

## Report on “Interpolative Fixed Point Theory in Perturbed Metric Spaces via Exact Metric Decomposition”

I have carefully reviewed the manuscript entitled “Interpolative Fixed Point Theory in Perturbed Metric Spaces via Exact Metric Decomposition.” The paper studies fixed point results for several classes of interpolative contractions in perturbed metric spaces, including interpolative perturbed Kannan contractions, interpolative perturbed Reich–Rus–Ćirić contractions, Suzuki-type trigger contractions, and a generalized interpolative perturbed contractive mapping. Examples are provided, and applications are discussed for a nonlinear integral equation and a Bellman-type functional equation arising in dynamic programming.

While the topic is relevant to fixed point theory, the manuscript has several structural, mathematical, and conceptual shortcomings, detailed below.

- Subsection 1.1 (“Some classical and recent fixed point principles”) should be moved to the Preliminaries section, as it consists entirely of background material rather than introductory motivation.
- Theorem 4.1 is a particular case of Theorem 4.2 and therefore should not be presented as an independent main result. More generally, the manuscript could be significantly streamlined by proving only the unified result (Theorem 4.4) and deriving Theorems 4.1–4.3 as corollaries.
- In Theorem 4.2, the authors claim that a recursion of the form  $a_{k+1} \geq ca_k^\vartheta$  with  $\vartheta \in (0, 1)$  guarantees the convergence of  $\sum a_k$ . This claim is not justified. While such a recursion implies  $a_k \rightarrow 0$  summability does not follow without an explicit comparison or domination argument. A rigorous lemma or a detailed justification (for instance, eventual geometric decay) is required.
- Most examples rely on constant mappings, which satisfy nearly all contractive conditions automatically. Such examples do not demonstrate the sharpness or genuine novelty of the interpolative assumptions. At least one nontrivial nonlinear example should be included, preferably one that fails to satisfy classical contraction conditions but satisfies the proposed interpolative framework.
- In the application to the nonlinear integral equation, the authors claim that  $(C[0, 1], \theta)$  equipped with  $L^1$ -metric, is complete. This is false:  $C[0, 1]$  is not complete under the  $L^1$ -metric. The authors must either replace  $C[0, 1]$  with  $L^1[0, 1]$  or use the supremum norm as the exact metric. This is a serious mathematical error, as completeness is essential for the fixed point arguments.

The manuscript contains a potentially interesting unified result (Theorem 4.4), but in its current form it suffers from structural redundancy, insufficient justification of some arguments, overly trivial examples, and at least one serious mathematical inaccuracy in the applications. I therefore recommend major revision.