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Spectral and Energy Analysis of the Rook Hypergraph Derived from the 8×8 Chessboard

Original Research Article

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Abstract

Graph and hypergraph models derived from chessboard movements provide an effective framework for studying structural and spectral properties of discrete mathematical systems. In this work, we investigate the Rook hypergraph associated with the standard 8×8 chessboard. In this construction, each square of the chessboard is considered as a vertex, while hyperedges are formed by combining all vertices lying in the same row and column as a given square, reflecting the legal movement of a Rook. We develop the adjacency, Laplacian, and Seidel matrix representations corresponding to this hypergraph and examine their spectral characteristics. The eigenvalues and their multiplicities are obtained through numerical computation using Python. Based on these spectra, the adjacency energy, Laplacian energy, and Seidel energy of the Rook hypergraph are determined. The analysis shows that the structure is regular of degree 14 and highly symmetric due to the row–column configuration of the chessboard. In particular, the adjacency and Laplacian energies are both equal to 196, while the Seidel energy is 364. These results illustrate how chessboard-based constructions yield structured spectral behavior and provide a useful model for studying grid-based networks and combinatorial structures.

Keywords: Rook Graph; Rook Hypergraph; Spectral Graph Theory; Adjacency Energy; Laplacian Energy; Seidel Energy; Chessboard Graphs.

2010 Mathematics Subject Classification: 05C65.

1 Introduction

In recent years, graph theory and hypergraph theory have attracted considerable attention from researchers due to their wide range of applications in mathematics, computer science, and related disciplines. These structures serve as powerful mathematical models for representing relationships and interactions among objects in complex systems. In particular, the study of spectral properties of graphs and hypergraphs has become an important area of investigation in modern mathematical research. Spectral graph theory focuses on the analysis of eigenvalues and eigenvectors of matrices associated with graphs, such as adjacency, Laplacian, and Seidel matrices, which reveal importa structural characteristics of the underlying graphs. These spectral parameters often provide useful insights into connectivity, symmetry, and other structural properties of graphs and hypergraphs (14; 15; 16; 17; 15; 22). Another important concept arising from spectral graph theory is graph energy, which is defined in terms of the eigenvalues of matrices associated with graphs. The notion of adjacency energy, Laplacian energy, and Seidel energy has received considerable attention in recent years due to their significance in understanding the structural behavior of graphs(12; 13; 2; 5; 23; 16; 17). These energy measures provide valuable information about the global properties of graphs and have been widely studied in various classes of graphs and graph-related structures. Hypergraphs consist of collections of finite sets and form one of the most general structures studied in discrete mathematics. Although the concept existed earlier, hypergraph theory developed as an independent discipline during the 1960s, as documented in (3; 24). Since then, the theory has expanded significantly, driven by its wide range of applications in computer science, combinatorics, and network modeling (2; 5; 7; 10).

Graphs and hypergraphs constructed from chessboard movements constitute an important class of combinatorial models with both theoretical and practical relevance. Classical problems such as the Knight's tour have been studied extensively, leading to efficient algorithms and deeper mathematical understanding (8; 25; 26). These investigations have motivated the study of chessboard-based graphs, in which the squares of a chessboard are treated as vertices and adjacency is determined by the legal moves of a specific chess piece. Within this framework, the Rook graph is obtained by considering the squares of a chessboard as vertices, where two vertices are adjacent if a Rook can move between them in a single move along the same row or column. Owing to the horizontal and vertical movement of the Rook, the resulting graph exhibits a highly regular structure characterized by row–column connectivity. A natural extension of this construction leads to the Rook hypergraph, in which a hyperedge consists of a vertex together with all vertices that are reachable from it by a single Rook move. Such constructions are closely related to classes of uniform and regular hypergraphs studied in (5; 20). In this paper, we construct the 64×64 adjacency matrix of the Rook hypergraph corresponding to an 8×8 chessboard and examine its associated Laplacian and Seidel matrices. The eigenvalues of these matrices and their corresponding energies are computed numerically using Python programming. The study of graph energies follows established developments in spectral graph theory (1; 12; 13; 23; 19), and is further motivated by recent investigations on the spectral properties of chessboard-based graphs and hypergraphs (26). In contemporary applications, efficient routing and movement strategies play a central role in areas such as automation, robotics, and intelligent logistics systems. Movement patterns analogous to those of a Rook on a chessboard naturally arise in grid-based layouts, where horizontal and vertical motions dominate. The spectral analysis of the Rook hypergraph presented in this work provides a mathematical framework that can be adapted to such routing and optimization problems.

2 Preliminaries

This section presents the fundamental definitions and matrix representations associated with the Rook graph defined on an $n \times n$ chessboard. These concepts form the basis for the spectral analysis carried out in the subsequent sections.

2.1 Rook Graph

Let n be a positive integer. The *Rook graph*, denoted by G_R, is a simple undirected graph derived from an $n \times n$ chessboard. The vertex set of G_R is given by

$$V(G_R) = \{(i, j) \mid 1 \leq i, j \leq n\},$$

where each ordered pair (i, j) corresponds to a square of the chessboard. Two distinct vertices (i, j) and (k, ℓ) are adjacent in G_R if and only if a Rook can move from one square to the other in a single move. Equivalently, adjacency is defined by

$$i = k \quad \text{or} \quad j = \ell.$$

Thus, edges connect pairs of squares lying in the same row or the same column. The order of the Rook graph R_n is n^2. For a vertex $(i, j) \in V(R_n)$, the degree, denoted by $d(i, j)$, is equal to the total number of squares in the same row and column as (i, j), excluding the vertex itself.

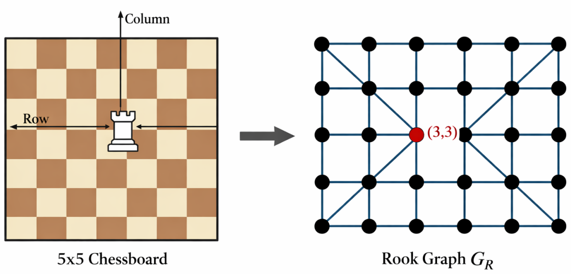

Figure 1: Chessboard to Rook Graph

2.2 Rook hypergraph

The *Rook hypergraph* associated with an 8×8 chessboard is the hypergraph $H_R = (V, E)$, where V is the set of all board squares and E consists of hyperedges corresponding to each row and each column of the board, with each hyperedge containing all vertices lying in that row or column.

2.3 Adjacency Matrix

The adjacency matrix of the Rook graph G_R is defined as

$$A(G_R) = [a_{uv}],$$

where $u = (i, j)$ and $v = (k, \ell)$ are vertices of G_R, and

$$a_{uv} = \begin{cases} 1, & \text{if } u \neq v \text{ and } (i = k \text{ or } j = \ell), \\ 0, & \text{otherwise.} \end{cases}$$

Since $A(G_R)$ is a real symmetric matrix, all of its eigenvalues are real. The collection of these eigenvalues together with their multiplicities is referred to as the *adjacency spectrum* of the Rook graph.

2.4 Laplacian Matrix

For a vertex $u \in V(G_R)$, the Laplacian degree is defined by

$$\delta_l(u) = \sum_{v \in V(G_R)} a_{uv}.$$

The Laplacian matrix of the Rook graph, denoted by $L(G_R)$, is given by

$$L(G_R) = D(G_R) - A(G_R),$$

where

$$D(G_R) = \mathrm{diag}(\delta_l(v_1), \delta_l(v_2), \dots, \delta_l(v_{n^2}))$$

is the diagonal matrix of Laplacian degrees. The matrix $L(G_R)$ is real and symmetric, and therefore all its eigenvalues are real and non-negative. Moreover, zero is always an eigenvalue of $L(G_R)$. The set of Laplacian eigenvalues, together with their multiplicities, is known as the *Laplacian spectrum* of the Rook graph.

2.5 Seidel Matrix

The Seidel matrix of the Rook graph G_R is defined as

$$S(G_R) = [s_{uv}],$$

where for vertices $u = (i, j)$ and $v = (k, \ell)$,

$$s_{uv} = \begin{cases} 0, & \text{if } u = v, \\ -1, & \text{if } u \neq v \text{ and } (i = k \text{ or } j = \ell), \\ 1, & \text{if } u \neq v \text{ and } (i \neq k \text{ and } j \neq \ell). \end{cases}$$

Hence, adjacent vertices are assigned the value -1, non-adjacent distinct vertices are assigned the value 1, and diagonal entries are zero. Since $S(G_R)$ is a real symmetric matrix, all its eigenvalues are real. The eigenvalues of $S(G_R)$ together with their multiplicities form the *Seidel spectrum* of the Rook graph.

3 Main Results

3.1 The Rook Hypergraph H_R

This section presents the hypergraph structure that arises naturally from the movement of a Rook on the standard 8×8 chessboard. Throughout this section, we employ the classical algebraic notation of chess to identify the squares of the board.

3.1.1 Notation and Vertex Labeling

The chessboard is composed of eight vertical files denoted by

$$a, b, c, d, e, f, g, h,$$

and eight horizontal ranks labeled by the integers

$$1, 2, \dots, 8.$$

Each square is uniquely identified by a symbol of the form x_i, where $x \in \{a, b, c, d, e, f, g, h\}$ and $1 \leq i \leq 8$.

Accordingly, the vertex set of the Rook hypergraph is defined as

$$V = \{x_i \mid x \in \{a, \ldots, h\},\ 1 \leq i \leq 8\}.$$

For instance, the vertex h_8 represents the square located at the upper-right corner of the chessboard.

In the Rook hypergraph, hyperedges correspond to the rows and columns of the board. Two distinct vertices x_i and y_j are incident with a common hyperedge if and only if they lie in the same file or the same rank, that is,

$$x = y \quad \text{or} \quad i = j.$$

This incidence rule faithfully reflects the horizontal and vertical movement of a Rook.

3.1.2 Local Structure: Interior Vertices

We begin by examining the incidence behavior of vertices corresponding to interior squares of the board. Consider the square a_1. The vertices incident with a_1 through row and column hyperedges consist of all squares lying in the same file or the same rank, namely

$$\{a_2, a_3, a_4, a_5, a_6, a_7, a_8,\ b_1, c_1, d_1, e_1, f_1, g_1, h_1\}.$$

Thus, the vertex a_1 is incident with fourteen distinct vertices in the associated adjacency structure. Since a_1 lies strictly within the interior of the board, both its row and column extend fully across the chessboard. Consequently, the degree of a_1 in the adjacency representation of the Rook hypergraph is equal to 14.

An identical argument applies to the symmetric interior square e_5. The set of vertices incident with e_5 is given by

$$\{e_1, e_2, e_3, e_4, e_6, e_7, e_8,\ a_5, b_5, c_5, d_5, f_5, g_5, h_5\},$$

which again yields a degree of 14.

Hence, every interior square of the chessboard possesses the same degree in the Rook hypergraph, reflecting the uniform row–column incidence pattern induced by Rook movement.

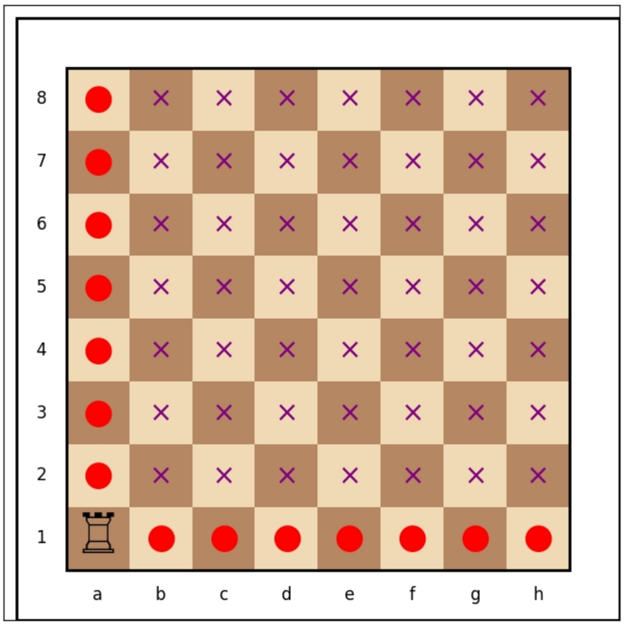

Figure 2: Rook Reachability Structure

We now formalize the Rook movement structure within a hypergraph framework that captures its full row and column reach.

The *Rook hypergraph* is defined as

$$H_R = (V, E),$$

where V is the set of all 64 squares of the 8×8 chessboard, and each hyperedge corresponds to the complete row and column passing through a fixed vertex.

For a given vertex $x_i \in V$, the associated hyperedge is defined by

$$E_{x_i} = \{x_i\} \cup \{y_j \in V \mid y = x \text{ or } j = i\}.$$

Thus, each hyperedge consists of the chosen square together with all squares lying in the same file or the same rank.

Accordingly, the hyperedge set is

$$E = \{E_{x_i} \mid x_i \in V\}.$$

3.1.3 Explicit Description of Hyperedges

The form of the hyperedges depends on the location of the corresponding square on the chessboard. For each $i = 1, 2, \ldots, 8$, the hyperedges may be described explicitly.

For the a-file, the hyperedge associated with the vertex a_i is

$$E_{a_i} = \{a_i,\ a_1, a_2, \ldots, a_8,\ b_i, c_i, d_i, e_i, f_i, g_i, h_i\}.$$

Similarly, for the b-file, the hyperedge corresponding to b_i is

$$E_{b_i} = \{b_i,\ b_1, b_2, \ldots, b_8,\ a_i, c_i, d_i, e_i, f_i, g_i, h_i\}.$$

In the same manner, for the files $c, d, e, f, g,$ and $h,$ each hyperedge is obtained by combining all squares in the same file and all squares in the same rank as the given vertex, subject to the boundaries of the board.

This construction provides a complete and systematic description of the Rook hypergraph on the standard 8×8 chessboard.

3.1.4 Structural Observations

In graph-theoretic terms, the *rook graph* on an 8×8 chessboard has 64 vertices (the squares) and an ordinary edge between any two squares in the same row or column. Each vertex has degree 14 (a rook moves to 7 other squares in its row and 7 in its column), and edges are simply 2-element subsets of the vertices. In contrast, the *rook hypergraph* on the same board also has 64 vertices but uses hyperedges instead: there are 64 hyperedges (one for each square), and the hyperedge for square x is the union of x's entire row and entire column. This union contains $8 + 8 - 1 = 15$ squares, so each hyperedge has size 15 (the hypergraph is 15-uniform) and each vertex lies in 15 hyperedges (its own plus those of the other 14 squares in its row or column). In summary, the rook graph records pairwise adjacency of squares (14 neighbors per vertex) while the rook hypergraph records full rank/file incidence (each hyperedge is a complete row and column). Both structures admit the same symmetries (permuting rows or columns and transposing the board), but the rook graph's edges are 2-element links whereas the rook hypergraph's edges are 15-element sets spanning an entire row and column. This hypergraph representation offers a natural setting for the investigation of combinatorial and spectral properties associated with Rook movement, and it serves as the foundation for the analysis developed in the subsequent sections.

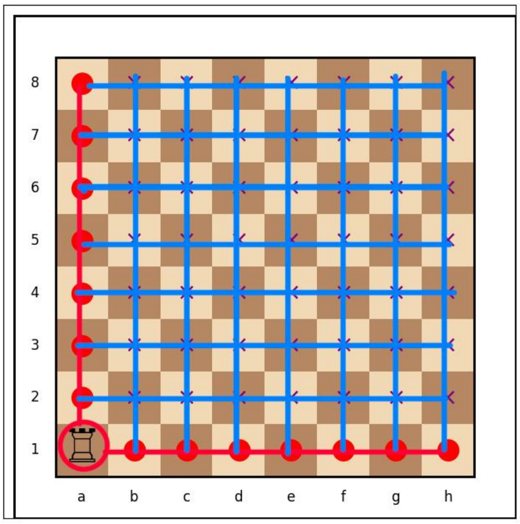

Figure 3: Adjacency Graph of Rook Moves on the 8×8 Chessboard

3.1.5 Uniform Structure of the Rook Hypergraph

A hypergraph is called k-uniform if every hyperedge contains the same number of vertices. In the Rook hypergraph, each hyperedge corresponds to the movement of a rook from a particular square of the chessboard. From any square, a rook can move along its entire row and column. There are 8

squares in the same column and 8 squares in the same row as the chosen square. Since the square itself belongs to both the row and the column, it would be counted twice if we simply added these numbers. Removing this duplication gives

$$8 + 8 - 1 = 15.$$

Therefore, every hyperedge of the Rook hypergraph contains exactly 15 vertices. Hence the Rook hypergraph is 15-uniform.

3.1.6 Hyperedge Incidence of Each Vertex

Let x_i be any vertex of the hypergraph. A hyperedge contains x_i whenever it is generated by a square lying in the same row or the same column as x_i. There are 8 vertices in the same column as x_i and 8 vertices in the same row. Since the vertex x_i belongs to both sets, it is counted twice when these two groups are combined. After removing the repeated count, the number of distinct vertices that generate hyperedges containing x_i is

$$8 + 8 - 1 = 15.$$

Hence each vertex appears in exactly 15 hyperedges.

3.1.7 Total Incidence in the Hypergraph

The chessboard contains 64 squares, and therefore the Rook hypergraph has 64 vertices. In the construction of the hypergraph, one hyperedge is associated with each vertex, so the number of hyperedges is also 64. From the previous theorem, each hyperedge contains 15 vertices. Therefore, the total number of incidences between vertices and hyperedges is obtained by multiplying the number of hyperedges by the number of vertices in each hyperedge:

$$64 \times 15 = 960.$$

Thus the Rook hypergraph contains 960 vertex–hyperedge incidences.

3.2 The Adjacency Matrix of the Rook Hypergraph H_R

In the Rook hypergraph associated with the standard 8×8 chessboard, two vertices are defined to be adjacent if they belong to at least one common hyperedge. Since each hyperedge consists of all squares lying in the same row or the same column, two distinct vertices are adjacent precisely when they share a common row or a common column.

Accordingly, for vertices $u = (i, j)$ and $v = (k, \ell)$, adjacency is given by

$$u \sim v \iff i = k \text{ or } j = \ell.$$

As any two distinct squares can share at most one row or one column, each adjacent pair contributes exactly one unit to the corresponding entry of the adjacency matrix.

Consequently, the adjacency matrix of the Rook hypergraph (which coincides with the adjacency matrix of the Rook graph H_R) is a real symmetric matrix of order 64×64 with all diagonal entries equal to zero.

To illustrate this structure, consider the vertex e_5. The set of squares adjacent to e_5 consists of all squares in the same row and the same column, namely

$$\{e_1, e_2, e_3, e_4, e_6, e_7, e_8\}$$

and

$$\{a_5, b_5, c_5, d_5, f_5, g_5, h_5\}.$$

Hence, the degree of the vertex e_5 in the Rook hypergraph is

$$\deg(e_5) = 14,$$

and all remaining vertices of the chessboard have zero adjacency with e_5.

3.2.1 The Adjacency Matrix of the Rook Hypergraph H_R

Let $A_d = A(H_R)$ denote the adjacency matrix of the Rook hypergraph on the 8×8 chessboard. We decompose A_d into 8×8 block matrices according to the eight files of the chessboard, namely A, B, C, D, E, F, G, and H.

Thus, the matrix A can be written in block form as

$$A_d = \begin{pmatrix} 0 & I & I & I & I & I & I & I \\ I & 0 & I & I & I & I & I & I \\ I & I & 0 & I & I & I & I & I \\ I & I & I & 0 & I & I & I & I \\ I & I & I & I & 0 & I & I & I \\ I & I & I & I & I & 0 & I & I \\ I & I & I & I & I & I & 0 & I \\ I & I & I & I & I & I & I & 0 \end{pmatrix},$$

Let X and Y be two files, and define

$$d = |X - Y|.$$

Then the entries of the block matrix $[XY]$ are given by

$$[XY]_{ij} = \begin{cases} 1, & \text{if } i = j \text{ and } d \neq 0, \\ 0, & \text{otherwise.} \end{cases}$$

This reflects the fact that vertices lying in different files are adjacent only when they share the same rank, while vertices within the same file are never adjacent through column-based connections.

Consequently, the adjacency matrix of the Rook graph is fully described by a finite collection of block matrices determined by the relative positions of the files.

3.2.2 Explicit Block Types

The block structure of the adjacency matrix of the Rook graph is significantly simpler than that of diagonal-move graphs. Adjacency between vertices in different files occurs only when they lie in the same rank. As a consequence, all non-diagonal blocks share an identical structure.

Block Type $d = 0$

When $X = Y$, the block $[XX]$ corresponds to vertices lying in the same file. Since a Rook cannot move between two distinct squares in the same file without changing ranks, there is no adjacency within a file. Hence, the diagonal blocks are zero matrices of order 8:

$$B_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Block Type $d \neq 0$

For two distinct files $X \neq Y$, the block $[XY]$ represents adjacency between squares lying in different files. In this case, a Rook can move between two squares if and only if they share the same rank. Therefore, each such block is an identity matrix of order 8:

$$B_1 = I = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Thus, the adjacency matrix of the Rook hypergraph consists of zero diagonal blocks and identical identity matrices in all off-diagonal positions. This block uniformity reflects the regular row–column connectivity induced by Rook movements on the chessboard.

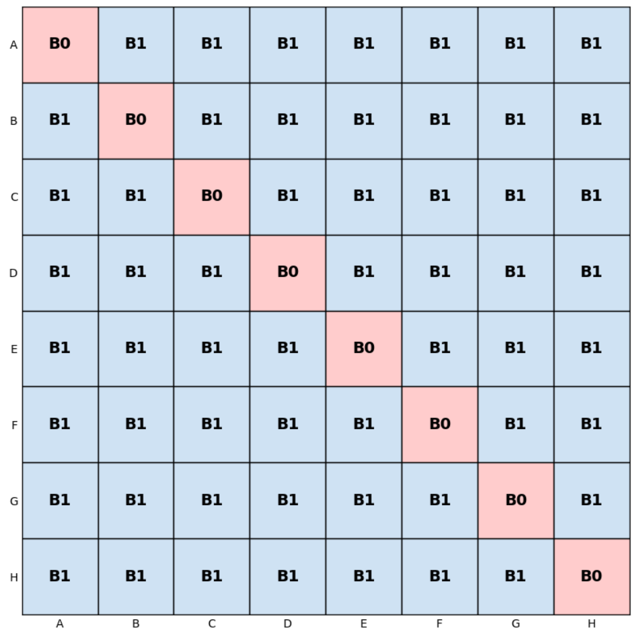

Figure 4: Adjacency Matrix Representation of block type

3.2.3 Symmetry Properties

The adjacency matrix A_d of the Rook hypergraph is symmetric, since the hypergraph is undirected. In block form, all diagonal blocks are zero matrices, while every off-diagonal block is an identity matrix of order 8, reflecting row-based adjacency. Each row of $A(H_R)$ has 14 nonzero entries, confirming that the Rook graph is regular.

3.3 The Adjacency Energy of Rook Hypergraph H_R

The adjacency matrix $A(H_R)$ of the Rook hypergraph on the 8×8 chessboard is a real symmetric matrix. Consequently, all eigenvalues of $A(H_R)$ are real.

Using numerical computation, the eigenvalues of the adjacency matrix were obtained along with their multiplicities. The resulting adjacency spectrum of the Rook hypergraph is presented in Table 1.

Table 1: Adjacency Eigenvalues of the Rook hypergraph H_R

Eigenvalue	Multiplicity
14	1
6	14
-2	49

The energy of the Bishop's graph is defined by

$$E(A_d) = \sum_{i=1}^{64} |\lambda_i|.$$

Using the computed spectrum,

$$E(A_d) = 196.$$

This quantity was evaluated numerically using Python.

3.4 The Laplacian Matrix of the Rook Hypergraph H_R

Let H_R denote the Rook hypergraph associated with the standard 8×8 chessboard. The vertex set consists of the 64 squares of the board. For convenience, the vertices are indexed row-wise as

$$v_k = (i, j), \qquad k = 8(i-1) + j,$$

where $i, j \in \{1, 2, \ldots, 8\}$. In the Rook hypergraph, two vertices are adjacent if they belong to at least one common hyperedge. Since each hyperedge consists of all squares lying in the same row or the same column, adjacency occurs precisely when two squares share a common row or a common column.

For a square (i, j), the number of adjacent vertices is determined by the remaining squares in its row and column. Hence, the degree of the vertex (i, j) is given by

$$\delta(i, j) = (8-1) + (8-1) = 14,$$

independent of the position of the square on the board. Thus, the degree distribution of the Rook hypergraph is uniform and may be represented by the constant matrix

$$\begin{pmatrix} 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 & 14 & 14 & 14 & 14 \end{pmatrix}.$$

Let $A(H_R)$ denote the adjacency matrix of the Rook hypergraph and let

$$D(H_R) = \operatorname{diag}(\delta(v_1), \delta(v_2), \dots, \delta(v_{64}))$$

be the corresponding diagonal degree matrix.

The Laplacian matrix of the Rook hypergraph is then defined as

$$L(H_R) = D(H_R) - A(H_R).$$

Accordingly, for $1 \le p, q \le 64$, the entries of $L(H_R)$ are given by

$$\ell_{pq} = \begin{cases} \delta(v_p), & \text{if } p = q, \\ -1, & \text{if } v_p \text{ and } v_q \text{ are adjacent}, \\ 0, & \text{otherwise}. \end{cases}$$

Hence, for $1 \le p, q \le 64$, the entries of the Laplacian matrix of the Rook hypergraph H_R are given by

$$L_{pq} = \begin{cases} \delta(v_p), & \text{if } p = q, \\ -1, & \text{if } v_p \text{ and } v_q \text{ lie in the same row or the same column}, \\ 0, & \text{otherwise}. \end{cases}$$

Thus, the Laplacian matrix $L(H_R)$ is a real symmetric matrix of order 64×64 whose diagonal entries are equal to the vertex degrees, while the off-diagonal entries are equal to -1 whenever two vertices are adjacent through a common row or column. A schematic form of the Laplacian matrix is illustrated below:

$$L(H_R) = \begin{bmatrix} 14 & -1 & -1 & \cdots & -1 & 0 & \cdots & 0 \\ -1 & 14 & 0 & \cdots & 0 & -1 & \cdots & 0 \\ -1 & 0 & 14 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & & \ddots & & & & \vdots \\ -1 & 0 & 0 & \cdots & 14 & 0 & \cdots & -1 \\ 0 & -1 & 0 & \cdots & 0 & 14 & \cdots & -1 \\ \vdots & & & & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & -1 & \cdots & 14 \end{bmatrix}_{64\times 64}.$$

Since any square on the chessboard can be reached from any other square using a sequence of Rook moves, the Rook hypergraph is connected. Consequently, the Laplacian matrix of the Rook hypergraph has exactly one zero eigenvalue.

3.5 The Laplacian Energy of Rook Hypergraph H_R

The Laplacian spectrum of the Rook hypergraph H_R consists of 64 real eigenvalues. Since the Rook hypergraph is connected, the Laplacian matrix has exactly one zero eigenvalue. Owing to the regular row–column structure of the chessboard, the remaining eigenvalues appear with multiplicities determined by the symmetry of the graph.

The Laplacian eigenvalues of the Rook hypergraph are listed in Table 2.

Table 2: Laplacian Eigenvalues of the Rook Hypergraph

Eigenvalue	Multiplicity
0	1
8	14
16	49

For a graph or hypergraph G with n vertices and m edges, the Laplacian energy is defined as

$$LE(G) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right|,$$

where $\mu_1, \mu_2, \ldots, \mu_n$ denote the Laplacian eigenvalues. For the Rook hypergraph on the 8×8 chessboard, we have

$$n = 64, \qquad m = 448, \qquad \frac{2m}{n} = 14.$$

Using the Laplacian spectrum given above, the Laplacian energy is computed as

$$LE(H_R) = \sum_{i=1}^{64} |\mu_i - 14| .$$

Hence, the Laplacian energy of the Rook hypergraph is

$$LE(H_R) = 196.$$

3.6 The Seidel Matrix of the Rook Hypergraph H_R

Let H_R denote the Rook hypergraph associated with the standard 8×8 chessboard, with vertex set

$$V(H_R) = \{v_1, v_2, \ldots, v_{64}\}.$$

Let $A(H_R)$ be the adjacency matrix of H_R. For a simple graph or hypergraph G of order n, the Seidel matrix is defined by

$$S(G) = J - I - 2A(G),$$

where J denotes the all–ones matrix and I is the identity matrix of order n. In the case of the Rook hypergraph ($n = 64$), this gives

$$S(H_R) = J_{64} - I_{64} - 2A(H_R).$$

Equivalently, the entries of the Seidel matrix $S(H_R) = [s_{ij}]$ are given by

$$s_{ij} = \begin{cases} 0, & i = j, \\ -1, & \text{if } v_i \text{ and } v_j \text{ are adjacent}, \\ 1, & \text{otherwise}, \end{cases} \qquad 1 \leq i, j \leq 64.$$

Thus, $S(H_R)$ is a real symmetric matrix of order 64×64 with zero diagonal entries. The off-diagonal entries are equal to -1 whenever two vertices lie in the same row or the same column of the chessboard, and take the value 1 otherwise. This representation captures the contrast between adjacency and non-adjacency induced by Rook movements.

A schematic form of the Seidel matrix is shown below:

$$S(H_R) = \begin{bmatrix} 0 & -1 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ -1 & 0 & 1 & \cdots & 1 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & -1 & 1 & \cdots & 1 \\ \vdots & & & \ddots & & & & \vdots \\ 1 & 1 & -1 & \cdots & 0 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 & \cdots & -1 \\ \vdots & & & & & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & -1 & \cdots & 0 \end{bmatrix}_{64\times 64}.$$

3.7 The Seidel Energy of Rook Hypergraph H_R

Since the Seidel matrix $S(H_R)$ of the Rook hypergraph is real and symmetric, all its eigenvalues are real. The Seidel spectrum consists of 64 eigenvalues, whose multiplicities reflect the high degree of symmetry inherent in the 8×8 chessboard and the uniform row–column adjacency induced by Rook moves.

Using the relation $S = J - I - 2A$, the Seidel eigenvalues of the Rook hypergraph are obtained from the adjacency spectrum. The resulting eigenvalues and their multiplicities are presented in Table 3.

Table 3: Seidel Eigenvalues of the Rook Hypergraph

Eigenvalue	Multiplicity
35	1
-13	14
3	49

The large multiplicities arise from the regular structure of the Rook hypergraph and the symmetry of the chessboard. The Seidel energy of a graph or hypergraph H_R is defined as

$$E_S(H_R) = \sum_{i=1}^{n} |\sigma_i|,$$

where $\sigma_1, \sigma_2, \ldots, \sigma_n$ denote the Seidel eigenvalues.

For the Rook hypergraph on the 8×8 chessboard, the Seidel energy is therefore

$$E_S(H_R) = |35| + 14|-13| + 49|3|.$$

Hence, the Seidel energy of the Rook hypergraph is

$$E_S(H_R) = 364.$$

4 Conclusion

In this study, we examined the Rook hypergraph constructed from the standard 8×8 chessboard by treating each square as a vertex and forming hyperedges from complete rows and columns. This approach naturally reflects the horizontal and vertical movement of a Rook and results in a highly symmetric and uniformly structured hypergraph.

We developed the adjacency, Laplacian, and Seidel matrix representations of this hypergraph and analyzed their spectral properties. The results show that the structure is regular of degree 14 and connected. The eigenvalue distributions obtained for all three matrices clearly demonstrate how the row–column arrangement of the chessboard governs the algebraic behavior of the model. The calculated adjacency energy and Laplacian energy are both 196, while the Seidel energy is 364, highlighting consistent structural symmetry across different matrix representations.

Overall, this work illustrates how a simple movement rule on a grid can lead to rich and well-organized spectral properties. The Rook hypergraph provides a clear mathematical framework for studying grid-based connectivity patterns and can serve as a reference model for related chessboard graphs, larger boards, or other piece-movement hypergraphs. Future research may extend these ideas to generalized $n \times n$ boards, weighted structures, or applications in routing, network optimization, and combinatorial design.

References

[1] Balakrishnan R. The energy of a graph. Linear Algebra and its Applications. 2004;387:287–295.

[2] Banerjee A. On the spectrum of hypergraphs. arXiv preprint arXiv:1711.09356; 2017.

[3] Bretto A. Introduction to hypergraph theory and its use in engineering and image processing. Advances in Imaging and Electron Physics. 2004;131:3–64.

[4] Cai, D., Song, M., Sun, C., Zhang, B., Hong, S., and Li, H. Hypergraph structure learning for hypergraph neural networks. In *Proceedings of the 31st International Joint Conference on Artificial Intelligence (IJCAI 2022)*, pp. 1923–1929, 2022.

[5] Cardoso K, Hoppen C, Trevisan V. The spectrum of a class of uniform hypergraphs. Linear Algebra and its Applications. 2020;590:243–257.

[6] Chan, T. H. H., Louis, A., Tang, Z. G., and Zhang, C. Spectral properties of hypergraph Laplacian and approximation algorithms. *Journal of the ACM*, 65(3):1–48, 2018.

[7] Dai Q, Gao Y. Hypergraph computation. Springer Nature; 2023.

[8] Elkies ND, Stanley RP. The mathematical knight. The Mathematical Intelligencer. 2003;25(1):22–34.

[9] Feng Y, You H, Zhang Z, Ji R, Gao Y. Hypergraph neural networks. Proceedings of the AAAI Conference on Artificial Intelligence. 2019;33:3558–3565.

[10] Gao Y, Ji S, Han X, Dai Q. Hypergraph computation. Engineering. 2024;40:188–201.

[11] Gao, Y., Zhang, Z., Lin, H., Zhao, X., Du, S., and Zou, C. Hypergraph learning: Methods and practices. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 44(5):2548–2566, 2020.

[12] Gutman I. The energy of a graph: Old and new results. In: Algebraic Combinatorics and Applications. Springer; 2001. p.196–211.

[13] Gutman I, Zhou B. Laplacian energy of a graph. Linear Algebra and its Applications. 2006;414(1):29–37.

[14] Jakkewad, S. G., Metkar, R. G., Dhanorkar, G. A., & Tekalkar, P. N. (2025). A generalized study of zero divisor graphs of Boolean rings $\mathbb{Z}_2^n$. *Communications on Applied Nonlinear Analysis*, 32(7S).

[15] Jakkewad, S. G., Metkar, R. G., Nalawade, N. B., Murugan, V., & Toker, K. (2025). Structural properties of zero divisor graphs of $\mathbb{Z}_r \times \mathbb{Z}_s$. *Panamerican Mathematical Journal*, 35(2S).

[16] Jakkewad, S. G., Metkar, R. G., Murugan, V., & Tekalkar, P. (2025). Spectral analysis of the adjacency matrix and Hamiltonian properties of zero-divisor graphs. *Utilitas Mathematica*, 122(1), 195–206.

[17] Jakkewad SG, Metkar RG, Yadav YA. Spectral analysis of compressed zero divisor graphs over $\prod_{k=1}^{n} \mathbb{Z}_{p_k}$ for $2 \leq n \leq 5$, where each p_k is a prime. *Journal of Advances in Mathematics and Computer Science*. 2026;41(3):91–109. https://doi.org/10.9734/jamcs/2026/v41i32110.

[18] Jakkewad SG, Metkar RG. On the graph-theoretic properties of compressed zero divisor graphs of product rings over $\prod_{k=1}^{n} \mathbb{Z}_{p_k}$. *Asian Research Journal of Mathematics*. 2026;22(2):150–165. https://doi.org/10.9734/arjom/2026/v22i21051.

[19] Kumar JS, Archana B, Muralidharan K, Srija R. Spectral graph theory: Eigenvalues, Laplacians and graph connectivity. Metallurgical and Materials Engineering. 2025;31(3):78–84.

[20] Kumar KR, Varghese RP. Spectrum of (k, r)-regular hypergraphs. International Journal of Mathematical Combinatorics. 2017;2:52–59.

[21] Lee, G., Bu, F., Eliassi-Rad, T., and Shin, K. A survey on hypergraph mining: Patterns, tools, and generators. *ACM Computing Surveys*, 57(8):1–36, 2025.

[22] Nalawade, N. B., Bapat, M. S., Jakkewad, S. G., Dhanorkar, G. A., & Bhosale, D. J. (2025). Structural properties of zero-divisor hypergraph and superhypergraph over $\mathbb{Z}_n$. *Panamerican Mathematical Journal*, 35(4S), 485.

[23] Nikiforov V. The energy of graphs and matrices. Journal of Mathematical Analysis and Applications. 2007;326(2):1472–1475.

[24] Ouvrard X. Hypergraphs: An introduction and review. arXiv preprint arXiv:2002.05014; 2020.

[25] Parberry I. An efficient algorithm for the knight's tour problem. Discrete Applied Mathematics. 1997;73(3):251–260.

[26] Santhosh Kumar N, Suma P, Jasmine Mathew. A study of the energy and spectral characteristics of the knight's hypergraph. Asian Research Journal of Mathematics. 2026;22(2):20–31.

[27] Schlag, S., Heuer, T., Gottesbüren, L., Akhremtsev, Y., Schulz, C., and Sanders, P. High-quality hypergraph partitioning. *ACM Journal of Experimental Algorithmics*, 27:1–39, 2023.

[28] Shetty, S. S., and Bhat, K. A. Spectral theory of hypergraphs: A survey. *arXiv preprint arXiv:2507.13664*, 2025.