Original Research Article

Spectral and Energy Analysis of the Queen Hypergraph Derived from the 8×8 Chessboard

Abstract

Hypergraphs induced by chessboard movements offer a natural setting for examining structural and spectral features of discrete systems. In this paper, we study the Queen hypergraph associated with an 8×8 chessboard, where each square is regarded as a vertex and adjacency is defined by the legal moves of a queen in a single step. We construct the adjacency, Laplacian and seidel matrices of the Queen hypergraph and investigate their spectral behaviour. The eigenvalues of these matrices are obtained through numerical computation using Python, and the corresponding energy measures are evaluated. The results demonstrate how the combined row, column, and diagonal connectivity inherent to queen movements shapes the spectrum and energy of the hypergraph, and they provide a basis for comparison with other chessboard-based graph and hypergraph models.

Keywords: Queen Hypergraph;Adjacency Matrix; Laplacian Matrix; Seidel Matrix; Graph Energy

2010 Mathematics Subject Classification: 05C65

1 Introduction

Hypergraphs consist of families of finite sets and constitute one of the most general structures studied in discrete mathematics. Although the notion of hypergraphs appeared earlier, the subject gained independent recognition during the 1960s, as noted in (4; 25). Since then, hypergraph theory has expanded rapidly, driven by its wide-ranging applications in areas such as computer science, combinatorics, and network modelling (3; 6; 8; 13).

Graphs and hypergraphs generated from chessboard movements form an important class of combinatorial models with both theoretical and practical relevance. Well-known problems such as the Knight's tour have been studied extensively, leading to efficient algorithms and deeper mathematical understanding (9; 26; 27). These classical investigations naturally motivated the construction of chessboard-based graphs, where the squares of a chessboard are treated as vertices and adjacency is defined according to the legal moves of a given chess piece.

In this setting, the Queen hypergraph arises by considering the squares of a chessboard as vertices, with adjacency determined by the movement of a queen in a single move. Since a queen combines the movement patterns of both the rook and the bishop, the resulting structure exhibits row-wise, column-wise, and diagonal connectivity. A corresponding hypergraph model is obtained by associating each vertex with a hyperedge that consists of the vertex itself together with all vertices reachable from it in one queen move. Such constructions naturally fall within the class of regular and uniform hypergraphs studied in (6; 21). We construct the 64×64 adjacency matrix of the Queen hypergraph associated with the standard 8×8 chessboard and examine the corresponding Laplacian and Seidel matrices. The eigenvalues of these matrices and their associated energy measures are computed numerically using Python. Our analysis is guided by classical results in spectral graph theory and graph energy (2; 15; 16; 24; 20; 29), and is further motivated by recent studies on chessboard-based hypergraphs (27). From an applied perspective, efficient movement and routing strategies are central to modern applications such as robotics, automation, and intelligent logistics. The combined row, column, and diagonal movement patterns of the queen provide a natural abstraction for modelling routing and optimisation problems on grid-based systems. The spectral analysis of the Queen hypergraph developed in this work offers a mathematical framework that may be adapted to such applications.

2 Preliminaries

In this section, we introduce the fundamental definitions and matrix representations associated with the Queen graph on an $n \times n$ chessboard. These concepts will be used throughout the paper.

2.1 Queen Graph

Let n be a positive integer. The *Queen graph*, denoted by G_Q, is constructed from an $n \times n$ chessboard by taking each square as a vertex. The vertex set of G_Q is given by

$$V(G_Q) = \{(i, j) \mid 1 \leq i, j \leq n\},$$

where each ordered pair (i, j) represents a square of the chessboard.

Two distinct vertices (i, j) and (k, ℓ) are said to be adjacent if and only if a queen can move from one square to the other in a single move. Equivalently,

$$i = k \quad \text{or} \quad j = \ell \quad \text{or} \quad |i - k| = |j - \ell|.$$

Thus, adjacency arises from row, column, or diagonal alignment. The order of G_Q is n^2.

For a vertex $(i, j) \in V(G_Q)$, its degree, denoted by $d(i, j)$, equals the total number of squares lying in the same row, the same column, and the two diagonals passing through (i, j), excluding the vertex itself.

2.2 Adjacency Matrix

The adjacency matrix of the Queen graph G_Q is defined as

$$A(G_Q) = [a_{uv}],$$

where $u = (i, j)$ and $v = (k, \ell)$ are vertices of G_Q, and

$$a_{uv} = \begin{cases} 1, & \text{if } u \neq v \text{ and } (i = k \text{ or } j = \ell \text{ or } |i - k| = |j - \ell|), \\ 0, & \text{otherwise.} \end{cases}$$

Since $A(G_Q)$ is a real symmetric matrix, all its eigenvalues are real. The collection of eigenvalues of $A(G_Q)$ together with their multiplicities is referred to as the *adjacency spectrum* of G_Q.

2.3 Laplacian Matrix

For a vertex $u \in V(G_Q)$, the Laplacian degree is defined by

$$\delta_l(u) = \sum_{v \in V(G_Q)} a_{uv}.$$

The Laplacian matrix of G_Q, denoted by $L(G_Q)$, is defined as

$$L(G_Q) = D - A,$$

where $A = A(G_Q)$ and

$$D = \operatorname{diag}(\delta_l(v_1), \delta_l(v_2), \dots, \delta_l(v_{n^2}))$$

is the diagonal matrix of Laplacian degrees.

The matrix $L(G_Q)$ is real and symmetric; hence all its eigenvalues are real and non-negative. Moreover, 0 is always an eigenvalue of $L(G_Q)$. The eigenvalues together with their multiplicities constitute the *Laplacian spectrum* of G_Q.

2.4 Seidel Matrix

The Seidel matrix of the Queen graph G_Q is defined as

$$S(G_Q) = [s_{uv}],$$

where for vertices $u = (i, j)$ and $v = (k, \ell)$,

$$s_{uv} = \begin{cases} 0, & \text{if } u = v, \\ -1, & \text{if } u \neq v \text{ and } (i = k \text{ or } j = \ell \text{ or } |i-k| = |j-\ell|), \\ 1, & \text{if } u \neq v \text{ and no adjacency condition holds.} \end{cases}$$

Thus, adjacent vertices receive the value -1, non-adjacent distinct vertices receive 1, and diagonal entries are zero. Since $S(G_Q)$ is real and symmetric, all its eigenvalues are real. The set of eigenvalues of $S(G_Q)$ together with their multiplicities is called the *Seidel spectrum* of G_Q.

3 Main Results

3.1 The Queen Hypergraph H_Q

In this section, we describe the hypergraph structure induced by the movement of a queen on a standard 8×8 chessboard. Throughout this section, we use the conventional algebraic notation of chess to label the squares of the board.

3.1.1 Notation and Vertex Labeling

The chessboard is composed of eight vertical files labeled

$$A, B, C, D, E, F, G, H,$$

and eight horizontal ranks numbered

$$1, 2, 3, 4, 5, 6, 7, 8.$$

Each square is uniquely denoted by a symbol of the form x_i, where $x \in \{A, B, C, D, E, F, G, H\}$ and $1 \leq i \leq 8$.

Accordingly, the vertex set of the Queen hypergraph H_Q is

$$V = \{x_i \mid x \in \{A, \ldots, H\},\ 1 \leq i \leq 8\}.$$

For example, the square H_8 represents the upper-right corner of the chessboard.

Two distinct squares x_i and y_j are adjacent in the Queen hypergraph if and only if a queen can move from one square to the other in a single legal move. Equivalently,

$$x = y \quad \text{or} \quad i = j \quad \text{or} \quad |x - y| = |i - j|,$$

which corresponds to movement along a common file, rank, or diagonal.

3.1.2 Local Structure: Interior Vertices

We first analyze the reachability of interior squares under queen movement.

Consider a queen placed at the square D_5 on the 8×8 chessboard. The set of squares reachable in a single move consists of all squares lying in the same rank, the same file, and the two diagonals passing through D_5. Consequently,

$$\deg(D_5) = 27.$$

Indeed, from D_5, the queen can move horizontally to all other squares in rank 5, vertically to all squares in file D, and diagonally along both diagonals until the boundary of the board is reached. Since D_5 is an interior square, all such movements are unobstructed, yielding twenty-seven distinct neighboring vertices.

A similar argument applies to the symmetric interior square E_5.

For a queen placed at E_5, all squares lying in the same rank, file, and diagonals are reachable in one move. Hence,

$$\deg(E_5) = 27.$$

Thus, every interior square of the chessboard that is sufficiently far from the boundary has degree 27 in the Queen hypergraph.

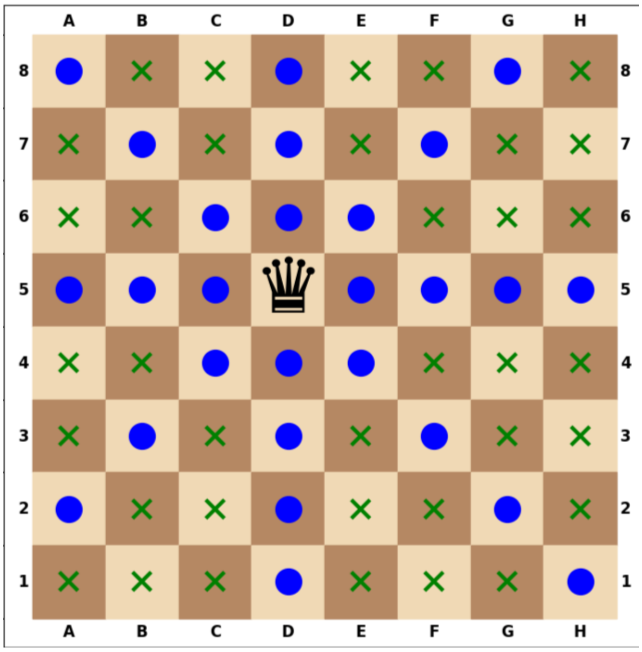

Figure 1: Queen Reachability Structure

We now extend the graph framework associated with queen movement to a hypergraph setting. The *Queen Hypergraph* H_Q is defined as

$$H_Q = (V, E),$$

where V denotes the set of all 64 squares of the chessboard, and each hyperedge corresponds to the complete movement neighbourhood of a fixed square under queen moves. For a given vertex $x_i \in V$, the associated hyperedge is defined by

$$E_{x_i} = \{x_i\} \cup \{y_j \in V \mid x = y \text{ or } i = j \text{ or } |x - y| = |i - j|\}.$$

Thus, each hyperedge consists of the chosen square together with all squares lying in the same file, the same rank, and the two diagonals passing through it.

Accordingly, the hyperedge set is given by

$$E_{H_Q} = \{E_{x_i} \mid x_i \in V\}.$$

3.1.3 Explicit Hyperedge Structure

The structure of the hyperedges depends on the position of the square on the chessboard. For each $i = 1, 2, \ldots, 8$, the hyperedges may be described as follows.

For the a-file, the hyperedge corresponding to a_i is

$$E_{A_i} = \{A_i,\ A_1, A_2, \ldots, A_8,\ B_i, C_i, D_i, E_i, F_i, G_i, H_i,\ B(i \pm 1), C(i \pm 2), \ldots, H(i \pm 7)\},$$

whenever the indices satisfy $1 \leq i \pm k \leq 8$.

For the B-file, the hyperedge associated with B_i is

$$E_{B_i} = \{B_i,\ B_1, B_2, \ldots, B_8,\ A_i, C_i, D_i, E_i, F_i, G_i, H_i,\ A(i \pm 1), C(i \pm 1), D(i \pm 2), \ldots, H(i \pm 6)\},$$

subject to the boundary restrictions of the board.

In a similar manner, for the files C, D, E, F, G, and H, each hyperedge is obtained by extending horizontally, vertically, and diagonally from the given square, while respecting the constraints

$$1 \leq i \pm k \leq 8.$$

This description provides a complete characterization of the Queen Hypergraph H_Q on the standard 8×8 chessboard.

3.1.4 Structural Observation

Each hyperedge of the Queen hypergraph H_Q corresponds to the union of a maximal row, a maximal column, and the two maximal diagonals intersecting at a single vertex. Interior vertices generate hyperedges of maximum size, while vertices near the boundary or corners give rise to smaller hyperedges due to truncation by the edges of the chessboard.

This hypergraph formulation forms the basis for the spectral and combinatorial analysis carried out in the subsequent sections.

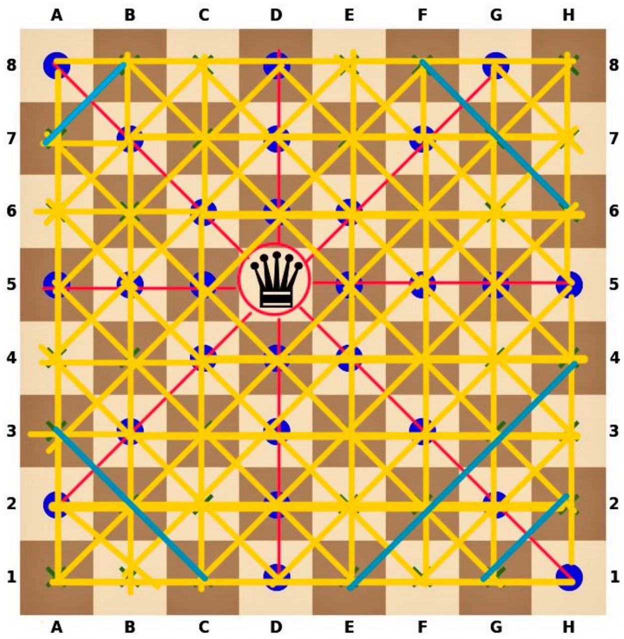

Figure 2: Queen Moves on the 8×8 Chessboard

3.2 The Adjacency Matrix of the Queen Hypergraph H_Q

In the Queen hypergraph H_Q associated with the standard 8×8 chessboard, two vertices are considered adjacent if they belong to at least one common hyperedge. Since each hyperedge represents the complete set of squares reachable by a queen in a single move, adjacency occurs whenever two distinct squares lie in the same row, the same column, or the same diagonal.

Accordingly, for vertices $u = (i, j)$ and $v = (k, \ell)$, adjacency is characterized by

$$u \sim v \quad \Longleftrightarrow \quad (i = k) \text{ or } (j = \ell) \text{ or } (|i - k| = |j - \ell|).$$

As no pair of distinct squares can share more than one of these relationships simultaneously, each adjacent pair contributes exactly one unit to the adjacency matrix.

Hence, the adjacency matrix of the Queen hypergraph (equivalently, the Queen graph H_Q) is a real symmetric matrix of order 64×64 with all diagonal entries equal to zero.

As an illustration, consider the vertex E_5. All squares lying in the same rank, the same file, and along the two diagonals through e_5 are adjacent to it. Consequently, the degree of E_5 is

$$\deg(E_5) = 27,$$

and all remaining vertices have zero adjacency with E_5.

3.2.1 Block Structure of the Adjacency Matrix

Let $A_d(H_Q)$ denote the adjacency matrix of the Queen hypergraph. We partition A_d into 8×8 blocks corresponding to the chessboard files

$$A, B, C, D, E, F, G, H.$$

Thus, the adjacency matrix $A_d(H_Q)$ admits a block representation in terms of the block matrices $B_0, B_1, \ldots, B_7$ as

$$A_d(H_Q) = \begin{pmatrix} B_0 & B_1 & B_2 & B_3 & B_4 & B_5 & B_6 & B_7 \\ B_1 & B_0 & B_1 & B_2 & B_3 & B_4 & B_5 & B_6 \\ B_2 & B_1 & B_0 & B_1 & B_2 & B_3 & B_4 & B_5 \\ B_3 & B_2 & B_1 & B_0 & B_1 & B_2 & B_3 & B_4 \\ B_4 & B_3 & B_2 & B_1 & B_0 & B_1 & B_2 & B_3 \\ B_5 & B_4 & B_3 & B_2 & B_1 & B_0 & B_1 & B_2 \\ B_6 & B_5 & B_4 & B_3 & B_2 & B_1 & B_0 & B_1 \\ B_7 & B_6 & B_5 & B_4 & B_3 & B_2 & B_1 & B_0 \end{pmatrix}.$$

For convenience, the files are indexed numerically as

$$A = 1,\ B = 2,\ C = 3,\ D = 4,\ E = 5,\ F = 6,\ G = 7,\ H = 8.$$

Let X and Y denote two files and define

$$d = |X - Y|.$$

Then the entries of the block $[XY]$ are given by

$$[XY]_{ij} = \begin{cases} 1, & \text{if } i = j \text{ or } |i - j| = d, \\ 0, & \text{otherwise.} \end{cases}$$

Thus, each block combines row-based adjacency and diagonal adjacency, and the full adjacency matrix of the Queen hypergraph is completely determined by the eight block types corresponding to $d = 0, 1, \ldots, 7$.

3.2.2 Explicit Block Types

Block Type $d = 0$

$$B_0 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Block Type $d = 1$

$$B_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Block Type $d = 2$

$$B_2 = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Block Type $d = 3$

$$B_3 = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Block Type $d = 4$

$$B_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Block Type $d = 5$

$$B_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Block Type $d = 6$

$$B_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Block Type $d = 7$

$$B_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

3.2.3 Symmetry and Structural Characterization

The adjacency matrix of the Queen hypergraph H_Q satisfies the following properties:

1. The matrix $A_d(H_Q)$ is symmetric, that is, $A_d(H_Q) = A_d(H_Q)^T$.
2. The block associated with the pair of files (X, Y) is the transpose of the block associated with (Y, X), namely $[XY] = [YX]^T$.

3. Each block depends only on the file separation $d = |X - Y|$ and not on the specific file labels.

Consequently, the adjacency matrix $A_d(H_Q)$ exhibits a block Toeplitz structure and is completely determined by the block matrices

$$B_0, B_1, \ldots, B_7.$$

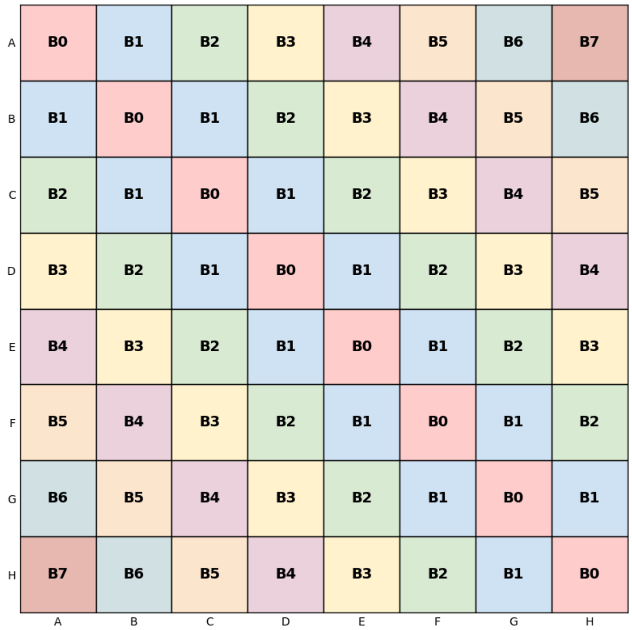

Figure 3: Adjacency Matrix of Queen Moves on the 8×8 Chessboard

3.3 The Adjacency Energy of Queen Hypergraph H_Q

The adjacency matrix $A_d(H_Q)$ of the Queen hypergraph is real and symmetric. Hence, all its eigenvalues are real. The eigenvalues of $A_d(H_Q)$, together with their multiplicities, were computed numerically and are listed below.

Table 1: Eigenvalues of the Queen Hypergraph

λ_1	λ_2	λ_3	λ_4
-4.000000	-4.000000	-4.000000	-4.000000
-4.000000	-4.000000	-4.000000	-4.000000
-4.000000	-4.000000	-4.000000	-4.000000
-4.000000	-4.000000	-4.000000	-4.000000
-4.000000	-4.000000	-4.000000	-4.000000
-4.000000	-4.000000	-4.000000	-4.000000
-4.000000	-3.170447	-2.942465	-2.942465
-2.935550	-2.362221	-1.858571	-1.858571
-1.841092	-1.465496	-0.697791	-0.697791
0.505777	0.505777	1.068750	1.423772
1.423772	1.522694	1.916808	2.389433
2.389433	2.572498	3.168127	3.741697
4.000000	4.000000	4.000000	4.265573
4.265573	4.442102	4.442102	4.637216
4.875296	4.875296	5.011792	6.037928
7.161275	7.596872	7.596872	22.936022

The adjacency energy of the Queen hypergraph H_Q is defined as

$$E(A_d(H_Q)) = \sum_{i=1}^{64} |\lambda_i|.$$

Using the computed spectrum, the adjacency energy of the Queen hypergraph H_Q is

$$E(A_d(H_Q)) = 245.54492.$$

3.4 The Laplacian Matrix of the Queen Hypergraph H_Q

Let H_Q denote the Queen hypergraph corresponding to the standard 8×8 chessboard. The vertex set consists of the 64 squares of the board. We index the vertices row-wise as

$$v_k = (i, j), \qquad k = 8(i-1) + j,$$

where $i, j \in \{1, 2, \dots, 8\}$. In the Queen hypergraph, two vertices are adjacent if and only if the corresponding squares lie in the same row, the same column, or on the same diagonal. Hence, the degree of a vertex (i, j) equals the total number of squares reachable by a queen in one legal move.

For any square (i, j), the degree is given by

$$\delta(i, j) = 14 + \min(i-1, j-1) + \min(i-1, 8-j) + \min(8-i, j-1) + \min(8-i, 8-j),$$

where the constant term 14 accounts for horizontal and vertical moves, and the remaining terms represent diagonal reachability.

Accordingly, the degree distribution over the chessboard is

$$\begin{pmatrix} 21 & 23 & 25 & 25 & 25 & 25 & 23 & 21 \\ 23 & 25 & 27 & 27 & 27 & 27 & 25 & 23 \\ 25 & 27 & 29 & 29 & 29 & 29 & 27 & 25 \\ 25 & 27 & 29 & 31 & 31 & 29 & 27 & 25 \\ 25 & 27 & 29 & 31 & 31 & 29 & 27 & 25 \\ 25 & 27 & 29 & 29 & 29 & 29 & 27 & 25 \\ 23 & 25 & 27 & 27 & 27 & 27 & 25 & 23 \\ 21 & 23 & 25 & 25 & 25 & 25 & 23 & 21 \end{pmatrix}.$$

Let

$$D(H_Q) = \operatorname{diag}\big(\delta(v_1), \delta(v_2), \dots, \delta(v_{64})\big)$$

denote the degree matrix of the Queen hypergraph. The matrix $D(G)$ is a 64×64 diagonal matrix whose diagonal entries correspond to the degrees of the vertices. Let $A_d(G)$ denote the adjacency matrix of the Queen graph. The Laplacian matrix of G is defined by

$$L(H_Q) = D(H_Q) - A_d(H_Q).$$

Thus, for $1 \leq p, q \leq 64$, the entries of the Laplacian matrix are given by

$$L_{pq} = \begin{cases} \delta(v_p), & \text{if } p = q, \\ -1, & \text{if } v_p \text{ and } v_q \text{ lie in the same row, column, or diagonal,} \\ 0, & \text{otherwise.} \end{cases}$$

Hence, $L(H_Q)$ is a real symmetric matrix of order 64×64 with diagonal entries equal to the vertex degrees and off-diagonal entries equal to -1 whenever two vertices are adjacent under queen movement.

A schematic representation of the Laplacian matrix is

$$L(H_Q) = \begin{bmatrix} 21 & -1 & 0 & \cdots & -1 \\ -1 & 23 & -1 & \cdots & 0 \\ 0 & -1 & 25 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ -1 & 0 & 0 & \cdots & 21 \end{bmatrix}_{64\times 64}.$$

Since the Queen hypergraph H_Q is connected, the Laplacian matrix possesses exactly one zero eigenvalue.

3.5 The Laplacian Energy of Queen hypergraph H_Q

The Laplacian spectrum of the Queen hypergraph H_Q associated with the standard 8×8 chessboard consists of 64 real eigenvalues. Since the Queen hypergraph is connected, the Laplacian matrix possesses exactly one zero eigenvalue. All remaining eigenvalues are positive and occur with multiplicities arising from the geometric symmetry of the chessboard and the uniform movement pattern of the queen. For clarity, the Laplacian eigenvalues are arranged in a 4×16 format and listed in Table 2.

Table 2: Laplacian Eigenvalues of the Queen Hypergraph

μ_1	μ_2	μ_3	μ_4
0.00000	14.21829	14.21829	16.00000
16.27210	16.82980	17.00000	17.16611
17.16611	17.58522	17.95309	17.95309
18.39445	18.65086	18.65086	18.99708
19.12914	19.54011	19.87238	20.09494
20.68415	20.68415	21.23230	21.38922
21.38922	21.93693	21.93693	22.48227
22.98977	22.98977	23.66242	23.66242
23.94230	24.11302	24.11302	24.69830
25.00000	25.00000	25.00000	25.34936
25.42125	25.59947	25.60555	25.92520
26.15414	26.15414	26.30026	26.30026
26.97140	26.97140	27.00000	27.00000
27.45312	27.45312	27.53222	27.78848
27.94567	28.50701	28.50701	28.70522
29.00000	29.85023	29.85023	30.05721

For a Queen hypergraph H_Q with n vertices and m edges, the Laplacian energy is defined by

$$LE(H_Q) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right|,$$

where $\mu_1, \mu_2, \ldots, \mu_n$ are the Laplacian eigenvalues of H_Q.

For the Queen graph on the 8×8 chessboard,

$$n = 64, \qquad \frac{2m}{n} = \overline{d} = 22.75,$$

where $\overline{d}$ denotes the average vertex degree.

Hence, the Laplacian energy of the Queen hypergraph H_Q is

$$LE(H_Q) = \sum_{i=1}^{64} |\mu_i - 25| \,.$$

Using numerical computation carried out in Python, the Laplacian energy is obtained as

$$LE(H_Q) = 259.14589$$

This value reflects the dense and highly symmetric connectivity induced by the queen's combined horizontal, vertical, and diagonal movements on the chessboard.

3.6 The Seidel Matrix of the Queen Hypergraph H_Q

The Queen hypergraph H_Q associated with the standard 8×8 chessboard, whose vertex set is

$$V(G) = \{v_1, v_2, \ldots, v_{64}\}.$$

Let $A_d(H_Q)$ represent the adjacency matrix of H_Q. In the case of the Queen hypergraph H_Q, where $n = 64$, this becomes

$$S(H_Q) = J_{64} - I_{64} - 2A(H_Q).$$

Equivalently, the entries of the Seidel matrix $S(H_Q) = [s_{ij}]$ are given by

$$s_{ij} = \begin{cases} 0, & i = j, \\ -1, & \text{if } v_i \text{ is adjacent to } v_j, \\ 1, & \text{if } v_i \text{ is not adjacent to } v_j, \end{cases} \qquad 1 \leq i, j \leq 64.$$

Thus, $S(H_Q)$ is a real symmetric matrix of order 64×64 with all diagonal entries equal to zero. The off–diagonal entry equals -1 whenever two vertices correspond to squares lying in the same row, the same column, or on the same diagonal of the chessboard, and equals 1 otherwise. This construction captures the complementary relationship between adjacency and non–adjacency inherent in the Queen graph.

A schematic form of the Seidel matrix is given by

$$S(H_Q) = \begin{bmatrix} 0 & 1 & 1 & \cdots & -1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 & -1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 & \cdots & 1 \\ \vdots & & & \ddots & & & & \vdots \\ -1 & 1 & 1 & \cdots & 0 & 1 & \cdots & 1 \\ 1 & -1 & 1 & \cdots & 1 & 0 & \cdots & 1 \\ \vdots & & & & & & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 1 & \cdots & 0 \end{bmatrix}_{64\times 64}.$$

3.7 The Seidel Energy of Queen Hypergraph H_Q

Since the Seidel matrix $S(H_Q)$ of the Queen hypergraph is real and symmetric, all its eigenvalues are real. The Seidel spectrum of the Queen hypergraph on the 8×8 chessboard consists of 64 eigenvalues. Numerical computation reveals a wide spread in the spectrum, reflecting the dense connectivity induced by the queen's horizontal, vertical, and diagonal moves.

The computed Seidel eigenvalues, arranged in a 4×16 format, are presented in Table **??**.

Table 3: Seidel Eigenvalues of the Queen hypergraph

σ_1	σ_2	σ_3	σ_4
-16.19374	-16.19374	-15.32255	-13.43638
-11.05625	-10.75059	-10.75059	-10.27505
-9.88420	-9.88420	-9.53115	-9.53115
-9.00000	-9.00000	-9.00000	-8.48339
-7.70183	-6.14500	-5.77887	-5.77887
-4.91535	-4.04539	-3.84754	-3.84754
-3.17104	-2.01155	-2.01155	0.39558
0.39558	1.93099	2.68081	2.71714
2.71714	3.72444	4.86903	4.88493
4.88493	5.34089	7.00000	7.00000
7.00000	7.00000	7.00000	7.00000
7.00000	7.00000	7.00000	7.00000
7.00000	7.00000	7.00000	7.00000
7.00000	7.00000	7.00000	7.00000
7.00000	7.00000	7.00000	7.00000
7.00000	7.00000	7.00000	18.00605

The multiplicities appearing in the Seidel spectrum are a direct consequence of the rotational and reflective symmetries of the chessboard, together with the uniform movement pattern of the queen.

The Seidel energy of a hypergraph H_Q with Seidel eigenvalues $\sigma_1, \sigma_2, \ldots, \sigma_n$ is defined by

$$E_S(H_Q) = \sum_{i=1}^{n} |\sigma_i|.$$

For the Queen hypergraph on the 8×8 chessboard, the Seidel energy was evaluated numerically using Python. The resulting value is

$$E_S(H_Q) = 455.09506.$$

Thus, the relatively large Seidel energy reflects the strong contrast between adjacency and non-adjacency created by the queen's combined row, column, and diagonal connectivity.

4 Conclusion

In this work, we examined the spectral properties of the Queen hypergraph H_Q associated with the standard 8×8 chessboard using its adjacency, Laplacian, and Seidel matrices. Each matrix representation captures a distinct aspect of the structure induced by the queen's movement. The adjacency spectrum reflects the dense connectivity generated by the queen's ability to move horizontally, vertically, and diagonally. The eigenvalue multiplicities observed in the spectrum arise from the high degree of symmetry present in the chessboard. The Laplacian spectrum encodes the variation in vertex degrees across the board and confirms that the Queen graph is connected. In addition, the computed Laplacian energy provides a quantitative measure of the deviation of the spectrum from the average degree, highlighting the strong global connectivity of the graph.

The Seidel matrix offers a complementary viewpoint by incorporating both adjacency and non-adjacency relations. Its spectrum accentuates the contrast between connected and non-connected vertex pairs, leading to a comparatively larger Seidel energy. This observation emphasizes the sensitivity of the Seidel framework to the overall structural balance of the Queen hypergraph.

Overall, these spectral investigations show that the Queen hypergraph possesses a rich algebraic structure governed by the combined movement patterns of the queen. The results contribute to the spectral study of chessboard-based graphs and hypergraphs and provide a foundation for future work on generalized $n \times n$ configurations and related combinatorial models.

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