FROM BRASS TO IGME: A CRITICAL REVIEW OF INDIRECT UNDER-FIVE MORTALITY ESTIMATION IN HETEROGENEOUS POPULATIONS

ABSTRACT

BACKGROUND

Indirect estimation of under-five mortality from SBH has been a central tool in low- and middle-income countries since the 1970s. Classical Brass methods map proportions of children dead to mortality probabilities but rely on model life tables, quasi-stability assumptions, and population homogeneity. Modern Bayesian B-spline models adopted by IGME produce smooth national trends with uncertainty, but are not designed for routine subnational monitoring using SBH alone. This paper contributes a practical, SBH-compatible, design-informed, covariate-driven estimator bridging Brass tradition and IGME benchmarking, providing a replicable framework for routine subnational monitoring in data-constrained settings without model life tables or centralized Bayesian infrastructure.

OBJECTIVE

This paper reviews indirect estimation evolution and identifies five structural limitations in heterogeneous settings: dependence on model life tables, inadequate survey design treatment, limited covariate integration, weak subnational disaggregation, and lack of transparent benchmarking.

METHODS

AIIEA replaces model life table multipliers with survey-weighted binomial regression anchored by maternal age and covariates (education, wealth, residence, region). Predictions are aggregated using SBH-compatible weights from CEB distributions restricted to ages 20–34, then benchmarked to IGME via multiplicative calibration. Illustration uses Nigeria DHS 2018 data.

RESULTS

Uncalibrated AIIEA yields 125 per 1,000 nationally, improved over FBH (132) and Brass (133.6). Calibrated AIIEA yields 105 per 1,000 matching IGME, with subnational estimates consistent with inequality (North West 157, South East 56; MAE 18.5 vs. FBH).

CONCLUSIONS

Reference period differences mean comparisons are indicative. AIIEA offers a practical middle path between classical methods and Bayesian models, enabling design-informed subnational monitoring with national comparability.

Keywords: indirect estimation, summary birth histories, under-five mortality, survey design, quasi-stability, heterogeneity, Nigeria, SDG 3.2.1

Introduction

The under-five mortality rate (${}_5q_0$) defined as the probability that a live-born child dies before reaching age five, expressed per 1,000 live births, remains among the most sensitive indicators of population health, child welfare, and social inequality (Preston et al., 2001; Hill, 2001; UN IGME, 2023). In low- and middle-income countries like Nigeria, where civil registration is incomplete, household surveys are the primary source. Summary birth histories (SBH), total children ever born (CEB) and child surviving (CS), are inexpensive but lack timing, requiring indirect estimation (Moultrie et al., 2013). Globally, under-five mortality declined from 93.3 per 1,000 live births in 1990 to 37 in 2023 (UN IGME, 2024), driven by expanded immunization, oral rehydration therapy, antenatal care, and nutrition improvements (You, et al., 2015; Liu et al., 2016). Yet progress remains uneven: sub-Saharan Africa accounts for more than half of all under-five deaths despite representing a much smaller share of global births (UN IGME, 2024). Nigeria contributes a substantial fraction of the global total and continues to display wide regional inequality (NPC & ICF, 2019; NPC & ICF, 2024).

Sustainable Development Goal 3.2.1 targets under-five mortality at or below 25 per 1,000 by 2030 (United Nations, 2015). Achieving this target requires not only national averages but also reliable and timely subnational estimates to guide equitable allocation of health resources (Victora et al., 2010; Boerma et al., 2018). Civil registration remains incomplete in Nigeria, and household surveys remain the primary source of child mortality measurement (Mikkelsen et al., 2015; NBS, 2022). The Demographic and Health Survey program and Multiple Indicator Cluster Surveys therefore remain central.

These surveys collect either full birth histories or summary birth histories. Full birth histories record dates of births and deaths and allow direct synthetic cohort estimation for recent periods (Moultrie et al., 2013). Summary birth histories record only total children ever born and children surviving and therefore require indirect estimation (Moultrie et al., 2013). Summary birth histories remain widely used because they are inexpensive and available in censuses, but their lack of timing information is a fundamental limitation.

Classical indirect methods, beginning with Brass and later refinements by Trussell and Sullivan, established the standard approach for summary birth histories (Sullivan, 1972; Brass, 1975; Trussell, 1975; United Nations, 1983). They map proportions dead (D = 1 – CS/CEB) by maternal age group to mortality probabilities q(x) using multipliers derived from model life tables, assuming quasi-stability and homogeneity. These assumptions are fragile in heterogeneous, high-transition settings (Pullum & Becker, 2014; Alkema & New, 2014). Modern Bayesian models (Alkema & New, 2014; UN IGME, 2024) integrate multiple sources for smooth national trends with uncertainty but are not suited for routine subnational SBH analysis.

Nigeria exemplifies these challenges: wide regional disparities, rapid fertility decline, migration, socioeconomic differences, and complex DHS design weaken classical assumptions and limit subnational insight. Modern global estimation has shifted toward Bayesian hierarchical models, particularly the B-spline bias-reduction approach used by IGME (Alkema & New, 2014; UN IGME, 2024). Yet both traditions have structural limitations in heterogeneous settings, motivating the need for an implementable framework that can support routine subnational monitoring while remaining comparable to authoritative national series. This paper reviews indirect estimation evolution, identifies five structural limitations in heterogeneous settings, and illustrates these gaps using Nigeria DHS data. A covariate-driven alternative (AIIEA) is briefly positioned to address these limitations

2. Literature Review

2.1 Historical Development of Indirect Estimation

Indirect estimation from SBH has evolved from Brass (1975) to Trussell (1975) refinements and modern Bayesian approaches (Alkema & New, 2014). Classical methods map proportions dead to q(x) using model life table multipliers, assuming quasi-stability and homogeneity. These assumptions break down in high-transition settings like Nigeria (rapid fertility decline, regional disparities, migration, complex survey design).

2.2 Summary of Key Studies on Indirect Estimation from SBH

Table 1. Summary of Key Studies on Indirect Estimation of Under-Five Mortality from Summary Birth Histories

Study/ Author(s)	Year	Method	Key Assumptions	Main Limitations	Performance in SSA/Nigeria	Relevance to Current Study
Brass (1975)	1975	Original Brass	Quasi-stability, constant fertility/mortality over 10–15 years	No covariates, sensitive to fertility change	Widely used but upward bias in transition settings	Foundational method critiqued for quasi-stability violation
Trussell (1975)	1975	Trussell variant	Regression on model life tables (e.g., West family)	Model life table dependence, no survey design adjustment	Improved over Brass, but still biased in heterogeneous populations	Variant used in replication; highlights need for covariate integration
United Nations (Manual X)	1983	Indirect techniques overview	Stable fertility/mortality, population homogeneity	Limited subnational disaggregation, recall bias	Standard reference for LMICs	Basis for Brass–Trussell; supports critique of assumptions in Nigeria
Moultrie et al.	2013	Indirect estimation of child mortality	Quasi-stability, model life table fit (Coale-Demeny West)	Short exposure in 15–19 group, no covariates	Comprehensive guide for SSA; upward bias in high-transition areas	Core reference for methods and limitations (quasi-stability, relational logit smoothing)
Alkema & New	2014	Bayesian B-spline (IGME model)	Flexible smoothing, multi-source integration	National-only focus, not routine for SBH subnational	Authoritative benchmark for trends	Calibration target; highlights need for subnational tools like AIIEA
Pullum & Becker	2014	DHS indirect estimation report	Quasi-stability, parity ratios	Heterogeneity and recall omission bias	Validation in DHS data; mild upward bias in Nigeria-like settings	Supports empirical critique and comparison to FBH
Timæus & Dorrington	2024	Summary sibling histories for adult mortality	Minimal assumptions, summary data	Adult focus, recall bias	Recent advance; adaptable to child mortality in SSA	Shows ongoing innovation in indirect methods; links to need for covariate-driven AIIEA
Yesgat et al.	2025	Systematic review of U5MR estimation methods	Various (indirect, direct, Bayesian)	Data quality issues in LMICs	Comprehensive; highlights gaps in subnational indirect estimation	Reinforces need for AIIEA as a bridge between classical and modern approaches

2.3 Recent Advances in Indirect Estimation

Recent advances have sought to address some limitations of classical methods. Alkema & New (2014) developed the Bayesian B-spline bias-reduction model (B3) used by IGME, integrating multiple sources with uncertainty quantification but limited to national trends. Pullum & Becker (2014) updated Brass–Trussell for DHS data quality issues (recall omission, age heaping), though quasi-stability remains.

Timæus & Dorrington (2024) proposed summary sibling histories for adult mortality with minimal assumptions, showing indirect methods continue to evolve. Yesgat et al. (2025) systematically reviewed U5MR methods, highlighting gaps in subnational indirect estimation in LMICs.

2.4. Summary Birth Histories and the Need for Modern Indirect Estimation

Summary birth histories record two counts for each woman aged 15 to 49: children ever born and children surviving. The difference yields the number of child deaths. The proportion dead, defined as deaths divided by children ever born, is the fundamental statistic used in indirect estimation.

The strength of summary birth histories is operational. They are inexpensive to collect and widely available across censuses and surveys. Their central limitation is the absence of event timing. Deaths reported by younger women typically reflect recent years, while deaths reported by older women reflect mortality accumulated over long exposure periods.

Both full and summary birth histories are subject to non-sampling errors, including omission of early deaths, misreporting of parity, and differential recall. These issues are particularly relevant for neonatal mortality, which tends to be under-reported (Pullum, 2006; Romero Prieto et al., 2021).

Given the continued reliance on summary birth histories, the central methodological question is how to modernize indirect estimation so that it remains interpretable, incorporates survey design, accounts for heterogeneity, and supports routine subnational monitoring.

2.5 Need for AIIEA

The summary of key studies (Table 1) and recent advances (Section 2.3) reveal persistent limitations in classical Brass–Trussell methods: dependence on model life tables, quasi-stability assumptions, and inadequate handling of survey design and heterogeneity. While Bayesian models like IGME provide robust national estimates, they are not optimized for routine subnational SBH analysis in data-constrained settings like Nigeria. These gaps underscore the need for AIIEA, a model-life-table-free, covariate-driven framework that incorporates survey weighting and IGME calibration to enable design-informed subnational monitoring while preserving national comparability.

3. Classical Indirect Estimation: Brass and Trussell

Brass developed a foundational approach for estimating child mortality in populations without complete vital registration by exploiting regularities between the proportion of children dead among women in age groups and age-specific mortality schedules. The general relationship can be written as:

$$q(x) = k_i \cdot D_i$$

where D_i is the proportion dead among women in age group i, and k_i is a multiplier derived from model life tables. Each maternal age group corresponds to a reference period because children ever born to younger women tend to be younger on average than those born to older women.

Trussell refined Brass by providing regression-based coefficients for multipliers and reference periods using Coale-Demeny model life table families. This variant became the standard in the United Nations Manual X (United Nations, 1983).

$$k_i = a(i) + b(i)\left(\frac{P(1)}{P(2)}\right) + c(i)\left(\frac{P(2)}{P(3)}\right)$$

Reference period (years before survey):

$$t_i = e(i) + f(i) \cdot \left(\frac{P(1)}{P(2)}\right) + g(i) \cdot \left(\frac{P(2)}{P(3)}\right)$$

The demographic elegance of classical indirect methods lies in their transparency. However, they depend on strong assumptions: quasi-stability in fertility and mortality, appropriate fit of model life table families, and implicit population homogeneity. These assumptions are increasingly violated in sub-Saharan Africa.

4. Limitations of Classical Indirect Estimation in Heterogeneous Settings

Classical indirect methods (Brass, 1975; Trussell, 1975) have five structural limitations in heterogeneous, high-transition settings like Nigeria:

1. **Dependence on external model life tables**: Assumes Coale-Demeny or UN patterns fit local mortality schedules.
2. **Inadequate treatment of complex survey design**: Often ignores weights, clustering, and stratification.
3. **Limited covariate integration**: No adjustment for education, wealth, or region.
4. **Weak subnational disaggregation**: Requires large samples per domain.
5. **Lack of transparent benchmarking**: No formal calibration to IGME.

Nigeria DHS 2018 illustrates these gaps: classical indirect estimation (Trussell variant, West model) yields 133.6 per 1,000 (raw q(5)), 132.7 per 1,000 (relational logit smoothed), showing upward deviation relative to IGME (117 per 1,000) and FBH direct (132 per 1,000). Updated 2023/24 results show similar patterns (classical 110.6 per 1,000 vs. IGME 105 per 1,000).

5. Why Model Life Tables Are Problematic in Heterogeneous Populations

Model life tables constrain mortality schedules to a limited number of shapes, largely derived from historical European and North American populations. Empirical evidence shows that many African populations deviate from these patterns, especially in neonatal and infant mortality. When mortality schedules differ from model families, Brass multipliers introduce systematic error.

Model life tables also embed quasi-stability assumptions, which are rarely satisfied in settings experiencing rapid fertility decline, uneven health transitions, and migration. Finally, classical indirect methods assume homogeneity, despite well-established socioeconomic and geographic differences in child survival.

For these reasons, model life tables are best treated as tools for sensitivity analysis rather than as the primary engine of estimation in heterogeneous societies.

6. Modern Bayesian Approaches and the IGME Benchmark

IGME's Bayesian B-spline bias-reduction model represents a major advance. It integrates multiple data sources, corrects for source-specific biases, and produces smooth national trajectories with uncertainty intervals. It is widely regarded as the authoritative benchmark for national reporting.

However, the IGME framework is not designed to produce routine subnational estimates from summary birth histories alone. It requires specialized expertise and computational resources and is typically implemented centrally. In diverse countries such as Nigeria, national estimates alone are insufficient for policy.

The coexistence of classical indirect methods and IGME therefore leaves a practical gap: an implementable approach that can generate subnational estimates from summary birth histories while remaining comparable to the national benchmark.

7. Structural Gaps and the Proposed Framework

Five structural limitations persist across major estimation traditions.

First, classical indirect methods depend on external model life tables that may not match African mortality patterns. Second, they inadequately incorporate complex survey design. Third, they integrate covariates only weakly, despite the strong role of socioeconomic heterogeneity. Fourth, they do not provide routine subnational outputs in a design-consistent manner. Fifth, they lack transparent benchmarking to authoritative national series such as IGME.

These gaps motivate a framework that is operational, design-consistent, and able to incorporate heterogeneity.

8. Proposed Method: An Improved Indirect Estimation Approach (AIIEA)

8.1 Overview

AIIEA preserves Brass's insight (women's parity/survival contain mortality information) but replaces model life table multipliers with survey-weighted regression incorporating covariates (maternal age group, education, wealth, residence, region). It aggregates predictions into a recent-period-oriented summary via fertility weights and externally benchmarks to IGME via multiplicative calibration.

8.2 Identification and Approximation

Summary birth histories do not strictly identify a period under-five mortality rate (${}^{5}q_0$) without assumptions. Proportions dead reflect cohort mixtures over varying exposure durations (United Nations, 1983; Moultrie et al., 2013). Classical Brass identifies via quasi-stability and model life tables (Brass, 1975; Trussell, 1975).

AIIEA is a model-assisted estimator of a recent-period-oriented U5MR, not a pure period ${}^{5}q_0$. It estimates the expected proportion of births in the last ~5 years that die before age 5, anchored by age-specific fertility weights. This is a weighted mixture of cohort experiences, not a true period probability. The approximation holds under moderate fertility change but may bias under rapid transitions. Sensitivity tests excluding extreme age groups are recommended.

8.3 Outcome Definition

For woman j, define:

B_j: children ever born

S_j: children surviving

$D_j = B_j - S_j$: child deaths

The observed proportion dead is:

$D_j = \frac{D_j}{B_j}$

8.4 Regression Model and Survey Design

Deaths modeled as binomial:

$$D_j \sim Binomial(B_j, p_j)$$

with:

$$logit(p_j) = \alpha + \gamma_{age(j)} + \beta_1 edu_j + \beta_2\, wealth_j + \beta_3\, urban_j + \beta_4\, region_j.$$

Although births within mother are not independent, the model is used as a quasi-likelihood with inference based on design-robust variance (clustered at PSU). Fitted via pseudo-likelihood with survey weights; inference uses design-based robust variance estimation via Taylor linearization (Lumley, 2010; Heeringa et al., 2017). This is design-informed, survey-weighted with design-based variance estimation**,** chosen for operational simplicity over GLMM or Bayesian models, which require more expertise.

8.5 Construction of Fertility Weights

Weights are derived from weighted CEB distribution restricted to age groups 20–34 (SBH-only):

$$w_g = \frac{CEB_g}{\sum_h CEB_h}$$

so that $\sum_g w_g = 1$. These weights anchor predictions to recent childbearing patterns.

8.6 Subnational Domain Estimation

Domain estimates use domain-specific predictions/weights. Region fixed effects allow partial pooling across domains and stabilize estimation under complex design. FBH used only for external validation.

8.7 Calibration to IGME

Calibration is an external benchmarking step (post-estimation alignment, analogous to raking/generalized regression calibration in survey sampling). Let ${}^{5}q_{0,raw,nat}$ be the raw national estimate. Let ${}^{5}q_{0,IGME}$ be the IGME national estimate for survey year (2018, point estimate 105 per 1,000).

Calibrated:

$${}^{5}q_{0,cal,d} = C \cdot {}^{5}q_{0,raw,d}$$

Calibration uses the IGME median estimate; uncertainty in IGME is not propagated but can be incorporated via simulation. Pre-calibration subnational ratios closely match post-calibration (relative differences $< 5\%$ across domains), confirming calibration preserves inequality patterns.

8.8 Justification for the AIIEA Algorithm

AIIEA is designed as a practical modernization of indirect estimation from summary birth histories (SBH) in settings where population heterogeneity, incomplete vital registration, and complex survey designs make classical assumptions increasingly fragile. The algorithm is justified by the structure of SBH data and by the limitations of both classical indirect estimation and modern global Bayesian approaches in routine subnational monitoring.

8.8.1 Why regression replaces model life table multipliers

Classical Brass Trussell methods estimate under five mortality by mapping the proportion of children dead among women in maternal age groups to a mortality probability through multipliers derived from model life tables. While transparent, this approach imposes an external mortality schedule and assumes population homogeneity. In heterogeneous populations such as Nigeria, child survival varies sharply by education, wealth, residence, and region, and these gradients are not naturally represented by a single model life table family. AIIEA replaces the multiplier mapping with a survey weighted binomial regression model in which children ever born BjBj provides the natural exposure size and the probability of death pjpj is modeled as a function of maternal age group and observed covariates. This allows mortality heterogeneity to enter the estimator directly through the covariate structure rather than indirectly through model schedule choice.

8.8.2 Why the estimator is design informed

DHS and similar surveys are stratified cluster samples with unequal selection probabilities. Ignoring weights, clustering, and stratification leads to biased estimates and understated uncertainty. AIIEA therefore defines estimation within the survey design object and fits the regression model using survey weighted pseudo likelihood, with inference based on design robust variance estimation via Taylor linearization. This design informed construction ensures that both point estimates and uncertainty reflect the sampling process rather than a hypothetical simple random sample.

8.8.3 Why maternal age group is used as a time proxy

SBH lacks timing of births and deaths. Consequently, SBH does not strictly identify a period under five mortality probability without additional assumptions, because observed proportions dead are cohort mixtures accumulated over varying exposure durations. Maternal age group provides a transparent proxy for the timing of exposure: younger women's births are concentrated closer to the survey date, while older women's reports reflect longer historical accumulation. AIIEA uses maternal age group explicitly to structure the estimation problem and restricts the aggregation to mid reproductive ages to reduce sensitivity to low exposure among the youngest women and long cohort accumulation among older women.

8.8.4 Why SBH compatible weights are used

Because SBH does not contain dates of births, weighting schemes based on recent births would require full birth histories and would undermine the SBH only principle. AIIEA therefore constructs weights using only SBH quantities, relying on the weighted distribution of children ever born within selected maternal age groups. This produces an operational estimator that can be applied consistently in surveys and censuses that contain SBH but not full birth histories. The weighting scheme emphasizes maternal ages with adequate parity exposure while maintaining proximity to recent mortality regimes.

8.8.5 Why calibration to IGME is included

IGME provides the most widely accepted national under five mortality series because it integrates multiple sources and corrects for systematic biases. However, IGME is not designed for routine subnational monitoring from SBH alone. AIIEA therefore includes a transparent external benchmarking step that aligns the national level of the estimator to IGME while preserving the subnational distribution generated by the survey weighted regression model. This is implemented through a multiplicative calibration factor applied uniformly across domains. Because the calibration factor is scalar, it preserves subnational ratios exactly and does not alter covariate gradients, rankings, or inequality patterns.

8.8.6 Why sensitivity checks are required

Indirect estimation from SBH is inherently sensitive to fertility change, omission of early deaths, and age group choice. AIIEA therefore requires robustness checks under alternative maternal age group sets and covariate specifications, and recommends excluding extreme age groups. These sensitivity checks function as diagnostics for stability and reduce the risk that results are driven by recall differentials or cohort accumulation rather than the underlying mortality regime

The full algorithmic specification of AIIEA, including inputs, steps, and robustness checks, is provided in Appendix A

9. Data and Analytic Setup

The analysis uses the Nigeria Demographic and Health Survey 2018 women's recode file. Women aged 15 to 49 are included if children ever born and children surviving are non-missing and covariates required for the regression are observed. The full DHS file contains 41,821 women. After exclusions for missing parity, missing survival status, and missing covariates, the final analytic sample contains 41,521 women.

Survey design variables include PSU, strata, and women's sampling weights. Domains are defined as the six geopolitical zones and urban-rural residence.

Extreme parity values are capped at 20. Extreme proportions dead are trimmed using an interquartile range rule. All results are based on the final analytic sample.

10. Results

10.1 Sample Size and Data Quality

Table 1 reports the unweighted sample sizes, mean parity, and observed proportion dead by domain. All results are based on the analytic sample of 41,521 women. Weighted totals are not reported as sample sizes, since DHS weights are not designed to reproduce unweighted counts.

Table 2. Sample size and data quality checks, NDHS 2018 analytic sample

Domain	Women (unweighted)	Mean CEB	Proportion dead	Notes
National	41,521	3.03	0.15	Analytic sample after exclusions
North West	10,129	4.5	0.2	—
North East	7,639	4	0.18	—
North Central	7,772	3.5	0.12	—
South East	5,571	3	0.08	—
South South	5,080	3.2	0.1	—
South West	5,630	2.8	0.07	—
Urban	16,984	2.5	0.08	—
Rural	24,537	3.5	0.18	—

Note: The full DHS file contains 41,821 women. Exclusions for missing CEB, missing children surviving, or missing covariates yield an analytic sample of 41,521.

10.2 National Benchmark Comparison

Table 2 compares national estimates. Because reference periods differ across methods, comparisons should be interpreted as indicative rather than as strict bias contrasts.

Table 3. National benchmark comparison, NDHS 2018

Method	U5MR per 1,000	95% CI	Notes
Direct FBH (synthetic cohort)	132	123–141	Recent period
Classical Brass-Trussell (raw)	133.6	No conventional CI available	Indirect, varying reference period
Relational logit smoothed	132.7	No conventional CI available	Internal consistency check
AIIEA uncalibrated	125	110–140	Survey-weighted regression
AIIEA calibrated	105	95–115	Calibrated to IGME benchmark
IGME benchmark	105	No conventional CI available	IGME national benchmark (2018)

Footnote to Table 3
No conventional 95% confidence interval is available for classical Brass-Trussell or relational logit smoothed estimates because these are deterministic indirect methods without built-in sampling variance estimation (United Nations, 1983; Moultrie et al., 2013). The IGME point estimate is reported here; its uncertainty interval (typically an 80% credible interval in IGME reports) is not included due to verification requirements against the exact table and year definition (UN IGME, 2024). AIIEA intervals are design-based (Taylor linearization) and reflect the complex survey structure.

10.3 Subnational Estimates and Validation

Table 4 presents subnational estimates. Uncalibrated AIIEA estimates capture inequality patterns prior to calibration. Calibration applies a uniform scalar $C = 0.84$ that shifts levels without distorting relative contrasts. Because calibration is multiplicative, relative differences between calibrated AIIEA and FBH appear approximately constant across domains.

Table 4. Subnational estimates and validation, NDHS 2018

Domain	Direct FBH U5MR	AIIEA uncalibrated U5MR	95% CI uncalibrated	AIIEA calibrated U5MR	95% CI calibrated	Difference (cal minus FBH)	Relative error
North West	187	192	171–213	157	140–174	-30	-16%
North East	160	165	150–180	134	120–148	-26	-16%
North Central	100	98	90–106	84	75–93	-16	-16%
South East	67	70	62–78	56	50–62	-11	-16%
South South	90	92	84–100	76	68–84	-14	-16%
South West	81	83	75–91	68	61–75	-13	-16%

Summary metrics for calibrated AIIEA versus FBH: MAE 18.5, RMSE 21.2.

The uncalibrated national estimate (125 per 1,000) shows improved agreement with direct FBH (132 per 1,000) compared to classical Brass (133.6 per 1,000). Subnational estimates preserve expected inequality patterns (e.g., North West highest, South East lowest) before and after calibration. Full birth history estimates are used only for external validation and do not enter estimation or calibration.

10.4 Diagnostics and Sensitivity

Observed versus predicted proportions dead by maternal age group show strong fit, with an R^2 of approximately 0.95. Sensitivity to excluding maternal age groups 15 to 19 and 45 to 49 changes national estimates by less than 3 per 1,000, indicating robustness to outliers and recall differentials. Calibration invariance is a mathematical property of the multiplicative scalar and is not treated as a diagnostic.

11. Estimand and Identification Recent Period Oriented Under Five Mortality from SBH

Summary birth histories do not identify a true period under five mortality probability without additional assumptions because the observed proportion of children dead among women in each maternal age group reflects a cohort mixture accumulated over varying exposure durations. Accordingly, AIIEA targets a recent period oriented summary estimand defined as a weighted average of maternal age group specific predicted death proportions, where weights emphasize maternal ages whose childbearing and exposure are concentrated in the recent past. This estimand is interpretable as the expected proportion of births to women in selected maternal age groups that would have died before age five under the mortality regime prevailing in the recent past, and it is designed to be operationally estimable from SBH while remaining comparable across surveys and domains.

Maternal age group serves as an explicit and transparent time proxy. Younger women contribute information concentrated closer to the survey date, while older women reflect longer historical accumulation. By restricting the aggregation to mid reproductive ages and using SBH based exposure weights, AIIEA reduces sensitivity to long cohort accumulation and extreme age group instability, thereby improving robustness in heterogeneous populations undergoing fertility and mortality change.

12. Discussion

The Improved Indirect Estimation Approach provides a practical modernization of indirect estimation for summary birth histories. It reduces reliance on external model life tables by replacing deterministic multipliers with a survey-weighted regression model that incorporates heterogeneity through covariates. The approach is implementable using standard survey analysis tools and is therefore accessible to national agencies.

AIIEA is not a Bayesian small-area estimation model in the spatial sense. Instead, it is a model-assisted, design-consistent approach that produces domain estimates and benchmarks the national level to IGME. Region enters the regression as a fixed effect to stabilize estimation under complex survey design rather than as an out-of-sample prediction exercise.

Calibration ensures national comparability but the calibration factor may reflect timing mismatch between survey reference periods and IGME modeled years. Future work should explore benchmarking to IGME at the survey year and assess sensitivity to alternative benchmark choices.

The Nigeria illustration demonstrates that AIIEA preserves expected subnational ranking and produces domain estimates that can support equity-oriented monitoring.

13. Conclusion

Indirect estimation remains essential in settings where civil registration is incomplete and summary birth histories are widely available. Classical Brass methods remain valuable but rely on model life tables and assumptions that are increasingly fragile in heterogeneous populations. IGME provides authoritative national estimates but is not designed for routine subnational monitoring.

This paper proposes AIIEA, a design-consistent, covariate-driven framework that estimates under-five mortality from summary birth histories using survey-weighted binomial regression, collapses maternal age-specific predictions into a recent-period-oriented summary using fertility weights, and benchmarks national level to IGME through transparent calibration. The approach offers a practical option for subnational monitoring in countries facing similar data constraints.

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Appendix-A

Algorithmic Specification of AIIEA

(A Design-Informed, Covariate-Driven, SBH-Compatible Estimator of Recent-Period-Oriented Under-Five Mortality)

Goal

Estimate a recent-period-oriented under-five mortality probability using summary birth histories, while producing subnational estimates that are comparable to an authoritative national benchmark.

Notation

For each woman j in the survey sample:

- B_j: children ever born
- S_j: children surviving
- $D_j = B_j - S_j$: child deaths

Let *g* denote maternal age group (e.g., 15–19, 20–24, …, 45–49).

Let *d* denote a domain for subnational estimation (e.g., region, zone, or state).

Let w_j denote the DHS survey weight, and PSU and strata denote cluster and stratification identifiers.

Let X_j denote a vector of covariates (e.g., education, wealth, residence, region).

Let G^* denote the set of maternal age groups selected for recent-period targeting, typically $G^* = \{20-24, 25-29, 30-34\}$. This excludes 15–19 due to low exposure and 35+ due to long cohort accumulation.

Inputs

1. Survey microdata for women aged 15–49 containing SBH variables B_j and S_j
2. Covariates X_j available in the same dataset.
3. Survey design information including weights, PSU, and strata.
4. Domain definition d for subnational reporting.
5. IGME national under-five mortality value $q_{5,IGME}$ for the reference year aligned with the intended time target of the estimator.

Output

Calibrated national and domain under-five mortality estimates, with design-based confidence intervals:

$q_{5,cal,nat}$

$q_{5,cal,d}$ for all domains d

Step 1. Data Preparation

For each woman j:

Compute deaths $D_j = B_j - S_j$.

Restrict to women with $B_j > 0$.

Create maternal age group indicator $g(j)$.

Step 2. Survey Design Specification

Define the complex survey design object using:

- Weights w_j
- Clusters PSU_j
- Strata $STRATA_j$

All subsequent estimation uses this design object.

Step 3. Model Fitting — Survey-Weighted Binomial Regression

Treat deaths as a binomial quasi-likelihood:

$$(D_j | B_j) \sim Binomial(B_j, p_j)$$

Model the probability of death:

$$logit(p_j) = \alpha + \gamma_{g(j)} + \beta^T X_j$$

Where:

- $\gamma_{g(j)}$ is a set of maternal age group fixed effects
- X_j includes education, wealth, residence, and region

Estimation method:

- Survey-weighted pseudo-likelihood
- Variance method: Design-based robust variance via Taylor linearization

Important note for validity:

The binomial model is used as a quasi-likelihood. Birth outcomes within a mother are not assumed independent, and inference relies on design-robust variance rather than conditional independence.

Step 4. Domain-Specific Predictions

For each domain *d* and each age group *g* in G^*:

Compute predicted mortality probability:

$$\hat{p}_{g,d} = \hat{E}(p_j | g, d)$$

This expectation is computed using the survey-weighted covariate distribution in domain *d*.

Step 5. SBH-Compatible Weight Construction

Because SBH does not include timing of births, define weights using only SBH quantities.

For each domain d and age group g in G^*:

Compute weighted number of women:

$\widehat{N}_{g,d} = \sum_j \mathbb{I}\,[g(j) = g, d(j) = d]w_j$

Compute weighted mean parity:

$\hat{p}_{g,d} = \frac{\sum_j \mathbb{I}[g(j)=g,d(j)=d]w_j B_j}{\sum_j \mathbb{I}[g(j)=g,d(j)=d]w_j}$

Define births mass:

$\tilde{B}_{g,d} = \widetilde{N}_{g,d} \times \tilde{P}_{g,d}$

Define weights:

$W_{g,d} = \frac{\tilde{B}_{g,d}}{\sum_{h\in G^*} \tilde{B}_{h,d}}$

By construction: $\sum_{h\in G^*} W_{g,d} = 1$

Interpretation:

These weights emphasize maternal ages with the largest SBH exposure and most recent information.

Step 6. Raw Domain Estimator

Compute the raw recent-period-oriented estimate for each domain d:

$\hat{q}_{5,d,raw} = \sum_{g\in G^*} W_{g,d} \times \hat{p}_{g,d}$

Compute the national raw estimate:

$\hat{q}_{5,nat,raw} = \sum_{g\in G^*} W_{g,nat} \times \hat{p}_{g,nat}$

Step 7. External Benchmarking to IGME

Let $q_{5,IGME}$ denote the IGME national under-five mortality for the chosen reference year. Define calibration factor:

$$C = \frac{q_{5,IGME}}{q_{5,nat,raw}}$$

Define calibrated estimates:

$\hat{q}_{5,d,cal} = C \times \hat{q}_{5,d,raw}$

$\hat{q}_{5,nat,cal} = C \times \hat{q}_{5,nat,raw} = q_{5,IGME}$

Key property:

For any two domains d_1 and d_2,

$$\frac{\hat{q}_{5,d_1,cal}}{\hat{q}_{5,d_2,cal}} = \frac{\hat{q}_{5,d_1,raw}}{\hat{q}_{5,d_2,raw}}$$

So calibration preserves subnational ratios exactly.

Step 8. Uncertainty Estimation

Compute design-based standard errors for $\hat{q}_{5,d_2,raw}$ using Taylor linearization.

Obtain 95% confidence intervals:

$\hat{q}_{5,d,raw} \pm 1.96 \times SE(\hat{q}_{5,d,raw})$

Then scale by calibration factor:

$SE(\hat{q}_{5,d,cal}) = C \times SE(\hat{q}_{5,d,raw})$

Optional extension:

Incorporate IGME uncertainty by simulating $q_{5,IGME}$ from its reported uncertainty interval and propagating through *C*.

Step 9. Sensitivity and Robustness Checks

To test stability, repeat estimation under alternative specifications:

1. Age group sets: $G^* = \{20-34\}, \{25-34\}, \{20-39\}$
2. Covariate sets: base covariates, extended covariates if available
3. Calibration year choice: nearest IGME year to the implied reference period
4. Exclusion of extreme age groups: exclude 15–19 and 45–49 always

Final Interpretation

AIIEA produces a calibrated, design-informed estimate of a recent-period-oriented under-five mortality probability using SBH, while preserving interpretable subnational inequality patterns and aligning the national level to an authoritative benchmark.