Generalized Class of Estimators for Mean of Population through Imputation Technique

Abstract

In the current study, we have developed two generalized class of estimators for estimating the mean of population based on imputation technique under simple random sampling without replacement (SRSWOR), and we have also found its properties. Traditional imputation techniques such as mean and ratio method are found to be the particular cases of the developed imputation techniques. Also, some estimators discussed by Ahmed et al.(2006) are found as the members of the developed generalized class of estimators. Developed estimators outperform under the given mathematical conditions. It is then vindicated with the help of empirical study using two authentic datasets.

Keywords: Estimators; Imputation; Bias; Missing Data

1 Introduction

Occasionally we obtain data with missing observations in sample surveys. Using such data to estimate the mean of the population does not give an appropriate estimate. In those circumstances, imputation techniques have been used to estimate the mean for the population. These techniques have been used to enhance the precision and truthfulness of data analysis. Many imputation techniques, including the mean, ratio, regression and power transformation methods have been utilized to substitute missing observations.

The imputation technique based on ancillary variable was initiated by Lee et al.(1994). Further, Singh and Horn(2000), Singh and Deo(2003), Ahmed et al.(2006), Prasad(2018), Bhushan and Pandey(2018), Bhushan et al.(2023) and Pandey et al.(2025) including others have formed a variety of imputation techniques based on information on ancillary variable.

In the current research, we have developed two generalized class of estimators for estimating the mean of population based on imputation technique under SRSWOR, and we have also examined its properties. Some of the estimators existing in literature are found as the particular cases of the developed estimators. Developed generalized class of estimators outperform under the given mathematical conditions. Empirical analysis using two authentic data sets have been done to prove the dominance of the developed estimators over other corresponding estimators found as their special cases.

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2 Terminologies and sampling method used

Considering a finite population Υ=$\{1,2,...,N\}$ having size N. A random sample s of size n is chosen by using SRSWOR from Υ for the estimation of mean of population $\overline{Y}$. (y,x) represent the study and ancillary variables. Further, let r denotes the count of responding elements taken from n and A indicates the collection of elements that respond. Also, A^c represents the collection of non-responding elements. For units $j \in A$, values of sample on study variables y are gotten while for lost values as $j \in A^c$, suitable values are acquired. It is supposed that through the help of an ancillary variable x, the imputation technique is carried out and $x_s = \{x_j : j \in s\}$ are known.

We indicate,

$\overline{Y}$: Mean of population for N observations on y.

$\overline{X}$: Mean of population for N observations on x.

Let $\overline{y}_r = \frac{1}{r}\sum_{j=1}^{r} y_j$, $\overline{y}_n = \frac{1}{n}\sum_{j=1}^{n} y_j$ are the means of response and sample for study variable y and $\overline{x}_r = \frac{1}{r}\sum_{j=1}^{r} x_j$, $\overline{x}_n = \dfrac{1}{n}\sum_{j=1}^{n} x_j$ are the means of response and sample for ancillary variable x.

The information about mean square for population, population covariance and coefficient of variation for Y and X are given as follows:

$S_y^2 = \frac{1}{(N-1)}\sum_{j=1}^{N}\left(y_j - \overline{Y}\right)^2, S_x^2 = \frac{1}{(N-1)}\sum_{j=1}^{N}\left(x_j - \overline{X}\right)^2$

$S_{yx} = \frac{1}{(N-1)}\sum_{j=1}^{N}\left(y_j - \overline{Y}\right)\left(x_j - \overline{X}\right), C_y = \frac{S_y}{\overline{Y}}, C_x = \frac{S_x}{\overline{X}}, C_{yx} = \frac{S_{yx}}{\overline{Y}\overline{X}}$

Following notations are used in order to acquire the mean square error (MSE) of the developed generalized class of imputation methods: $\overline{y}_r = \overline{Y}(1+\epsilon_0)$, $\overline{x}_r = \overline{X}(1+\epsilon_1)$, $\overline{x}_n = \overline{X}(1+\epsilon_2)$ such that $E(\epsilon_0) = 0, E(\epsilon_1) = 0, E(\epsilon_2) = 0$, and $E(\epsilon_0^2) = f_r C_y^2$, $E(\epsilon_1^2) = f_r C_x^2$, $E(\epsilon_2^2) = f_n C_x^2$, $E(\epsilon_0\epsilon_1) = f_r \rho C_y C_x$, $E(\epsilon_1\epsilon_2) = f_n C_x^2$ and $E(\epsilon_0\epsilon_2) = f_n \rho C_y C_x$, where ρ is the coefficient of correlation between study and ancillary variables.
And, $f_r = \left(\frac{1}{r} - \frac{1}{N}\right)$ and $f_n = \left(\frac{1}{n} - \frac{1}{N}\right)$.

3 Suggested Imputation Methods

By using information on $\overline{X}$ and $\overline{x}_n$, we have developed the following generalized class of estimator for the mean of population using imputation method as:

$$y_{.j_1} = \begin{cases} y_j & if \;\; j \in A \\ \frac{1}{n-r}\left[n\overline{y}_r\left(\frac{q_1\overline{x}_n + (1-q_1)\overline{X}}{\overline{X}}\right)^{\alpha} - r\overline{y}_r\right] & if \;\; j \in A^c \end{cases} \tag{3.1}$$

Estimator for the mean of population using above imputation method is derived as:

$$\begin{aligned}
{P_1}^* =& \frac{1}{n}\sum_{j=1}^{n} y_{.j_1} = \frac{1}{n}\left[\sum_{j\in A} y_{.j_1} + \sum_{j\in A^c} y_{.j_1}\right] \\
=& \frac{1}{n}\left[\sum_{j=1}^{r} y_j + \sum_{j=1}^{n-r}\frac{1}{n-r}\left\{n\overline{y}_r\left(\frac{q_1\overline{x}_n+(1-q_1)\overline{X}}{\overline{X}}\right)^{\alpha} - r\overline{y}_r\right\}\right] \\
=& \frac{1}{n}\left[r\overline{y}_r + (n-r)\frac{1}{n-r}\left\{n\overline{y}_r\left(\frac{q_1\overline{x}_n+(1-q_1)\overline{X}}{\overline{X}}\right)^{\alpha} - r\overline{y}_r\right\}\right] \\
=& \frac{1}{n}\left[r\overline{y}_r + n\overline{y}_r\left(\frac{q_1\overline{x}_n+(1-q_1)\overline{X}}{\overline{X}}\right)^{\alpha} - r\overline{y}_r\right] \\
=& \frac{1}{n}\left[n\overline{y}_r\left(\frac{q_1\overline{x}_n+(1-q_1)\overline{X}}{\overline{X}}\right)^{\alpha}\right] = \overline{y}_r\left(\frac{q_1\overline{x}_n+(1-q_1)\overline{X}}{\overline{X}}\right)^{\alpha}
\end{aligned} \tag{3.2}$$

By using the above imputation technique, estimator for the mean of population is obtained as:

$${P_1}^* = \overline{y}_r\left(\frac{q_1\overline{x}_n+(1-q_1)\overline{X}}{\overline{X}}\right)^{\alpha}\ , \tag{3.3}$$

where 0$\leq q_1 \leq$1 and α is a chosen constant.

Now by using information on $\overline{X}$ and $\overline{x}_r$, we have developed another generalized class of estimator for the mean of population using imputation method as:

$$y_{.j_2} = \begin{cases} y_j & if\ \ j\in A \\ \frac{1}{n-r}\left[n\overline{y}_r\left(\frac{q_2\overline{x}_r+(1-q_2)\overline{X}}{\overline{X}}\right)^{\beta} - r\overline{y}_r\right] & if\ \ j\in A^c \end{cases} \tag{3.4}$$

By using the above imputation technique, estimator for the mean of population is obtained as:

$$\begin{aligned}
{P_2}^* =& \frac{1}{n}\sum_{j=1}^{n} y_{.j_2} = \frac{1}{n}\left[\sum_{j\in A} y_{.j_2} + \sum_{j\in A^c} y_{.j_2}\right] \\
=& \frac{1}{n}\left[\sum_{j=1}^{r} y_j + \sum_{j=1}^{n-r}\frac{1}{n-r}\left\{n\overline{y}_r\left(\frac{q_2\overline{x}_r+(1-q_2)\overline{X}}{\overline{X}}\right)^{\beta} - r\overline{y}_r\right\}\right] \\
=& \frac{1}{n}\left[r\overline{y}_r + (n-r)\frac{1}{n-r}\left\{n\overline{y}_r\left(\frac{q_2\overline{x}_r+(1-q_2)\overline{X}}{\overline{X}}\right)^{\beta} - r\overline{y}_r\right\}\right] \\
=& \frac{1}{n}\left[r\overline{y}_r + n\overline{y}_r\left(\frac{q_2\overline{x}_r+(1-q_2)\overline{X}}{\overline{X}}\right)^{\beta} - r\overline{y}_r\right] \\
=& \frac{1}{n}\left[n\overline{y}_r\left(\frac{q_2\overline{x}_r+(1-q_2)\overline{X}}{\overline{X}}\right)^{\beta}\right] = \overline{y}_r\left(\frac{q_2\overline{x}_r+(1-q_2)\overline{X}}{\overline{X}}\right)^{\beta}
\end{aligned} \tag{3.5}$$

By using the above imputation technique, estimator for the mean of population is obtained as:

$$P_2{}^* = \overline{y}_r \left(\frac{q_2 \overline{x}_r + (1-q_2)\overline{X}}{\overline{X}} \right)^{\beta} , \tag{3.6}$$

where 0$\leq q_2 \leq$1 and β is a chosen constant.

4 Developed Estimators' Bias and MSE

Considering the developed generalized class of estimator $P_1{}^* = \overline{y}_r \left(\frac{q_1 \overline{x}_n + (1-q_1)\overline{X}}{\overline{X}} \right)^{\alpha}$

Making use of the notations specified in section **2**, the estimator previously indicated is expressed as:

$$\begin{aligned} P_1{}^* &= \overline{Y}\left(1+\epsilon_0\right) \left(\frac{q_1 \overline{X}(1+\epsilon_2) + (1-q_1)\overline{X}}{\overline{X}} \right)^{\alpha} \\ &= \overline{Y}\left(1+\epsilon_0\right)\left(1+q_1\epsilon_2\right)^{\alpha} \\ &= \overline{Y}\left(1+\epsilon_0+q_1\alpha\epsilon_2+q_1\alpha\epsilon_0\epsilon_2+\frac{\alpha(\alpha-1)}{2}q_1^2\epsilon_2^2\right) \end{aligned} \tag{4.1}$$

Subtracting $\overline{Y}$ from the expression (4.1), we get

$$P_1{}^* - \overline{Y} = \overline{Y}\left(\epsilon_0+q_1\alpha\epsilon_2+q_1\alpha\epsilon_o\epsilon_2+\frac{\alpha(\alpha-1)}{2}q_1^2\epsilon_2^2\right) \tag{4.2}$$

Assuming expectation on expression (4.2), we get Bias($P_1{}^*$) as:

$$\text{Bias}(P_1{}^*) = \overline{Y} f_n \left(\frac{\alpha(\alpha-1)}{2} q_1^2 C_x^2 + q_1\alpha\rho C_y C_x \right) \tag{4.3}$$

Taking square and expectation on equation (4.2), MSE of the estimator $P_1{}^*$ to the 1^{st} order of approximation is acquired as:

$$\text{MSE}(P_1{}^*) = \overline{Y}^2 \left[f_r C_y^2 + f_n \left(q_1^2\alpha^2 C_x^2 + 2q_1\alpha\rho C_y C_x \right) \right] \tag{4.4}$$

Partially differentiating MSE($P_1{}^*$) with respect to (w.r.t.) α and equating it to zero, we get $\alpha_{opt.}$ as:

$$\alpha_{opt.} = -\frac{\rho C_y}{C_x}\frac{1}{q_1} \tag{4.5}$$

Now, putting $\alpha_{opt.}$ in MSE($P_1{}^*$), we acquire minimum MSE of $P_1{}^*$ as:

$$\text{min. MSE}(P_1{}^*) = \overline{Y}^2 \left(f_r C_y^2 - f_n \rho^2 C_y^2 \right) \tag{4.6}$$

In the similar manner, considering the developed generalized class of estimator $P_2{}^* = \overline{y}_r \left(\frac{q_2 \overline{x}_r + (1-q_2)\overline{X}}{\overline{X}} \right)^{\beta}$

Making use of the notations specified in section **2**, the estimator previously indicated is expressed as:

$$\begin{aligned} P_2{}^* &= \overline{Y}\,(1+\epsilon_0)\left(\frac{q_2\overline{X}(1+\epsilon_1)+(1-q_2)\overline{X}}{\overline{X}}\right)^{\beta} \\ &= \overline{Y}\,(1+\epsilon_0)\,(1+q_2\epsilon_1)^{\beta} \\ &= \overline{Y}\left(1+\epsilon_0+q_2\beta\epsilon_1+q_2\beta\epsilon_0\epsilon_1+\frac{\beta(\beta-1)}{2}q_2^2\epsilon_1^2\right) \end{aligned} \tag{4.7}$$

Subtracting $\overline{Y}$ from the expression (4.7), we get

$$P_2{}^*-\overline{Y}=\overline{Y}\left(\epsilon_0+q_2\beta\epsilon_1+q_2\beta\epsilon_o\epsilon_1+\frac{\beta(\beta-1)}{2}q_2^2\epsilon_1^2\right) \tag{4.8}$$

Assuming expectation on expression (4.8), we get Bias($P_2{}^*$) as:

$$\text{Bias}(P_2{}^*)=\overline{Y}f_r\left(\frac{\beta(\beta-1)}{2}q_2^2C_x^2+q_2\beta\rho C_yC_x\right) \tag{4.9}$$

Taking square and expectation on equation (4.8), MSE of the estimator $P_2{}^*$ to the 1^{st} order of approximation is acquired as:

$$\text{MSE}(P_2{}^*)=\overline{Y}^2f_r\left(C_y^2+q_2^2\beta^2C_x^2+2q_2\beta\rho C_yC_x\right) \tag{4.10}$$

Partially differentiating MSE($P_2{}^*$) w.r.t. β and equating it to zero, we get $\beta_{opt.}$ as:

$$\beta_{opt.}=-\frac{\rho C_y}{C_x}\frac{1}{q_2} \tag{4.11}$$

Now, putting $\beta_{opt.}$ in MSE($P_2{}^*$), we acquire minimum MSE of $P_2{}^*$ as:

$$\text{min. MSE}(P_2{}^*)=\overline{Y}^2f_r(1-\rho^2)C_y^2 \tag{4.12}$$

5 Generalized class of estimators' particular cases

Particular cases of the developed generalized class of estimators are shown in Table 1 and 2:

Table 1: Particular cases of the developed generalized class of estimator $P_1{}^*$

Estimators	α	q_1
$P_1=\overline{y}_r$ Given by Lee et al.(1994)	0	q_1
$P_2=\overline{y}_r\left(\frac{\overline{X}}{\overline{x}_n}\right)$ Given by Ahmed et al.(2006)	-1	1
$P_3=\overline{y}_r\left(\frac{\overline{X}}{q_1\overline{x}_n+(1-q_1)\overline{X}}\right)$ Given by Ahmed et al.(2006)	-1	q_1

Table 2: Particular cases of the developed generalized class of estimator ${P_2}^*$

Estimators	β	q_2
$P_1 = \overline{y}_r$ Given by Lee et al.(1994)	0	q_2
$P_4 = \overline{y}_r \left(\frac{\overline{X}}{\overline{x}_r}\right)$ Given by Ahmed et al.(2006)	-1	1
$P_5 = \overline{y}_r \left(\frac{\overline{X}}{q_2\overline{x}_r + (1-q_2)\overline{X}}\right)$ Given by Ahmed et al.(2006)	-1	q_2

MSEs of the particular cases of the developed generalized class of estimators are:

(i) MSE(P_1)=$\overline{Y}^2 f_r C_y^2$
(ii) MSE(P_2)=$\overline{Y}^2 \left[f_r C_y^2 + f_n \left(C_x^2 - 2\rho C_y C_x\right)\right]$
(iii) MSE(P_3)=$\overline{Y}^2 \left[f_r C_y^2 + f_n \left(q_1^2 C_x^2 - 2q_1\rho C_y C_x\right)\right]$
(iv) MSE(P_4)=$\overline{Y}^2 f_r \left(C_y^2 + C_x^2 - 2\rho C_y C_x\right)$
(v) MSE(P_5)=$\overline{Y}^2 f_r \left(C_y^2 + q_2^2 C_x^2 - 2q_2\rho C_y C_x\right)$

6 Analysis of Comparisons

Within this section, MSEs of the developed generalized class of estimators are compared with its particular cases.

(i) Compared to P_1, ${P_1}^*$ is more proficient if $MSE({P_1}^*) < MSE(P_1)$
$\implies \overline{Y}^2 \left[f_r C_y^2 + f_n \left(q_1^2\alpha^2 C_x^2 + 2q_1\alpha\rho C_y C_x\right)\right] < \overline{Y}^2 f_r C_y^2$
Solving it further, we get
$\implies 2\rho C_y < -q_1\alpha C_x$
$\implies \rho < -\frac{q_1\alpha C_x}{2C_y}$.

(ii) Compared to P_2, ${P_1}^*$ is more proficient if $MSE({P_1}^*) < MSE(P_2)$
$\implies \overline{Y}^2 \left[f_r C_y^2 + f_n \left(q_1^2\alpha^2 C_x^2 + 2q_1\alpha\rho C_y C_x\right)\right] < \overline{Y}^2 \left[f_r C_y^2 + f_n \left(C_x^2 - 2\rho C_y C_x\right)\right]$
Solving it further, we get
$\implies 2\rho C_y < C_x(1 - q_1\alpha)$
$\implies \rho < \frac{C_x(1-q_1\alpha)}{2C_y}$.

(iii) Compared to P_3, ${P_1}^*$ is more proficient if $MSE({P_1}^*) < MSE(P_3)$
$\implies \overline{Y}^2 \left[f_r C_y^2 + f_n \left(q_1^2\alpha^2 C_x^2 + 2q_1\alpha\rho C_y C_x\right)\right] < \overline{Y}^2 \left[f_r C_y^2 + f_n \left(q_1^2 C_x^2 - 2q_1\rho C_y C_x\right)\right]$
Solving it further, we get
$\implies 2\rho C_y < (1-\alpha)q_1 C_x$
$\implies \rho < \frac{(1-\alpha)q_1 C_x}{2C_y}$.

(iv) Compared to P_1, ${P_2}^*$ is more proficient if $MSE({P_2}^*) < MSE(P_1)$
$\implies \overline{Y}^2 f_r \left(C_y^2 + q_2^2\beta^2 C_x^2 + 2q_2\beta\rho C_y C_x\right) < \overline{Y}^2 f_r C_y^2$
Solving it further, we get
$\implies 2\rho C_y < -q_2\beta C_x$

$\implies \rho < \frac{-q_2\beta C_x}{2C_y}$.

(v) Compared to P_4, ${P_2}^*$ is more proficient if $MSE({P_2}^*) < MSE(P_4)$
$\implies \overline{Y}^2 f_r \left(C_y^2 + q_2^2\beta^2 C_x^2 + 2q_2\beta\rho C_y C_x\right) < \overline{Y}^2 f_r \left(C_y^2 + C_x^2 - 2\rho C_y C_x\right)$
Solving it further, we get
$\implies 2\rho C_y < C_x(1 - q_2\beta)$
$\implies \rho < \frac{C_x(1-q_2\beta)}{2C_y}$.

(vi) Compared to P_5, ${P_2}^*$ is more proficient if $MSE({P_2}^*) < MSE(P_5)$
$\implies \overline{Y}^2 f_r \left(C_y^2 + q_2^2\beta^2 C_x^2 + 2q_2\beta\rho C_y C_x\right) < \overline{Y}^2 f_r \left(C_y^2 + q_2^2 C_x^2 - 2q_2\rho C_y C_x\right)$
Solving it further, we get
$\implies 2\rho C_y < (1 - \beta)q_2 C_x$
$\implies \rho < \frac{(1-\beta)q_2 C_x}{2C_y}$.

7 Empirical Study

For empirical study, two different datasets have been taken into consideration.
Data set 1 [Source: Sarndal et al. (1992)]
Values of the population parameters regarding dataset 1 are given as follows:
N=284, $\overline{Y} = 3077.53$, $\overline{X} = 1779.07$, $S_y^2 = 22520027$, $S_x^2 = 18089114$, $C_y = 1.54$, $C_x = 2.39$, q_1=0.5, q_2=0.5, $\alpha_{opt.}$=-1.21, $\beta_{opt.}$=-1.21 and $\rho_{yx} = 0.94$.

Data set 2 [Source: Das (1988)]
Values of the population parameters regarding dataset 2 are given as follows:
N=278, $\overline{Y} = 39.06$, $\overline{X} = 25.11$, $S_y^2 = 3163.65$, $S_x^2 = 1634.35$, $C_y = 1.44$, $C_x = 1.61$, q_1=0.5, q_2=0.5, $\alpha_{opt.}$=-1.28, $\beta_{opt.}$=-1.28 and $\rho_{yx} = 0.72$.

Based on two cases, the empirical investigation of the developed generalized class of estimators and its members are examined:
Case I: SRSWOR is used to take a 20% sample from the population, and 15% of the sample's values are then presumed to be missing.
For dataset 1: N=284, n=57, r=48
For dataset 2: N=278, n=56, r=48

Case II: Again SRSWOR is used to take a 20% sample from the population, and 25% of the sample's values are then presumed to be missing.
For dataset 1: N=284, n=57, r=43
For dataset 2: N=278, n=56, r=42

The formula below is used to determine the Percentage Relative Efficiency(PRE) of the developed generalized class of estimators and its particular cases w.r.t. $P_1 = \overline{y}_r$:
$PRE(P_i) = \frac{MSE(P_1)}{MSE(P_i)} * 100$,
where P_i=$P_1, P_2, P_3, P_4, P_5, P_1^*$ and P_2^*.

Based on Case I and II, table 3 and table 4 present the empirical outcomes of the developed generalized class of estimators and its members for dataset 1 and 2.

Table 3: Estimators' MSE and PRE w.r.t. $P_1 = \overline{y}_r$ (Dataset 1)

Estimators	Case I		Case II	
	MSE	PRE	MSE	PRE
P_1	388602.3	100.00	442979.0	100.00
P_2	229290.6	169.48	283667.4	156.16
P_3	119025.3	326.48	173402.1	255.46
P_4	191919.0	202.48	218774.1	202.48
P_5	55787.55	696.57	63593.85	696.57
${P_1}^*$	**110673.6**	**351.12**	**165050.4**	**268.39**
${P_2}^*$	**45476.71**	**854.51**	**51840.22**	**854.51**

Table 4: Estimators' MSE and PRE w.r.t. $P_1 = \overline{y}_r$ (Dataset 2)

Estimators	Case I		Case II	
	MSE	PRE	MSE	PRE
P_1	54.52	100.00	63.94	100.00
P_2	38.18	142.78	47.60	134.32
P_3	32.26	169.02	41.67	153.43
P_4	34.77	156.78	40.78	156.78
P_5	27.61	197.47	32.38	197.47
${P_1}^*$	**31.07**	**175.46**	**40.49**	**157.91**
${P_2}^*$	**26.18**	**208.26**	**30.70**	**208.26**

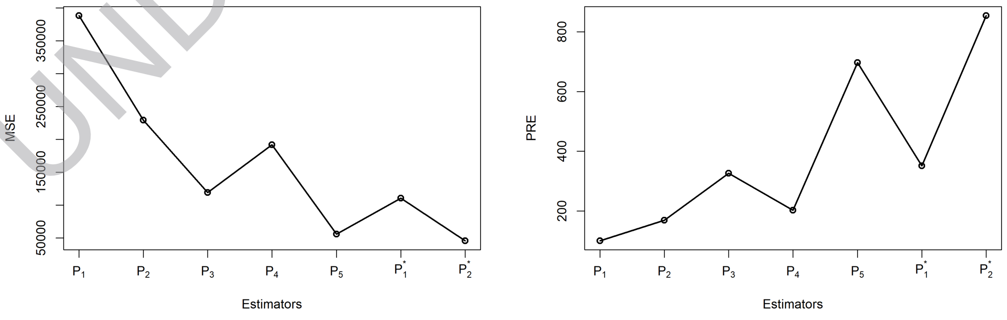

Figure 1: Estimators' MSE and PRE graph for dataset 1 based on Case I

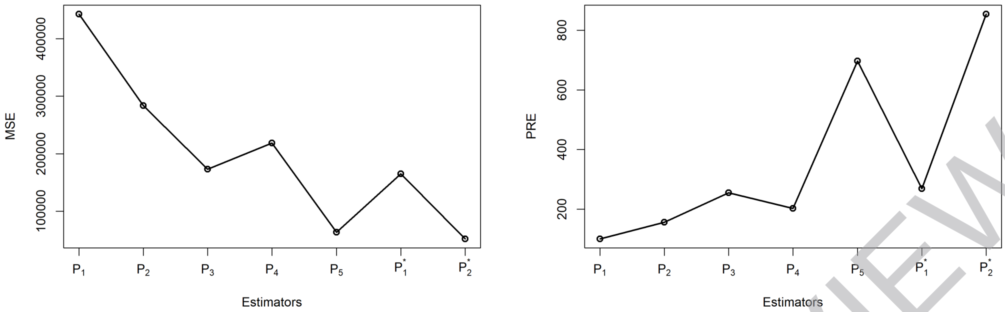

Figure 2: Estimators' MSE and PRE graph for dataset 1 based on Case II

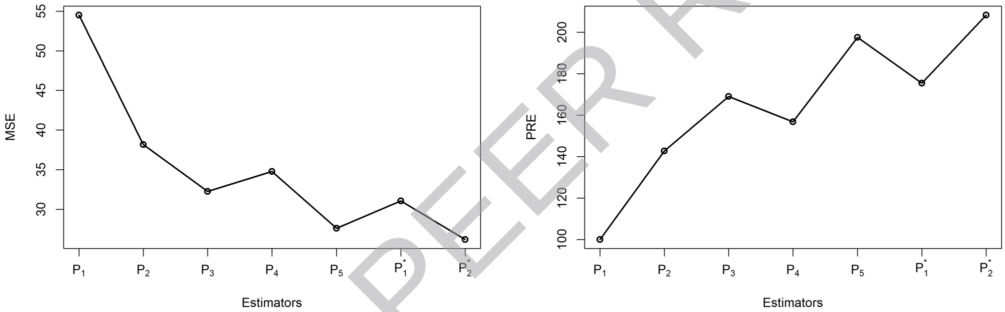

Figure 3: Estimators' MSE and PRE graph for dataset 2 based on Case I

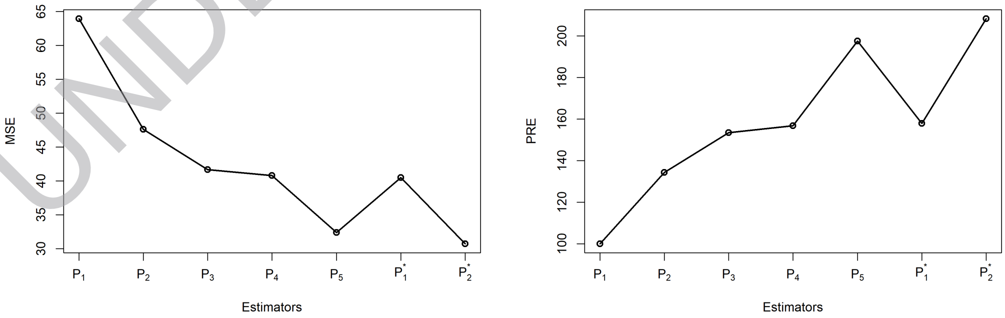

Figure 4: Estimators' MSE and PRE graph for dataset 2 based on Case II

8 Conclusion

In the current paper, generalized class of estimators for population mean estimation based on imputation technique under SRSWOR using ancillary variable have been recommended. The MSE and PRE results of the recommended estimators are presented in tables 4 and 5. These results are also exhibited through graph in figures 1, 2, 3 and 4. Tables and graphs show that the suggested generalized class of estimator P_1^* has lesser MSE than its particular cases, i.e., P_1, P_2 and P_3 and thus higher PRE than them. Similarly, the suggested generalized class of estimator P_2^* has lesser MSE than its particular cases, i.e., P_1, P_4 and P_5 and thus higher PRE than them.

Therefore, from tables and figures, it can be concluded that the developed generalized class of estimators has smaller MSE than the traditional mean and ratio method of imputation. The developed generalized class of estimators also outperform estimators given by Ahmed et al. (2006). Hence, it might be advised to the surveyors to further use the discussed general class of estimators as it has greater efficiency than other traditional estimators considered as their members.

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