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Spectral Properties and Energy Analysis of the Bishop Hypergraph on the 8×8 Chessboard

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Abstract

In this paper, we investigate the Bishop hypergraph H_B obtained from an 8×8 chessboard, where each square is treated as a vertex and two vertices are adjacent if a Bishop can move between them in a single move. We construct the adjacency and Laplacian matrices of the Bishop hypergraph H_B and analyze their spectral properties. The eigenvalues of these matrices are computed numerically using Python, and the corresponding graph energies are evaluated. The results highlight the influence of diagonal-based connectivity on the spectrum and energy of the Bishop hypergraph H_B and provide a comparative perspective with other chessboard based graphs.

Keywords: Bishop Hypergraph H_B; Adjacency Energy; Laplacian Energy;Siedal Energy

2010 Mathematics Subject Classification : 05C65

1 Introduction

Hypergraphs are collections of finite sets and represent one of the most general frameworks studied in discrete mathematics. Although the concept was introduced earlier, hypergraph theory emerged as an independent area of research during the 1960s, as documented in (3; 22). Since then, hypergraph theory has developed rapidly due to its wide applications in computer science, combinatorics, and network modeling (2; 5; 7; 10). Graphs and hypergraphs derived from chessboard movements form an important class of combinatorial models of theoretical and applied significance. Classical problems such as Knight's tour have been extensively studied, leading to efficient algorithms and deeper mathematical insights (8; 23; 24). These studies motivated the construction of chessboard-based graphs, where the squares of a chessboard are considered as vertices, and adjacency is defined according to the legal moves of a given chess piece. In this context, the Bishop hypergraph H_B is obtained by considering the squares of a chessboard as vertices, where two vertices are adjacent if a Bishop can move between them in a single diagonal move. Due to the strictly diagonal movement of the Bishop, the resulting graph admits a structural decomposition governed by diagonal connectivity patterns. A natural extension of this construction leads to the Bishop hypergraph H_B, in which a hyperedge consists of a vertex together with all vertices that are reachable from it by a single Bishop move. Such constructions fall within the broader framework of uniform and regular hypergraphs studied in (5; 18).

In this paper, we construct the 64×64 adjacency matrix of the Bishop Hypergraph H_B corresponding to an 8×8 chessboard and examine its associated Laplacian and Siedal matrices. The eigenvalues of these matrices and their corresponding energies are computed numerically using Python programming. The study of graph energies follows the classical developments in spectral graph theory (1; 6; 12; 13; 17; 21; 26). Recent investigations on the spectral characteristics of chessboard hypergraphs further motivate this work (24). In modern technological applications, efficient movement and routing strategies are essential in areas such as automation, robotics, and intelligent logistics systems. Diagonal movement patterns, analogous to those of the Bishop on a chessboard, can be employed to model routing and optimization problems in grid-based environments. The spectral analysis of the Bishop hypergraph H_B presented in this work provides a mathematical framework that can be adapted to such applications.

2 Preliminaries

In this section, we introduce the basic definitions and matrix representations associated with the Bishop hypergraph H_B on a $n \times n$ chessboard. These notions will be used throughout the paper.

2.1 Bishop graph

Let n be a positive integer. The Bishop graph, denoted by G_B, is the simple undirected graph constructed from $n \times n$ chessboard. The vertex set of G_B is defined as

$$V(G_B) = \{(i,j) \mid 1 \le i,j \le n\},$$

where each ordered pair (i,j) represents a square of the chessboard. Two distinct vertices (i,j) and (k,ℓ) are adjacent in G_B if and only if a Bishop can move from one square to the other in a single move. Equivalently,

$$|i-k| = |j-\ell|.$$

Thus, edges connect pairs of squares lying on the same diagonal. The order of G_B is n^2.

For any vertex $(i,j) \in V(G_B)$, its degree, denoted by $d(i,j)$, equals the number of squares lying on the two diagonals passing through (i,j), excluding the vertex itself.

2.2 Adjacency Matrix

The adjacency matrix of the Bishop graph G_B is the matrix

$$A(G_B) = [a_{uv}],$$

where $u = (i,j)$ and $v = (k,\ell)$ are vertices of G_B, and

$$a_{uv} = \begin{cases} 1, & \text{if } u \neq v \text{ and } |i-k| = |j-\ell|, \\ 0, & \text{otherwise.} \end{cases}$$

Since $A(G_B)$ is a real symmetric matrix, all its eigenvalues are real. The set of eigenvalues of $A(G_B)$ together with their multiplicities is called the adjacency spectrum of G_B.

2.3 Laplacian Matrix

For a vertex $u \in V(G_B)$, the Laplacian degree is defined by

$$\delta_l(u) = \sum_{v \in V(G_B)} a_{uv}.$$

The Laplacian matrix of G_B, denoted by $L(G_B)$, is defined as

$$L(G_B) = D(G_B) - A(G_B),$$

where $A(G_B)$ is adjacency matrix and

$$D = \operatorname{diag}(\delta_l(v_1), \delta_l(v_2), \ldots, \delta_l(v_{n^2}))$$

is the diagonal matrix of Laplacian degrees.

The matrix $L(G_B)$ is real and symmetric; hence all its eigenvalues are real and non-negative. Moreover, 0 is always an eigenvalue of $L(G_B)$. The eigenvalues together with their multiplicities form the Laplacian spectrum of G_B.

2.4 Seidel Matrix

The Seidel matrix of the Bishop graph G_B is the matrix

$$S(G_B) = [s_{uv}],$$

where for vertices $u = (i, j)$ and $v = (k, \ell)$,

$$s_{uv} = \begin{cases} 0, & \text{if } u = v, \\ -1, & \text{if } u \neq v \text{ and } |i-k| = |j-\ell|, \\ 1, & \text{if } u \neq v \text{ and } |i-k| \neq |j-\ell|. \end{cases}$$

Thus, adjacent vertices receive the entry -1, non-adjacent distinct vertices receive 1, and diagonal entries are zero.

Since $S(G_B)$ is real and symmetric, its eigenvalues are real. The set of eigenvalues of $S(G_B)$ together with their multiplicities is called the Seidel spectrum of G_B.

3 The Bishop Hypergraph H_B

In this section, we formalize the hypergraph structure naturally induced by the movement of a Bishop on a standard 8×8 chessboard. Throughout this section, we adopt the classical algebraic chess notation for labeling the squares.

3.1 Notation and Vertex Labeling

The chessboard consists of eight vertical files denoted by

$$a, b, c, d, e, f, g, h,$$

and eight horizontal ranks indexed by

$$1, 2, \ldots, 8.$$

Each square of the board is uniquely represented by a symbol of the form x_i, where $x \in \{a, b, c, d, e, f, g, h\}$ and $1 \leq i \leq 8$.

Thus, the vertex set is

$$V = \{x_i \mid x \in \{a, \ldots, h\},\ 1 \leq i \leq 8\}.$$

In particular, the square h_8 corresponds to the upper-right corner of the board.

Two distinct squares x_i and y_j are adjacent in the Bishop graph if and only if

$$|x - y| = |i - j|,$$

that is, the absolute difference between their file indices equals the absolute difference between their rank indices. This condition precisely characterizes diagonal movement.

3.2 Local Structure: Interior Vertices

We first examine the diagonal reachability of central squares.

Let a Bishop be placed at the square d_4 on the 8×8 chessboard. Then the set of all squares reachable in a single legal move is

$$\{a_1, b_2, c_3, e_3, f_2, g_1, c_5, b_6, a_7, e_5, f_6, g_7, h_8\}.$$

Consequently,

$$\deg(d_4) = 13.$$

Indeed, the Bishop moves along two diagonals passing through d_4. Each diagonal extends maximally until the boundary of the board is reached. Since d_4 is an interior square, both diagonals attain their full possible lengths, yielding thirteen distinct neighboring vertices.

An analogous argument applies to the symmetric interior square e_5.

For a Bishop placed at e_5, the set of squares reachable in one move is

$$\{a_1, b_2, c_3, d_4, f_4, g_3, h_2, d_6, c_7, b_8, f_6, g_7, h_8\}.$$

Hence,

$$\deg(e_5) = 13.$$

Thus, every interior square lying sufficiently far from the boundary has degree 13 in the Bishop hypergraph.

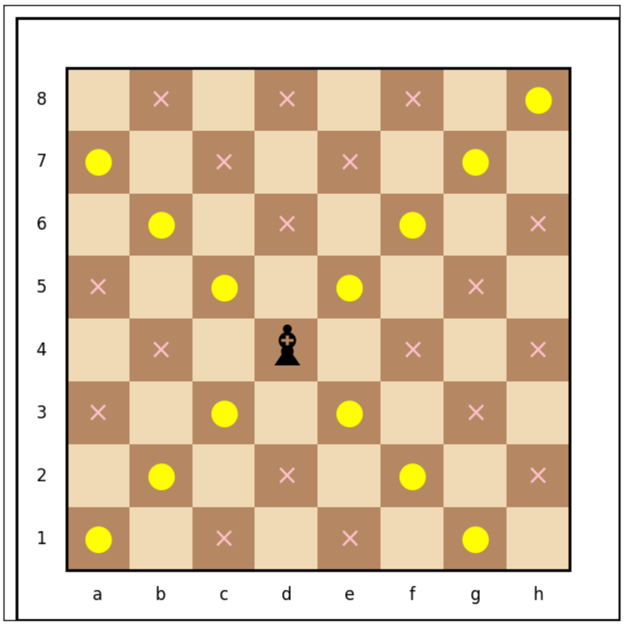


Figure 1: Bishop Reachability Structure

We now extend the graph structure to a hypergraph model. The *Bishop hypergraph* H_B is defined as

$$H_B = (V, E),$$

where V is the set of all 64 squares and each hyperedge corresponds to the complete diagonal neighborhood of a fixed square. For a given vertex $x_i \in V$, we define the associated hyperedge by

$$E_{x_i} = \{x_i\} \cup \{y_j \in V \mid |x - y| = |i - j|\}.$$

Thus, each hyperedge consists of the chosen square together with all squares lying on the two diagonals passing through it.

Consequently,

$$E = \{E_{x_i} \mid x_i \in V\}.$$

3.3 Explicit Hyperedge Structure

The diagonal structure varies depending on the position of the file. For each $i = 1, 2, \ldots, 8$, the hyperedges can be described as follows.

For the a-file:

$$E_{a_i} = \{a_i,\ b_{(i\pm1)},\ c_{(i\pm2)},\ d_{(i\pm3)},\ e_{(i\pm4)},\ f_{(i\pm5)},\ g_{(i\pm6)},\ h_{(i\pm7)}\},$$

whenever $1 \leq i \pm k \leq 8$.

For the b-file:

$$E_{b_i} = \{b_i,\ a_{(i\pm1)},\ c_{(i\pm1)},\ d_{(i\pm2)},\ e_{(i\pm3)},\ f_{(i\pm4)},\ g_{(i\pm5)},\ h_{(i\pm6)}\},$$

subject to board boundary constraints.

Similarly, for c, d, e, f, g, and h, the hyperedges are determined by extending diagonally in both directions while respecting

$$1 \leq i \pm k \leq 8.$$

This formulation provides a complete characterization of the Bishop hypergraph H_B on the standard chessboard.

3.4 Structural Observation

Each hyperedge corresponds to the union of two maximal diagonals intersecting at a single vertex. Interior vertices generate hyperedges of size 14, whereas boundary vertices produce smaller hyperedges due to truncation by the board limits.

This hypergraph representation will serve as the foundation for the spectral and combinatorial analysis developed in the subsequent sections.

4 The Adjacency Matrix of the Bishop Hypergraph H_B

In the Bishop hypergraph H_B associated with the standard 8×8 chessboard, two vertices are said to be adjacent whenever they belong to at least one common hyperedge. Since each hyperedge corresponds to a complete diagonal,two distinct squares are adjacent precisely when they lie on the same diagonal.

Thus, for vertices $u = (i, j)$ and $v = (k, \ell)$,

$$u \sim v \quad \Longleftrightarrow \quad |i - k| = |j - \ell|.$$

Because two distinct squares can lie on at most one common diagonal, each adjacent pair contributes exactly one unit in the adjacency matrix.

Hence, the adjacency matrix of the Bishop hypergraph H_B (equivalently, the Bishop graph G_B) is a symmetric matrix of order 64×64 with all diagonal entries equal to zero.

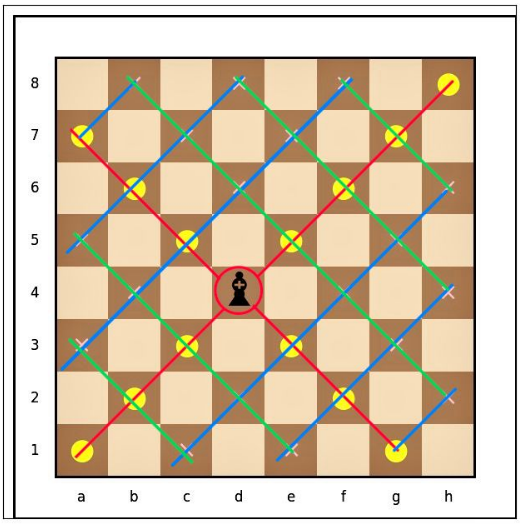


Figure 2: Adjacency Graph of Bishop Moves on the 8×8 Chessboard

As an illustration, consider the vertex e_5. The squares lying on its two diagonals are

$$\{a_1, b_2, c_3, d_4, f_6, g_7, h_8\}$$

and

$$\{b_8, c_7, d_6, f_4, g_3, h_2\}.$$

Thus, $\deg(e_5) = 13$, and all remaining vertices have zero adjacency with e_5.

4.1 Block Structure of the Adjacency Matrix $A(H_B)$

Let $A(H_B)$ denote the adjacency matrix. Partition $A(H_B)$ into 8×8 blocks corresponding to the files A, B, C, D, E, F, G, H:

$$A(H_B) = \begin{pmatrix} [AA] & [AB] & [AC] & [AD] & [AE] & [AF] & [AG] & [AH] \\ [BA] & [BB] & [BC] & [BD] & [BE] & [BF] & [BG] & [BH] \\ [CA] & [CB] & [CC] & [CD] & [CE] & [CF] & [CG] & [CH] \\ [DA] & [DB] & [DC] & [DD] & [DE] & [DF] & [DG] & [DH] \\ [EA] & [EB] & [EC] & [ED] & [EE] & [EF] & [EG] & [EH] \\ [FA] & [FB] & [FC] & [FD] & [FE] & [FF] & [FG] & [FH] \\ [GA] & [GB] & [GC] & [GD] & [GE] & [GF] & [GG] & [GH] \\ [HA] & [HB] & [HC] & [HD] & [HE] & [HF] & [HG] & [HH] \end{pmatrix}.$$

Index files numerically by

$$A = 1,\ B = 2,\ C = 3,\ D = 4,\ E = 5,\ F = 6,\ G = 7,\ H = 8.$$

For two files X and Y, define

$$d = |X - Y|.$$

Then

$$[XY]_{ij} = \begin{cases} 1, & \text{if } |i - j| = d, \\ 0, & \text{otherwise.} \end{cases}$$

Thus, the adjacency matrix is completely determined by eight distinct block types corresponding to $d = 0, 1, \ldots, 7$.

4.2 Explicit Block Types

Block Type $d = 0$

$$B_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Block Type $d = 1$

$$B_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Block Type $d = 2$

$$B_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Block Type $d = 3$

$$B_3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Block Type $d = 4$

$$B_4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Block Type $d = 5$

$$B_5 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Block Type $d = 6$

$$B_6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Block Type $d = 7$

$$B_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

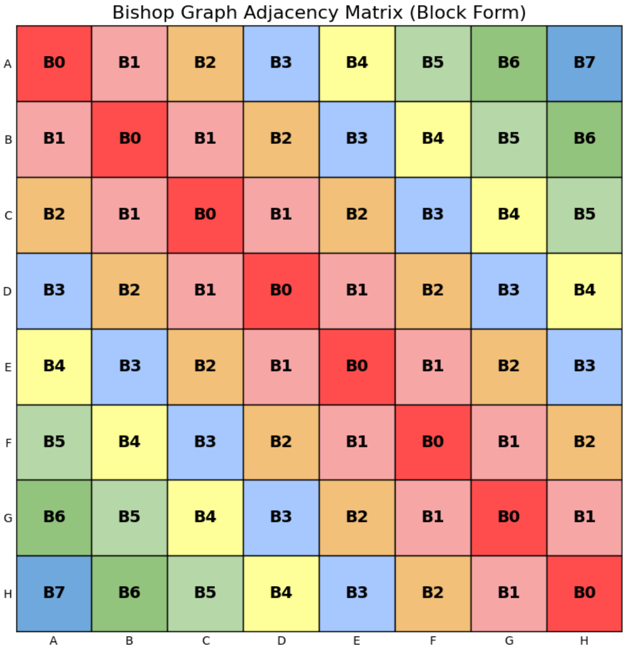
4.3 Symmetry and Structural Characterization

The adjacency matrix satisfies:

1. The matrix $A(H_B)$ is symmetric, that is, $A(H_B) = A(H_B)^T$.
2. The block corresponding to XY is the transpose of the block corresponding to YX, i.e., $[XY] = [YX]^T$.
3. Every block is determined solely by the distance parameter $d = |X - Y|$.

Hence, $A(H_B)$ is a block Toeplitz matrix completely determined by

$$B_0, B_1, \ldots, B_7.$$



Bishop Graph Adjacency Matrix (Block Form)

	A	B	C	D	E	F	G	H
A	B0	B1	B2	B3	B4	B5	B6	B7
B	B1	B0	B1	B2	B3	B4	B5	B6
C	B2	B1	B0	B1	B2	B3	B4	B5
D	B3	B2	B1	B0	B1	B2	B3	B4
E	B4	B3	B2	B1	B0	B1	B2	B3
F	B5	B4	B3	B2	B1	B0	B1	B2
G	B6	B5	B4	B3	B2	B1	B0	B1
H	B7	B6	B5	B4	B3	B2	B1	B0

Figure 3: Adjacency Matrix of Bishop Moves on the 8×8 Chessboard

5 The Adjacency Energy of Bishop Hypergraph H_B

The adjacency matrix $A(H_B)$ is real and symmetric; therefore, all eigenvalues are real. The computed spectrum (with multiplicities) is presented below.

Table 1: Eigenvalues of the Bishop hypergraph

λ_1	λ_2	λ_3	λ_4
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-2.000000	-2.000000	-2.000000	-2.000000
-1.170450	-1.170450	-1.000000	-1.000000
-0.362220	-0.362220	0.000000	0.000000
0.534500	0.534500	1.000000	1.000000
2.000000	2.000000	3.000000	3.000000
3.522690	3.522690	4.000000	4.000000
4.572500	4.572500	5.000000	5.000000
5.741700	5.741700	9.161270	9.161270

The energy of the Bishop hypergraph is defined by

$$E(H_B) = \sum_{i=1}^{64} |\lambda_i|.$$

Using the computed spectrum,

$$E(H_B) = 154.13066.$$

This concludes the structural and spectral characterization of the Bishop graph on the 8×8 chessboard.

6 The Laplacian Matrix of the Bishop Hypergraph H_B

Let H_B be the Bishop hypergraph defined on the standard 8×8 chessboard. The vertex set consists of the 64 squares of the board. We index the vertices row-wise as

$$v_k = (i, j), \qquad k = 8(i-1) + j,$$

where $i, j \in \{1, 2, \dots, 8\}$.

Two vertices are adjacent if and only if the corresponding squares lie on the same diagonal. For a square (i, j), the number of diagonally reachable squares determines its degree:

$$\delta(i,j) = \min(i-1, j-1) + \min(i-1, 8-j) + \min(8-i, j-1) + \min(8-i, 8-j).$$

Accordingly, the degree distribution over the chessboard is

$$\begin{pmatrix} 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\ 7 & 9 & 9 & 9 & 9 & 9 & 9 & 7 \\ 7 & 9 & 11 & 11 & 11 & 11 & 9 & 7 \\ 7 & 9 & 11 & 13 & 13 & 11 & 9 & 7 \\ 7 & 9 & 11 & 13 & 13 & 11 & 9 & 7 \\ 7 & 9 & 11 & 11 & 11 & 11 & 9 & 7 \\ 7 & 9 & 9 & 9 & 9 & 9 & 9 & 7 \\ 7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \end{pmatrix}.$$

Let $A(H_B)$ denote the adjacency matrix of H_B and

$$D(H_B) = \operatorname{diag}(\delta(v_1), \delta(v_2), \dots, \delta(v_{64}))$$

be the diagonal degree matrix.

The Laplacian matrix of the Bishop graph is defined by

$$L(H_B) = D(H_B) - A(H_B).$$

Hence, for $1 \le p, q \le 64$,

$$L_{pq} = \begin{cases} \delta(v_p), & \text{if } p = q, \\ -1, & \text{if } v_p \text{ and } v_q \text{ lie on the same diagonal}, \\ 0, & \text{otherwise}. \end{cases}$$

Thus, $L(H_B)$ is a symmetric matrix of order 64×64 with diagonal entries equal to the vertex degrees and off-diagonal entries equal to -1 whenever two vertices share a diagonal.

A schematic representation of the matrix is given by

$$L(H_B) = \begin{bmatrix} 7 & 0 & 0 & \cdots & -1 & 0 & \cdots & 0 \\ 0 & 7 & 0 & \cdots & 0 & -1 & \cdots & 0 \\ 0 & 0 & 7 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & & \ddots & & & & \vdots \\ -1 & 0 & 0 & \cdots & 9 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 & 9 & \cdots & 0 \\ \vdots & & & & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 7 \end{bmatrix}_{64 \times 64}.$$

Since a Bishop remains on squares of the same colour, the graph decomposes into two connected components. Consequently, the Laplacian matrix possesses exactly two zero eigenvalues.

The Laplacian spectrum of H_B consists of 64 real eigenvalues. Numerical computation shows that two eigenvalues are equal to zero, corresponding to the two connected components of the graph. The remaining eigenvalues occur with multiplicities arising from board symmetry.

Table 2: Laplacian Eigenvalues of the Bishop graph

μ_1	μ_2	μ_3	μ_4
0.00000	0.00000	2.99708	2.99708
3.54011	3.54011	4.00750	4.00750
4.09494	4.09494	4.57136	4.57136
5.20304	5.20304	5.76393	5.76393
6.43845	6.43845	7.00000	7.00000
7.94230	7.94230	8.00000	8.00000
8.69830	8.69830	9.00000	9.00000
9.00000	9.00000	9.00000	9.00000
9.34936	9.34936	10.03123	10.03123
10.07838	10.07838	10.15799	10.15799
10.23607	10.23607	10.56155	10.56155
11.00000	11.00000	11.00000	11.00000
11.53222	11.53222	11.78848	11.78848
12.28495	12.28495	12.35026	12.35026
12.63897	12.63897	13.00000	13.00000
14.05721	14.05721	14.67632	14.67632

7 The Laplacian Energy of Bishop Hypergraph H_B

For a graph G with n vertices and m edges, the Laplacian energy is defined by

$$LE(G) = \sum_{i=1}^{n} \left| \lambda_i - \frac{2m}{n} \right|,$$

where $\lambda_1, \dots, \lambda_n$ denote the Laplacian eigenvalues.
For the Bishop hypergraph H_B,

$$n = 64, \qquad \frac{2m}{n} = 8.75.$$

Therefore,

$$LE(H_B) = \sum_{i=1}^{64} |\lambda_i - 8.75| \,.$$

The computed Laplacian energy is

$$LE(H_B) = 181.97196.$$

Hence, the Laplacian energy of the Bishop graph on the 8×8 chessboard equals 181.97196.

8 The Seidel Matrix of the Bishop Hypergraph H_B

The Seidel matrix of a simple graph G of order n is defined by

$$S(G) = J - I - 2A(G),$$

where J is the all–ones matrix and I is the identity matrix of order n. For the Bishop hypergraph H_B ($n = 64$), we have

$$S(H_B) = J_{64} - I_{64} - 2A(H_B).$$

Equivalently, the entries of $S(H_B) = [s_{ij}]$ are given by

$$s_{ij} = \begin{cases} 0, & i = j, \\ -1, & \text{if } v_i \text{ is adjacent to } v_j, \\ 1, & \text{if } v_i \text{ is not adjacent to } v_j, \end{cases} \qquad 1 \le i, j \le 64.$$

Thus, the Seidel matrix is a real symmetric matrix of order 64×64 with zero diagonal entries. The off-diagonal entries take the value -1 whenever two vertices lie on the same diagonal of the chessboard and 1 otherwise. This structure reflects the complementarity between adjacency and non-adjacency in the Bishop graph.

A schematic representation of $S(H_B)$ is

$$S(G) = \begin{bmatrix} 0 & 1 & 1 & \cdots & -1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 & -1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 & \cdots & 1 \\ \vdots & & & \ddots & & & & \vdots \\ -1 & 1 & 1 & \cdots & 0 & 1 & \cdots & 1 \\ 1 & -1 & 1 & \cdots & 1 & 0 & \cdots & 1 \\ \vdots & & & & & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 1 & \cdots & 0 \end{bmatrix}_{64 \times 64}.$$

Since $S(G)$ is symmetric, all its eigenvalues are real. The Seidel spectrum of the Bishop graph consists of 64 eigenvalues. The extreme eigenvalues obtained numerically are

Table 3: Seidel Eigenvalues of the Bishop graph

σ_1	σ_2	σ_3	σ_4
-19.32255	-12.56820	-12.48339	-11.00000
-11.00000	-10.21559	-10.14500	-9.00000
-9.00000	-8.08039	-8.04539	-7.00000
-7.00000	-5.00000	-5.00000	-3.00000
-3.00000	-2.21491	-2.06901	-1.00000
-1.00000	-0.71364	-0.27556	1.00000
1.00000	1.00000	1.34089	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	3.00000
3.00000	3.00000	3.00000	45.79273

The multiplicities observed in the spectrum arise from the symmetry of the chessboard and the regular diagonal connectivity pattern.

9 The Seidel Energy of Bishop Hypergraph H_B

The Seidel energy of a graph G is defined as

$$E_S(G) = \sum_{i=1}^{n} |\sigma_i|,$$

where $\sigma_1, \sigma_2, \ldots, \sigma_n$ are the Seidel eigenvalues.

For the Bishop graph on the 8×8 chessboard, the computed Seidel energy is

$$E_S(G_B) = 316.26725.$$

Hence, the Seidel energy of the Bishop graph equals 316.26725.

10 Conclusion

In this work, the spectral characteristics of the Bishop graph on the 8×8 chessboard were examined through its adjacency, Laplacian, and Seidel matrices. Each matrix representation reveals a distinct aspect of the underlying structure.

The adjacency spectrum reflects the inherent diagonal connectivity of the graph and exhibits eigenvalue multiplicities that arise from the symmetry of the chessboard. The Laplacian spectrum encodes the variation in vertex degrees across the board and confirms the presence of two connected components, corresponding to the partition of squares by colour. Furthermore, the computed Laplacian energy quantifies the overall deviation of the spectrum from the average degree.

The Seidel matrix, constructed from both adjacency and non-adjacency relations, provides a complementary perspective. Its spectrum emphasizes the contrast between connected and non-connected vertex pairs, leading to a comparatively larger energy value. This highlights the sensitivity of the Seidel framework to global structural balance.

Collectively, these spectral investigations demonstrate that the Bishop graph possesses a rich algebraic structure governed by geometric symmetry. The results contribute to the spectral analysis of chessboard graphs and may serve as a foundation for further studies on generalized $n \times n$ configurations.

References

[1] Balakrishnan R. The energy of a graph. Linear Algebra and its Applications. 2004;387:287–295.

[2] Banerjee A. On the spectrum of hypergraphs. arXiv preprint arXiv:1711.09356; 2017.

[3] Bretto A. Introduction to hypergraph theory and its use in engineering and image processing. Advances in Imaging and Electron Physics. 2004;131:3–64.

[4] Cai, D., Song, M., Sun, C., Zhang, B., Hong, S., and Li, H. Hypergraph structure learning for hypergraph neural networks. In *Proceedings of the 31st International Joint Conference on Artificial Intelligence (IJCAI 2022)*, pp. 1923–1929, 2022.

[5] Cardoso K, Hoppen C, Trevisan V. The spectrum of a class of uniform hypergraphs. Linear Algebra and its Applications. 2020;590:243–257.

[6] Chan, T. H. H., Louis, A., Tang, Z. G., and Zhang, C. Spectral properties of hypergraph Laplacian and approximation algorithms. *Journal of the ACM*, 65(3):1–48, 2018.

[7] Dai Q, Gao Y. Hypergraph computation. Springer Nature; 2023.

[8] Elkies ND, Stanley RP. The mathematical knight. The Mathematical Intelligencer. 2003;25(1):22–34.

[9] Feng Y, You H, Zhang Z, Ji R, Gao Y. Hypergraph neural networks. Proceedings of the AAAI Conference on Artificial Intelligence. 2019;33:3558–3565.

[10] Gao Y, Ji S, Han X, Dai Q. Hypergraph computation. Engineering. 2024;40:188–201.

[11] Gao, Y., Zhang, Z., Lin, H., Zhao, X., Du, S., and Zou, C. Hypergraph learning: Methods and practices. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(5):2548–2566, 2020.

[12] Gutman I. The energy of a graph: Old and new results. In: Algebraic Combinatorics and Applications. Springer; 2001. p.196–211.

[13] Gutman I, Zhou B. Laplacian energy of a graph. Linear Algebra and its Applications. 2006;414(1):29–37.

[14] Jakkewad, S. G., Metkar, R. G., Dhanorkar, G. A., & Tekalkar, P. N. (2025). A generalized study of zero divisor graphs of Boolean rings $\mathbb{Z}_2^n$. *Communications on Applied Nonlinear Analysis*, 32(7S).

[15] Jakkewad, S. G., Metkar, R. G., Nalawade, N. B., Murugan, V., & Toker, K. (2025). Structural properties of zero divisor graphs of $\mathbb{Z}_r \times \mathbb{Z}_s$. *Panamerican Mathematical Journal*, 35(2S).

[16] Jakkewad, S. G., Metkar, R. G., Murugan, V., & Tekalkar, P. (2025). Spectral analysis of the adjacency matrix and Hamiltonian properties of zero-divisor graphs. *Utilitas Mathematica*, 122(1), 195–206.

[17] Kumar JS, Archana B, Muralidharan K, Srija R. Spectral graph theory: Eigenvalues, Laplacians and graph connectivity. Metallurgical and Materials Engineering. 2025;31(3):78–84.

[18] Kumar KR, Varghese RP. Spectrum of (k, r)-regular hypergraphs. International Journal of Mathematical Combinatorics. 2017;2:52–59.

[19] Lee, G., Bu, F., Eliassi-Rad, T., and Shin, K. A survey on hypergraph mining: Patterns, tools, and generators. *ACM Computing Surveys*, 57(8):1–36, 2025.

[20] Nalawade, N. B., Bapat, M. S., Jakkewad, S. G., Dhanorkar, G. A., & Bhosale, D. J. (2025). Structural properties of zero-divisor hypergraph and superhypergraph over $\mathbb{Z}_n$. *Panamerican Mathematical Journal*, 35(4S), 485.

[21] Nikiforov V. The energy of graphs and matrices. Journal of Mathematical Analysis and Applications. 2007;326(2):1472–1475.

[22] Ouvrard X. Hypergraphs: An introduction and review. arXiv preprint arXiv:2002.05014; 2020.

[23] Parberry I. An efficient algorithm for the knight's tour problem. Discrete Applied Mathematics. 1997;73(3):251–260.

[24] Santhosh Kumar N, Suma P, Jasmine Mathew. A study of the energy and spectral characteristics of the knight's hypergraph. Asian Research Journal of Mathematics. 2026;22(2):20–31.

[25] Schlag, S., Heuer, T., Gottesbüren, L., Akhremtsev, Y., Schulz, C., and Sanders, P. High-quality hypergraph partitioning. *ACM Journal of Experimental Algorithmics*, 27:1–39, 2023.

[26] Shetty, S. S., and Bhat, K. A. Spectral theory of hypergraphs: A survey. *arXiv preprint arXiv:2507.13664*, 2025.