transportation costs. The problem was first introduced by Hitchcock [1] and then expanded by Koopmans [2] and Dantzig [3]. The TP has limits based on how much availability is at the sources and how much is needed at the destinations. While TP can be solved using the simplex method as an LP model, this can become very complicated as the number of variables and constraints increases.

To simplify the challenge of solving large transportation problems (TPs), specialized algorithms like the Stepping Stone and Modified Distribution (MODI) methods have been developed. The objective in a TP is to minimize total transportation costs, with constraints defined by the availability and requirement at each point in the network.

Several techniques exist to find an initial basic feasible solution (IBFS), including Vogel's Approximation Method (VAM), the North-West corner rule, least cost and matrix minima methods. Over time, newer approaches have enhanced the accuracy of these initial solutions. For instance, Korukoğlu and Ballı [4] proposed an improved version of VAM in 2011 to better solve transportation problems.

Real-life problems often include uncertainties in availability, requirement, and transportation costs. Fuzzy logic, originally introduced by Zadeh [5] and further applied to linear programming by Zimmermann [6], has become a valuable tool to model and address such uncertainty effectively.

The foundations of fuzzy linear programming in the context of transportation were laid by Chanas, et al. [7] in 1984, where costs remained fixed but availability and requirement were uncertain. This was extended by Chanas and Kuchta [8] to include other imprecise parameters. Since then, researchers have expanded the field with single and multi-objective transportation problem (MOTP) models incorporating various fuzzy parameters.

Significant contributions to single-objective fuzzy TPs come from Singh, et al. [9], Ritu, et al. [10], Gahlawat, et al. [11], Saikia, Dutta and Talukdar [12], Tada and Ishii [13], Kaur and Kumar [14], Liu, et al. [15], Singh and Yadav [16], Gupta and Anupum [17], Kumar [18], and Hashmi, et al. [19]. For MOTPs, imprecise compromise programming methods were introduced by Li and Lai [20], with important developments from Ahmad and Adhami [21], Das and Roy [22], Ghosh, et al. [23], and Ghosh and Roy [24].

Atanassov [25] advanced classical fuzzy theory in 1986 by introducing intuitionistic fuzzy sets (IFSs), which unlike classical fuzzy sets, include degrees of membership, non-membership, and indeterminacy. While in IFS, the sum of memberships degree must remain at or below one , many real-world cases exceed this bound. To manage such situations, extended fuzzy sets like Pythagorean, Fermatean, q-rung orthopair, and n, m-rung orthopair fuzzy sets have been proposed. These refined models have since become essential tools for decision-making in diverse application areas [26, 27, 28, 29, 30, 31, 32, 33].

In 2021, Al-Shami, et al. [34] presented the square-root fuzzy set (SR-FS) as a new way to handle uncertainty. This model operates under a distinct boundary where the squared membership and the square root of non-membership when added is limited to a maximum value of one. For example, if membership is 0.2 and non-membership is 0.9, the sum $0.2 + 0.9 = 1.1 > 1$ exceeds IFS limits; however, $(0.2)^2 + \sqrt{0.9} = 0.04 + 0.9486 \approx 0.9886 < 1$ fits SR-FSs. This flexibility allows SR-FSs to capture greater uncertainty than prior fuzzy frameworks.

1.1. *Motivation and contribution*

At present, there are no research studies that specifically solve transportation problems in the context of square-root fuzzy environments. To address this gap, our paper presents an innovative approach for handling transportation problems (TPs) using SR-fuzzy sets (SR-FSs). In this method, the parameters for cost, availability, and requirement are modeled as SR-fuzzy numbers (SR-FNs) like (μ, ν), which follow the condition $0 \leq \mu^2 + \sqrt{\nu} \leq 1$. We introduce a novel score function for SR-FSs which better suits to dealing with the SR-fuzzy transportation problems (SRFTPs). By applying this specially designed score function to SR-FNs, we transform the fuzzy TP into a standard (crisp) TP, making it possible to find optimal solutions efficiently.

1.2. *Organization of the paper*

The remainder of the paper is structured as follows. Section 2 deals with basic concepts and prerequisites for the subsequent sections. We introduce a novel score function for SR-fuzzy sets in Section 3. In Section 4, we develop three different transportation problem models in SR-fuzzy environment. Section 5 discusses the algorithm for solving the SRFTPs. Section 6, illustrates the solution algorithm by numerical examples. Section 7 is devoted to the discussion. Section 8 deals with the conclusion and future research outlook.

2. Preliminaries

This section provides a detailed definition of Square-root Fuzzy Sets (SR-FSs) and discusses their fundamental properties. We also introduce and define score and accuracy functions for SR-FSs.

Definition 1. *[34] Let X be a universal set. A SR-FS S in X is formally represented as* $S = \{\langle x, \mu_S(x), \nu_S(x)\rangle : x \in X\}$, *where* $\mu_S, \nu_S(x) : X \rightarrow [0,1]$, *subject to the boundary* $0 \leq (\mu_S(x))^2 + \sqrt{\nu_S(x)} \leq 1, \forall x \in X$. *Here,* $\mu_S(x)$ *and* $\nu_S(x)$ *represent the membership and non-membership degrees, respectively, of the element* $x \in X$ *in the SR-FS S. The indeterminacy is expressed as:*

$$\pi_S(x) = 1 - [(\mu_S(x))^2 + \sqrt{\nu_S(x)}] \tag{1}$$

It is clear that the sum $(\mu_S(x))^2 + \sqrt{\nu_S(x)} + \pi_S(x)$ always equals one. Furthermore, when $(\mu_S(x))^2 + \sqrt{\nu_S(x)} = 1$ the indeterminacy degree naturally vanishes. For simplicity, we denote an SR-FS as an SR-FN represented by the pair $S = (\mu_S(x), \nu_S(x))$.

Definition 2. *[34] Let* $S = (\mu_S(x), \nu_S(x))$, $S_1 = (\mu_{S_1}(x), \nu_{S_1}(x))$ *and* $S_2 = (\mu_{S_2}(x), \nu_{S_2}(x))$ *be three SR-FSs over the universal set X and* $\lambda > 0$ *. Then some primary mathematical operations are formulated as:*

(i) Addition: $S_1 \oplus S_2 = \left(\sqrt{(\mu_{S_1})^2 + (\mu_{S_2})^2 - (\mu_{S_1})^2.(\mu_{S_2})^2}, \nu_{S_1}.\nu_{S_2}\right)$

(ii) Multiplication: $S_1 \otimes S_2 = \left(\mu_{S_1}.\mu_{S_2}, \left(\sqrt{\nu_{S_1}} + \sqrt{\nu_{S_2}} - \sqrt{\nu_{S_1}}.\sqrt{\nu_{S_2}}\right)^2\right)$

(iii) Scalar Multiplication: $\lambda S = \left(\sqrt{1 - (1 - \mu_S{}^2)^\lambda}, \nu_S{}^\lambda\right)$

(iv) Exponent: $S^\lambda = \left(\mu_S{}^\lambda, \left(1 - \left(1 - \sqrt{\nu_S}\right)^\lambda\right)^2\right)$

Definition 3. *[34] Let* $S = (\mu_S(x), \nu_S(x))$, $S_1 = (\mu_{S_1}(x), \nu_{S_1}(x))$ *and* $S_2 = (\mu_{S_2}(x), \nu_{S_2}(x))$ *be three SR-FSs over the universal set X. We characterize their fundamental set-theoretic operations through the following definitions:*

(i) Union: $S_1 \cup S_2 = (\max(\mu_{S_1}, \mu_{S_2}), \min(\nu_{S_1}, \nu_{S_2}))$

(ii) Intersection: $S_1 \cap S_2 = (\min(\mu_{S_1}, \mu_{S_2}), \max(\nu_{S_1}, \nu_{S_2}))$

(iii) Complement: $S^c = \left(\sqrt[4]{\nu_S}, (\mu_S)^4\right)$

Definition 4. *[34] Let* $S = (\mu_S(x), \nu_S(x))$ *be any SR-FS. The score function of S, denoted by Score(S), can be represented as:*

$$\text{Score}(S) = \mu_S{}^2 - \sqrt{\nu_S}, \tag{2}$$

For example, for an SR-FS $S = (0.4, 0.7)$, its score is $\text{Score}(S) = 0.4^2 - \sqrt{0.7} = 0.16 - 0.83666 \approx -0.67666$.

Definition 5. *[34] The score function is contained within the interval* $[-1, 1]$, *i.e.,* $\text{Score}(S) \in [-1, 1]$. *The score function is considered positive when* $Score(S) \in [0, 1]$ *and negative when* $Score(S) \in [-1, 0]$.

Definition 6. *[34] Let* $S = (\mu_S(x), \nu_S(x))$ *be any SR-FS. The accuracy function of S, denoted by acc(S), can be represented as:*

$$acc(S) = \mu_S{}^2 + \sqrt{\nu_S}, \tag{3}$$

For example, for an SR-FS $S = (0.4, 0.7)$, *its accuracy is* $acc(S) = 0.4^2 + \sqrt{0.7} = 0.16 + 0.83666 \approx 0.99666$.

The value of the accuracy function lies within the interval $[0, 1]$, *i.e.,* $acc(S) \in [0, 1]$. *When the degree of indeterminacy is added to the accuracy function, the sum is one. This implies that a lower degree of indeterminacy leads to higher accuracy for the SR-FS.*

Theorem 1. *[34] Let $S_1 = (\mu_{S_1}(x), \nu_{S_1}(x))$ and $S_2 = (\mu_{S_2}(x), \nu_{S_2}(x))$ be two SR-FSs. Then, the comparison between S_1 and S_2 is governed by the following rules:*

$$S_1 < S_2 \iff \begin{cases} \mathsf{Score}(S_1) < \mathsf{Score}(S_2) \\ \mathsf{Score}(S_1) = \mathsf{Score}(S_2) \text{ and } \mathsf{acc}(S_1) < \mathsf{acc}(S_2) \end{cases}$$

Furthermore, $S_1 = S_2$ iff both $\mathsf{Score}(S_1) = \mathsf{Score}(S_2)$ and $\mathsf{acc}(S_1) = \mathsf{acc}(S_2)$.

3. A novel score function for SR-fuzzy sets

Let us consider an SR-FS $S = (\mu_S(x), \nu_S(x))$. The range of the score function for this SR-FS given by (2) is $[-1, 1]$. Because transportation costs are always positive in a real-world setting, we can't just use any scoring method. We have to design a specialized function for the SR-fuzzy framework that specifically ensures the results remain non-negative and grounded in reality. So, here we introduce the following score function to evaluate the SR-FS $S = (\mu_S(x), \nu_S(x))$:

$$\mathfrak{Sc}(S) = \frac{1}{2}\left(1 + \mu_S{}^2 - \sqrt{\nu_S}\right), \tag{4}$$

We now examine the fundamental characteristics of this score function.

Theorem 2. *For any SR-FS $S = (\mu_S, \nu_S)$, the score function $\mathfrak{Sc}(S)$ holds the following five properties:*

(a) $\mathfrak{Sc}(S) = 1$ *at* $S = (1, 0)$.

(b) $\mathfrak{Sc}(S) = 0$ *at* $S = (0, 1)$.

(c) As the membership degree μ increases $\mathfrak{Sc}(S)$ increases monotonically .

(d) As the non-membership degree ν increases $\mathfrak{Sc}(S)$ decreases monotonically.

(e) $0 \le \mathfrak{Sc}(S) \le 1$.

Proof. (a) For $S = (1, 0)$,

$$\mathfrak{Sc}(S) = \frac{1}{2}(1 + 1^2 - \sqrt{0}) = \frac{1}{2} \cdot 2 = 1.$$

(b) For $S = (0, 1)$,

$$\mathfrak{Sc}(S) = \frac{1}{2}(1 + 0^2 - \sqrt{1}) = \frac{1}{2} \cdot 0 = 0.$$

(c) Differentiating $\mathfrak{Sc}(S)$ partially with respect to μ, we get

$$\frac{\partial \mathfrak{Sc}(S)}{\partial \mu} = \frac{1}{2}\frac{\partial}{\partial \mu}(1 + \mu^2 - \sqrt{\nu}) = \frac{1}{2} \cdot 2\mu = \mu \ge 0$$

since $\mu \in [0, 1]$.
This implies that the score function $\mathfrak{Sc}(S)$ increases monotonically whenever the degree of membership μ increases.
(d) Differentiating $\mathfrak{Sc}(S)$ partially with respect to ν, we get,

$$\frac{\partial \mathfrak{Sc}(S)}{\partial \nu} = \frac{1}{2}\frac{\partial}{\partial \nu}(1 + \mu^2 - \sqrt{\nu}) = \frac{1}{2} \cdot \frac{-1}{2.\sqrt{\nu}} = \frac{-1}{4\sqrt{\nu}} \le 0$$

since $\nu \in [0, 1]$. This implies that the score function $\mathfrak{Sc}(S)$ decreases monotonically when the non-membership degree ν increases.

(e) We know that

$$\begin{aligned}
& \mu^2 - \sqrt{\nu} \in [-1, 1] \\
\Rightarrow\ & -1 \leq \mu^2 - \sqrt{\nu} \leq 1 \\
\Rightarrow\ & -1 + 1 \leq \mu^2 - \sqrt{\nu} + 1 \leq 1 + 1 \\
\Rightarrow\ & 0 \leq 1 + \mu^2 - \sqrt{\nu} \leq 2 \\
\Rightarrow\ & 0 \leq \frac{1}{2}(1 + \mu^2 - \sqrt{\nu}) \leq 1 \\
\Rightarrow\ & \mathfrak{Sc}(S) \in [0, 1].
\end{aligned}$$

□

Thus, we have shown that $\mathfrak{Sc}(S)$ is always positive. It attains maximum value for $(\mu, \nu) = (1, 0)$ and vanishes for $(\mu, \nu) = (0, 1)$. Furthermore, the function $\mathfrak{Sc}(S)$ increases monotonically when the membership degree increases and decreases monotonically when the non-membership degree increases.

4. SR-fuzzy transportation problem models

Consider a transportation scenario consisting of m locations providing supplies, labeled as $\mathbb{S}_1, \mathbb{S}_2, \ldots, \mathbb{S}_m$, and n endpoints consuming these supplies. By a_i we denote the amount available at the ith supplier, and by b_j the amount required at the jth destination. The expense incurred in transporting a single unit from supplier i to destination j is c_{ij}. Let x_{ij} specify the number of units shipped along this route. The primary aim is to allocate the shipments x_{ij} so that the total distribution expenses are minimized, all availability limits are respected, and every requirement is fulfilled.

Formally, the general TP is expressed as:

$$\begin{aligned}
& \text{Minimize } z_0 \text{ (aggregate cost)} = \sum_{\mathrm{i}=1}^{m} \sum_{\mathrm{j}=1}^{n} c_{\mathrm{ij}} \,.\, x_{\mathrm{ij}} \text{ subject to} \\
& \sum_{\mathrm{j}=1}^{n} x_{\mathrm{ij}} \leq a_{\mathrm{i}}, \quad \mathrm{i} = 1, 2, \ldots, m \quad \text{(Availability restrictions)}, \\
& \sum_{\mathrm{i}=1}^{m} x_{\mathrm{ij}} \geq b_{\mathrm{j}}, \quad \mathrm{j} = 1, 2, \ldots, n \quad \text{(Requirement restrictions)}, \\
& x_{\mathrm{ij}} \geq 0, \quad \forall \mathrm{i}, \mathrm{j}.
\end{aligned} \tag{5}$$

When any of the parameters, such as c_{ij}, a_i, or b_j, are specified using square root fuzzy sets (SR-FSs), or more specifically as square root fuzzy numbers, the task morphs into a Square Root Fuzzy Transportation Problem (SRFTP).

Each SR-FS is represented as a pair (μ, ν), where $0 \leq \mu^2 + \sqrt{\nu} \leq 1$. Thus, the fuzzy forms for availability, requirement, and per-unit cost are given as (μ_{a_i}, ν_{a_i}), (μ_{b_j}, ν_{b_j}), and $(\mu_{c_{ij}}, \nu_{c_{ij}})$, respectively.

If only the cost parameters c_{ij} in (5) are replaced with the fuzzy versions ${c_{ij}}^{SR} = (\mu_{c_{ij}}, \nu_{c_{ij}})$, the problem is identified as a *1st Type Square Root Fuzzy Transportation Problem (1st Ty-SRFTP)*.

The formulation for the 1st Ty-SRFTP is as follows:

$$\begin{aligned}
&\text{Minimize } (\mu_{z_0}, \nu_{z_0}) = \sum_{i=1}^{m} \sum_{j=1}^{n} (\mu_{c_{ij}}, \nu_{c_{ij}}) \,.\, x_{ij} \text{ subject to} \\
&\sum_{j=1}^{n} x_{\mathrm{ij}} \leq a_{\mathrm{i}}, \quad \mathrm{i} = 1, 2, \ldots, m \quad \text{(Availability restrictions)}, \\
&\sum_{i=1}^{m} x_{\mathrm{ij}} \geq b_{\mathrm{j}}, \quad \mathrm{j} = 1, 2, \ldots, n \quad \text{(Requirement restrictions)}, \\
&x_{\mathrm{ij}} \geq 0, \quad \forall \mathrm{i}, \mathrm{j}, \text{ and } 0 \leq (\mu_{c_{ij}})^2 + \sqrt{\nu_{c_{ij}}} \leq 1.
\end{aligned} \tag{6}$$

When availability and requirement reach parity, i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, this problem is called balanced. The 1st Ty-SRFTP is presented in the tabular form by Table 1.

	$\mathbb{T}_1$	$\mathbb{T}_2$	...	$\mathbb{T}_n$	**Availability**
$\mathbb{S}_1$	(μ_{11}, ν_{11})	(μ_{12}, ν_{12})	...	(μ_{1n}, ν_{1n})	a_1
$\mathbb{S}_2$	(μ_{21}, ν_{21})	(μ_{22}, ν_{22})	...	(μ_{2n}, ν_{2n})	a_2
...	...	...	...	...	...
$\mathbb{S}_m$	(μ_{m1}, ν_{m1})	(μ_{m2}, ν_{m2})	...	(μ_{mn}, ν_{mn})	a_m
Requirement	b_1	b_2	...	b_n	

Table 1: Tabular form of 1st Ty-SRFTP

Alternatively, if the availability a_i and the requirement b_j are modeled using square root fuzzy numbers, replacing them with ${a_i}^{\mathrm{SR}} = (\mu_{a_i}, \nu_{a_i})$ and ${b_j}^{\mathrm{SR}} = (\mu_{b_j}, \nu_{b_j})$ in the model, then we have the *2nd Type Square Root Fuzzy Transportation Problem (2nd Ty-SRFTP)*.
This case is framed as:

$$\begin{aligned}
&\text{Minimize } (\mu_{z_0}, \nu_{z_0}) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \odot x_{ij} \text{ with constraints:} \\
&\sum_{j=1}^{n} x_{ij} \leq (\mu_{a_i}, \nu_{a_i}),\ i = 1, 2, \ldots, m, \\
&\sum_{i=1}^{m} x_{ij} \geq (\mu_{b_j}, \nu_{b_j}),\ j = 1, 2, \ldots, n, \\
&x_{ij} \geq 0\ \forall i, j,\ 0 \leq (\mu_{a_i})^2 + \sqrt{\nu_{a_i}} \leq 1,\ 0 \leq (\mu_{b_j})^2 + \sqrt{\nu_{b_j}} \leq 1.
\end{aligned} \tag{7}$$

This problem is **balanced** if $\sum_{i=1}^{m} \oplus(\mu_{a_i}, \nu_{a_i}) = \sum_{j=1}^{n} \oplus(\mu_{b_j}, \nu_{b_j})$, where $\oplus$ denotes the sum of SR-FNs. The 2nd Ty-SRFTP is demonstrated by the Table 2.

	$\mathbb{T}_1$	$\mathbb{T}_2$	...	$\mathbb{T}_n$	**Availability**
$\mathbb{S}_1$	c_{11}	c_{12}	...	c_{1n}	(μ_{a_1}, ν_{a_1})
$\mathbb{S}_2$	c_{21}	c_{22}	...	c_{2n}	(μ_{a_2}, ν_{a_2})
...	...	...	...	...	...
$\mathbb{S}_m$	c_{m1}	c_{m2}	...	c_{mn}	(μ_{a_m}, ν_{a_m})
Requirement	(μ_{b_1}, ν_{b_1})	(μ_{b_2}, ν_{b_2})	...	(μ_{b_n}, ν_{b_n})	

Table 2: Tabular form of 2nd Ty-SRFTP

If, on the other hand, every parameter including availability, requirement, and cost, is given as a square root fuzzy number, then the model is called the *3rd Type Square Root Fuzzy Transportation Problem (3rd Ty-SRFTP)*.

The mathematical structure of the 3rd Ty-SRFTP is:

$$\begin{aligned}
&\text{Minimize } (\mu_{z_0}, \nu_{z_0}) = \sum_{i=1}^{m}\sum_{j=1}^{n}(\mu_{c_{ij}}, \nu_{c_{ij}}) \,.\, x_{ij} \text{ subject to} \\
&\sum_{j=1}^{n} x_{ij} \leq (\mu_{a_i}, \nu_{a_i}),\ i = 1, 2, \ldots, m, \\
&\sum_{i=1}^{m} x_{ij} \geq (\mu_{b_j}, \nu_{b_j}),\ j = 1, 2, \ldots, n, \\
&x_{ij} \geq 0\ \forall i, j, \\
&0 \leq (\mu_{a_i})^2 + \sqrt{\nu_{a_i}} \leq 1,\ 0 \leq (\mu_{b_j})^2 + \sqrt{\nu_{b_j}} \leq 1, \\
&0 \leq (\mu_{c_{ij}})^2 + \sqrt{\nu_{c_{ij}}} \leq 1.
\end{aligned} \tag{8}$$

The 3rd Ty-SRFTP is demonstrated by the Table 3.

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\ldots$	$\mathbb{T}_n$	**Availability**
$\mathbb{S}_1$	(μ_{11}, ν_{11})	(μ_{12}, ν_{12})	$\ldots$	(μ_{1n}, ν_{1n})	(μ_{a_1}, ν_{a_1})
$\mathbb{S}_2$	(μ_{21}, ν_{21})	(μ_{22}, ν_{22})	$\ldots$	(μ_{2n}, ν_{2n})	(μ_{a_2}, ν_{a_2})
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathbb{S}_m$	(μ_{m1}, ν_{m1})	(μ_{m2}, ν_{m2})	$\ldots$	(μ_{mn}, ν_{mn})	(μ_{a_m}, ν_{a_m})
Requirement	(μ_{b_1}, ν_{b_1})	(μ_{b_2}, ν_{b_2})	$\ldots$	(μ_{b_n}, ν_{b_n})	

Table 3: Tabular form of 3rd Ty-SRFTP

We defuzzify the SR-FNs into crisp values (*see* Table 4) by employing our novel score function defined in equation (4).

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\ldots$	$\mathbb{T}_n$	**Availability**
$\mathbb{S}_1$	c_{11}	c_{12}	$\ldots$	c_{1n}	a_1
$\mathbb{S}_2$	c_{21}	c_{22}	$\ldots$	c_{2n}	a_2
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathbb{S}_m$	c_{m1}	c_{m2}	$\ldots$	c_{mn}	a_m
Requirement	b_1	b_2	$\ldots$	b_n	

Table 4: Crisp Transportation Problem

5. Algorithm for solving SR-fuzzy transportation problems

This section outlines a five-step procedure for solving TPs where costs, availability, and requirement are expressed as SR-FNs. The approach converts fuzzy parameters into deterministic values, allowing for the application of classical optimization methods.

Here, we explain an easy step-by-step method to solve the TP when costs, availability, and requirement are given as SR-FNs.

Step 1: We construct the transportation tableau using the given SR-FNs for all unit costs, source availability, and destination requirements.

Step 2: We use our novel score function to transform all the SR-FNs into crisp (real) values. This conversion produces a standard TP.

Step 3: Check if the TP is balanced by comparing total availability and total requirement:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \tag{9}$$

where a_i denotes the availability (supply quantity) at source i and b_j denotes the requirement (demand quantity) at destination j.

Balanced TPs are processed immediately; however, unbalanced cases require the addition of a dummy source or destination (with zero transportation costs) to achieve the parity between total availability and requirements before further analysis.

Step 4: We obtain an IBFS using the VAM. After that, we calculate the total transportation cost associated with this IBFS.

Step 5: We implement the MODI method for optimality test and improve the solution iteratively until the optimal allocation is reached and evaluate the corresponding optimal transportation cost.

These steps ensure that the defuzzified crisp problem is solved efficiently and accurately using well-established classical transportation techniques. Figure 1 illustrates the complete procedural logic of the proposed algorithm.

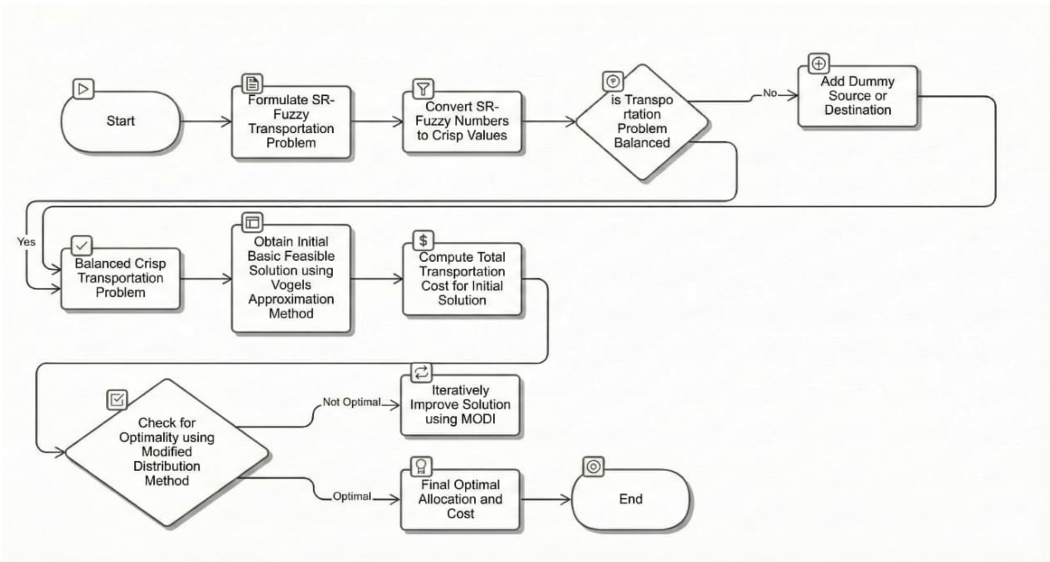

Figure 1: Flowchart showing the steps for solving SRFTP

6. Numerical examples

A prominent cement manufacturing firm manages four production plants, located in Ahmedabad ($\mathbb{T}_1$), Indore ($\mathbb{T}_2$), Nagpur ($\mathbb{T}_3$) , and Jaipur ($\mathbb{T}_4$), which are considered storage and distribution centers. The essential raw material for cement, limestone, is procured from suppliers in three regions: Gujarat($\mathbb{S}_1$), Madhya Pradesh ($\mathbb{S}_2$), and Rajasthan ($\mathbb{S}_3$). Due to fluctuating market conditions, precise figures for costs, available supplies, and requirements are hard to pin down. To account for this variability, these factors are modeled using Square Root Fuzzy Numbers (SR-FNs).

To optimize the logistics of transporting limestone from these availability regions to the plants, the analysis employs several specialized models. These approaches simulate various scenarios, balancing availability constraints against requirement needs while minimizing transportation costs under fuzzy conditions. By applying these models, the company can make informed decisions to ensure efficient, resilient operations despite market unpredictability.

This section presents the detailed computational outcomes achieved by implementing the proposed SR-Fuzzy Transportation Problem (SRFTP) frameworks and corresponding procedures. We analyze three separate variants: the 1st Ty-SRFTP (where costs are fuzzy), the 2nd Ty-SRFTP (with fuzzy availability and requirement), and the 3rd Ty-SRFTP (incorporating fuzzy values for all parameters). For each variant, we use the proposed algorithm established in Section 5 for solving the TP. We deal with each type of problem, one by one, in the following subsections.

6.1. *Solution of 1st Type SRFTP (fuzzy costs)*

Table 5 presents the initial formulation of the 1st Type Square Root Fuzzy Transportation Problem (1st Ty-SRFTP), where the transportation costs alone are represented as SR-FNs, while both availability and requirement remain crisp values. The subsequent tables illustrate the solution steps.

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**
$\mathbb{S}_1$	(0.2, 0.9)	(0.35, 0.75)	(0.7, 0.2)	(0.6, 0.1)	20
$\mathbb{S}_2$	(0.6, 0.3)	(0.4, 0.7)	(0.5, 0.55)	(0.8, 0.1)	18
$\mathbb{S}_3$	(0.3, 0.82)	(0.7, 0.2)	(0.2, 0.65)	(0.55, 0.47)	22
Requirement	18	16	17	9	

Table 5: Input data for 1st Ty-SRFTP

Using our proposed novel score function defined in equation (4), we get

$$\begin{aligned}
\mathfrak{Sc}(0.2, 0.9) &= 0.04565, & \mathfrak{Sc}(0.35, 0.75) &= 0.1282,\\
\mathfrak{Sc}(0.7, 0.2) &= 0.5213, & \mathfrak{Sc}(0.6, 0.1) &= 0.5218,\\
\mathfrak{Sc}(0.6, 0.3) &= 0.4061, & \mathfrak{Sc}(0.4, 0.7) &= 0.1616,\\
\mathfrak{Sc}(0.5, 0.55) &= 0.2542, & \mathfrak{Sc}(0.8, 0.1) &= 0.6618,\\
\mathfrak{Sc}(0.3, 0.82) &= 0.0922, & \mathfrak{Sc}(0.7, 0.2) &= 0.5214,\\
\mathfrak{Sc}(0.2, 0.65) &= 0.1169, & \mathfrak{Sc}(0.55, 0.47) &= 0.3084.
\end{aligned}$$

These defuzzified values are listed in Table 6.

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**
$\mathbb{S}_1$	0.04565	0.1282	0.5213	0.5218	20
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	18
$\mathbb{S}_3$	0.0922	0.5214	0.1169	0.3084	22
Requirement	18	16	17	9	

Table 6: Crisp transportation problem corresponding to 1st Ty-SRFTP

Since

$$\sum_{i=1}^{3} a_i = \sum_{j=1}^{4} b_j = 60\,, \tag{10}$$

this is a balanced TP.

Now, we obtain an IBFS using the VAM. All the steps are shown in Table 7, Table 8, Table 9, Table 10, Table 11, Table 12 and Table 13 below:

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.04565	0.1282	0.5213	0.5218	20	0.08
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	18	0.09
$\mathbb{S}_3$	0.0922	0.5214	0.1169	0.3084	22	0.02
Requirement	18	16	17	9		
Penalty Cost	0.05	0.03	0.14	0.21		

Table 7: First allocation

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.04565	0.1282	0.5213	0.5218	20	0.08
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	18	0.09
$\mathbb{S}_3$	0.0922	0.5214	0.1169	0.3084 (9)	13	0.02
Requirement	18	16	17	0		
Penalty Cost	0.05	0.03	0.14	-		

Table 8: Second allocation

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.04565	0.1282	0.5213	0.5218	20	0.08
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	18	0.09
$\mathbb{S}_3$	0.0922	0.5214	0.1169 (13)	0.3084 (9)	0	-
Requirement	18	16	4	0		
Penalty Cost	0.36	0.03	0.27	-		

Table 9: Third allocation

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.04565 (18)	0.1282	0.5213	0.5218	2	0.39
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	18	0.09
$\mathbb{S}_3$	0.0922	0.5214	0.1169 (13)	0.3084 (9)	0	-
Requirement	0	16	4	0		
Penalty Cost	-	0.03	0.27	-		

Table 10: Fourth allocation

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.04565 (18)	0.1282,(2)	0.5213	0.5218	0	-
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	18	0.09
$\mathbb{S}_3$	0.0922	0.5214	0.1169 (13)	0.3084 (9)	0	-
Requirement	0	14	4	0		
Penalty Cost	-	0.16	0.25	-		

Table 11: Fifth allocation

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.04565 (18)	0.1282 (2)	0.5213	0.5218	0	-
$\mathbb{S}_2$	0.4061	0.1616	0.2542 (4)	0.6618	14	0.16
$\mathbb{S}_3$	0.0922	0.5214	0.1169 (13)	0.3084 (9)	0	-
Requirement	0	14	0	0		
Penalty Cost	-	0.16	-	-		

Table 12: Sixth allocation

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**
$\mathbb{S}_1$	0.04565 (18)	0.1282 (2)	0.5213	0.5218	-
$\mathbb{S}_2$	0.4061	0.1616 (14)	0.2542 (4)	0.6618	-
$\mathbb{S}_3$	0.0922	0.5214	0.1169 (13)	0.3084 (9)	-
Requirement	-	-	-	-	

Table 13: IBFS for 1st Ty-SRFTP

From the Table 13, it is clear that the IBFS so obtained using VAM are:

$$(\mathbb{S}_1, \mathbb{T}_1): \ x_{11} = 18, \qquad (\mathbb{S}_1, \mathbb{T}_2): \ x_{12} = 2,$$
$$(\mathbb{S}_2, \mathbb{T}_2): \ x_{22} = 14, \qquad (\mathbb{S}_2, \mathbb{T}_3): \ x_{23} = 4,$$
$$(\mathbb{S}_3, \mathbb{T}_3): \ x_{33} = 13, \qquad (\mathbb{S}_3, \mathbb{T}_4): \ x_{34} = 9.$$

Based on the IBFS, the resulting total cost is evaluated as $(0.04565).18+(0.1282).2+(0.1616).14+(0.2542).4+(0.1169).13+(0.3084).9 = 8.75$.

Now, the MODI method is applied to verify optimality of this 1st Ty-SRFTP and the obtained results are shown as follows:

$$(\mathbb{S}_1, \mathbb{T}_1): \ x_{11} = 18, \qquad (\mathbb{S}_1, \mathbb{T}_2): \ x_{12} = 2,$$
$$(\mathbb{S}_2, \mathbb{T}_2): \ x_{22} = 14, \qquad (\mathbb{S}_2, \mathbb{T}_3): \ x_{23} = 4,$$
$$(\mathbb{S}_3, \mathbb{T}_3): \ x_{33} = 13, \qquad (\mathbb{S}_3, \mathbb{T}_4): \ x_{34} = 9.$$

and the final optimal transportation cost is 8.75 units.

6.2. *Solution of 2nd Type SRFTP (fuzzy availability and requirement)*

Table 14 presents the initial formulation of the 2nd Type Square Root Fuzzy Transportation Problem (2nd Ty-SRFTP), where the availability and requirement are represented as SR-FNs, while the transportation costs remain crisp values.

The subsequent tables illustrate the solution steps.

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**
$\mathbb{S}_1$	0.0332	0.0289	0.071	0.0418	(0.7, 0.2)
$\mathbb{S}_2$	0.05	0.063	0.082	0.0739	(0.5, 0.55)
$\mathbb{S}_3$	0.0912	0.0612	0.0574	0.0432	(0.6, 0.3)
Requirement	(0.2,0.9)	(0.3, 0.8)	(0.4, 0.7)	(0.81628, 0.0654)	

Table 14: Input data for 2nd Ty-SRFTP

Using our proposed novel score function defined in equation (4), we get

$$\mathfrak{Sc}(0.7, 0.2) = 0.52139, \qquad \mathfrak{Sc}(0.5, 0.55) = 0.25419,$$
$$\mathfrak{Sc}(0.6, 0.3) = 0.406138, \qquad \mathfrak{Sc}(0.2, 0.9) = 0.0456,$$
$$\mathfrak{Sc}(0.3, 0.8) = 0.09778, \qquad \mathfrak{Sc}(0.4, 0.7) = 0.16167,$$
$$\mathfrak{Sc}(0.81628, 0.0654) = 0.70529.$$

These defuzzified values are listed in Table 15.

Since

$$\sum_{i=1}^{3} a_i > \sum_{j=1}^{4} b_j, \tag{11}$$

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**
$\mathbb{S}_1$	0.0332	0.0289	0.071	0.0418	0.52139
$\mathbb{S}_2$	0.05	0.063	0.082	0.0739	0.25419
$\mathbb{S}_3$	0.0912	0.0612	0.0574	0.0432	0.406138
Requirement	0.0456	0.09778	0.16167	0.70529	

Table 15: Crisp transportation problem corresponding to 2nd Ty-SRFTP

it is unbalanced TP. Here, a dummy destination is introduced to equalize supply and demand. This dummy destination is assigned a requirement of 0.17138 units and a unit cost of zero across all sources.

Now, we obtain an IBFS using the VAM. All the steps are shown in Table 16 and Table 17 below:

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	$\mathbb{T}_{Dummy}$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.0332	0.0289	0.071	0.0418	0	0.52139	0.03
$\mathbb{S}_2$	0.05	0.063	0.082	0.0739	0	0.25419	0.05
$\mathbb{S}_3$	0.0912	0.0612	0.0574	0.0432	0	0.406138	0.41
Requirement	0.0456	0.09778	0.16167	0.70529	0.17138		
Penalty Cost	0.02	0.03	0.01	0	0		

Table 16: Balanced transportation problem corresponding to 2nd Ty-SRFTP

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	T_{Dummy}	**Availability**
$\mathbb{S}_1$	0.0332	0.0289(0.01)	0.071	0.0418(0.42)	0	0.52139
$\mathbb{S}_2$	0.05(0.05)	0.063	0.082(0.04)	0.0739	0(0.17)	0.25419
$\mathbb{S}_3$	0.0912	0.0612	0.0574(0.12)	0.0432(0.28)	0	0.406138
Requirement	0.0456	0.09778	0.16167	0.70529	0.17138	

Table 17: IBFS for 2nd Ty-SRFTP

From the Table 17, it is clear that the IBFS so obtained using VAM are:

$$\begin{aligned} (\mathbb{S}_1, \mathbb{T}_2): \ x_{12} &= 0.01, & (\mathbb{S}_1, \mathbb{T}_4): \ x_{14} &= 0.42, \\ (\mathbb{S}_2, \mathbb{T}_1): \ x_{21} &= 0.05, & (\mathbb{S}_2, \mathbb{T}_3): \ x_{23} &= 0.04, \\ (\mathbb{S}_2, \mathbb{T}_{dummy}): \ x_{2dummy} &= 0.17, & (\mathbb{S}_3, \mathbb{T}_3): \ x_{33} &= 0.12, \\ (\mathbb{S}_3, \mathbb{T}_4): \ x_{34} &= 0.28. \end{aligned}$$

Based on the IBFS, the resulting total cost is evaluated as, (0.0289).(0.1) + (0.0418).(0.42) + (0.05).(0.05) + (0.082).(0.04) + (0).(0.17) + (0.0574).(0.12) + (0.0432).(0.28) = 0.05.

Now, the MODI method is applied to verify optimality of this 2nd Ty-SRFTP and the obtained results are shown as follows:

$$\begin{aligned} (\mathbb{S}_1, \mathbb{T}_2): \ x_{12} &= 0.01, & (\mathbb{S}_1, \mathbb{T}_4): \ x_{14} &= 0.42, \\ (\mathbb{S}_2, \mathbb{T}_1): \ x_{21} &= 0.05, & (\mathbb{S}_2, \mathbb{T}_3): \ x_{23} &= 0.04, \\ (\mathbb{S}_2, \mathbb{T}_{dummy}): \ x_{2dummy} &= 0.17, & (\mathbb{S}_3, \mathbb{T}_3): \ x_{33} &= 0.12, \\ (\mathbb{S}_3, \mathbb{T}_4): \ x_{34} &= 0.28. \end{aligned}$$

and the minimum transportation cost is 0.05.

6.3. *Solution of 3rd Type SRFTP Analysis (all fuzzy parameters)*

Table 18 presents the initial formulation of the 3rd Type Square Root Fuzzy Transportation Problem (3rd Ty-SRFTP), in which all parameters, *viz.*, transportation cost, availability, and requirement are specified as SR-FNs. The subsequent tables illustrate the solution steps.

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**
$\mathbb{S}_1$	(0.2, 0.9)	(0.35, 0.75)	(0.7, 0.2)	(0.6, 0.1)	(0.7,0.2)
$\mathbb{S}_2$	(0.6, 0,3)	(0.4, 0.7)	(0.5, 0.55)	(0.8,0.1)	(0.5, 0.55)
$\mathbb{S}_3$	(0.3, 0.82)	(0.7, 0.2)	(0.2, 0.65)	(0.55, 0.47)	(0.6, 0.3)
Requirement	(0.2,0.9)	(0.3, 0.8)	(0.4, 0.7)	(0.81628, 0.0654)	

Table 18: Input data for 3rd Ty-SRFTP

Using our proposed novel score function defined in (4), we get

$$\mathfrak{Sc}(0.2, 0.9) = 0.04565, \quad \mathfrak{Sc}(0.35, 0.75) = 0.1282,$$
$$\mathfrak{Sc}(0.7, 0.2) = 0.5213, \quad \mathfrak{Sc}(0.6, 0.1) = 0.5218,$$
$$\mathfrak{Sc}(0.6, 0.3) = 0.4061, \quad \mathfrak{Sc}(0.4, 0.7) = 0.1616,$$
$$\mathfrak{Sc}(0.5, 0.55) = 0.2542, \quad \mathfrak{Sc}(0.8, 0.1) = 0.6618,$$
$$\mathfrak{Sc}(0.3, 0.82) = 0.0922, \quad \mathfrak{Sc}(0.7, 0.2) = 0.5214,$$
$$\mathfrak{Sc}(0.2, 0.65) = 0.1169, \quad \mathfrak{Sc}(0.55, 0.47) = 0.3084,$$
$$\mathfrak{Sc}(0.7, 0.2) = 0.52139, \quad \mathfrak{Sc}(0.5, 0.55) = 0.25419,$$
$$\mathfrak{Sc}(0.6, 0.3) = 0.406138, \quad \mathfrak{Sc}(0.2, 0.9) = 0.0456,$$
$$\mathfrak{Sc}(0.3, 0.8) = 0.09778, \quad \mathfrak{Sc}(0.4, 0.7) = 0.16167,$$
$$\mathfrak{Sc}(0.81628, 0.0654) = 0.70529.$$

These defuzzified values are listed in Table 19.

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	**Availability**
$\mathbb{S}_1$	0.04565	0.1282	0.5213	0.5218	0.52139
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	0.25419
$\mathbb{S}_3$	0.0922	0.5214	0.1169	0.3084	0.406138
Requirement	0.0456	0.09778	0.16167	0.70529	

Table 19: Crisp transportation problem corresponding to 3rd Ty-SRFTP

To ensure a feasible solution, the crisp transportation problem is checked for balance. Here,

$$\sum_{i=1}^{3} a_i > \sum_{j=1}^{4} b_j, \tag{12}$$

it is unbalanced TP. As the current TP is unbalanced, we added a dummy destination to equalize supply and demand. This dummy destination is assigned a requirement of 0.17138 units and a unit cost of zero across all sources .

Now, we obtain an IBFS using the VAM. All the steps are shown in Table 20 and Table 21 below:

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	$\mathbb{T}_{Dummy}$	**Availability**	**Penalty Cost**
$\mathbb{S}_1$	0.04565	0.1282	0.5213	0.5218	0	0.52139	0.05
$\mathbb{S}_2$	0.4061	0.1616	0.2542	0.6618	0	0.25419	0.16
$\mathbb{S}_3$	0.0922	0.5214	0.1169	0.3084	0	0.406138	0.09
Requirement	0.0456	0.09778	0.16167	0.70529	0.17138		
Penalty Cost	0.05	0.03	0.14	0.21	0		

Table 20: Balanced transportation problem corresponding to 3rd Ty-SRFTP

	$\mathbb{T}_1$	$\mathbb{T}_2$	$\mathbb{T}_3$	$\mathbb{T}_4$	$\mathbb{T}_{Dummy}$	**Availability**
$\mathbb{S}_1$	0.04565(0.05)	0.1282(0.01)	0.5213	0.5218(0.3)	0(0.08)	0.52139
$\mathbb{S}_2$	0.4061	0.1616	0.2542(0.16)	0.6618	0(0.09)	0.25419
$\mathbb{S}_3$	0.0922	0.5214	0.1169	0.3084(0.41)	0	0.406138
Requirement	0.0456	0.09778	0.16167	0.70529	0.17138	

Table 21: IBFS for 3rd Ty-SRFTP

From the Table 21, it is clear that the IBFS so obtained using VAM are:

$$(\mathbb{S}_1, \mathbb{T}_1): \ x_{11} = 0.05, \qquad (\mathbb{S}_1, \mathbb{T}_2): \ x_{12} = 0.01,$$
$$(\mathbb{S}_1, \mathbb{T}_4): \ x_{14} = 0.3, \qquad (\mathbb{S}_1, \mathbb{T}_{dummy}): \ x_{1dummy} = 0.08,$$
$$(\mathbb{S}_2, \mathbb{T}_3): \ x_{23} = 0.16, \qquad (\mathbb{S}_2, \mathbb{T}_{dummy}): \ x_{2dummy} = 0.09,$$
$$(\mathbb{S}_3, \mathbb{T}_4): \ x_{34} = 0.41.$$

Based on the IBFS, the resulting total cost is evaluated as (0.04565).(0.05) + (0.1282).(0.1) + (0.5218).(0.3) + 0.(0.8) + (0.2542).(0.16) + 0.(0.09) + (0.3084).(0.41) = 0.34.

Now, the MODI method is applied to verify optimality of this 3rd Ty-SRFTP and the obtained results are shown as follows:

$$(\mathbb{S}_1, \mathbb{T}_1): \ x_{11} = 0.05, \qquad (\mathbb{S}_1, \mathbb{T}_2): \ x_{12} = 0.01,$$
$$(\mathbb{S}_1, \mathbb{T}_4): \ x_{14} = 0.3, \qquad (\mathbb{S}_1, \mathbb{T}_{dummy}): \ x_{1dummy} = 0.08,$$
$$(\mathbb{S}_2, \mathbb{T}_3): \ x_{23} = 0.16, \qquad (\mathbb{S}_2, \mathbb{T}_{dummy}): \ x_{2dummy} = 0.09,$$
$$(\mathbb{S}_3, \mathbb{T}_4): \ x_{34} = 0.41.$$

and the final optimal transportation cost in 0.34.

7. Discussion

This research presents a novel and vital approach to solving the Square Root Fuzzy Transportation Problems (SRFTPs). The core contribution lies in utilizing Square Root Fuzzy Numbers (SR-FNs) to model uncertainty in availability, requirement, and transportation costs. Unlike traditional intuitionistic or Pythagorean fuzzy models, SR-FNs offer greater flexibility by allowing cases where the membership and non-membership degrees when added exceeds one, provided the square root fuzzy set condition $0 \leq \mu^2 + \sqrt{\nu} \leq 1$ is satisfied. A crucial innovation is the development of a novel positive score function $\mathfrak{Sc}(S) = \frac{1}{2}\left(1 + \mu_S^2 - \sqrt{\nu_S}\right)$. Since transportation costs must be non-negative, this function guarantees that the defuzzified (crisp) values lie in the required interval $[0, 1]$, in contrast to the standard score function which ranges in $[-1, 1]$. The proposed function exhibits monotonic growth relative to the membership degree μ, while remaining monotonically decreasing in non-membership degree ν, confirming its logical ranking properties for SR-FNs. The robust five-step methodology efficiently transforms the fuzzy SRFTP into an equivalent crisp TP. By applying the novel score function to three distinct models, viz., Fuzzy transportation costs only, fuzzy availability and requirement only, all parameters (costs, availability, and requirement) fuzzy, the study successfully computes the optimal transportation cost in each case. Numerical results from the illustrative example yielded minimum costs of 8.75, 0.05 and 0.34 for the 1st, 2nd, and 3rd type of SRFTP, respectively. This consistent applicability across all models validates both the effectiveness of the SR-FN framework and the superiority of the proposed positive score function in mitigating practical logistical uncertainties.

8. Conclusion

We have developed transportation problem models under SR-fuzzy environment. The study successfully introduced, formulated, and provided an efficient algorithmic solution for these models. The key accomplishment is the design of a new positive score function that accurately defuzzifies Square Root Fuzzy Numbers (SR-FNs) for cost-minimization purposes. This enables seamless conversion of the inherently uncertain fuzzy problem into a standard crisp transportation model, allowing the use of well-established methods such as the VAM and the MODI method to obtain optimal allocations and minimum total cost. The proposed methodology represents a significant advancement by extending the toolkit available for optimization under uncertainty, particularly due to the increased modeling flexibility of SR-FNs provide compared with classical fuzzy set theories.

Future research avenues

Building upon this foundation, promising directions for future research include:

1. *Multi-objective SRFTP*: Extending the current single-objective cost-minimization model to a Multi-Objective Transportation Problem (MOTP) that simultaneously optimizes additional criteria such as time, risk, carbon emissions, or reliability, all represented by SR-FNs.
2. *Methodological comparisons*: Conducting comprehensive comparative studies between the proposed SR-FN score function and alternative ranking/aggregation operators (e.g., accuracy functions, entropy measures, or other defuzzification techniques) when applied to SRFTP and related problems.
3. *Generalization of SR-FN applications*: Applying the developed SR-FN framework and score function to a wider class of optimization problems under uncertainty, including assignment problems, transshipment problems, network flow models, vehicle routing, and availability chain design.

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