A Study of the Energy and Spectral Characteristics of the Knight's Hypergraph

Abstract

Hypergraphs are generalization of graphs, introduced by Berge. In an ordinary graph, an edge connects exactly two vertices, whereas in hypergraphs, a hyperedge can join any number of vertices. Hypergraphs have applications in the field of Computer Science, Machine learning, Neural networks etc . In this paper, we focus on the Knight's hypergraph in which the squares of a chessboard are taken as vertices and each hyperedge include a vertex and all the vertices which are reachable by a knight in one move. We find the Adjacency matrix, Laplacian matrix, their eigenvalues and corresponding energies of the Knight's hypergraph with the help of Python programming.

Keywords: Hypergraph, Adjacency matrix, Laplacian matrix, Eigen spectrum, Energy, Knight's hypergraph

1 Introduction

Hypergraphs are systems of finite sets and form the most general concept in Discrete Mathematics. It was only in 1960s that Hypergraph become an independent theory. It was mostly in Hungary and France under the leadership of Mathematicians like Paul Erdos, Laszlo Lovasz, Paul Turan and C Berge. This branch of Mathematics has developed very rapidly in the later part of twentieth century, influenced by the advent of Computer science. The Knight's tour problem [8], [3] is a problem to find the path of a knight which traces all the squares of a chessboard exactly once. The Knight's tour problem has been extensively studied and many solutions are obtained. The Knight's graph is a graph in which squares of a chessboard are taken as vertices and two vertices are adjacent if they are reachable by a knight in one move. The Knight's hypergraph is the generalization of the knight's graph where one vertex can be adjacent to many vertices. More precisely a hyperedge include a vertex and all the vertices which are reachable from that vertex by a knight in one move. In this paper, we find out 64×64 adjacency matrix of the Knight's hypergraph and thereafter the energy of the Knight's hypergraph with the help of Python programming.

In the modern era , softwares, robotics and smart technologies have replaced human labour to enhance speed and efficiency in e-commerce and logistics. Legal knight movements or the 'L' shaped movements can be applied in warehouse management to improve storage layout, product movement and to model routing problems. The analysis of the Knight's hypergraph carried out in this paper and the matrix obtained can be made useful in framing such routing optimization models .

2 Preliminaries

Definition 1. *[1]:- A hypergraph H is a pair (V, E), where $V = \{v_1, v_2, ..., v_n\}$ is a finite set of vertices and $E = \{E_1, E_2, ..., E_m\}$ is a finite set of non empty subsets of V called hyperedges of H, such that $\cup_{j=1}^{m} E_j = V$. The number of vertices is called the order of H and the number of hyperedges is called the size of H. In a hypergraph, the vertices in a hyperedge are said to be adjacent. The degree of a vertex v_i of a hypergraph H is the number of hyperedges which contains the vertex v_i and is denoted by $d(v_i)$.*

$$\begin{aligned} V &= \{v_1, v_2, v_3, v_4, v_5\} \\ E &= \{E_1, E_2, E_3\} \\ E_1 &= \{v_2, v_4, v_5\} \\ E_2 &= \{v_1, v_2\} \\ E_3 &= \{v_1, v_3, v_4\} \end{aligned}$$

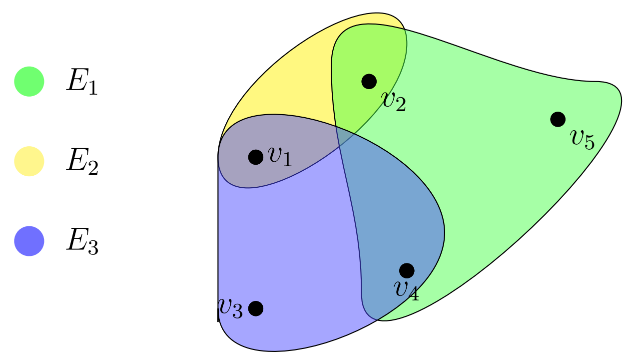

Figure 1: A hypergraph on 5 vertices and 3 edges

Definition 2. *[1]:- The adjacency matrix of a hypergraph H is denoted by $A(H) = [a_{ij}]$ where*

$$a_{ij} = \begin{cases} |E_k \in E : \{v_i, v_j\} \subseteq E_k| & if \quad i \neq j \\ 0 & otherwise \end{cases}$$

The eigenvalues of $A(H)$ are called the eigenvalues of hypergraph H. Since $A(H)$ is a real symmetric matrix, all the eigenvalues are real. The spectrum of H is the set of all eigenvalues of $A(H)$ together with their multiplicities.
If $\lambda_1, \lambda_2, ..., \lambda_s$ are distinct eigenvalues of H with multiplicities $m_1, m_2, ..., m_s$, then

$$spec(H) = \begin{pmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_s \\ m_1 & m_2 & \dots & m_s \end{pmatrix}$$

Definition 3. *[1]:- The Laplacian degree of a vertex $v_i \in V(H)$ is $\delta_l(v_i) = \sum_{j=1}^{m} a_{ij}$. The Laplacian matrix of a hypergraph H is denoted by $L = L(H)$ and is defined as $L = D - A$ where $D = diag(\delta_l(v_1), \delta_l(v_2), ..., \delta_l(v_n))$. The matrix L is symmetric and positive definite. So all eigenvalues are real and non-negative with smallest eigenvalue zero. These eigenvalues with their multiplicities collectively called the Laplacian spectrum.*

3 The Knight's Hypergraph

Definition 4 (The Knight's Graph)**.** *:- The Knight's graph is the graph in which 64 squares of chessboard are taken as vertices and two squares or vertices are connected in the graph if they are reachable by a knight in one move.*

Definition 5 (The Knight's Hypergraph)**.** *:-The Knight's hypergraph is the hypergraph in which 64 squares in a chessboard are taken as vertices and every hyperedge consists of a vertex and all the vertices which are reachable by a knight in one move.*
This graph contains 64 vertices and 64 hyperedges. Hence pictorial representation is not so easy . So for analysing the graph and to obtain the adjacency matrix, we follow certain notations. These are the notations used for the 64 squares in a chess game.
In a chessboard, we call the first column as A file and 8 squares in A file as a_1,a_2,..a_8 *, the second column as B file and the squares in B file as* b_1,b_2...b_8 *and so on up to H file and the last square* h_8.
Figure 2 shows all the possibilities that a knight can move from the square d_4 *and they are* b_3, b_5, c_2, c_6, e_2, e_6, f_3*and* f_5. *So let us call this hyperedge as hyperedge with centre at* d_4 *denoted by* E_{d_4}.
$E_{d_4} = \{d_4, b_3, b_5, c_2, c_6, e_2, e_6, f_3, f_5\}$

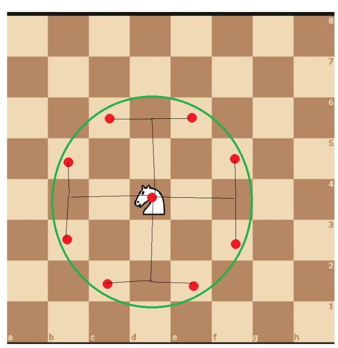

Figure 2: Knight movement possibilities

Hence the Knight's hypergraph is $H = (V, E)$

Where

$$
\begin{aligned}
V &= \{a_8, b_8, c_8, d_8, e_8, f_8, g_8, h_8, \\
. &\quad a_7, b_7, c_7, d_7, e_7, f_7, g_7, h_7, \\
&\quad \ldots \\
&\quad \ldots \\
&\quad a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1\}
\end{aligned}
$$

And

$$
\begin{aligned}
E &= \{E_{a_8}, E_{b_8}, E_{c8}, E_{d8}, E_{e_8}, E_{f_8}, E_{g_8}, E_{h_8}, \\
&\quad E_{a_7}, E_{b_7}, E_{c_7}, E_{d_7}, E_{e_7}, E_{f_7}, E_{g_7}, E_{h_7}, \\
&\quad \ldots \\
&\quad \ldots \\
&\quad E_{a_1}, E_{b_1}, E_{c_1}, E_{d_1}, E_{e_1}, E_{f_1}, E_{g_1}, E_{h_1}\}
\end{aligned}
$$

Where,

$$
\begin{aligned}
E_{a_i} &= \{a_i, b_{(i-2)}, b_{(i+2)}, c_{(i-1)}, c_{(i+1)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

$$
\begin{aligned}
E_{b_i} &= \{b_i, a_{(i-2)}, a_{(i+2)}, c_{(i-2)}, c_{(i+2)}, d_{(i-1)}, d_{(i+1)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

$$
\begin{aligned}
E_{c_i} &= \{c_i, a_{(i-1)}, a_{(i+1)}, b_{(i-2)}, b_{(i+2)}, d_{(i-2)}, d_{(i+2)}, e_{(i-1)}, e_{(i+1)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

$$
\begin{aligned}
E_{d_i} &= \{d_i, b_{(i-1)}, b_{(i+1)}, c_{(i-2)}, c_{(i+2)}, e_{(i-2)}, e_{(i+2)}, f_{(i-1)}, f_{(i+1)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

$$
\begin{aligned}
E_{e_i} &= \{e_i, c_{(i-1)}, c_{(i+1)}, d_{(i-2)}, d_{(i+2)}, f_{(i-2)}, f_{(i+2)}, g_{(i-1)}, g_{(i+1)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

$$
\begin{aligned}
E_{f_i} &= \{f_i, d_{(i-1)}, d_{(i+1)}, e_{(i-2)}, e_{(i+2)}, g_{(i-2)}, g_{(i+2)}, h_{(i-1)}, h_{(i+1)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

$$
\begin{aligned}
E_{g_i} &= \{g_i, e_{(i-1)}, e_{(i+1)}, f_{(i-2)}, f_{(i+2)}, h_{(i-2)}, h_{(i+2)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

$$
\begin{aligned}
E_{h_i} &= \{h_i, f_{(i-1)}, f_{(i+1)}, g_{(i-2)}, g_{(i+2)} : \\
&\quad 0 \leq i-2, i-1, i+1, i+2 \leq 8\}, \\
&\quad i = 1, 2, ..., 8
\end{aligned}
$$

Definition 6 (The adajcency matrix of the Knight's hypergraph)**.** *:- In a Hypergraph , two vertices are adjacent if they are common in one hyperedge and the corresponding entry in the adjacency matrix is the number of hyperedges containing both vertices.*
In the Knight's hypergraph, two vertices are adjacent if they are reachable by a knight in one move or in two moves and the corresponding entry in the adjacency matrix is the number of possibilities by which these two vertices are reachable by a knight in one or in two moves.

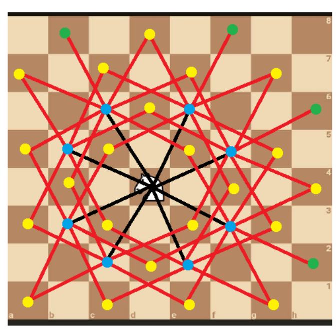

Figure 3: Adjacency

Hence the adjacency matrix of the Knight's Hypergraph is a square matrix of order 64.
Figure 3 shows how d_4 vertex is adjacent to other vertices .
The vertex d_4 is adjacent to blue and green labelled vertices in one way.

The vertex d_4 is adjacent to the yellow labelled vertices in two ways and all other vertices in zero ways.
Using figure 3 we find out the adjacency possibilities of 64 vertices with each other, and we observe that

1. *This matrix is symmetric matrix with all diagonal entries zero.*

2. 64×64 *matrix can be treated as a combination of 64 block matrices of order* 8×8*. These block matrices are the adjacency matrices of A file with A file, A file with B file, ...H file with H file.*

3. *Due to symmetry of the chessboard, many of these block matrices are same and are given the same notations and we get the following different blocks.*

   (a) *The adjacency matrix of A file with A file (say [AA]), which is same as the adjacency matrix of H file with H file*

$$[AA] = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

   (b) *The adajacency matrix of A file with B file (say [AB]),which is same as the adjacency matrix of H file with G file.*
   Due to symmetry, the adjacency matrices of B file with A file and G file with H file are transpose of [AB].

$$[AB] = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(c) *The adjacency matrix of B file with B file (say [BB]) ,which is same as the adjacency matrix of G file with G file.*

$$[BB] = \begin{bmatrix} 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 2 \\ 2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(d) *The adjacency matrix of A file with C file [AC] = the adjacency matrix of B file with D file [BD]= the adjacency matrix of C file with E file [CE]= the adjacency matrix of D file with F file [DF]= the adjacency matrix of E file with G file [EG]= the adjacency matrix of F file with H file [FH]. SImilarly due to symmetry we get these matrices are same as [CA],[DB],[EC],FD,[GE],[HF].*

We call these block matrices as [Q]

$$[Q] = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(e) *. [BC]=[CD]=[DE]=[EF]=[FG]=[CB]=[DC]=[ED]=[FE]=[GF].*

We call these block matrices as [P].

$$[P] = \begin{bmatrix} 0 & 1 & 1 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 & 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 2 & 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 & 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 & 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 1 & 1 & 0 \end{bmatrix}$$

(f) . [AD]=[BE]=[CF]=[DG]=[EH]=[DA]=[EB]=[FC]=[GD]=[HE].

We call all these block matrices as [R].

$$[R] = \begin{bmatrix} 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \end{bmatrix}$$

(g) . [AE]=[BF]=[CG]=[DH]=[EA]=[FB]=[GC]=[HD].

We call these block matrices as [S].

$$[S] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(h) . [CC]=[DD]=[EE]=[FF].

We call these block matrices as [D].

$$[D] = \begin{bmatrix} 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \end{bmatrix}$$

(i) .

$$[AF] = [AG] = [AH] = [BG] = [BH] = [CH] = [FA] = [GA] = [GB] = [HA] = [HB] = [HC].$$

All these block matrices are zero matrices denoted by [0].

Combining all these , we get the adjacency matrix as the 8×8 matrix with entries are the 8×8 block matrices obtained above.

And the Adjacency matrix is as follows.

	A	B	C	D	E	F	G	H
A	[AA]	[AB]	[Q]	[R]	[S]	[0]	[0]	[0]
B	[AB]'	[BB]	[P]	[Q]	[R]	[S]	[0]	[0]
C	[Q]	[P]	[D]	[P]	[Q]	[R]	[S]	[0]
D	[R]	[Q]	[P]	[D]	[P]	[Q]	[R]	[S]
E	[S]	[R]	[Q]	[P]	[D]	[P]	[Q]	[R]
F	[0]	[S]	[R]	[Q]	[P]	[D]	[P]	[Q]
G	[0]	[0]	[S]	[R]	[Q]	[P]	[BB]	[AB]'
H	[0]	[0]	[0]	[S]	[R]	[Q]	[AB]	[AA]

4 The Energy of the knight's hypergraph

Definition 7. *The Energy of a graph/hypergraph:-*
The energy of a graph/hypergraph is the sum of the absolute values of the eigenvalues of its adjacency matrix.
For the knight's hypergraph, the adjacency matrix is a 64×64 *matrix. So we use Python programming to find its eigenvalues and hence graph energy. Since the adjacency matrix is a symmetric matrix of order 64, we get 64 real numbers as its eigenvalues . The collection of eigenvalues or the eigen spectrum obtained using Python is listed below.*

The eigenvalues of the Knight's hypergraph are

$$\left\{\begin{array}{llll}
-6.76629207, & -6.76629207, & -6.0536517, & -5.96629206, \\
-5.96629206, & -5.83056446, & -5.80505071, & -5.52267497, \\
-5.3983357, & -5.3983357, & -5.31093628, & -5.13028815, \\
-4.93057489, & -4.78337845, & -4.73877083, & -4.73877083, \\
-4.60585049, & -4.38763183, & -4.19935141, & -4.19935141, \\
-3.8189587, & -3.8189587, & -3.73713403, & -3.56269421, \\
-3.29045932, & -3.29045932, & -2.82285377, & -2.77046619, \\
-2.6664803, & -2.34367463, & -2.34367463, & -2.25830655, \\
-1.57235606, & -1.33264629, & -0.97495884, & -0.95073415, \\
-0.72000298, & -0.60433008, & -0.60433008, & -0.35385596, \\
-0.35385596, & -0.30121246, & 0.24745974, & 0.42622305, \\
0.42622305, & 1.53815809, & 1.53815809, & 2.11331664, \\
2.66257058, & 2.67917741, & 2.84503095, & 3.28174244, \\
3.98504752, & 3.98504752, & 4.56654951, & 4.56654951, \\
7.91874659, & 7.91874659, & 9.46778072, & 9.54778863, \\
11.04559601, & 11.04559601, & 23.59996913, & 35.5856115,
\end{array}\right\}$$

the energy of a hypergraph is the sum of absolute values of all its eigenvalues.

Energy= $\sum \mid \lambda_i \mid=$ ***301.9821785666297***

Definition 8 (The Laplacian energy of the Knight's Hypergraph)**.** *The Laplacian degree of a vertex* $v_i \in V(H)$ *is* $\delta_l(v_i) = \sum_{j=1}^{m} a_{ij}$*. The Laplacian matrix of a hypergraph* H *is denoted by* $L = L(H)$ *and is defined as* $L = D - A$ *where* $D = diag(\delta_l(v_1), \delta_l(v_2), ..., \delta_l(v_n))$*.*

Here, from the adjacency matrix

We get

$$\delta_l(v_i) = \sum_{j=1}^{m} a_{ij}$$

$$= \left\{ \begin{array}{cccccccc} 12, & 18, & 23, & 26, & 26, & 23, & 18, & 12, \\ 18, & 24, & 32, & 37, & 37, & 32, & 24, & 18, \\ 23, & 32, & 42, & 48, & 48, & 42, & 32, & 23, \\ 26, & 37, & 48, & 56, & 56, & 48, & 37, & 26, \\ 26, & 37, & 48, & 56, & 56, & 48, & 37, & 26, \\ 23, & 32, & 42, & 48, & 48, & 42, & 32, & 23, \\ 18, & 24, & 32, & 37, & 37, & 32, & 24, & 18, \\ 12, & 18, & 23, & 26, & 26, & 23, & 18, & 12 \end{array} \right\}$$

Now,
$L = D - A$ *where* $D = diag(\delta_l(v_1), \delta_l(v_2), ..., \delta_l(v_n))$.

$$L = \begin{bmatrix} 12 & 0 & -1 & . & . & . & 0 & 0 & 0 \\ 0 & 18 & 0 & . & . & . & 0 & 0 & 0 \\ -1 & 0 & 23 & . & . & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 23 & 0 & -1 \\ 0 & 0 & 0 & . & . & . & 0 & 18 & 0 \\ 0 & 0 & 0 & . & . & . & -1 & 0 & 12 \end{bmatrix}$$

Now using Python programming , we find the Laplacian eigenvalues and hence Laplacian energy . Laplacian eigenvalues are

$$\left\{\begin{array}{llll}
0, & 9.16738846, & 10.9649665, & 10.9649665. \\
12.4932678, & 12.8641821, & 14.6345557, & 14.6345557, \\
17.0886644, & 18.0450611, & 18.0450611, & 18.4121099, \\
18.7205134, & 18.8615491, & 20.0355417, & 20.0355417, \\
22.4650656, & 22.4650656, & 22.5866526, & 22.8944773, \\
22.9819833, & 23.3428841, & 23.3428841, & 24.6519214, \\
25.6647137, & 25.7132628, & 25.7132628, & 26.7049696, \\
26.7482473, & 27.4884118, & 28.3998092, & 28.5706526, \\
28.5706526, & 29.0547667, & 30.5561609, & 30.5561609, \\
33.7916809, & 33.8355291, & 33.9952090, & 33.9952090, \\
36.4063523, & 36.4063523, & 37.3230872, & 38.0299265, \\
39.5749735, & 39.7243451, & 39.7243451, & 40.4252155, \\
42.7317928, & 44.1613902, & 44.8193037, & 44.8193037, \\
48.0742861, & 48.7552547, & 48.9079869, & 48.9079869, \\
50.3525104, & 50.5428064, & 52.7263458, & 52.7263458, \\
57.1899290, & 58.4279898, & 59.0923061, & 59.0923061
\end{array}\right\}$$

$$\begin{aligned}
\textit{Laplacian energy} &= \textit{Sum of Laplacian eigenvalues} \\
&= \boldsymbol{\mathit{2008.0000000000016}}
\end{aligned}$$

5 Conclusion

As we all know that knight is one of the most dangerous pieces in a chessboard as it can jump over the pieces and due to its ’L’ shape pattern of movements. The same pattern makes interesting and challenging while analysing its graph theoretical aspects. Throughout this paper we focused on knight’s movements as a hypergraph and finally successful in analysing and evaluating its eigen spectrum and energy.

6 Open Problems

1. Compare the eigen spectrum and connectedness of the Knight's graph and the Knight's hypergraph.
2. Obtain the eigen spectrum of the hypergraphs of movements of Bishop, Rook, Queen in a chessboard and compare their connectedness with that of the Knight's hypergraph.

References

[1] Bretto A. Introduction to hypergraph theory and its use in engineering and image processing. ADVANCES IN IMAGING AND ELECTRON PHYSICS.. 2004 Jan 1;131:3-64.

[2] Dai Q, Gao Y. Hypergraph computation. Springer Nature; 2023.

[3] Elkies ND, Stanley RP. The mathematical knight. The Mathematical Intelligencer. 2003 Dec;25(1):22-34.

[4] Gao Y, Ji S, Han X, Dai Q. Hypergraph computation. Engineering. 2024 Sep 1;40:188-201.

[5] Guo H, Zhou B, Lin H. The Wiener index of uniform hypergraphs. MATCH Commun. Math. Comput. Chem. 2017 Jan 1;78(1):133-52.

[6] Kumar JS, Archana B, Muralidharan K, Srija R. Spectral Graph Theory: Eigen Values Laplacians and Graph Connectivity. Metallurgical and Materials Engineering. 2025 Mar 13;31(3):78-84.

[7] Kumar KR, Varghese RP. Spectrum of (k, r)-regular hypergraphs. International J. Math. Combin. 2017 Jun 1;2:52-9.

[8] Parberry I. An efficient algorithm for the Knight's tour problem. Discrete Applied Mathematics. 1997 Mar 21;73(3):251-60.

[9] Xavier O, Hypergraphs: an introduction and review, arXiv preprint arXiv:2002.05014, 2020

[10] Yifan Feng, Haoxauan You, Zizhao Zhang, Rongrong Ji, Yue Gao; Hypergraph Nueral Networks, arXiv preprint arXiv:1809.09401