USE OF WEIGHTS SUM OF SQUARES RATIO AS AN ALTERNATIVE TO RELATIVE EFFICIENCY IN CHOICE OF BEST LINEAR UNBIASED ESTIMATOR OF THE POPULATION MEAN FOR CORRELATED RANDOM VARIABLES

Abstract

When choosing the best linear unbiased estimator (BLUE) of the population mean for correlated random variables with a known covariance structure, the weights sum of squares ratio (WSSR) is recommended as an alternative to relative efficiency. In statistical computing, the weights of the linear unbiased estimator (LUE) of population mean have a big impact on how stable the estimator is. The weights guarantee that LUE are unbiased and have minimal or reduced variance across comparable classes of estimators. For correlated random variables with a moving average process of order one structure, the Langrangian Multiplier Symmetric Based Modified (LMSBM) approach was used to calculate the efficient weights of the generalized linear unbiased estimator (GLUE) of the population mean. The geometric features of the vector of weights were used to establish the sufficient condition for algebraic equivalence of two LUE's (GLUE and arithmetic mean (AM)) of the population mean. Theoretical insights from the result led to the development of a new statistical method that uses $WSSR$ instead of nariance based relative efficiency to measure GLUE's efficiency in comparison to AM. The findings demonstrate that the correlation point's interval across the different sample sizes converges approximately to $[-0.20, 0.20]$ if either $WSSR$ or variance based relative efficiency is used as the basis for efficiency judgement. This corresponds with the correlation point's interval across the various sample sizes obtained using $WSSR$ for judgement. $WSSR$ is therefore preferred for use as an alternative to check the efficient performance of GLUE in relation to AM since it is simple to compute and the correlation points interval across the various sample sizes for the equi-efficiency of GLUE in relation to AM corresponds with the results of WSSR evaluation. The applicability of WSSR as a foundation for efficiency evaluation was illustrated through real-life application. Therefore, WSSR is recommended for use as a suitable alternative to relative efficiency.

Key words: Weights Sum of Squares Ratio (WSSR), Relative Efficiency, Generalized Linear Unbiased Estimator (GLUE), Arithmetic Mean (AM), Best Linear Unbiased Estimator (BLUE)

1.0 Introduction

Let

$$\tilde{X}_{\eta} = \sum_{i=1}^{\eta} \varpi_i X_i ; \forall \varpi_i \geq 0 \tag{1.1}$$

be the generalized linear unbiased estimator (GLUE) of the mean, whose vector space structural representation is;

$$\tilde{X}_{\eta} = \begin{pmatrix} \varpi_1 & \varpi_2 & . & . & . & \varpi_{\eta} \end{pmatrix} \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_{\eta} \end{bmatrix} = \varpi_{\eta}^{T} \mathbf{X}_{\eta} = \varpi_{\eta} \mathbf{X}_{\eta}^{T} \tag{1.2}$$

$$\varpi_n = \begin{bmatrix} \varpi_1 & \varpi_2 & . & . & . & \varpi_n \end{bmatrix}^T \tag{1.3}$$

where ϖ_i is the *ith* weights, $\varpi_1, \varpi_2, ..., \varpi_{\eta}$ are sequence of weights, η is the number of observations, X_i is the *ith* observations or variables, $X_1, X_2, ..., X_{\eta}$ are the sequence of observations or variables, ϖ_{η} is an $(\eta \times 1)$ vector of weights, ϖ_{η}^{T} is the transpose of ϖ_{η}, $\mathbf{X}_{\eta}$ is an $(\eta \times 1)$ vector of sequence of observations and $\mathbf{X}_{\eta}^{T}$ is the transpose of $\mathbf{X}_{\eta}$. It is evident that GLUE is a linear combination of observations and the corresponding unknown weights (John, 1965; Even & Binley, 1992; Montgomery et al, 2015). Again, GLUE is also a member of the class of linear unbiased estimators (LUEs), which are linear and unbiased (Montgomery et al, 2015). Because of the difficulty with GLUE weight computation, analysts in the literature have either ignored it or avoided utilizing it. Instead, utilize the arithmetic mean (AM), which is a geometric centroid metric of all LUEs and an example of GLUE to approximate the population mean of observations (Box et al, 2016; Wei, 2023). The computation of GLUE's unknown weights is a crucial statistical computing step which has drawn attention in the literature (Iwueze et al, 2011; 2012; 2015a,b; Onyemachi et al, 2026).

To guarantee GLUE's stability and turn it into BLUE, efficient weights are crucial. When the total of its efficient weights equals unity, the GLUE expectation produces an unbiased approximation of the population mean. However, when effectively calculated weights are used, GLUE produces the most stable population mean estimate in that class with the least amount of variance; this desirable estimator will be referred to as BLUE in this study. "So, how do we choose a desirable and well-informed estimator among its similar class with minimal or reduced variance?" is the question. In particular, how can we determine which estimator is BLUE by making an informed decision between GLUE and AM? Determining the rival estimators' (GLUE and AM) efficiency is one method of making such a decision.

The ideas of first and second order efficiency as well as relative efficiency have been used in the literature to explore efficiency, which is a measure of desirability when choosing between similar classes of linear unbiased estimators (Everitt, 2002; Michael, 1986). It is challenging to choose one of a similar class of linear unbiased estimators based on first order efficiency if the difference between their estimates does not converge to zero in probability (Shao, 2003). Second, in order to find second order efficiency asymptotically, the estimator's variance must equal the Cramer-Rao lower limit up to second order (Michael, 1986; Rao, 1961; Rao, 1962). Furthermore, second order efficiency focuses on the variance of estimators while ignoring the dependence structure between the variables (Michael, 1986). Relative efficiency (RE) has been favored in the literature to assess how well two similar classes of linear estimators compare in terms of precision in order to make an informed choice of the estimator to use when estimating a parameter in finite samples because of the shortcomings of the aforementioned efficiency measures (Shabbir & Gupta, 2011). However, when the variables have a specified structure, calculating the variance of some estimators appears to be practically challenging, time-consuming, and requiring a certain level of skill, which may limit or impede the choice of the best estimator to use. In particular, think about calculating the variance of GLUE when the variables are associated with a moving average process of order one $[MA(1)]$ that has a defined covariance structure (Iwueze et al, 2015 a,b; Box et al., 2016). The variance of the observed correlated random variables multiplied by the sum of the

squares of the weights and twice the sum of the product of the correlation index with the corresponding weights is the variance of GLUE, as we can see in (1.4);

$$\operatorname{var}\left(\tilde{X}_{\eta}\right) = R(0)\left\{\sum_{i=1}^{\eta}\varpi_i^{\,2} + 2\sum_{k=1}^{\eta-1}\sum_{i=1}^{\eta-k}\rho_k\,\varpi_i\,\varpi_{i+k}\right\} \tag{1.4}$$

where $R(0)$ is the variance of the $X_1, X_2, \ldots, X_{\eta}$ and ρ_k is the kth lag correlation index of the observed correlated random variables. It appears to be practically challenging, time-consuming, and requiring a certain amount of experience to compute this variance in (1.4). When the correlated variables have a moving average process of order one covariance structure or a comparable complex variance structure, we recommend that the analyst use an alternative measure of relative efficiency for straightforward statistical evaluation that does not compute variances but produces comparable results in differentiating among LUEs of mean. Consequently, such alternative measure of relative efficiency will be simpler to evaluate, analyze, and make inferences when deciding which BLUE to use for mean estimate. A novel method utilizing the weight sum of squares ratio (WSSR) of the competing estimators (GLUE and AM) will be explicitly presented in this paper as an alternative to the relative efficiency in making an informed choice of BLUE to use for estimating the population mean of correlated variables with $MA(1)$ covariance structure, because the efficient weights of LUE play significant roles on the estimator's stability.

It is important to remember that mathematicians utilize linear algebra to study the vector space and mathematical structures of linear estimators and comprehend their behavior. This is crucial to keep in mind while studying linear estimators for efficiency's sake (Fraleigh and Beauregard, 1990). This approach can lead to the development of novel statistical methods and provides theoretical insights into the fundamental statistical concepts (Shao, 2003; Fraleigh and Beauregard, 1990). Based on this notion, Section 2.0 presents the methodology, which includes the novel approach that measures GLUE's efficiency in relation to AM without calculating their variances to determine the BLUE. Section 3.0 discusses the results, Section 4.0 discusses the application to real-life data, and Section 5.0 provides the conclusion.

2.0 Methodology

The structural form, autocovariance and autocorrelation functions of moving average process of order one is discussed in Section 2.1. The structural form, vector space structure and variance of arithmetic mean (AM) for correlated variables are presented in Section 2.2. The efficient method for computing BLUE's weights for correlated variables with moving average process of order one, $MA(1)$, is given due attention in Section 2.3.The study will look at the condition within which AM and GLUE are algebraically equivalent. In our context, algebraic equivalent of the AM and GLUE are determined by the equality of their weights in magnitude and both being in the same direction as discussed in Section 2.4. The use of estimator's weights sum of squares ratio is a new but, simpler novel approach or method proposed in the study as an alternative method for measurement of efficiency. The new method is described in Section 2.5. For completeness, the traditional means of measuring efficiency based on variances is dealt with in Section 2.6.

2.1 The structural form and Covariance Structure of Moving Average Process of order One

The structural form representation of the moving average rocess of order one $(\mathrm{MA}(1))$ process is (Chatfield, 2004; Box et al, 2016; Brockwell & Davis, 2016; Marcel, 2020; Wei, 2023; Balakrishna, 2023);

$$X_t = \psi_0 + \psi_1 a_{t-1} + a_t = \psi_0 + (1+\psi_1 B) a_t \; ; \tag{2.1}$$

where a_t is a white noise process with zero mean and constant variance $\sigma_a^{\,2}$, ψ_1 is a real coefficient, ψ_0 is a constant and B is the backward shift operator. $\mathrm{MA}(1)$ process is always stationary but it is invertible if $|\psi_1|<1$. The expectation of $\mathrm{MA}(1)$ process is;

$$E(X_t) = \psi_0 \tag{2.2}$$

The autocovariance $(R(k))$ of (2.1) is (Marcel, 2020; Balakrishna, 2023);

$$R(k) = \begin{cases} (1+\psi_1^{\,2})\sigma_a^2, k=0 \\ \psi_1 \sigma_a^2, \qquad k=\pm 1 \\ 0, \qquad\qquad k \neq \pm 1 \end{cases} \tag{2.3}$$

while, the autocorreation (ρ_k) function of (2.1) is (Marcel, 2020; Balakrishna, 2023);

$$\rho_k = \begin{cases} 1, & k=0 \\ \dfrac{\psi_1}{1+\psi_1^{\,2}}, & k=\pm 1 \\ 0, & k \neq \pm 1 \end{cases} \tag{2.4}$$

The covariance structure of moving average process of order $q, (\mathrm{MA(q)}, q=1)$ will be utilized while deriving the weights of the best linear unbiased estimator (BLUE). Throughout the discussion, we shall assume that $\rho_1 = \rho$ while deriving the BLUE 's weights.

2.2 The structural form, vector space structure and variance of arithmetic mean (AM)

The mathematical structural form of AM $(\overline{X}_\eta)$ is (Montgomery et al, 2015);

$$\overline{X}_\eta = \frac{1}{\eta}\sum_{i=1}^{\eta} X_i = \sum_{i=1}^{\eta} \beta_i X_i \tag{2.5}$$

while, its vector space structural representation is;

$$\overline{X}_\eta = \begin{pmatrix} \frac{1}{\eta} & \frac{1}{\eta} & . & . & . & \frac{1}{\eta} \end{pmatrix} \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_\eta \end{bmatrix} = \boldsymbol{\beta}_\eta^{\,T} \mathbf{X}_\eta = \boldsymbol{\beta}_\eta \mathbf{X}_\eta^{\,T} \tag{2.6}$$

$$\boldsymbol{\beta}_\eta = \begin{bmatrix} \frac{1}{\eta} & \frac{1}{\eta} & . & . & . & \frac{1}{\eta} \end{bmatrix}^T = \begin{bmatrix} \beta_1 & \beta_2 & . & . & . & \beta_\eta \end{bmatrix}^T \tag{2.7}$$

$$\mathbf{X}_\eta = \begin{bmatrix} X_1 & X_2 & . & . & . & X_\eta \end{bmatrix}^T \tag{2.8}$$

The variance of $\overline{X}_\eta$ is given as (Chatfield, 2004; Box et al, 2016; Wei, 2023);

$$\mathrm{var}\,(\overline{X}_\eta) = \frac{R(0)}{\eta}\left[1 + 2\sum_{k=1}^{\eta-1}\left(1-\frac{k}{\eta}\right)\rho_k\right] \tag{2.9}$$

where η is the number of observations, $X_1, X_2, \ldots, X_\eta$ are the sequence of correlated observations or variables, $\boldsymbol{\beta}_\eta$ is an $(\eta \times 1)$ vector of weights, $\boldsymbol{\beta}_\eta^{\,T}$ is the transpose of $\boldsymbol{\beta}_\eta$, $\mathbf{X}_\eta$ is an $(\eta \times 1)$ vector of sequence of correlated observations, $\mathbf{X}_\eta^{\,T}$ is the transpose of $\mathbf{X}_\eta$, $R(0)$ is the variance of the correlated variables and ρ_k is the kth lag correlation index of

the correlated observations.

2.3 Method for Computing Generalized Linear Unbiased Estimator (GLUE) Weights

Weights of generalized linear unbiased estimator (GLUE) will be computed in this paper using the Langrangian multiplier symmetric based modified (LMSBM) method discussed in Onyemachi et al (2026). The method involves, first, minimizing the variance (1.4) of GLUE subject to linear constraints (the generalized symmetric pattern of the vector of weights and unbiasedness property) (Onyemachi et al, 2026). The resulting quadratic programming problem reduces to standard condensed form as;

$$\left.\begin{aligned} &Min\ S(\varpi_h) = \varpi_h^{\ T}\ \rho_h\ \varpi_h \\ &S.t.:\ \mathbf{J}_h^{\ T}\ \varpi_h = 1 \\ &\qquad \varpi_h \geq 0\ \forall\ h \end{aligned}\right\} \tag{2.10}$$

where ϖ_h is the $(h\times 1)$ vector of unknown weights of GLUE, $\mathbf{J}_h$ is the $(h\times 1)$ vector of constants, $\boldsymbol{\rho}_h$ is the $(h\times h)$ correlation matrix of the correlated random variables with the covariance structure of moving average process and the dimension h is given as;

$$h = \begin{cases} \dfrac{\eta}{2};\eta = 2\varsigma\ \ for\ \text{fixed positive interger}\ \varsigma > 0 \\ \dfrac{\eta+1}{2};\eta = 2\varsigma + 1\ \text{for fixed positive interger}\ \varsigma > 0 \end{cases} \tag{2.11}$$

The LMSBM model for computing the weights of GLUE was obtained as;

$$L(\varpi_h, \lambda) = \varpi_h^{\ T}\ \boldsymbol{\rho}_h\ \varpi_h - \lambda\left(\mathbf{J}_h^{\ T}\ \varpi_h - 1\right) = \varpi_h^{\ T}\ \boldsymbol{\rho}_h\ \varpi_h - \lambda\left(\varpi_h^{\ T}\ \mathbf{J}_h - 1\right) \tag{2.12}$$

where λ is the Langrangian Multiplier constant and $\mathbf{1}_h^{\ T}\ \varpi_h$ is a scalar. The LMSBM model was minimized with respect to ϖ_h using the method of Calculus to obtain;

$$\frac{\partial L(\varpi_h, \lambda)}{\partial \varpi_h} = 2\boldsymbol{\rho}_h\ \varpi_h - \lambda\ \mathbf{J}_h \tag{2.13}$$

Then, mathematical expressions for calculating the estimates of ϖ_h as ϖ_{h0} were obtained by equating (2.13) to zero as;

$$\varpi_{h0} = \frac{\boldsymbol{\rho}_h^{\ -1}\ \mathbf{J}_h}{\mathbf{J}_h^{\ T}\ \boldsymbol{\rho}_h^{\ -1}\ \mathbf{J}_h} \tag{2.15}$$

where $\boldsymbol{\rho}_h^{-1}$ is the inverse of Matrix $\boldsymbol{\rho}_h$.

2.4 Sufficient Condition for Algebraic Equivalence of Two Linear Unbiased Estimators of the Population Mean

In the paper, the sufficient condition for algebraic equivalence of two linear unbiased estimators (the arithmetic mean (AM) $\left(\overline{X}_\eta\right)$ and the generalised linear unbiased estimator (GLUE) $\left(\widetilde{X}_\eta\right)$) of the population mean via their vector of weights are defined using the geometric properties (the magnitude or length of the vector, the distance from one vector of weights to another, and the implication of the angle indicating the direction between the two nonzero vectors); $\boldsymbol{\beta}_\eta$ for AM and ϖ_η for GLUE as stated in Definition 2.1 (Fraleigh and Beauregard, 1990).

Definition 2.1: Two nonzero vectors $\boldsymbol{\beta}_\eta$ and ϖ_η in $\Re^\eta$ are said to be equal in magnitude and same direction iff

i. $\left\|\boldsymbol{\beta}_\eta\right\| = \left\|\varpi_\eta\right\|$ for any fixed positive interger η .

ii. $\left\|\boldsymbol{\beta}_\eta - \varpi_\eta\right\| = \mathrm{d}\left(\boldsymbol{\beta}_\eta, \varpi_\eta\right) = \sqrt{\sum_{i=1}^{\eta}\left(\beta_i - \varpi_i\right)^2} = 0$.

iii. $\left|\boldsymbol{\beta}_\eta \bullet \varpi_\eta\right| = \left\|\boldsymbol{\beta}_\eta\right\|\left\|\varpi_\eta\right\|$.

The sufficient condition for the algebraic equivalence of AM and GLUE is established in Theorem 2.1.

Theorem 2.1: Consider the Generalized Linear Unbiased Estimator (GLUE) and Arithmetic Mean (AM)) of correlated variables with the first order moving average process autocorrelation structure. Let $\varpi_i, i = 1, 2, \ldots, \eta$ be the weights associated with the GLUE of the population mean (μ) and $\frac{1}{\eta}$ be the weights associated with the AM of the population mean (μ). Then, $\sum_{i=1}^{\eta}\varpi_i^{\,2} = \frac{1}{\eta}$, if the two estimators are equal or algebraically equivalent.

Proof: From Definition (2.1i):

$$\|\boldsymbol{\beta}_\eta\| = \sqrt{\sum_{i=1}^{\eta}\left(\frac{1}{\eta}\right)^2} = \sqrt{\sum_{i=1}^{\eta}\beta_i^{\,2}} = \sqrt{\frac{1}{\eta}} \Rightarrow \|\boldsymbol{\beta}_\eta\|^2 = \frac{1}{\eta} \tag{2.17}$$

$$\|\varpi_n\| = \sqrt{\sum_{i=1}^{\eta}\varpi_i^{\,2}} = \left(\varpi_\eta^{\,T}\varpi_\eta\right)^{\frac{1}{2}} \Rightarrow \|\varpi_\eta\|^2 = \sum_{i=1}^{\eta}\varpi_i^{\,2} = \left(\varpi_\eta^{\,T}\varpi_\eta\right) \tag{2.18}$$

Equating (2.17) and (2.18), we obtain;

$$\|\boldsymbol{\beta}_\eta\|^2 = \frac{1}{\eta} = \left(\varpi_\eta^{\,T}\varpi_\eta\right) = \sum_{i=1}^{\eta}\varpi_i^{\,2} \tag{2.19}$$

From Definition (2.1ii):

$$\mathrm{d}\left(\boldsymbol{\beta}_\eta, \varpi_\eta\right) = \sqrt{\sum_{i=1}^{\eta}\left(\beta_i - \varpi_i\right)^2} = \sqrt{\sum_{i=1}^{\eta}\left(\frac{1}{\eta} - \varpi_i\right)^2} = 0$$

$$\Rightarrow \quad \sum_{i=1}^{\eta}\left(\frac{1}{\eta} - \varpi_i\right)^2 = \sum_{i=1}^{\eta}\left(\left(\frac{1}{\eta}\right)^2 - 2\frac{\varpi_i}{\eta} + \varpi_i^{\,2}\right) = 0$$

$$\Rightarrow \quad \sum_{i=1}^{\eta}\varpi_i^{\,2} - \frac{1}{\eta} = 0 \text{ provided } \sum_{i=1}^{\eta}\varpi_i = 1.$$

$$\sum_{i=1}^{\eta}\varpi_i^{\,2} = \frac{1}{\eta} \tag{2.20}$$

However, an alternative way of proving Theorem 2.1 is to apply Cauchy Schwartz lower bound in Definition (2.1iii) (Earl, 1988).

2.5 Weight Sum of Squares Ratio as Alternative to Relative Efficiency

The weight sum of squares ratio (WSSR) is proposed for use as an alternative measure of relative efficiency. The WSSR of the vector of coefficients of Generalized linear unbiased estimator (GLUE) relative to Arithmetic mean (AM), $WSSR\left(\overline{X}_\eta, \tilde{X}_\eta\right)$ is given as;

$$WSSR\left(\overline{X}_\eta, \tilde{X}_\eta\right) = \frac{\sum_{i=1}^{\eta}\beta_i^{\,2}}{\sum_{i=1}^{\eta}\varpi_i^{\,2}} = \frac{1}{\eta\sum_{i=1}^{\eta}\varpi_i^{\,2}} \tag{2.21}$$

Then the following decisions hold; if

(a) $WSSR\left(\overline{X}_\eta, \tilde{X}_\eta\right) > 1; \sum_{i=1}^{\eta}\varpi_i^{\,2} < \frac{1}{\eta} \Rightarrow$ AM is more efficient than GLUE and AM is the BLUE.

(b) $WSSR\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)<1;\sum_{i=1}^{\eta}\varpi_i^{\ 2}>\frac{1}{\eta}\Rightarrow$ GLUE is more efficient than AM and GLUE is the BLUE. **(c)** $WSSR\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)=1;\sum_{i=1}^{\eta}\varpi_i^{\ 2}=\frac{1}{\eta}\Rightarrow$ the two estimators are 100% similar and/or equi-efficient. Throughout the paper $WSSR\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)$ will be simply written as $WSSR$.

Proof: See that from Section (2.4);

$$\begin{cases}\sum_{i=1}^{\eta}\varpi_i^{\ 2}=\sum_{i=1}^{\eta}\beta_i^{\ 2}=\frac{1}{\eta};\text{GLUE is algebraica lly equivalent to AM}\\ \sum_{i=1}^{\eta}\varpi_i^{\ 2}\neq\sum_{i=1}^{\eta}\beta_i^{\ 2}=\frac{1}{\eta};\text{GLUE is not algebraica lly equivalent to AM}\end{cases}$$

$$\sum_{i=1}^{\eta}\varpi_i^{\ 2}\neq\sum_{i=1}^{\eta}\beta_i^{\ 2}=\frac{1}{\eta}\Rightarrow\sum_{i=1}^{\eta}\varpi_i^{\ 2}>\sum_{i=1}^{\eta}\beta_i^{\ 2}=\frac{1}{\eta}\ or\ \sum_{i=1}^{\eta}\varpi_i^{\ 2}<\sum_{i=1}^{\eta}\beta_i^{\ 2}=\frac{1}{\eta}$$

Following the trichotomy property of real numbers (Earl, 1988),

$$WSSR\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)=\frac{\sum_{i=1}^{\eta}\beta_i^{\ 2}}{\sum_{i=1}^{\eta}\varpi_i^{\ 2}}=\frac{1}{\eta\sum_{i=1}^{\eta}\varpi_i^{\ 2}}=\vartheta_0=\begin{cases}>1\Rightarrow\sum_{i=1}^{\eta}\varpi_i^{\ 2}<\frac{1}{\eta}\\ <1\Rightarrow\sum_{i=1}^{\eta}\varpi_i^{\ 2}>\frac{1}{\eta}\\ =1\Rightarrow\sum_{i=1}^{\eta}\varpi_i^{\ 2}=\frac{1}{\eta}\end{cases}$$

2.6 Efficiency of the Estimating Methods using Variances

The variance-based relative efficiency, $\mathrm{Re}l\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)$, of AM relative to GLUE is defined as (Iwueze et al, 2015a, b);

$$\mathrm{Re}l\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)=\frac{\mathrm{var}\left(\tilde{X}_{\eta}\right)}{\mathrm{var}\left(\overline{X}_{\eta}\right)}\tag{2.22}$$

Then the following decisions hold; if

(a) $\mathrm{Re}l\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)<1\Rightarrow\mathrm{var}\left(\overline{X}_{\eta}\right)>\mathrm{var}\left(\tilde{X}_{\eta}\right)$; GLUE is more efficient than AM and GLUE is the BLUE. (b) $\mathrm{Re}l\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)>1\Rightarrow\mathrm{var}\left(\overline{X}_{\eta}\right)<\mathrm{var}\left(\tilde{X}_{\eta}\right)$; AM is more efficient than GLUE and AM is the BLUE. (c) $\mathrm{Re}l\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)=1\Rightarrow\mathrm{var}\left(\overline{X}_{\eta}\right)=\mathrm{var}\left(\tilde{X}_{\eta}\right)$; the two estimators are 100% similar and/or equi-efficient. Throughout the paper, $\mathrm{Re}l\left(\overline{X}_{\eta},\ \tilde{X}_{\eta}\right)$ will be simply written as $\mathrm{Re}l$.

3.0 Results and Discussion

According to Onyemachi et al. (2026), when the sample size (η) is even or odd; $\eta = 2\varsigma$ *or* $\eta = 2\varsigma + 1 \forall \varsigma = 1, 2, ..., 8$, Tables 1 and 2 display the algebraic expressions derived from evaluating (2.15) required for calculating the estimate of weights of GLUE for correlated variables that admits the covariance structure of the moving average process of order one, obtained using the efficient method. When sample sizes are equal to four, thirteen, and sixteen at the various correlational points in Tables 3, 4, and 5, respectively, estimates of GLUE's weights were calculated and purposefully displayed for illustration with the corresponding algebraic expressions of these weights. The findings in Tables 3, 4, and 5 demonstrate that the estimations of GLUE's weights at each correlational point are positive definite, suggesting that GLUE's weights are comparable to AM's weights along the same Cartesian plane coordinates. Additionally, the results demonstrate that each set of estimates resulted in an unbiased estimate of the population mean because the sum of the estimates at each correlation point equals one. Further, when the correlation index value is either zero or doubtful, the results in Tables .3, 4, and 5 demonstrate that GLUE's weights are equal to AM's weights. The equi-efficient of GLUE and AM results from this range of values where the correlation index is either zero or dubious. GLUE's weights sum of squares $\left[\sum_{i=1}^{\eta} \varpi_i^{\,2}\right]$, AM's weights sum of squares $\left[\frac{1}{\eta}\right]$, weights sum of squares ratio $[WSSR]$, GLUE's and AM's variances, and variance-based relative efficiency $\left[\mathrm{Re}l\left(\bar{X}_\eta, \tilde{X}_\eta\right)\right]$ were calculated when sample sizes (η) were four and thirteen, as demonstrated in Tables 6 and 7, in order to determine the bounds within which GLUE and AM are equi-efficient at particular sample sizes and varying correlation points. The results in Tables 6 and 7 demonstrate that at all sample sizes and different correlation points, $\sum_{i=1}^{\eta} \varpi_i^{\,2} \geq \frac{1}{\eta}$ *and* $\mathrm{var}\left(\bar{X}_\eta\right) \geq \mathrm{var}\left(\tilde{X}_\eta\right)$ *for fixed* η. When the correlation points are not questionable (significantly different from zero) then $\sum_{i=1}^{\eta} \varpi_i^{\,2} > \frac{1}{\eta}$ and $\mathrm{var}\left(\bar{X}_\eta\right) > \mathrm{var}\left(\tilde{X}_\eta\right)$

$for\ fixed\eta$, but when the correlation points are questionable (significantly not different from zero) then $\sum_{i=1}^{\eta}\varpi_i^{\ 2}=\frac{1}{\eta}$ and $\mathrm{var}\left(\overline{X}_\eta\right)=\mathrm{var}\left(\tilde{X}_\eta\right)$ $for\ fixed\eta$. Consequently, the interval or region of the correlation points at different sample sizes where $\sum_{i=1}^{\eta}\varpi_i^{\ 2}>\frac{1}{\eta}$ and $\mathrm{var}\left(\overline{X}_\eta\right)>\mathrm{var}\left(\tilde{X}_\eta\right)$ $for\ fixed\eta$ produces the region where GLUE is BLUE, while when $\sum_{i=1}^{\eta}\varpi_i^{\ 2}=\frac{1}{\eta}$ and $\mathrm{var}\left(\overline{X}_\eta\right)=\mathrm{var}\left(\tilde{X}_\eta\right)$ $for\ fixed\eta$ produces the region of the correlation points where GLUE is equi-efficient with AM. Therefore, using $WSSR$ and $\mathrm{Re}l$ as a foundation for judgment, the interval of correlation points at various odd and even sample sizes (η); $\eta=2\varsigma\ and\ \eta=2\varsigma+1\forall\varsigma=1,2,...,8$; within which GLUE is equi-efficient with AM was chosen, as shown in Tables 8. According to Table 8's results, AM and BLUE have a favorable competition inside the correlation points interval throughout the different sample sizes, and when the sample size becomes closer to seventeen, the correlation points interval across the different sample sizes converges. The findings in Table 8 also demonstrate that when $WSSR$ is used as the basis for evaluating the estimators' efficiency, the correlation points interval across the various sample sizes converges to $[-0.20,0.20]$, whereas when $\mathrm{Re}l$ is used as the basis for evaluating the estimators' efficiency, the correlation points interval across the various sample sizes converges to $[-0.17,0.24]$. This result shows that if either $WSSR$ or $\mathrm{Re}l$ are used as the basis for efficiency judgement, the correlation points interval across the various sample sizes converges roughly to $[-0.20,0.20]$. This interval corresponds with the correlation points interval across the various sample sizes obtained using WSSR for judgement. Because it is easy to compute and the correlation points interval across the different sample sizes for the equi-efficiency of GLUE in relation to AM corresponds with the results of its evaluation, $WSSR$ is therefore favored for use as an alternative to check the efficient performance of GLUE in relation to AM. For correlated random variables with sample sizes of four and thirteen, Tables 9 and 10 provide estimates of the weights of the

Best Linear Unbiased Estimates (BLUE) chosen using $WSSR$ as a basis for evaluating efficiency.

4.0 APPLICATION

Real-world data on the monthly average exchange of Naira per unit of EURO currency for the period January 2004 to December 2018 that admits moving average process of order one (MA(1)) was used to empirically demonstrate the use of weights sum of squares, $WSSR$, as an alternative to relative efficiency in the choice of best linear unbiased estimates (BLUE) of the population mean of correlated variables (CBN, 2014; CBN, 2021; Onyemachi et al, 2016). The data's stationary series admits the MA(1) process, whose parameter estimates, residual sum of squares, and Modified Box-Pierce (Ljung-Box) Chi-Square Statistic were calculated using Minitab 22 series software as (Onyemachi et al, 2026).

Final Estimates of Parameters

Type	Coef	SE Coef	T-Value	P-Value
MA 1	-0.4001	0.0687	-5.82	0.000

Differencing: 1 regular difference

Residual Sums of Squares

DF	SS	MS
178	9761.46	54.8397

Back forecasts excluded

Modified Box-Pierce (Ljung-Box) Chi-Square Statistic

Lag	12	24	36	48
Chi-Square	5.84	16.76	21.04	29.37
DF	11	23	35	47
P-Value	0.884	0.821	0.970	0.979

Because the modified Box-Pierce (Ljung-Box) Chi-Square test statistic is less than $\chi^2(\ln n)$ tabulated value and the p-value at the seasonal lags is greater than the 0.05 level of significance, the modified Box-Pierce (Ljung-Box) Chi-Square test statistic and p-value at the seasonal lags—12, 24, 36, and 48—are not significant. Consequently, there is no rejection of the premise that the residual is independent and identically distributed. In a similar vein, the MA(1) model's estimated coefficient's p value is less than the 0.05 level of significance, suggesting that the estimated coefficient significantly influences the model's behavior. Therefore, the computation of the BLUE's weights for correlated variables that admits the covariance structure of the MA(1) model was demonstrated by purposefully choosing four distinct data sets from the real-life series, as shown in Table 11, with varying numbers of

observations equal to ten, eleven, twelve, and thirteen (Onyemachi et al., 2026). Because there is a cutoff at lag one of the autocorrelation coefficients, the estimates of autocorrelation coefficients across the different sample sizes for the four purposefully chosen real-life data sets displayed in Table 12 show that all four selected real-life data sets admit moving average processes of order one (Onyemachi et al, 2026). Thus, the estimations of BLUE's weights and population mean with its associated variance was computed following the algorithm;

i. Use the method of likelihood estimation (MLE) to estimate the coefficient of MA(1) process, teta $\hat{\theta}$, fitted to the correlated variables and using Newton Rapherson Method to implement its convergence (Jain et al, 2022;).

ii. Calculate the MA(1) process fitted to the correlated variables' error variance ($\hat{\sigma}^2$).

iii. Choose the initial value of ρ by estimating the theoretical autocorrelation coefficients of MA(1) process with;

$$\hat{\rho} = \frac{\hat{\psi}}{1+\hat{\psi}^2} \tag{4.1}$$

iv. To determine the estimates of GLUE's weights for selected sequence of correlated variables, $\langle X_t, t=1,2,\ldots,\eta \rangle$ at each samle size η, evaluate GLUE's algebraic expressions at the varied numbers of observations (n) with the value of $\hat{\rho}$.

v. Compute the population mean and variance of the correlated variables, $\langle X_t, t=1,2,\ldots,\eta \rangle$ with the computed estimates of GLUE's weights in step iv.

vi. Calculate weights sum of squares ratio (WSSR) and variance based relative efficiency $(\mathrm{Re}\,l)$ estimates.

vii. If $\sum_{i=1}^{\eta} \varpi_i^2 > \frac{1}{\eta}$ such that $WSSR < 1$ and $\mathrm{var}(\bar{X}_\eta) > \mathrm{var}(\tilde{X}_\eta)$ such that $\mathrm{Re}\,l < 1$, GLUE is more efficient than AM, and GLUE is the best linear unbiased estimator (BLUE) with its weights $\varpi_1, \varpi_2, \ldots, \varpi_\eta$ regarded as efficient weights estimates for mean estimation .

The findings in Table 13 show that the MA(1) process coefficients are highly negative, ranging from -0.806 to -0.884 across sample sizes. Table 14 demonstrates that the usage of the weights will result in the realization of an unbiased estimate of the population mean because the total of the estimates of GLUE's weights evaluated at different sample sizes equals one. Furthermore, the outcome in Table 15 demonstrates that the use of GLUE's weights, as indicated in Table 14, computes the mean estimate of the correlated variables more precisely than AM across the different sample sizes of the four purposively selected data sets. The results also show that $\sum_{i=1}^{\eta} \varpi_i^{\,2} > \frac{1}{\eta}$ such that $WSSR < 1$ and $\mathrm{var}\left(\bar{X}_{\eta}\right) > \mathrm{var}\left(\tilde{X}_{\eta}\right)$ such that $\mathrm{Re}\,l < 1$ across varying sample sizes (η) of the four purposively selected data sets. According to Table 15's results, GLUE is the best linear unbiased estimator (BLUE) and is therefore more efficient than AM. WSSR is therefore advised as an acceptable alternative for relative efficiency in selecting the best linear unbiased estimator (BLUE) of population mean of correlated variables with defined covariance structure after it was discovered that using $WSSR$ or $\mathrm{Re}\,l$ produced the same result.

5.0 Conclusion

When selecting the best linear unbiased estimator (BLUE) of the population mean for correlated random variables with moving average processes of order one covariance structure, the weights sum of squares ratio ($WSSR$) has been proposed as an alternative to relative efficiency. The study took into consideration two linear unbiased estimators (LUE) of the population mean for correlated random variables: the arithmetic mean (AM) and the generalized linear unbiased estimator (GLUE). In statistical computing, the weights of these LUEs of population mean have a major impact on their stability, ensuring that they are unbiased and have little to no variance. The sufficient condition for algebraic equivalence of AM and GLUE via their vector of weights was determined by applying linear algebra and the geometric qualities of the weights. In order to use $WSSR$ instead of relative efficiency when selecting the best linear unbiased estimator (BLUE) of the population mean for correlated random variables with moving average processes of order one covariance structure, new

statistical techniques were developed as a result of the established result in theorem 2.1. Unlike the traditional approach of measuring relative efficiency, $WSSR$ was used to measure the efficiency of GLUE relative AM. The results show that employing $WSSR$ or $\mathrm{Re}\,l$ produce the same outcomes when significant correlation indices are utilized in statistical study. Because $WSSR$ is easy to compute, it is recommended as an acceptable alternative for relative efficiency when choosing the best linear unbiased estimator (BLUE) of the population mean of correlated variables with a specific covariance structure.

Table 1: Algebraic Expression of GLUE's weights when sample size $n=2k, h=n/2$ and $h=k$ $\forall k=1,2,\ldots,8$

K	BLUE coefficients or weights
1	$\varpi_1=\left[\varpi_1=\varpi_2\right]=\left[\frac{1}{2}\right]$
2	$\varpi_2=\begin{bmatrix}\varpi_1=\varpi_4\\ \varpi_2=\varpi_3\end{bmatrix}=\frac{1}{2(\rho-2)}\begin{bmatrix}-1\\ \rho-1\end{bmatrix}$
3	$\varpi_3=\begin{bmatrix}\varpi_1=\varpi_6\\ \varpi_2=\varpi_5\\ \varpi_3=\varpi_4\end{bmatrix}=\frac{1}{4\rho^2+4\rho-6}\begin{bmatrix}\rho^2-1\\ \rho^2+\rho-1\\ \rho-1\end{bmatrix}$
4	$\varpi_4=\begin{pmatrix}\varpi_1=\varpi_8\\ \varpi_2=\varpi_7\\ \varpi_3=\varpi_6\\ \varpi_4=\varpi_5\end{pmatrix}=\frac{1}{4\rho^3-12\rho^2-6\rho+8}\begin{pmatrix}1-2\rho^2\\ 1-\rho-2\rho^2+\rho^3\\ 1-\rho-\rho^2+\rho^3\\ 1-\rho-\rho^2\end{pmatrix}$
5	$\varpi_5=\begin{pmatrix}\varpi_1=\varpi_{10}\\ \varpi_2=\varpi_9\\ \varpi_3=\varpi_8\\ \varpi_4=\varpi_7\\ \varpi_5=\varpi_6\end{pmatrix}=\frac{1}{\left[10-8\rho-24\rho^2+12\rho^3+6\rho^4\right]}\begin{pmatrix}1-3\rho^2+\rho^4\\ 1-\rho-3\rho^2+2\rho^3+\rho^4\\ 1-\rho-2\rho^2+2\rho^3\\ 1-\rho-2\rho^2+\rho^3\\ 1-\rho-2\rho^2+\rho^3+\rho^4\end{pmatrix}$
6	$\varpi_6=\begin{bmatrix}\varpi_1=\varpi_{12}\\ \varpi_2=\varpi_{11}\\ \varpi_3=\varpi_{10}\\ \varpi_4=\varpi_9\\ \varpi_5=\varpi_8\\ \varpi_6=\varpi_7\end{bmatrix}=\frac{1}{\begin{bmatrix}12-10\rho-40\rho^2+24\rho^3\\ +24\rho^4-6\rho^5\end{bmatrix}}\begin{bmatrix}1-4\rho^2+3\rho^4\\ 1-\rho-4\rho^2+3\rho^3+3\rho^4-\rho^5\\ 1-\rho-3\rho^2+3\rho^3+\rho^4-\rho^5\\ 1-\rho-3\rho^2+2\rho^3+\rho^4\\ 1-\rho-3\rho^2+2\rho^3+2\rho^4\\ 1-\rho-3\rho^2+2\rho^3+2\rho^4-\rho^5\end{bmatrix}$

Source: Onyemachi et al (2026) **Remark:** All algebraic expressions computed using (2.15).

Table 1 continues: Algebraic Expression of GLUE's weights when sample size $n=2k, h=n/2$ and $h=k \quad \forall k=1,2,...,8$

K	BLUE coefficients or weights
7	$\varpi_7=\begin{bmatrix}\varpi_1=\varpi_{14}\\ \varpi_2=\varpi_{13}\\ \varpi_3=\varpi_{12}\\ \varpi_4=\varpi_{11}\\ \varpi_5=\varpi_{10}\\ \varpi_6=\varpi_9\\ \varpi_7=\varpi_8\end{bmatrix}=\frac{1}{\begin{bmatrix}14-12\rho-60\rho^2+40\rho^3\\ +60\rho^4-24\rho^5-8\rho^6\end{bmatrix}}\begin{bmatrix}1-5\rho^2+6\rho^4-\rho^6\\ 1-\rho-5\rho^2+4\rho^3+6\rho^4-3\rho^5-\rho^6\\ 1-\rho-4\rho^2+4\rho^3+3\rho^4-3\rho^5\\ 1-\rho-4\rho^2+3\rho^3+3\rho^4-\rho^5\\ 1-\rho-4\rho^2+3\rho^3+4\rho^4-\rho^5-\rho^6\\ 1-\rho-4\rho^2+3\rho^3+4\rho^4-2\rho^5-\rho^6\\ 1-\rho-4\rho^2+3\rho^3+4\rho^4-2\rho^5\end{bmatrix}$
8	$\varpi_8=\begin{bmatrix}\varpi_1=\varpi_{14}\\ \varpi_2=\varpi_{13}\\ \varpi_3=\varpi_{12}\\ \varpi_4=\varpi_{11}\\ \varpi_5=\varpi_{10}\\ \varpi_6=\varpi_9\\ \varpi_7=\varpi_8\\ \varpi_7=\varpi_8\end{bmatrix}=\frac{1}{\begin{bmatrix}16-14\rho-84\rho^2+60\rho^3\\ +120\rho^4-60\rho^5-40\rho^6\\ +6\rho^7\end{bmatrix}}\begin{bmatrix}1-6\rho^2+10\rho^4-4\rho^6\\ 1-\rho-6\rho^2+5\rho^3+10\rho^4-6\rho^5-4\rho^6+\rho^7\\ 1-\rho-5\rho^2+5\rho^3+6\rho^4-6\rho^5-\rho^6+\rho^7\\ 1-\rho-5\rho^2+4\rho^3+6\rho^4-3\rho^5-\rho^6\\ 1-\rho-5\rho^2+4\rho^3+7\rho^4-3\rho^5-3\rho^6-\rho^7\\ 1-\rho-5\rho^2+4\rho^3+7\rho^4-4\rho^5-3\rho^6+\rho^7\\ 1-\rho-5\rho^2+4\rho^3+7\rho^4-4\rho^5-2\rho^6+\rho^7\\ 1-\rho-5\rho^2+4\rho^3+7\rho^4-4\rho^5-2\rho^6\end{bmatrix}$

Source: Onyemachi et al (2026) **Remark:** All algebraic expressions computed using (2.15).

Table 2: Algebraic Expression of GLUE's coefficients or weights when sample size , $n=2k+1(h=n+1)/2 \quad \forall k=1,2,...,8.$

K	BLUE coefficients or weights
1	$\varpi_2=\begin{bmatrix}\varpi_1=\varpi_3\\ \varpi_2\end{bmatrix}=\frac{1}{-(3-4\rho)}\begin{bmatrix}\rho-1\\ 2\rho-1\end{bmatrix}$
2	$\varpi_3=\begin{bmatrix}\varpi_1=\varpi_5\\ \varpi_2=v_4\\ \varpi_3\end{bmatrix}=\frac{1}{-(5-8\rho-\rho^2)}\begin{bmatrix}\rho^2+\rho-1\\ 2\rho-1\\ -(\rho-1)^2\end{bmatrix}$
3	$\varpi_4=\begin{pmatrix}\varpi_1=\varpi_7\\ \varpi_2=\varpi_6\\ \varpi_3=\varpi_5\\ \varpi_4\end{pmatrix}=\frac{1}{7-12\rho-6\rho^2+8\rho^3}\begin{pmatrix}1-\rho-2\rho^2+\rho^3\\ 1-2\rho-\rho^2+2\rho^3\\ 1-2\rho+\rho^3\\ 1-2\rho\end{pmatrix}$
4	$\varpi_5=\begin{pmatrix}\varpi_1=\varpi_9\\ \varpi_2=\varpi_8\\ \varpi_3=\varpi_7\\ \varpi_4=\varpi_6\\ \varpi_5\end{pmatrix}=\frac{1}{9-16\rho-15\rho^2+24\rho^3+\rho^4}\begin{pmatrix}1-\rho-3\rho^2+2\rho^3+\rho^4\\ 1-2\rho-2\rho^2+4\rho^3\\ 1-2\rho-\rho^2+3\rho^3-\rho^4\\ 1-2\rho-\rho^2+2\rho^3\\ 1-2\rho-\rho^2+2\rho^3+\rho^4\end{pmatrix}$

Source: Onyemachi et al (2026). **Remark:** All algebraic expressions computed using (2.15).

Table 2 continues: Algebraic Expression of GLUE's coefficients or weights when sample size $n=2k+1, h=(n+1)/2 \quad \forall k=1,2,\ldots,8.$

k	**BLUE coefficients or weights**
5	$\varpi_6=\begin{pmatrix}\varpi_1=\varpi_{11}\\ \varpi_2=\varpi_{10}\\ \varpi_3=\varpi_9\\ \varpi_4=\varpi_8\\ \varpi_5=\varpi_7\\ \varpi_6\end{pmatrix}=\frac{1}{\begin{bmatrix}11-20\rho-28\rho^2+48\rho^3\\ +9\rho^4-12\rho^5\end{bmatrix}}\begin{pmatrix}1-\rho-4\rho^2+3\rho^3+3\rho^4-\rho^5\\ 1-2\rho-3\rho^2+6\rho^3+\rho^4-2\rho^5\\ 1-2\rho-2\rho^2+5\rho^3-\rho^4-\rho^5\\ 1-2\rho-2\rho^2+4\rho^3\\ 1-2\rho-2\rho^2+4\rho^3+\rho^4-\rho^5\\ 1-2\rho-2\rho^2+4\rho^3+\rho^4-2\rho^5\end{pmatrix}$
6	$\varpi_7=\begin{pmatrix}\varpi_1=\varpi_{13}\\ \varpi_2=\varpi_{12}\\ \varpi_3=\varpi_{11}\\ \varpi_4=\varpi_{10}\\ \varpi_5=\varpi_9\\ \varpi_6=\varpi_8\\ \varpi_7\end{pmatrix}=\frac{1}{\begin{bmatrix}13-24\rho-45\rho^2+80\rho^3\\ +30\rho^4-48\rho^5-\rho^6\end{bmatrix}}\begin{pmatrix}1-\rho-5\rho^2+4\rho^3+6\rho^4-3\rho^5-\rho^6\\ 1-2\rho-4\rho^2+8\rho^3+3\rho^4-6\rho^5\\ 1-2\rho-3\rho^2+7\rho^3-4\rho^5+\rho^6\\ 1-2\rho-3\rho^2+6\rho^3+\rho^4-2\rho^5\\ 1-2\rho-3\rho^2+6\rho^3+2\rho^4-3\rho^5-\rho^6\\ 1-2\rho-3\rho^2+6\rho^3+2\rho^4-4\rho^5\\ 1-2\rho-3\rho^2+6\rho^3+2\rho^4-4\rho^5+\rho^6\end{pmatrix}$
7	$\varpi_8=\begin{pmatrix}\varpi_1=\varpi_{15}\\ \varpi_2=\varpi_{14}\\ \varpi_3=\varpi_{13}\\ \varpi_4=\varpi_{12}\\ \varpi_5=\varpi_{11}\\ \varpi_6=\varpi_{10}\\ \varpi_7=\varpi_9\\ \varpi_8\end{pmatrix}=\frac{1}{\begin{bmatrix}15-28\rho-66\rho^2\\ +120\rho^3+66\rho^4\\ -120\rho^5-4\rho^6\\ +16\rho^7\end{bmatrix}}\begin{pmatrix}1-\rho-6\rho^2+5\rho^3+10\rho^4-6\rho^5-4\rho^6+\rho^7\\ 1-2\rho-5\rho^2+10\rho^3+5\rho^4-12\rho^5+\rho^6+2\rho^7\\ 1-2\rho-4\rho^2+9\rho^3+2\rho^4-9\rho^5+2\rho^6+\rho^7\\ 1-2\rho-4\rho^2+8\rho^3+2\rho^4-6\rho^5+2\rho^6\\ 1-2\rho-4\rho^2+8\rho^3+4\rho^4-7\rho^5-2\rho^6+\rho^7\\ 1-2\rho-4\rho^2+8\rho^3+4\rho^4-8\rho^5-\rho^6+2\rho^7\\ 1-2\rho-4\rho^2+8\rho^3+4\rho^4-8\rho^5+\rho^7\\ 1-2\rho-4\rho^2+8\rho^3+4\rho^4-8\rho^5\end{pmatrix}$
8	$\varpi_9=\begin{pmatrix}\varpi_1=\varpi_{17}\\ \varpi_2=\varpi_{16}\\ \varpi_3=\varpi_{15}\\ \varpi_4=\varpi_{14}\\ \varpi_5=\varpi_{13}\\ \varpi_6=\varpi_{12}\\ \varpi_7=\varpi_{11}\\ \varpi_8=\varpi_{10}\\ \varpi_9\end{pmatrix}=\frac{1}{\begin{bmatrix}17-32\rho-91\rho^2\\ +168\rho^3+135\rho^4\\ -240\rho^5-50\rho^6\\ +76\rho^7+\rho^8\end{bmatrix}}\begin{pmatrix}1-\rho-7\rho^2+6\rho^3+15\rho^4-10\rho^5-10\rho^6+4\rho^7+\rho^8\\ 1-2\rho-6\rho^2+12\rho^3+10\rho^4-20\rho^5-4\rho^6+7\rho^7\\ 1-2\rho-5\rho^2+11\rho^3+5\rho^4-16\rho^5+2\rho^6+5\rho^7-\rho^8\\ 1-2\rho-5\rho^2+10\rho^3+6\rho^4-12\rho^5-\rho^6+2\rho^7\\ 1-2\rho-5\rho^2+10\rho^3+7\rho^4-13\rho^5-4\rho^6+4\rho^7+\rho^8\\ 1-2\rho-5\rho^2+10\rho^3+7\rho^4-14\rho^5-3\rho^6+6\rho^7\\ 1-2\rho-5\rho^2+10\rho^3+7\rho^4-14\rho^5-2\rho^6+4\rho^7-\rho^8\\ 1-2\rho-5\rho^2+10\rho^3+7\rho^4-14\rho^5-2\rho^6+4\rho^7\\ 1-2\rho-5\rho^2+10\rho^3+7\rho^4-14\rho^5-2\rho^6+4\rho^7+\rho^8\end{pmatrix}$

Source: Onyemachi et al (2026). **Remark:** All algebraic expressions computed using (2.15).

Table 3: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is four

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	$\sum_{i=1}^{4}\varpi_i$	ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	$\sum_{i=1}^{4}\varpi_i$	ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	$\sum_{i=1}^{4}\varpi_i$
-0.50	0.2000	0.3000	0.3000	0.2000	1	**-0.25**	0.2222	0.2778	0.2778	0.2222	1	**0.00**	0.2500	0.2500	0.2500	0.2500	1
-0.49	0.2008	0.2992	0.2992	0.2008	1	**-0.24**	0.2232	0.2768	0.2768	0.2232	1	**0.01**	0.2513	0.2487	0.2487	0.2513	1
-0.48	0.2016	0.2984	0.2984	0.2016	1	**-0.23**	0.2242	0.2758	0.2758	0.2242	1	**0.02**	0.2525	0.2475	0.2475	0.2525	1
-0.47	0.2024	0.2976	0.2976	0.2024	1	**-0.22**	0.2252	0.2748	0.2748	0.2252	1	**0.03**	0.2538	0.2462	0.2462	0.2538	1
-0.46	0.2033	0.2967	0.2967	0.2033	1	**-0.21**	0.2262	0.2738	0.2738	0.2262	1	**0.04**	0.2551	0.2449	0.2449	0.2551	1
-0.45	0.2041	0.2959	0.2959	0.2041	1	**-0.20**	0.2273	0.2727	0.2727	0.2273	1	**0.05**	0.2564	0.2436	0.2436	0.2564	1
-0.44	0.2049	0.2951	0.2951	0.2049	1	**-0.19**	0.2283	0.2717	0.2717	0.2283	1	**0.06**	0.2577	0.2423	0.2423	0.2577	1
-0.43	0.2058	0.2942	0.2942	0.2058	1	**-0.18**	0.2294	0.2706	0.2706	0.2294	1	**0.07**	0.2591	0.2409	0.2409	0.2591	1
-0.42	0.2066	0.2934	0.2934	0.2066	1	**-0.17**	0.2304	0.2696	0.2696	0.2304	1	**0.08**	0.2604	0.2396	0.2396	0.2604	1
-0.41	0.2075	0.2925	0.2925	0.2075	1	**-0.16**	0.2315	0.2685	0.2685	0.2315	1	**0.09**	0.2618	0.2382	0.2382	0.2618	1
-0.40	0.2083	0.2917	0.2917	0.2083	1	**-0.15**	0.2326	0.2674	0.2674	0.2326	1	**0.10**	0.2632	0.2368	0.2368	0.2632	1
-0.39	0.2092	0.2908	0.2908	0.2092	1	**-0.14**	0.2336	0.2664	0.2664	0.2336	1	**0.11**	0.2646	0.2354	0.2354	0.2646	1
-0.38	0.2101	0.2899	0.2899	0.2101	1	**-0.13**	0.2347	0.2653	0.2653	0.2347	1	**0.12**	0.2660	0.2340	0.2340	0.2660	1
-0.37	0.2110	0.2890	0.2890	0.2110	1	**-0.12**	0.2358	0.2642	0.2642	0.2358	1	**0.13**	0.2674	0.2326	0.2326	0.2674	1
-0.36	0.2119	0.2881	0.2881	0.2119	1	**-0.11**	0.2370	0.2630	0.2630	0.2370	1	**0.14**	0.2688	0.2312	0.2312	0.2688	1
-0.35	0.2128	0.2872	0.2872	0.2128	1	**-0.10**	0.2381	0.2619	0.2619	0.2381	1	**0.15**	0.2703	0.2297	0.2297	0.2703	1
-0.34	0.2137	0.2863	0.2863	0.2137	1	**-0.09**	0.2392	0.2608	0.2608	0.2392	1	**0.16**	0.2717	0.2283	0.2283	0.2717	1
-0.33	0.2146	0.2854	0.2854	0.2146	1	**-0.08**	0.2404	0.2596	0.2596	0.2404	1	**0.17**	0.2732	0.2268	0.2268	0.2732	1
-0.32	0.2155	0.2845	0.2845	0.2155	1	**-0.07**	0.2415	0.2585	0.2585	0.2415	1	**0.18**	0.2747	0.2253	0.2253	0.2747	1
-0.31	0.2165	0.2835	0.2835	0.2165	1	**-0.06**	0.2427	0.2573	0.2573	0.2427	1	**0.19**	0.2762	0.2238	0.2238	0.2762	1
-0.30	0.2174	0.2826	0.2826	0.2174	1	**-0.05**	0.2439	0.2561	0.2561	0.2439	1	**0.20**	0.2778	0.2222	0.2222	0.2778	1
-0.29	0.2183	0.2817	0.2817	0.2183	1	**-0.04**	0.2451	0.2549	0.2549	0.2451	1	**0.21**	0.2793	0.2207	0.2207	0.2793	1
-0.28	0.2193	0.2807	0.2807	0.2193	1	**-0.03**	0.2463	0.2537	0.2537	0.2463	1	**0.22**	0.2809	0.2191	0.2191	0.2809	1
-0.27	0.2203	0.2797	0.2797	0.2203	1	**-0.02**	0.2475	0.2525	0.2525	0.2475	1	**0.23**	0.2825	0.2175	0.2175	0.2825	1
-0.26	0.2212	0.2788	0.2788	0.2212	1	**-0.01**	0.2488	0.2512	0.2512	0.2488	1	**0.24**	0.2841	0.2159	0.2159	0.2841	1

Table 3 continues: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is four

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	$\sum_{i=1}^{4}\varpi_i$
0.25	0.2857	0.2143	0.2143	0.2857	1
0.26	0.2874	0.2126	0.2126	0.2874	1
0.27	0.2890	0.2110	0.2110	0.2890	1
0.28	0.2907	0.2093	0.2093	0.2907	1
0.29	0.2924	0.2076	0.2076	0.2924	1
0.30	0.2941	0.2059	0.2059	0.2941	1
0.31	0.2959	0.2041	0.2041	0.2959	1
0.32	0.2976	0.2024	0.2024	0.2976	1
0.33	0.2994	0.2006	0.2006	0.2994	1
0.34	0.3012	0.1988	0.1988	0.3012	1
0.35	0.3030	0.1970	0.1970	0.3030	1
0.36	0.3049	0.1951	0.1951	0.3049	1
0.37	0.3067	0.1933	0.1933	0.3067	1
0.38	0.3086	0.1914	0.1914	0.3086	1
0.39	0.3106	0.1894	0.1894	0.3106	1
0.40	0.3125	0.1875	0.1875	0.3125	1
0.41	0.3145	0.1855	0.1855	0.3145	1
0.42	0.3165	0.1835	0.1835	0.3165	1
0.43	0.3185	0.1815	0.1815	0.3185	1
0.44	0.3205	0.1795	0.1795	0.3205	1
0.45	0.3226	0.1774	0.1774	0.3226	1
0.46	0.3247	0.1753	0.1753	0.3247	1
0.47	0.3268	0.1732	0.1732	0.3268	1
0.48	0.3289	0.1711	0.1711	0.3289	1
0.49	0.3311	0.1689	0.1689	0.3311	1
0.50	0.3333	0.1667	0.1667	0.3333	1

Table 4: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Thirteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	$\sum_{i=1}^{13}\varpi_i$
-0.50	0.0286	0.0527	0.0725	0.0879	0.0989	0.1055	0.1077	0.1055	0.0989	0.0879	0.0725	0.0527	0.0286	1
-0.49	0.0310	0.0553	0.0739	0.0877	0.0972	0.1027	0.1045	0.1027	0.0972	0.0877	0.0739	0.0553	0.0310	1
-0.48	0.0331	0.0574	0.0751	0.0875	0.0957	0.1003	0.1018	0.1003	0.0957	0.0875	0.0751	0.0574	0.0331	1
-0.47	0.0351	0.0594	0.0760	0.0872	0.0943	0.0982	0.0995	0.0982	0.0943	0.0872	0.0760	0.0594	0.0351	1
-0.46	0.0368	0.0611	0.0768	0.0869	0.0931	0.0964	0.0975	0.0964	0.0931	0.0869	0.0768	0.0611	0.0368	1
-0.45	0.0385	0.0626	0.0775	0.0867	0.0920	0.0949	0.0957	0.0949	0.0920	0.0867	0.0775	0.0626	0.0385	1
-0.44	0.0401	0.0639	0.0781	0.0864	0.0911	0.0934	0.0942	0.0934	0.0911	0.0864	0.0781	0.0639	0.0401	1
-0.43	0.0415	0.0651	0.0785	0.0861	0.0902	0.0922	0.0928	0.0922	0.0902	0.0861	0.0785	0.0651	0.0415	1
-0.42	0.0429	0.0662	0.0789	0.0857	0.0893	0.0911	0.0916	0.0911	0.0893	0.0857	0.0789	0.0662	0.0429	1
-0.41	0.0442	0.0672	0.0792	0.0854	0.0886	0.0900	0.0905	0.0900	0.0886	0.0854	0.0792	0.0672	0.0442	1
-0.40	0.0454	0.0682	0.0795	0.0851	0.0879	0.0891	0.0895	0.0891	0.0879	0.0851	0.0795	0.0682	0.0454	1
-0.39	0.0466	0.0690	0.0797	0.0848	0.0872	0.0883	0.0886	0.0883	0.0872	0.0848	0.0797	0.0690	0.0466	1
-0.38	0.0478	0.0698	0.0799	0.0845	0.0866	0.0875	0.0878	0.0875	0.0866	0.0845	0.0799	0.0698	0.0478	1
-0.37	0.0489	0.0705	0.0800	0.0842	0.0861	0.0868	0.0870	0.0868	0.0861	0.0842	0.0800	0.0705	0.0489	1
-0.36	0.0499	0.0711	0.0801	0.0839	0.0856	0.0862	0.0864	0.0862	0.0856	0.0839	0.0801	0.0711	0.0499	1
-0.35	0.0509	0.0717	0.0802	0.0837	0.0851	0.0856	0.0857	0.0856	0.0851	0.0837	0.0802	0.0717	0.0509	1
-0.34	0.0519	0.0723	0.0802	0.0834	0.0846	0.0850	0.0852	0.0850	0.0846	0.0834	0.0802	0.0723	0.0519	1
-0.33	0.0529	0.0728	0.0803	0.0831	0.0842	0.0845	0.0846	0.0845	0.0842	0.0831	0.0803	0.0728	0.0529	1
-0.32	0.0538	0.0732	0.0803	0.0828	0.0837	0.0841	0.0841	0.0841	0.0837	0.0828	0.0803	0.0732	0.0538	1
-0.31	0.0547	0.0737	0.0803	0.0826	0.0834	0.0836	0.0837	0.0836	0.0834	0.0826	0.0803	0.0737	0.0547	1
-0.30	0.0556	0.0741	0.0802	0.0823	0.0830	0.0832	0.0833	0.0832	0.0830	0.0823	0.0802	0.0741	0.0556	1
-0.29	0.0564	0.0744	0.0802	0.0820	0.0826	0.0828	0.0829	0.0828	0.0826	0.0820	0.0802	0.0744	0.0564	1
-0.28	0.0573	0.0748	0.0802	0.0818	0.0823	0.0825	0.0825	0.0825	0.0823	0.0818	0.0802	0.0748	0.0573	1
-0.27	0.0581	0.0751	0.0801	0.0816	0.0820	0.0821	0.0821	0.0821	0.0820	0.0816	0.0801	0.0751	0.0581	1
-0.26	0.0589	0.0754	0.0800	0.0813	0.0817	0.0818	0.0818	0.0818	0.0817	0.0813	0.0800	0.0754	0.0589	1
-0.25	0.0597	0.0757	0.0799	0.0811	0.0814	0.0815	0.0815	0.0815	0.0814	0.0811	0.0799	0.0757	0.0597	1

Table 4 continues: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Thirteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	$\sum_{i=1}^{13}\varpi_i$
-0.24	0.0604	0.0759	0.0799	0.0809	0.0811	0.0812	0.0812	0.0812	0.0811	0.0809	0.0799	0.0759	0.0604	1
-0.23	0.0612	0.0761	0.0798	0.0806	0.0809	0.0809	0.0809	0.0809	0.0809	0.0806	0.0798	0.0761	0.0612	1
-0.22	0.0620	0.0763	0.0797	0.0804	0.0806	0.0807	0.0807	0.0807	0.0806	0.0804	0.0797	0.0763	0.0620	1
-0.21	0.0627	0.0765	0.0796	0.0802	0.0804	0.0804	0.0804	0.0804	0.0804	0.0802	0.0796	0.0765	0.0627	1
-0.20	0.0634	0.0767	0.0794	0.0800	0.0801	0.0802	0.0802	0.0802	0.0801	0.0800	0.0794	0.0767	0.0634	1
-0.19	0.0642	0.0768	0.0793	0.0798	0.0799	0.0799	0.0799	0.0799	0.0799	0.0798	0.0793	0.0768	0.0642	1
-0.18	0.0649	0.0770	0.0792	0.0796	0.0797	0.0797	0.0797	0.0797	0.0797	0.0796	0.0792	0.0770	0.0649	1
-0.17	0.0656	0.0771	0.0791	0.0794	0.0795	0.0795	0.0795	0.0795	0.0795	0.0794	0.0791	0.0771	0.0656	1
-0.16	0.0663	0.0772	0.0790	0.0793	0.0793	0.0793	0.0793	0.0793	0.0793	0.0793	0.0790	0.0772	0.0663	1
-0.15	0.0670	0.0773	0.0788	0.0791	0.0791	0.0791	0.0791	0.0791	0.0791	0.0791	0.0788	0.0773	0.0670	1
-0.14	0.0677	0.0773	0.0787	0.0789	0.0789	0.0789	0.0789	0.0789	0.0789	0.0789	0.0787	0.0773	0.0677	1
-0.13	0.0684	0.0774	0.0786	0.0787	0.0788	0.0788	0.0788	0.0788	0.0788	0.0787	0.0786	0.0774	0.0684	1
-0.12	0.0690	0.0774	0.0785	0.0786	0.0786	0.0786	0.0786	0.0786	0.0786	0.0786	0.0785	0.0774	0.0690	1
-0.11	0.0697	0.0775	0.0783	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0784	0.0783	0.0775	0.0697	1
-0.10	0.0704	0.0775	0.0782	0.0783	0.0783	0.0783	0.0783	0.0783	0.0783	0.0783	0.0782	0.0775	0.0704	1
-0.09	0.0710	0.0775	0.0781	0.0781	0.0781	0.0781	0.0781	0.0781	0.0781	0.0781	0.0781	0.0775	0.0710	1
-0.08	0.0717	0.0775	0.0779	0.0780	0.0780	0.0780	0.0780	0.0780	0.0780	0.0780	0.0779	0.0775	0.0717	1
-0.07	0.0724	0.0774	0.0778	0.0778	0.0778	0.0778	0.0778	0.0778	0.0778	0.0778	0.0778	0.0774	0.0724	1
-0.06	0.0730	0.0774	0.0777	0.0777	0.0777	0.0777	0.0777	0.0777	0.0777	0.0777	0.0777	0.0774	0.0730	1
-0.05	0.0737	0.0774	0.0775	0.0776	0.0776	0.0776	0.0776	0.0776	0.0776	0.0776	0.0775	0.0774	0.0737	1
-0.04	0.0743	0.0773	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0773	0.0743	1
-0.03	0.0750	0.0772	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0773	0.0772	0.0750	1
-0.02	0.0756	0.0771	0.0772	0.0772	0.0772	0.0772	0.0772	0.0772	0.0772	0.0772	0.0772	0.0771	0.0756	1
-0.01	0.0763	0.0770	0.0770	0.0770	0.0770	0.0770	0.0770	0.0770	0.0770	0.0770	0.0770	0.0770	0.0763	1
0.00	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	0.0769	1
0.01	0.0776	0.0768	0.0768	0.0768	0.0768	0.0768	0.0768	0.0768	0.0768	0.0768	0.0768	0.0768	0.0776	1

Table 4 continues: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Thirteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	$\sum_{i=1}^{13}\varpi_i$
0.02	0.0782	0.0767	0.0767	0.0767	0.0767	0.0767	0.0767	0.0767	0.0767	0.0767	0.0767	0.0767	0.0782	1
0.03	0.0789	0.0765	0.0766	0.0766	0.0766	0.0766	0.0766	0.0766	0.0766	0.0766	0.0766	0.0765	0.0789	1
0.04	0.0795	0.0763	0.0765	0.0765	0.0765	0.0765	0.0765	0.0765	0.0765	0.0765	0.0765	0.0763	0.0795	1
0.05	0.0802	0.0762	0.0764	0.0764	0.0764	0.0764	0.0764	0.0764	0.0764	0.0764	0.0764	0.0762	0.0802	1
0.06	0.0808	0.0760	0.0763	0.0763	0.0763	0.0763	0.0763	0.0763	0.0763	0.0763	0.0763	0.0760	0.0808	1
0.07	0.0815	0.0758	0.0762	0.0762	0.0762	0.0762	0.0762	0.0762	0.0762	0.0762	0.0762	0.0758	0.0815	1
0.08	0.0822	0.0756	0.0761	0.0760	0.0761	0.0761	0.0761	0.0761	0.0761	0.0760	0.0761	0.0756	0.0822	1
0.09	0.0828	0.0753	0.0760	0.0759	0.0760	0.0760	0.0760	0.0760	0.0760	0.0759	0.0760	0.0753	0.0828	1
0.10	0.0835	0.0751	0.0759	0.0758	0.0759	0.0759	0.0759	0.0759	0.0759	0.0758	0.0759	0.0751	0.0835	1
0.11	0.0842	0.0748	0.0759	0.0757	0.0758	0.0758	0.0758	0.0758	0.0758	0.0757	0.0759	0.0748	0.0842	1
0.12	0.0849	0.0745	0.0758	0.0756	0.0757	0.0757	0.0757	0.0757	0.0757	0.0756	0.0758	0.0745	0.0849	1
0.13	0.0856	0.0742	0.0757	0.0755	0.0756	0.0756	0.0756	0.0756	0.0756	0.0755	0.0757	0.0742	0.0856	1
0.14	0.0863	0.0739	0.0757	0.0754	0.0755	0.0755	0.0755	0.0755	0.0755	0.0754	0.0757	0.0739	0.0863	1
0.15	0.0870	0.0736	0.0757	0.0753	0.0754	0.0754	0.0754	0.0754	0.0754	0.0753	0.0757	0.0736	0.0870	1
0.16	0.0877	0.0733	0.0756	0.0752	0.0753	0.0753	0.0753	0.0753	0.0753	0.0752	0.0756	0.0733	0.0877	1
0.17	0.0884	0.0729	0.0756	0.0751	0.0752	0.0752	0.0752	0.0752	0.0752	0.0751	0.0756	0.0729	0.0884	1
0.18	0.0891	0.0725	0.0756	0.0750	0.0751	0.0751	0.0751	0.0751	0.0751	0.0750	0.0756	0.0725	0.0891	1
0.19	0.0898	0.0721	0.0756	0.0749	0.0750	0.0750	0.0750	0.0750	0.0750	0.0749	0.0756	0.0721	0.0898	1
0.20	0.0906	0.0717	0.0756	0.0748	0.0750	0.0749	0.0749	0.0749	0.0750	0.0748	0.0756	0.0717	0.0906	1
0.21	0.0913	0.0712	0.0756	0.0747	0.0749	0.0748	0.0748	0.0748	0.0749	0.0747	0.0756	0.0712	0.0913	1
0.22	0.0921	0.0707	0.0757	0.0745	0.0748	0.0747	0.0748	0.0747	0.0748	0.0745	0.0757	0.0707	0.0921	1
0.23	0.0929	0.0702	0.0758	0.0744	0.0747	0.0747	0.0747	0.0747	0.0747	0.0744	0.0758	0.0702	0.0929	1
0.24	0.0937	0.0697	0.0758	0.0743	0.0747	0.0746	0.0746	0.0746	0.0747	0.0743	0.0758	0.0697	0.0937	1
0.25	0.0945	0.0692	0.0759	0.0741	0.0746	0.0745	0.0745	0.0745	0.0746	0.0741	0.0759	0.0692	0.0945	1
0.26	0.0953	0.0686	0.0761	0.0740	0.0745	0.0744	0.0744	0.0744	0.0745	0.0740	0.0761	0.0686	0.0953	1
0.27	0.0961	0.0679	0.0762	0.0738	0.0745	0.0743	0.0744	0.0743	0.0745	0.0738	0.0762	0.0679	0.0961	1

Table 4 continues: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Thirteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	$\sum_{i=1}^{13}\varpi_i$
0.28	0.0970	0.0673	0.0764	0.0736	0.0744	0.0742	0.0743	0.0742	0.0744	0.0736	0.0764	0.0673	0.0970	1
0.29	0.0979	0.0666	0.0766	0.0734	0.0744	0.0741	0.0742	0.0741	0.0744	0.0734	0.0766	0.0666	0.0979	1
0.30	0.0988	0.0658	0.0768	0.0732	0.0744	0.0740	0.0741	0.0740	0.0744	0.0732	0.0768	0.0658	0.0988	1
0.31	0.0997	0.0651	0.0771	0.0729	0.0744	0.0738	0.0741	0.0738	0.0744	0.0729	0.0771	0.0651	0.0997	1
0.32	0.1006	0.0642	0.0774	0.0726	0.0744	0.0737	0.0740	0.0737	0.0744	0.0726	0.0774	0.0642	0.1006	1
0.33	0.1016	0.0633	0.0778	0.0723	0.0744	0.0736	0.0740	0.0736	0.0744	0.0723	0.0778	0.0633	0.1016	1
0.34	0.1027	0.0624	0.0782	0.0720	0.0744	0.0734	0.0739	0.0734	0.0744	0.0720	0.0782	0.0624	0.1027	1
0.35	0.1037	0.0614	0.0787	0.0716	0.0745	0.0732	0.0739	0.0732	0.0745	0.0716	0.0787	0.0614	0.1037	1
0.36	0.1048	0.0603	0.0792	0.0711	0.0746	0.0730	0.0739	0.0730	0.0746	0.0711	0.0792	0.0603	0.1048	1
0.37	0.1060	0.0591	0.0798	0.0706	0.0747	0.0728	0.0739	0.0728	0.0747	0.0706	0.0798	0.0591	0.1060	1
0.38	0.1072	0.0578	0.0805	0.0700	0.0750	0.0725	0.0740	0.0725	0.0750	0.0700	0.0805	0.0578	0.1072	1
0.39	0.1084	0.0564	0.0814	0.0693	0.0752	0.0722	0.0741	0.0722	0.0752	0.0693	0.0814	0.0564	0.1084	1
0.40	0.1098	0.0549	0.0824	0.0685	0.0756	0.0717	0.0743	0.0717	0.0756	0.0685	0.0824	0.0549	0.1098	1
0.41	0.1112	0.0532	0.0835	0.0676	0.0761	0.0712	0.0746	0.0712	0.0761	0.0676	0.0835	0.0532	0.1112	1
0.42	0.1127	0.0513	0.0848	0.0664	0.0768	0.0705	0.0750	0.0705	0.0768	0.0664	0.0848	0.0513	0.1127	1
0.43	0.1144	0.0492	0.0865	0.0649	0.0777	0.0696	0.0757	0.0696	0.0777	0.0649	0.0865	0.0492	0.1144	1
0.44	0.1162	0.0467	0.0884	0.0631	0.0789	0.0683	0.0767	0.0683	0.0789	0.0631	0.0884	0.0467	0.1162	1
0.45	0.1182	0.0439	0.0909	0.0607	0.0807	0.0665	0.0781	0.0665	0.0807	0.0607	0.0909	0.0439	0.1182	1
0.46	0.1206	0.0404	0.0941	0.0575	0.0833	0.0638	0.0804	0.0638	0.0833	0.0575	0.0941	0.0404	0.1206	1
0.47	0.1234	0.0360	0.0985	0.0529	0.0874	0.0596	0.0843	0.0596	0.0874	0.0529	0.0985	0.0360	0.1234	1
0.48	0.1270	0.0300	0.1050	0.0457	0.0943	0.0524	0.0911	0.0524	0.0943	0.0457	0.1050	0.0300	0.1270	1
0.49	0.1323	0.0205	0.1163	0.0326	0.1077	0.0381	0.1050	0.0381	0.1077	0.0326	0.1163	0.0205	0.1323	1
0.50	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429	1

Table 5: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Sixteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	ϖ_{14}	ϖ_{15}	ϖ_{16}	$\sum_{i=1}^{16}\varpi_i$
-0.50	0.0195	0.0366	0.0512	0.0634	0.0756	0.0805	0.0854	0.0878	0.0878	0.0854	0.0805	0.0756	0.0634	0.0512	0.0366	0.0195	1
-0.49	0.0221	0.0397	0.0537	0.0646	0.0746	0.0786	0.0824	0.0843	0.0843	0.0824	0.0786	0.0746	0.0646	0.0537	0.0397	0.0221	1
-0.48	0.0242	0.0423	0.0556	0.0654	0.0738	0.0771	0.0801	0.0815	0.0815	0.0801	0.0771	0.0738	0.0654	0.0556	0.0423	0.0242	1
-0.47	0.0262	0.0444	0.0572	0.0659	0.0730	0.0758	0.0782	0.0793	0.0793	0.0782	0.0758	0.0730	0.0659	0.0572	0.0444	0.0262	1
-0.46	0.0279	0.0463	0.0584	0.0663	0.0723	0.0747	0.0766	0.0775	0.0775	0.0766	0.0747	0.0723	0.0663	0.0584	0.0463	0.0279	1
-0.45	0.0294	0.0479	0.0594	0.0666	0.0717	0.0737	0.0752	0.0759	0.0759	0.0752	0.0737	0.0717	0.0666	0.0594	0.0479	0.0294	1
-0.44	0.0309	0.0493	0.0603	0.0668	0.0712	0.0729	0.0741	0.0746	0.0746	0.0741	0.0729	0.0712	0.0668	0.0603	0.0493	0.0309	1
-0.43	0.0322	0.0505	0.0610	0.0669	0.0707	0.0721	0.0731	0.0735	0.0735	0.0731	0.0721	0.0707	0.0669	0.0610	0.0505	0.0322	1
-0.42	0.0334	0.0516	0.0616	0.0669	0.0702	0.0714	0.0722	0.0726	0.0726	0.0722	0.0714	0.0702	0.0669	0.0616	0.0516	0.0334	1
-0.41	0.0346	0.0526	0.0621	0.0670	0.0698	0.0708	0.0715	0.0717	0.0717	0.0715	0.0708	0.0698	0.0670	0.0621	0.0526	0.0346	1
-0.40	0.0357	0.0535	0.0625	0.0669	0.0694	0.0702	0.0708	0.0710	0.0710	0.0708	0.0702	0.0694	0.0669	0.0625	0.0535	0.0357	1
-0.39	0.0367	0.0544	0.0628	0.0669	0.0690	0.0697	0.0702	0.0703	0.0703	0.0702	0.0697	0.0690	0.0669	0.0628	0.0544	0.0367	1
-0.38	0.0377	0.0551	0.0631	0.0668	0.0686	0.0693	0.0696	0.0697	0.0697	0.0696	0.0693	0.0686	0.0668	0.0631	0.0551	0.0377	1
-0.37	0.0387	0.0558	0.0634	0.0667	0.0683	0.0688	0.0691	0.0692	0.0692	0.0691	0.0688	0.0683	0.0667	0.0634	0.0558	0.0387	1
-0.36	0.0396	0.0564	0.0636	0.0666	0.0680	0.0684	0.0687	0.0687	0.0687	0.0687	0.0684	0.0680	0.0666	0.0636	0.0564	0.0396	1
-0.35	0.0405	0.0570	0.0637	0.0665	0.0677	0.0681	0.0683	0.0683	0.0683	0.0683	0.0681	0.0677	0.0665	0.0637	0.0570	0.0405	1
-0.34	0.0413	0.0575	0.0639	0.0664	0.0674	0.0677	0.0679	0.0679	0.0679	0.0679	0.0677	0.0674	0.0664	0.0639	0.0575	0.0413	1
-0.33	0.0421	0.0580	0.0640	0.0662	0.0671	0.0674	0.0675	0.0676	0.0676	0.0675	0.0674	0.0671	0.0662	0.0640	0.0580	0.0421	1
-0.32	0.0429	0.0585	0.0641	0.0661	0.0669	0.0671	0.0672	0.0672	0.0672	0.0672	0.0671	0.0669	0.0661	0.0641	0.0585	0.0429	1
-0.31	0.0437	0.0589	0.0641	0.0660	0.0666	0.0668	0.0669	0.0669	0.0669	0.0669	0.0668	0.0666	0.0660	0.0641	0.0589	0.0437	1
-0.30	0.0444	0.0593	0.0642	0.0658	0.0664	0.0666	0.0666	0.0667	0.0667	0.0666	0.0666	0.0664	0.0658	0.0642	0.0593	0.0444	1
-0.29	0.0452	0.0596	0.0642	0.0657	0.0662	0.0663	0.0664	0.0664	0.0664	0.0664	0.0663	0.0662	0.0657	0.0642	0.0596	0.0452	1
-0.28	0.0459	0.0599	0.0642	0.0656	0.0660	0.0661	0.0661	0.0661	0.0661	0.0661	0.0661	0.0660	0.0656	0.0642	0.0599	0.0459	1
-0.27	0.0466	0.0603	0.0643	0.0654	0.0658	0.0659	0.0659	0.0659	0.0659	0.0659	0.0659	0.0658	0.0654	0.0643	0.0603	0.0466	1
-0.26	0.0473	0.0605	0.0643	0.0653	0.0656	0.0657	0.0657	0.0657	0.0657	0.0657	0.0657	0.0656	0.0653	0.0643	0.0605	0.0473	1
-0.25	0.0479	0.0608	0.0642	0.0652	0.0654	0.0655	0.0655	0.0655	0.0655	0.0655	0.0655	0.0654	0.0652	0.0642	0.0608	0.0479	1

Table 5 continues: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Sixteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	ϖ_{14}	ϖ_{15}	ϖ_{16}	$\sum_{i=1}^{16}\varpi_i$
-0.24	0.0486	0.0610	0.0642	0.0650	0.0652	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0652	0.0650	0.0642	0.0610	0.0486	1
-0.23	0.0493	0.0613	0.0642	0.0649	0.0651	0.0651	0.0651	0.0651	0.0651	0.0651	0.0651	0.0651	0.0649	0.0642	0.0613	0.0493	1
-0.22	0.0499	0.0615	0.0641	0.0648	0.0649	0.0649	0.0649	0.0649	0.0649	0.0649	0.0649	0.0649	0.0648	0.0641	0.0615	0.0499	1
-0.21	0.0505	0.0616	0.0641	0.0646	0.0648	0.0648	0.0648	0.0648	0.0648	0.0648	0.0648	0.0648	0.0646	0.0641	0.0616	0.0505	1
-0.20	0.0511	0.0618	0.0640	0.0645	0.0646	0.0646	0.0646	0.0646	0.0646	0.0646	0.0646	0.0646	0.0645	0.0640	0.0618	0.0511	1
-0.19	0.0518	0.0620	0.0640	0.0644	0.0645	0.0645	0.0645	0.0645	0.0645	0.0645	0.0645	0.0645	0.0644	0.0640	0.0620	0.0518	1
-0.18	0.0524	0.0621	0.0639	0.0643	0.0643	0.0643	0.0643	0.0643	0.0643	0.0643	0.0643	0.0643	0.0643	0.0639	0.0621	0.0524	1
-0.17	0.0530	0.0622	0.0639	0.0641	0.0642	0.0642	0.0642	0.0642	0.0642	0.0642	0.0642	0.0642	0.0641	0.0639	0.0622	0.0530	1
-0.16	0.0535	0.0623	0.0638	0.0640	0.0641	0.0641	0.0641	0.0641	0.0641	0.0641	0.0641	0.0641	0.0640	0.0638	0.0623	0.0535	1
-0.15	0.0541	0.0624	0.0637	0.0639	0.0639	0.0639	0.0639	0.0639	0.0639	0.0639	0.0639	0.0639	0.0639	0.0637	0.0624	0.0541	1
-0.14	0.0547	0.0625	0.0636	0.0638	0.0638	0.0638	0.0638	0.0638	0.0638	0.0638	0.0638	0.0638	0.0638	0.0636	0.0625	0.0547	1
-0.13	0.0553	0.0626	0.0636	0.0637	0.0637	0.0637	0.0637	0.0637	0.0637	0.0637	0.0637	0.0637	0.0637	0.0636	0.0626	0.0553	1
-0.12	0.0559	0.0627	0.0635	0.0636	0.0636	0.0636	0.0636	0.0636	0.0636	0.0636	0.0636	0.0636	0.0636	0.0635	0.0627	0.0559	1
-0.11	0.0564	0.0627	0.0634	0.0635	0.0635	0.0635	0.0635	0.0635	0.0635	0.0635	0.0635	0.0635	0.0635	0.0634	0.0627	0.0564	1
-0.10	0.0570	0.0627	0.0633	0.0634	0.0634	0.0634	0.0634	0.0634	0.0634	0.0634	0.0634	0.0634	0.0634	0.0633	0.0627	0.0570	1
-0.09	0.0575	0.0628	0.0632	0.0633	0.0633	0.0633	0.0633	0.0633	0.0633	0.0633	0.0633	0.0633	0.0633	0.0632	0.0628	0.0575	1
-0.08	0.0581	0.0628	0.0632	0.0632	0.0632	0.0632	0.0632	0.0632	0.0632	0.0632	0.0632	0.0632	0.0632	0.0632	0.0628	0.0581	1
-0.07	0.0587	0.0628	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0631	0.0628	0.0587	1
-0.06	0.0592	0.0628	0.0630	0.0630	0.0630	0.0630	0.0630	0.0630	0.0630	0.0630	0.0630	0.0630	0.0630	0.0630	0.0628	0.0592	1
-0.05	0.0598	0.0628	0.0629	0.0629	0.0629	0.0629	0.0629	0.0629	0.0629	0.0629	0.0629	0.0629	0.0629	0.0629	0.0628	0.0598	1
-0.04	0.0603	0.0627	0.0628	0.0628	0.0628	0.0628	0.0628	0.0628	0.0628	0.0628	0.0628	0.0628	0.0628	0.0628	0.0627	0.0603	1
-0.03	0.0609	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0609	1
-0.02	0.0614	0.0626	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0627	0.0626	0.0614	1
-0.01	0.0620	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0626	0.0620	1
0.00	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	0.0625	1
0.01	0.0630	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0624	0.0630	1

Table 5 continues: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Sixteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	ϖ_{14}	ϖ_{15}	ϖ_{16}	$\sum_{i=1}^{16}\varpi_i$
0.02	0.0636	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0636	1
0.03	0.0641	0.0622	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0623	0.0622	0.0641	1
0.04	0.0647	0.0621	0.0622	0.0622	0.0622	0.0622	0.0622	0.0622	0.0622	0.0622	0.0622	0.0622	0.0622	0.0622	0.0621	0.0647	1
0.05	0.0652	0.0620	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0620	0.0652	1
0.06	0.0658	0.0618	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0621	0.0618	0.0658	1
0.07	0.0664	0.0617	0.0620	0.0620	0.0620	0.0620	0.0620	0.0620	0.0620	0.0620	0.0620	0.0620	0.0620	0.0620	0.0617	0.0664	1
0.08	0.0669	0.0615	0.0620	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0620	0.0615	0.0669	1
0.09	0.0675	0.0613	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0619	0.0613	0.0675	1
0.10	0.0680	0.0612	0.0619	0.0618	0.0618	0.0618	0.0618	0.0618	0.0618	0.0618	0.0618	0.0618	0.0618	0.0619	0.0612	0.0680	1
0.11	0.0686	0.0610	0.0618	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0618	0.0610	0.0686	1
0.12	0.0692	0.0607	0.0618	0.0616	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0617	0.0616	0.0618	0.0607	0.0692	1
0.13	0.0697	0.0605	0.0617	0.0616	0.0616	0.0616	0.0616	0.0616	0.0616	0.0616	0.0616	0.0616	0.0616	0.0617	0.0605	0.0697	1
0.14	0.0703	0.0603	0.0617	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0617	0.0603	0.0703	1
0.15	0.0709	0.0600	0.0617	0.0614	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0615	0.0614	0.0617	0.0600	0.0709	1
0.16	0.0715	0.0598	0.0617	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0617	0.0598	0.0715	1
0.17	0.0721	0.0595	0.0617	0.0613	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0614	0.0613	0.0617	0.0595	0.0721	1
0.18	0.0727	0.0592	0.0617	0.0612	0.0613	0.0613	0.0613	0.0613	0.0613	0.0613	0.0613	0.0613	0.0612	0.0617	0.0592	0.0727	1
0.19	0.0733	0.0589	0.0617	0.0611	0.0613	0.0612	0.0612	0.0612	0.0612	0.0612	0.0612	0.0613	0.0611	0.0617	0.0589	0.0733	1
0.20	0.0739	0.0585	0.0617	0.0611	0.0612	0.0612	0.0612	0.0612	0.0612	0.0612	0.0612	0.0612	0.0611	0.0617	0.0585	0.0739	1
0.21	0.0746	0.0582	0.0618	0.0610	0.0612	0.0611	0.0611	0.0611	0.0611	0.0611	0.0611	0.0612	0.0610	0.0618	0.0582	0.0746	1
0.22	0.0752	0.0578	0.0618	0.0609	0.0611	0.0611	0.0611	0.0611	0.0611	0.0611	0.0611	0.0611	0.0609	0.0618	0.0578	0.0752	1
0.23	0.0759	0.0574	0.0619	0.0608	0.0611	0.0610	0.0610	0.0610	0.0610	0.0610	0.0610	0.0611	0.0608	0.0619	0.0574	0.0759	1
0.24	0.0765	0.0570	0.0620	0.0607	0.0610	0.0609	0.0610	0.0609	0.0609	0.0610	0.0609	0.0610	0.0607	0.0620	0.0570	0.0765	1
0.25	0.0772	0.0565	0.0621	0.0606	0.0610	0.0609	0.0609	0.0609	0.0609	0.0609	0.0609	0.0610	0.0606	0.0621	0.0565	0.0772	1
0.26	0.0779	0.0561	0.0622	0.0605	0.0609	0.0608	0.0608	0.0608	0.0608	0.0608	0.0608	0.0609	0.0605	0.0622	0.0561	0.0779	1
0.27	0.0786	0.0556	0.0623	0.0603	0.0609	0.0607	0.0608	0.0608	0.0608	0.0608	0.0607	0.0609	0.0603	0.0623	0.0556	0.0786	1

Table 5 continues: Estimates of Weights of Generalized Linear Unbiased Estimator (GLUE) as well as their Weights Sum at varying autocorrelation (ρ) points, when sample size is Sixteen

ρ	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}	ϖ_{14}	ϖ_{15}	ϖ_{16}	$\sum_{i=1}^{16}\varpi_i$
0.28	0.0793	0.0550	0.0625	0.0602	0.0609	0.0607	0.0607	0.0607	0.0607	0.0607	0.0607	0.0609	0.0602	0.0625	0.0550	0.0793	1
0.29	0.0801	0.0545	0.0626	0.0600	0.0608	0.0606	0.0607	0.0607	0.0607	0.0607	0.0606	0.0608	0.0600	0.0626	0.0545	0.0801	1
0.30	0.0808	0.0539	0.0629	0.0599	0.0608	0.0605	0.0606	0.0606	0.0606	0.0606	0.0605	0.0608	0.0599	0.0629	0.0539	0.0808	1
0.31	0.0816	0.0532	0.0631	0.0597	0.0608	0.0604	0.0606	0.0605	0.0605	0.0606	0.0604	0.0608	0.0597	0.0631	0.0532	0.0816	1
0.32	0.0824	0.0526	0.0634	0.0595	0.0608	0.0604	0.0605	0.0605	0.0605	0.0605	0.0604	0.0608	0.0595	0.0634	0.0526	0.0824	1
0.33	0.0832	0.0519	0.0637	0.0592	0.0608	0.0603	0.0605	0.0604	0.0604	0.0605	0.0603	0.0608	0.0592	0.0637	0.0519	0.0832	1
0.34	0.0841	0.0511	0.0640	0.0590	0.0608	0.0602	0.0605	0.0604	0.0604	0.0605	0.0602	0.0608	0.0590	0.0640	0.0511	0.0841	1
0.35	0.0850	0.0503	0.0644	0.0587	0.0609	0.0601	0.0604	0.0603	0.0603	0.0604	0.0601	0.0609	0.0587	0.0644	0.0503	0.0850	1
0.36	0.0859	0.0494	0.0649	0.0583	0.0609	0.0599	0.0604	0.0602	0.0602	0.0604	0.0599	0.0609	0.0583	0.0649	0.0494	0.0859	1
0.37	0.0869	0.0484	0.0654	0.0579	0.0610	0.0598	0.0604	0.0602	0.0602	0.0604	0.0598	0.0610	0.0579	0.0654	0.0484	0.0869	1
0.38	0.0879	0.0474	0.0660	0.0575	0.0611	0.0596	0.0604	0.0601	0.0601	0.0604	0.0596	0.0611	0.0575	0.0660	0.0474	0.0879	1
0.39	0.0890	0.0463	0.0668	0.0569	0.0612	0.0594	0.0604	0.0600	0.0600	0.0604	0.0594	0.0612	0.0569	0.0668	0.0463	0.0890	1
0.40	0.0901	0.0450	0.0676	0.0563	0.0614	0.0592	0.0605	0.0599	0.0599	0.0605	0.0592	0.0614	0.0563	0.0676	0.0450	0.0901	1
0.41	0.0913	0.0437	0.0685	0.0556	0.0616	0.0589	0.0606	0.0599	0.0599	0.0606	0.0589	0.0616	0.0556	0.0685	0.0437	0.0913	1
0.42	0.0926	0.0422	0.0696	0.0547	0.0619	0.0585	0.0607	0.0598	0.0598	0.0607	0.0585	0.0619	0.0547	0.0696	0.0422	0.0926	1
0.43	0.0941	0.0405	0.0710	0.0537	0.0622	0.0580	0.0609	0.0597	0.0597	0.0609	0.0580	0.0622	0.0537	0.0710	0.0405	0.0941	1
0.44	0.0956	0.0386	0.0726	0.0524	0.0627	0.0574	0.0612	0.0595	0.0595	0.0612	0.0574	0.0627	0.0524	0.0726	0.0386	0.0956	1
0.45	0.0974	0.0364	0.0746	0.0508	0.0632	0.0566	0.0616	0.0594	0.0594	0.0616	0.0566	0.0632	0.0508	0.0746	0.0364	0.0974	1
0.46	0.0995	0.0338	0.0770	0.0487	0.0640	0.0555	0.0623	0.0592	0.0592	0.0623	0.0555	0.0640	0.0487	0.0770	0.0338	0.0995	1
0.47	0.1019	0.0307	0.0802	0.0460	0.0650	0.0540	0.0632	0.0589	0.0589	0.0632	0.0540	0.0650	0.0460	0.0802	0.0307	0.1019	1
0.48	0.1048	0.0268	0.0845	0.0424	0.0663	0.0518	0.0647	0.0585	0.0585	0.0647	0.0518	0.0663	0.0424	0.0845	0.0268	0.1048	1
0.49	0.1087	0.0217	0.0906	0.0370	0.0683	0.0485	0.0672	0.0580	0.0580	0.0672	0.0485	0.0683	0.0370	0.0906	0.0217	0.1087	1
0.50	0.1143	0.0143	0.1000	0.0286	0.0714	0.0429	0.0714	0.0571	0.0571	0.0714	0.0429	0.0714	0.0286	0.1000	0.0143	0.1143	1

Table6a: Estimates of Generalized Linear Unbiased Estimator (GLUE) and Arithmetic Mean (AM) Weights Sum of Squares, Weights Sum of Squares Ratio $(WSSR)$, AM and BLUE Variances and variance based relative efficiency (REL) at varying correlation points, when sample (η) size is four

ρ	$\sum_{i=1}^{4}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	Rel	ρ	$\sum_{i=1}^{4}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	Rel
-0.50	0.2600	0.2500	0.96	0.0625	0.0500	0.80	**-0.24**	0.2529	0.2500	0.99	0.1600	0.1568	0.98
-0.49	0.2597	0.2500	0.96	0.0663	0.0542	0.82	**-0.23**	0.2527	0.2500	0.99	0.1638	0.1608	0.98
-0.48	0.2594	0.2500	0.96	0.0700	0.0584	0.83	**-0.22**	0.2525	0.2500	0.99	0.1675	0.1648	0.98
-0.47	0.2591	0.2500	0.97	0.0738	0.0626	0.85	**-0.21**	0.2523	0.2500	0.99	0.1713	0.1688	0.99
-0.46	0.2587	0.2500	0.97	0.0775	0.0667	0.86	**-0.20**	0.2521	0.2500	0.99	0.1750	0.1727	0.99
-0.45	0.2584	0.2500	0.97	0.0813	0.0709	0.87	**-0.19**	0.2519	0.2500	0.99	0.1788	0.1767	0.99
-0.44	0.2581	0.2500	0.97	0.0850	0.0751	0.88	**-0.18**	0.2517	0.2500	0.99	0.1825	0.1806	0.99
-0.43	0.2578	0.2500	0.97	0.0888	0.0792	0.89	**-0.17**	0.2515	0.2500	0.99	0.1863	0.1846	0.99
-0.42	0.2575	0.2500	0.97	0.0925	0.0834	0.90	**-0.16**	0.2514	0.2500	0.99	0.1900	0.1885	0.99
-0.41	0.2572	0.2500	0.97	0.0963	0.0875	0.91	**-0.15**	0.2512	0.2500	1.00	0.1938	0.1924	0.99
-0.40	0.2569	0.2500	0.97	0.1000	0.0917	0.92	**-0.14**	0.2511	0.2500	1.00	0.1975	0.1964	0.99
-0.39	0.2567	0.2500	0.97	0.1038	0.0958	0.92	**-0.13**	0.2509	0.2500	1.00	0.2013	0.2003	1.00
-0.38	0.2564	0.2500	0.98	0.1075	0.0999	0.93	**-0.12**	0.2508	0.2500	1.00	0.2050	0.2042	1.00
-0.37	0.2561	0.2500	0.98	0.1113	0.1040	0.94	**-0.11**	0.2507	0.2500	1.00	0.2088	0.2080	1.00
-0.36	0.2558	0.2500	0.98	0.1150	0.1081	0.94	**-0.10**	0.2506	0.2500	1.00	0.2125	0.2119	1.00
-0.35	0.2555	0.2500	0.98	0.1188	0.1122	0.95	**-0.09**	0.2505	0.2500	1.00	0.2163	0.2158	1.00
-0.34	0.2553	0.2500	0.98	0.1225	0.1163	0.95	**-0.08**	0.2504	0.2500	1.00	0.2200	0.2196	1.00
-0.33	0.2550	0.2500	0.98	0.1263	0.1204	0.95	**-0.07**	0.2503	0.2500	1.00	0.2238	0.2235	1.00
-0.32	0.2548	0.2500	0.98	0.1300	0.1245	0.96	**-0.06**	0.2502	0.2500	1.00	0.2275	0.2273	1.00
-0.31	0.2545	0.2500	0.98	0.1338	0.1285	0.96	**-0.05**	0.2501	0.2500	1.00	0.2313	0.2311	1.00
-0.30	0.2543	0.2500	0.98	0.1375	0.1326	0.96	**-0.04**	0.2501	0.2500	1.00	0.2350	0.2349	1.00
-0.29	0.2540	0.2500	0.98	0.1413	0.1367	0.97	**-0.03**	0.2501	0.2500	1.00	0.2388	0.2387	1.00
-0.28	0.2538	0.2500	0.99	0.1450	0.1407	0.97	**-0.02**	0.2500	0.2500	1.00	0.2425	0.2425	1.00
-0.27	0.2535	0.2500	0.99	0.1488	0.1447	0.97	**-0.01**	0.2500	0.2500	1.00	0.2463	0.2462	1.00
-0.26	0.2533	0.2500	0.99	0.1525	0.1488	0.98	**0.00**	0.2500	0.2500	1.00	0.2500	0.2500	1.00
-0.25	0.2531	0.2500	0.99	0.1563	0.1528	0.98	**0.01**	0.2500	0.2500	1.00	0.2538	0.2537	1.00

Table6b: Estimates of Generalized Linear Unbiased Estimator (GLUE) and Arithmetic Mean (AM) Weights Sum of Squares, Weights Sum of Squares Ratio $(WSSR)$, AM and BLUE Variances and variance based relative efficiency $(\mathrm{Re}\,l)$ at varying correlation points, when sample size (η) is four

ρ	$\sum_{i=1}^{4}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	$\mathrm{Re}\,l$	ρ	$\sum_{i=1}^{4}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	$\mathrm{Re}\,l$
0.02	0.2500	0.2500	1.00	0.2575	0.2575	1.00	**0.27**	0.2561	0.2500	0.98	0.3513	0.3460	0.99
0.03	0.2501	0.2500	1.00	0.2613	0.2612	1.00	**0.28**	0.2566	0.2500	0.97	0.3550	0.3493	0.98
0.04	0.2501	0.2500	1.00	0.2650	0.2649	1.00	**0.29**	0.2572	0.2500	0.97	0.3588	0.3526	0.98
0.05	0.2502	0.2500	1.00	0.2688	0.2686	1.00	**0.30**	0.2578	0.2500	0.97	0.3625	0.3559	0.98
0.06	0.2502	0.2500	1.00	0.2725	0.2723	1.00	**0.31**	0.2584	0.2500	0.97	0.3663	0.3591	0.98
0.07	0.2503	0.2500	1.00	0.2763	0.2759	1.00	**0.32**	0.2591	0.2500	0.96	0.3700	0.3624	0.98
0.08	0.2504	0.2500	1.00	0.2800	0.2796	1.00	**0.33**	0.2598	0.2500	0.96	0.3738	0.3656	0.98
0.09	0.2506	0.2500	1.00	0.2838	0.2832	1.00	**0.34**	0.2605	0.2500	0.96	0.3775	0.3688	0.98
0.10	0.2507	0.2500	1.00	0.2875	0.2868	1.00	**0.35**	0.2612	0.2500	0.96	0.3813	0.3720	0.98
0.11	0.2508	0.2500	1.00	0.2913	0.2904	1.00	**0.36**	0.2620	0.2500	0.95	0.3850	0.3751	0.97
0.12	0.2510	0.2500	1.00	0.2950	0.2940	1.00	**0.37**	0.2629	0.2500	0.95	0.3888	0.3783	0.97
0.13	0.2512	0.2500	1.00	0.2988	0.2976	1.00	**0.38**	0.2638	0.2500	0.95	0.3925	0.3814	0.97
0.14	0.2514	0.2500	0.99	0.3025	0.3012	1.00	**0.39**	0.2647	0.2500	0.94	0.3963	0.3844	0.97
0.15	0.2516	0.2500	0.99	0.3063	0.3047	1.00	**0.40**	0.2656	0.2500	0.94	0.4000	0.3875	0.97
0.16	0.2519	0.2500	0.99	0.3100	0.3083	0.99	**0.41**	0.2666	0.2500	0.94	0.4038	0.3905	0.97
0.17	0.2522	0.2500	0.99	0.3138	0.3118	0.99	**0.42**	0.2677	0.2500	0.93	0.4075	0.3935	0.97
0.18	0.2524	0.2500	0.99	0.3175	0.3153	0.99	**0.43**	0.2688	0.2500	0.93	0.4113	0.3965	0.96
0.19	0.2528	0.2500	0.99	0.3213	0.3188	0.99	**0.44**	0.2699	0.2500	0.93	0.4150	0.3995	0.96
0.20	0.2531	0.2500	0.99	0.3250	0.3222	0.99	**0.45**	0.2711	0.2500	0.92	0.4188	0.4024	0.96
0.21	0.2534	0.2500	0.99	0.3288	0.3257	0.99	**0.46**	0.2723	0.2500	0.92	0.4225	0.4053	0.96
0.22	0.2538	0.2500	0.98	0.3325	0.3291	0.99	**0.47**	0.2736	0.2500	0.91	0.4263	0.4082	0.96
0.23	0.2542	0.2500	0.98	0.3363	0.3325	0.99	**0.48**	0.2749	0.2500	0.91	0.4300	0.4111	0.96
0.24	0.2546	0.2500	0.98	0.3400	0.3359	0.99	**0.49**	0.2763	0.2500	0.91	0.4338	0.4139	0.95
0.25	0.2551	0.2500	0.98	0.3438	0.3393	0.99	**0.50**	0.2778	0.2500	0.90	0.4375	0.4167	0.95
0.26	0.2556	0.2500	0.98	0.3475	0.3426	0.99							

Table 7: Estimates of Generalized Linear Unbiased Estimator (GLUE) and Arithmetic Mean (AM) Weights Sum of Squares, Weights Sum of Squares Ratio $(WSSR)$, AM and BLUE Variances and variance based relative efficiency $(\mathrm{Re}\,l)$ at varying correlation points, when sample size (η) is thirteen

ρ	$\sum_{i=1}^{13}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	$\mathrm{Re}\,l$	ρ	$\sum_{i=1}^{13}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	$\mathrm{Re}\,l$
-0.50	0.0866	0.0769	0.89	0.0059	0.0022	0.37	**-0.24**	0.0776	0.0769	0.99	0.0428	0.0422	0.99
-0.49	0.0853	0.0769	0.90	0.0073	0.0039	0.53	**-0.23**	0.0775	0.0769	0.99	0.0443	0.0437	0.99
-0.48	0.0842	0.0769	0.91	0.0088	0.0055	0.63	**-0.22**	0.0775	0.0769	0.99	0.0457	0.0452	0.99
-0.47	0.0833	0.0769	0.92	0.0102	0.0071	0.70	**-0.21**	0.0774	0.0769	0.99	0.0471	0.0466	0.99
-0.46	0.0825	0.0769	0.93	0.0116	0.0088	0.76	**-0.20**	0.0774	0.0769	0.99	0.0485	0.0481	0.99
-0.45	0.0819	0.0769	0.94	0.0130	0.0104	0.80	**-0.19**	0.0773	0.0769	0.99	0.0499	0.0496	0.99
-0.44	0.0814	0.0769	0.94	0.0144	0.0119	0.83	**-0.18**	0.0773	0.0769	1.00	0.0514	0.0510	0.99
-0.43	0.0809	0.0769	0.95	0.0159	0.0135	0.85	**-0.17**	0.0772	0.0769	1.00	0.0528	0.0525	0.99
-0.42	0.0806	0.0769	0.95	0.0173	0.0151	0.87	**-0.16**	0.0772	0.0769	1.00	0.0542	0.0539	1.00
-0.41	0.0802	0.0769	0.96	0.0187	0.0166	0.89	**-0.15**	0.0772	0.0769	1.00	0.0556	0.0554	1.00
-0.40	0.0799	0.0769	0.96	0.0201	0.0182	0.90	**-0.14**	0.0771	0.0769	1.00	0.0570	0.0568	1.00
-0.39	0.0796	0.0769	0.97	0.0215	0.0197	0.92	**-0.13**	0.0771	0.0769	1.00	0.0585	0.0583	1.00
-0.38	0.0794	0.0769	0.97	0.0230	0.0213	0.93	**-0.12**	0.0771	0.0769	1.00	0.0599	0.0597	1.00
-0.37	0.0792	0.0769	0.97	0.0244	0.0228	0.93	**-0.11**	0.0770	0.0769	1.00	0.0613	0.0612	1.00
-0.36	0.0790	0.0769	0.97	0.0258	0.0243	0.94	**-0.10**	0.0770	0.0769	1.00	0.0627	0.0626	1.00
-0.35	0.0788	0.0769	0.98	0.0272	0.0258	0.95	**-0.09**	0.0770	0.0769	1.00	0.0641	0.0641	1.00
-0.34	0.0786	0.0769	0.98	0.0286	0.0273	0.95	**-0.08**	0.0770	0.0769	1.00	0.0656	0.0655	1.00
-0.33	0.0785	0.0769	0.98	0.0301	0.0288	0.96	**-0.07**	0.0770	0.0769	1.00	0.0670	0.0669	1.00
-0.32	0.0784	0.0769	0.98	0.0315	0.0303	0.96	**-0.06**	0.0770	0.0769	1.00	0.0684	0.0684	1.00
-0.31	0.0782	0.0769	0.98	0.0329	0.0318	0.97	**-0.05**	0.0769	0.0769	1.00	0.0698	0.0698	1.00
-0.30	0.0781	0.0769	0.98	0.0343	0.0333	0.97	**-0.04**	0.0769	0.0769	1.00	0.0712	0.0712	1.00
-0.29	0.0780	0.0769	0.99	0.0357	0.0348	0.97	**-0.03**	0.0769	0.0769	1.00	0.0727	0.0727	1.00
-0.28	0.0779	0.0769	0.99	0.0372	0.0363	0.98	**-0.02**	0.0769	0.0769	1.00	0.0741	0.0741	1.00
-0.27	0.0778	0.0769	0.99	0.0386	0.0378	0.98	**-0.01**	0.0769	0.0769	1.00	0.0755	0.0755	1.00
-0.26	0.0778	0.0769	0.99	0.0400	0.0393	0.98	**0.00**	0.0769	0.0769	1.00	0.0769	0.0769	1.00
-0.25	0.0777	0.0769	0.99	0.0414	0.0408	0.98	**0.01**	0.0769	0.0769	1.00	0.0783	0.0783	1.00

Table 7 continues: Estimates of Generalized Linear Unbiased Estimator (GLUE) and Arithmetic Mean (AM) Weights Sum of Squares, Weights Sum of Squares Ratio $(WSSR)$, AM and BLUE Variances and variance based relative efficiency $(\mathrm{Re}\,l)$ at varying correlation points, when sample size (η) is thirteen

ρ	$\sum_{i=1}^{13}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	$\mathrm{Re}\,l$	ρ	$\sum_{i=1}^{13}\varpi_i^2$	$1/\eta$	$WSSR$	$\mathrm{var}(\bar{X}_n)$	$\mathrm{var}(\tilde{X}_n)$	$\mathrm{Re}\,l$
0.02	0.0769	0.0769	1.00	0.0798	0.0798	1.00	**0.27**	0.0779	0.0769	0.99	0.1153	0.1145	0.99
0.03	0.0769	0.0769	1.00	0.0812	0.0812	1.00	**0.28**	0.0780	0.0769	0.99	0.1167	0.1158	0.99
0.04	0.0769	0.0769	1.00	0.0826	0.0826	1.00	**0.29**	0.0781	0.0769	0.99	0.1181	0.1172	0.99
0.05	0.0769	0.0769	1.00	0.0840	0.0840	1.00	**0.30**	0.0782	0.0769	0.98	0.1195	0.1185	0.99
0.06	0.0770	0.0769	1.00	0.0854	0.0854	1.00	**0.31**	0.0783	0.0769	0.98	0.1209	0.1199	0.99
0.07	0.0770	0.0769	1.00	0.0869	0.0868	1.00	**0.32**	0.0785	0.0769	0.98	0.1224	0.1212	0.99
0.08	0.0770	0.0769	1.00	0.0883	0.0882	1.00	**0.33**	0.0786	0.0769	0.98	0.1238	0.1225	0.99
0.09	0.0770	0.0769	1.00	0.0897	0.0896	1.00	**0.34**	0.0788	0.0769	0.98	0.1252	0.1239	0.99
0.10	0.0770	0.0769	1.00	0.0911	0.0910	1.00	**0.35**	0.0790	0.0769	0.97	0.1266	0.1252	0.99
0.11	0.0770	0.0769	1.00	0.0925	0.0924	1.00	**0.36**	0.0792	0.0769	0.97	0.1280	0.1265	0.99
0.12	0.0771	0.0769	1.00	0.0940	0.0938	1.00	**0.37**	0.0794	0.0769	0.97	0.1295	0.1278	0.99
0.13	0.0771	0.0769	1.00	0.0954	0.0952	1.00	**0.38**	0.0797	0.0769	0.97	0.1309	0.1291	0.99
0.14	0.0771	0.0769	1.00	0.0968	0.0966	1.00	**0.39**	0.0800	0.0769	0.96	0.1323	0.1304	0.99
0.15	0.0772	0.0769	1.00	0.0982	0.0980	1.00	**0.40**	0.0803	0.0769	0.96	0.1337	0.1317	0.98
0.16	0.0772	0.0769	1.00	0.0996	0.0994	1.00	**0.41**	0.0807	0.0769	0.95	0.1351	0.1330	0.98
0.17	0.0772	0.0769	1.00	0.1011	0.1008	1.00	**0.42**	0.0812	0.0769	0.95	0.1366	0.1343	0.98
0.18	0.0773	0.0769	1.00	0.1025	0.1021	1.00	**0.43**	0.0818	0.0769	0.94	0.1380	0.1355	0.98
0.19	0.0773	0.0769	0.99	0.1039	0.1035	1.00	**0.44**	0.0826	0.0769	0.93	0.1394	0.1367	0.98
0.20	0.0774	0.0769	0.99	0.1053	0.1049	1.00	**0.45**	0.0837	0.0769	0.92	0.1408	0.1380	0.98
0.21	0.0774	0.0769	0.99	0.1067	0.1063	1.00	**0.46**	0.0852	0.0769	0.90	0.1422	0.1392	0.98
0.22	0.0775	0.0769	0.99	0.1082	0.1077	1.00	**0.47**	0.0875	0.0769	0.88	0.1437	0.1403	0.98
0.23	0.0776	0.0769	0.99	0.1096	0.1090	0.99	**0.48**	0.0919	0.0769	0.84	0.1451	0.1414	0.97
0.24	0.0776	0.0769	0.99	0.1110	0.1104	0.99	**0.49**	0.1021	0.0769	0.75	0.1465	0.1423	0.97
0.25	0.0777	0.0769	0.99	0.1124	0.1118	0.99	**0.50**	0.1429	0.0769	0.54	0.1479	0.1429	0.97
0.26	0.0778	0.0769	0.99	0.1138	0.1131	0.99							

Table 8: Interval of the correlation points at varying sample size (η); $\eta = 2\varsigma$ *and* $\eta = 2\varsigma + 1 \forall \varsigma = 1, 2, ..., 8$; within which GLUE is equi-efficient with AM selected using $WSSR$ and $\mathrm{Re}l$ as basis for judgement

η	Equi-Efficiency Report using $WSSR$ and $\mathrm{Re}l$	
	$WSSR$	$\mathrm{Re}l$
3	$[-0.18, 0.12]$	$[-0.14, 0.14]$
4	$[-0.15, 0.13]$	$[-0.13, 0.15]$
5	$[-0.14, 0.13]$	$[-0.12, 0.15]$
6	$[-0.15, 0.14]$	$[-0.13, 0.16]$
7	$[-0.15, 0.14]$	$[-0.13, 0.17]$
8	$[-0.16, 0.15]$	$[-0.14, 0.18]$
9	$[-0.16, 0.15]$	$[-0.14, 0.19]$
10	$[-0.17, 0.16]$	$[-0.14, 0.20]$
11	$[-0.17, 0.17]$	$[-0.14, 0.20]$
12	$[-0.18, 0.17]$	$[-0.15, 0.21]$
13	$[-0.18, 0.18]$	$[-0.16, 0.22]$
14	$[-0.19, 0.19]$	$[-0.16, 0.23]$
15	$[-0.19, 0.19]$	$[-0.16, 0.24]$
16	$[-0.20, 0.20]$	$[-0.17, 0.24]$
17	$[-0.20, 0.20]$	$[-0.17, 0.24]$

Table 9: Estimates of the weights of the Best linear unbiased estimates (BLUE) selected using WSSR, as basis for judgment, at varying correlation points $(\hat{\rho})$ when sample size (η) equal to four

$\hat{\rho}$	ϖ_1	ϖ_2	ϖ_3	ϖ_4	$\hat{\rho}$	ϖ_1	ϖ_2	ϖ_3	ϖ_4	$\hat{\rho}$	ϖ_1	ϖ_2	ϖ_3	ϖ_4
-0.50	0.2000	0.3000	0.3000	0.2000	**-0.26**	0.2212	0.2788	0.2788	0.2212	**0.27**	0.2890	0.2110	0.2110	0.2890
-0.49	0.2008	0.2992	0.2992	0.2008	**-0.25**	0.2222	0.2778	0.2778	0.2222	**0.28**	0.2907	0.2093	0.2093	0.2907
-0.48	0.2016	0.2984	0.2984	0.2016	**-0.24**	0.2232	0.2768	0.2768	0.2232	**0.29**	0.2924	0.2076	0.2076	0.2924
-0.47	0.2024	0.2976	0.2976	0.2024	**-0.23**	0.2242	0.2758	0.2758	0.2242	**0.30**	0.2941	0.2059	0.2059	0.2941
-0.46	0.2033	0.2967	0.2967	0.2033	**-0.22**	0.2252	0.2748	0.2748	0.2252	**0.31**	0.2959	0.2041	0.2041	0.2959
-0.45	0.2041	0.2959	0.2959	0.2041	**-0.21**	0.2262	0.2738	0.2738	0.2262	**0.32**	0.2976	0.2024	0.2024	0.2976
-0.44	0.2049	0.2951	0.2951	0.2049	**-0.20**	0.2273	0.2727	0.2727	0.2273	**0.33**	0.2994	0.2006	0.2006	0.2994
-0.43	0.2058	0.2942	0.2942	0.2058	**-0.19**	0.2283	0.2717	0.2717	0.2283	**0.34**	0.3012	0.1988	0.1988	0.3012
-0.42	0.2066	0.2934	0.2934	0.2066	**-0.18**	0.2294	0.2706	0.2706	0.2294	**0.35**	0.3030	0.1970	0.1970	0.3030
-0.41	0.2075	0.2925	0.2925	0.2075	**-0.17**	0.2304	0.2696	0.2696	0.2304	**0.36**	0.3049	0.1951	0.1951	0.3049
-0.40	0.2083	0.2917	0.2917	0.2083	**-0.16**	0.2315	0.2685	0.2685	0.2315	**0.37**	0.3067	0.1933	0.1933	0.3067
-0.39	0.2092	0.2908	0.2908	0.2092	**0.14**	0.2688	0.2312	0.2312	0.2688	**0.38**	0.3086	0.1914	0.1914	0.3086
-0.38	0.2101	0.2899	0.2899	0.2101	**0.15**	0.2703	0.2297	0.2297	0.2703	**0.39**	0.3106	0.1894	0.1894	0.3106
-0.37	0.2110	0.2890	0.2890	0.2110	**0.16**	0.2717	0.2283	0.2283	0.2717	**0.40**	0.3125	0.1875	0.1875	0.3125
-0.36	0.2119	0.2881	0.2881	0.2119	**0.17**	0.2732	0.2268	0.2268	0.2732	**0.41**	0.3145	0.1855	0.1855	0.3145
-0.35	0.2128	0.2872	0.2872	0.2128	**0.18**	0.2747	0.2253	0.2253	0.2747	**0.42**	0.3165	0.1835	0.1835	0.3165
-0.34	0.2137	0.2863	0.2863	0.2137	**0.19**	0.2762	0.2238	0.2238	0.2762	**0.43**	0.3185	0.1815	0.1815	0.3185
-0.33	0.2146	0.2854	0.2854	0.2146	**0.20**	0.2778	0.2222	0.2222	0.2778	**0.44**	0.3205	0.1795	0.1795	0.3205
-0.32	0.2155	0.2845	0.2845	0.2155	**0.21**	0.2793	0.2207	0.2207	0.2793	**0.45**	0.3226	0.1774	0.1774	0.3226
-0.31	0.2165	0.2835	0.2835	0.2165	**0.22**	0.2809	0.2191	0.2191	0.2809	**0.46**	0.3247	0.1753	0.1753	0.3247
-0.30	0.2174	0.2826	0.2826	0.2174	**0.23**	0.2825	0.2175	0.2175	0.2825	**0.47**	0.3268	0.1732	0.1732	0.3268
-0.29	0.2183	0.2817	0.2817	0.2183	**0.24**	0.2841	0.2159	0.2159	0.2841	**0.48**	0.3289	0.1711	0.1711	0.3289
-0.28	0.2193	0.2807	0.2807	0.2193	**0.25**	0.2857	0.2143	0.2143	0.2857	**0.49**	0.3311	0.1689	0.1689	0.3311
-0.27	0.2203	0.2797	0.2797	0.2203	**0.26**	0.2874	0.2126	0.2126	0.2874	**0.50**	0.3333	0.1667	0.1667	0.3333

Table 10: Estimates of the weights of the Best linear unbiased estimates (BLUE) selected using WSSR, as basis for judgment, at varying correlation points $(\hat{\rho})$ when sample size (η) equal to Thirteen

$\hat{\rho}$	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}
-0.50	0.0286	0.0527	0.0725	0.0879	0.0989	0.1055	0.1077	0.1055	0.0989	0.0879	0.0725	0.0527	0.0286
-0.49	0.0310	0.0553	0.0739	0.0877	0.0972	0.1027	0.1045	0.1027	0.0972	0.0877	0.0739	0.0553	0.0310
-0.48	0.0331	0.0574	0.0751	0.0875	0.0957	0.1003	0.1018	0.1003	0.0957	0.0875	0.0751	0.0574	0.0331
-0.47	0.0351	0.0594	0.0760	0.0872	0.0943	0.0982	0.0995	0.0982	0.0943	0.0872	0.0760	0.0594	0.0351
-0.46	0.0368	0.0611	0.0768	0.0869	0.0931	0.0964	0.0975	0.0964	0.0931	0.0869	0.0768	0.0611	0.0368
-0.45	0.0385	0.0626	0.0775	0.0867	0.0920	0.0949	0.0957	0.0949	0.0920	0.0867	0.0775	0.0626	0.0385
-0.44	0.0401	0.0639	0.0781	0.0864	0.0911	0.0934	0.0942	0.0934	0.0911	0.0864	0.0781	0.0639	0.0401
-0.43	0.0415	0.0651	0.0785	0.0861	0.0902	0.0922	0.0928	0.0922	0.0902	0.0861	0.0785	0.0651	0.0415
-0.42	0.0429	0.0662	0.0789	0.0857	0.0893	0.0911	0.0916	0.0911	0.0893	0.0857	0.0789	0.0662	0.0429
-0.41	0.0442	0.0672	0.0792	0.0854	0.0886	0.0900	0.0905	0.0900	0.0886	0.0854	0.0792	0.0672	0.0442
-0.40	0.0454	0.0682	0.0795	0.0851	0.0879	0.0891	0.0895	0.0891	0.0879	0.0851	0.0795	0.0682	0.0454
-0.39	0.0466	0.0690	0.0797	0.0848	0.0872	0.0883	0.0886	0.0883	0.0872	0.0848	0.0797	0.0690	0.0466
-0.38	0.0478	0.0698	0.0799	0.0845	0.0866	0.0875	0.0878	0.0875	0.0866	0.0845	0.0799	0.0698	0.0478
-0.37	0.0489	0.0705	0.0800	0.0842	0.0861	0.0868	0.0870	0.0868	0.0861	0.0842	0.0800	0.0705	0.0489
-0.36	0.0499	0.0711	0.0801	0.0839	0.0856	0.0862	0.0864	0.0862	0.0856	0.0839	0.0801	0.0711	0.0499
-0.35	0.0509	0.0717	0.0802	0.0837	0.0851	0.0856	0.0857	0.0856	0.0851	0.0837	0.0802	0.0717	0.0509
-0.34	0.0519	0.0723	0.0802	0.0834	0.0846	0.0850	0.0852	0.0850	0.0846	0.0834	0.0802	0.0723	0.0519
-0.33	0.0529	0.0728	0.0803	0.0831	0.0842	0.0845	0.0846	0.0845	0.0842	0.0831	0.0803	0.0728	0.0529
-0.32	0.0538	0.0732	0.0803	0.0828	0.0837	0.0841	0.0841	0.0841	0.0837	0.0828	0.0803	0.0732	0.0538
-0.31	0.0547	0.0737	0.0803	0.0826	0.0834	0.0836	0.0837	0.0836	0.0834	0.0826	0.0803	0.0737	0.0547
-0.30	0.0556	0.0741	0.0802	0.0823	0.0830	0.0832	0.0833	0.0832	0.0830	0.0823	0.0802	0.0741	0.0556
-0.29	0.0564	0.0744	0.0802	0.0820	0.0826	0.0828	0.0829	0.0828	0.0826	0.0820	0.0802	0.0744	0.0564
-0.28	0.0573	0.0748	0.0802	0.0818	0.0823	0.0825	0.0825	0.0825	0.0823	0.0818	0.0802	0.0748	0.0573
-0.27	0.0581	0.0751	0.0801	0.0816	0.0820	0.0821	0.0821	0.0821	0.0820	0.0816	0.0801	0.0751	0.0581
-0.26	0.0589	0.0754	0.0800	0.0813	0.0817	0.0818	0.0818	0.0818	0.0817	0.0813	0.0800	0.0754	0.0589
-0.25	0.0597	0.0757	0.0799	0.0811	0.0814	0.0815	0.0815	0.0815	0.0814	0.0811	0.0799	0.0757	0.0597
-0.24	0.0604	0.0759	0.0799	0.0809	0.0811	0.0812	0.0812	0.0812	0.0811	0.0809	0.0799	0.0759	0.0604
-0.23	0.0612	0.0761	0.0798	0.0806	0.0809	0.0809	0.0809	0.0809	0.0809	0.0806	0.0798	0.0761	0.0612
-0.22	0.0620	0.0763	0.0797	0.0804	0.0806	0.0807	0.0807	0.0807	0.0806	0.0804	0.0797	0.0763	0.0620
-0.21	0.0627	0.0765	0.0796	0.0802	0.0804	0.0804	0.0804	0.0804	0.0804	0.0802	0.0796	0.0765	0.0627

Table 10 continues: Estimates of the weights of the Best linear unbiased estimates (BLUE) selected using WSSR, as basis for judgment, at varying correlation points $(\hat{\rho})$ when sample size (η) equal to Thirteen

$\hat{\rho}$	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}
-0.20	0.0634	0.0767	0.0794	0.0800	0.0801	0.0802	0.0802	0.0802	0.0801	0.0800	0.0794	0.0767	0.0634
-0.19	0.0642	0.0768	0.0793	0.0798	0.0799	0.0799	0.0799	0.0799	0.0799	0.0798	0.0793	0.0768	0.0642
0.19	0.0898	0.0721	0.0756	0.0749	0.0750	0.0750	0.0750	0.0750	0.0750	0.0749	0.0756	0.0721	0.0898
0.20	0.0906	0.0717	0.0756	0.0748	0.0750	0.0749	0.0749	0.0749	0.0750	0.0748	0.0756	0.0717	0.0906
0.21	0.0913	0.0712	0.0756	0.0747	0.0749	0.0748	0.0748	0.0748	0.0749	0.0747	0.0756	0.0712	0.0913
0.22	0.0921	0.0707	0.0757	0.0745	0.0748	0.0747	0.0748	0.0747	0.0748	0.0745	0.0757	0.0707	0.0921
0.23	0.0929	0.0702	0.0758	0.0744	0.0747	0.0747	0.0747	0.0747	0.0747	0.0744	0.0758	0.0702	0.0929
0.24	0.0937	0.0697	0.0758	0.0743	0.0747	0.0746	0.0746	0.0746	0.0747	0.0743	0.0758	0.0697	0.0937
0.25	0.0945	0.0692	0.0759	0.0741	0.0746	0.0745	0.0745	0.0745	0.0746	0.0741	0.0759	0.0692	0.0945
0.26	0.0953	0.0686	0.0761	0.0740	0.0745	0.0744	0.0744	0.0744	0.0745	0.0740	0.0761	0.0686	0.0953
0.27	0.0961	0.0679	0.0762	0.0738	0.0745	0.0743	0.0744	0.0743	0.0745	0.0738	0.0762	0.0679	0.0961
0.28	0.0970	0.0673	0.0764	0.0736	0.0744	0.0742	0.0743	0.0742	0.0744	0.0736	0.0764	0.0673	0.0970
0.29	0.0979	0.0666	0.0766	0.0734	0.0744	0.0741	0.0742	0.0741	0.0744	0.0734	0.0766	0.0666	0.0979
0.30	0.0988	0.0658	0.0768	0.0732	0.0744	0.0740	0.0741	0.0740	0.0744	0.0732	0.0768	0.0658	0.0988
0.31	0.0997	0.0651	0.0771	0.0729	0.0744	0.0738	0.0741	0.0738	0.0744	0.0729	0.0771	0.0651	0.0997
0.32	0.1006	0.0642	0.0774	0.0726	0.0744	0.0737	0.0740	0.0737	0.0744	0.0726	0.0774	0.0642	0.1006
0.33	0.1016	0.0633	0.0778	0.0723	0.0744	0.0736	0.0740	0.0736	0.0744	0.0723	0.0778	0.0633	0.1016
0.34	0.1027	0.0624	0.0782	0.0720	0.0744	0.0734	0.0739	0.0734	0.0744	0.0720	0.0782	0.0624	0.1027
0.35	0.1037	0.0614	0.0787	0.0716	0.0745	0.0732	0.0739	0.0732	0.0745	0.0716	0.0787	0.0614	0.1037
0.36	0.1048	0.0603	0.0792	0.0711	0.0746	0.0730	0.0739	0.0730	0.0746	0.0711	0.0792	0.0603	0.1048
0.37	0.1060	0.0591	0.0798	0.0706	0.0747	0.0728	0.0739	0.0728	0.0747	0.0706	0.0798	0.0591	0.1060
0.38	0.1072	0.0578	0.0805	0.0700	0.0750	0.0725	0.0740	0.0725	0.0750	0.0700	0.0805	0.0578	0.1072
0.39	0.1084	0.0564	0.0814	0.0693	0.0752	0.0722	0.0741	0.0722	0.0752	0.0693	0.0814	0.0564	0.1084
0.40	0.1098	0.0549	0.0824	0.0685	0.0756	0.0717	0.0743	0.0717	0.0756	0.0685	0.0824	0.0549	0.1098
0.41	0.1112	0.0532	0.0835	0.0676	0.0761	0.0712	0.0746	0.0712	0.0761	0.0676	0.0835	0.0532	0.1112
0.42	0.1127	0.0513	0.0848	0.0664	0.0768	0.0705	0.0750	0.0705	0.0768	0.0664	0.0848	0.0513	0.1127

Table 10 continues: Estimates of the weights of the Best linear unbiased estimates (BLUE) selected using WSSR, as basis for judgment, at varying correlation points $(\hat{\rho})$ when sample size (η) equal to Thirteen

$\hat{\rho}$	ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6	ϖ_7	ϖ_8	ϖ_9	ϖ_{10}	ϖ_{11}	ϖ_{12}	ϖ_{13}
0.43	0.1144	0.0492	0.0865	0.0649	0.0777	0.0696	0.0757	0.0696	0.0777	0.0649	0.0865	0.0492	0.1144
0.44	0.1162	0.0467	0.0884	0.0631	0.0789	0.0683	0.0767	0.0683	0.0789	0.0631	0.0884	0.0467	0.1162
0.45	0.1182	0.0439	0.0909	0.0607	0.0807	0.0665	0.0781	0.0665	0.0807	0.0607	0.0909	0.0439	0.1182
0.46	0.1206	0.0404	0.0941	0.0575	0.0833	0.0638	0.0804	0.0638	0.0833	0.0575	0.0941	0.0404	0.1206
0.47	0.1234	0.0360	0.0985	0.0529	0.0874	0.0596	0.0843	0.0596	0.0874	0.0529	0.0985	0.0360	0.1234
0.48	0.1270	0.0300	0.1050	0.0457	0.0943	0.0524	0.0911	0.0524	0.0943	0.0457	0.1050	0.0300	0.1270
0.49	0.1323	0.0205	0.1163	0.0326	0.1077	0.0381	0.1050	0.0381	0.1077	0.0326	0.1163	0.0205	0.1323
0.50	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429	0.0000	0.1429

Table 11: Four purposively selected data sets on monthly average exchange of Naira per unit of EURO currency (X_t)

Data 1			**Data 2**			**Data 3**			**Data 4**		
Year	Month	X_t	Year	Month	X_t	Year	Month	X_t	Year	Month	X_t
2004	November	172.67	2010	July	189.83	2017	May	337.72	2011	April	220.08
2004	December	177.95	2010	August	191.9	2017	June	343.24	2011	May	219.66
2005	January	174.13	2010	September	195.91	2017	July	358.50	2011	June	220.22
2005	February	173.05	2010	October	208.34	2017	August	360.93	2011	July	216.08
2005	March	176.26	2010	November	203.64	2017	September	364.53	2011	August	216.79
2005	April	171.8	2010	December	197.27	2017	October	359.34	2011	September	211.73
2005	May	168.52	2011	January	200.57	2017	November	359.07	2011	October	208.22
2005	June	161.67	2011	February	205.58	2017	December	362.36	2011	November	208.78
2005	July	160.00	2011	March	211.17	2018	January	373.00	2011	December	206.52
2005	August	162.81	2011	April	220.08	2018	February	377.84	2012	January	202.52
			2011	May	219.66	2018	March	377.19	2012	February	206.71
						2018	April	374.22	2012	March	206.05
									2012	April	205.00

Source: Onyemachi et al, 2026

Table 12: Estimates of Autocorrelation Coefficients across the sample sizes for Data sets 1, 2, 3 and 4

Sample Size	lags	Estimates of Autocorrelation Coefficients	T value
n	k	$\hat{\rho}_k$	
10	**1**	**0.7239**	**2.29**
	2	**0.3248**	**0.72**
	3	**0.0066**	**0.01**
11	**1**	**0.6032**	**2.00**
	2	**0.1333**	**0.34**
	3	**-0.0925**	**-0.23**
12	**1**	**0.6490**	**2.25**
	2	**0.2420**	**0.62**
	3	**0.0309**	**0.08**
13	**1**	**0.7800**	**2.81**
	2	**0.5904**	**1.43**
	3	**0.3743**	**0.79**

Source: Onyemachi et al, 2026

Table 13: Estimate of MA(1) coefficients with its associated error variance across sample sizes for Data sets 1, 2, 3 and 4

η	$\hat{\psi}_1$	$\hat{\sigma}^2$
10	-0.884	19.217
11	-0.837	40.533
12	-0.867	69.924
13	-0.806	17.255

Table 14: Estimates of GLUE Weights at each correlation points ($\hat{\rho}$) of the varying sample sizes for Data sets 1, 2, 3 and 4

η	$\hat{\rho}$	Estimate of GLUE Weights													$\sum_{i=1}^{\eta}\varpi_i$
		$\hat{\varpi}_1$	$\hat{\varpi}_2$	$\hat{\varpi}_3$	$\hat{\varpi}_4$	$\hat{\varpi}_5$	$\hat{\varpi}_6$	$\hat{\varpi}_7$	$\hat{\varpi}_8$	$\hat{\varpi}_9$	$\hat{\varpi}_{10}$	$\hat{\varpi}_{11}$	$\hat{\varpi}_{12}$	$\hat{\varpi}_{13}$	
10	**-0.4962**	0.0463	0.0825	0.1091	0.1267	0.1354	0.1354	0.1267	0.1091	0.0825	0.0463				1.00
11	**-0.4922**	0.0402	0.0715	0.0949	0.1110	0.1205	0.1237	0.1205	0.1110	0.0949	0.0715	0.0402			1.00
12	**-0.495**	0.0342	0.0616	0.0829	0.0986	0.1088	0.1139	0.1139	0.1088	0.0986	0.0829	0.0616	0.0342		1.00
13	**-0.4886**	0.0298	0.0541	0.0733	0.0878	0.0980	0.104	0.1060	0.1040	0.0980	0.0878	0.0733	0.0541	0.0298	1.00

Table 15: Arithmetic mean $(\overline{X}_n)$ and Mean $(\tilde{X}_n)$ computed using GLUE's calculated Weights, Weights Sum of Squares for AM and GLUE, AM and GLUE Variances, Weights Sum of Squares Ratio $(WSSR)$ and variance based relative efficiency $(\mathrm{Re}\,l)$ at each correlation point ($\hat{\rho}$) of the varying sample sizes (η)for Data sets 1, 2, 3 and 4

η	$\hat{\rho}$	$\tilde{X}_n$	$\overline{X}_n$	$\sum_{i=1}^{\eta}\varpi_i^{\,2}$	$\frac{1}{\eta}$	$v(\tilde{X}_n)$	$v(\overline{X}_n)$	$WSSR$	$\mathrm{Re}\,l$
10	-0.4962	170.45	169.89	0.1105	0.1000	0.1028	0.2052	0.91	0.50
11	-0.4922	203.63	204.00	0.1005	0.0909	0.2034	0.3873	0.90	0.53
12	-0.495	363.05	362.33	0.0927	0.0833	0.2557	0.5394	0.90	0.47
13	-0.4886	211.03	211.41	0.0859	0.0769	0.0525	0.1144	0.90	0.46

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