Radiation Effect on Natural Convection Flow Past an Oscillatory Moving Infinite Vertical Plate

Abstract

The interplay of free convection with mass transfer on an unsteady, viscous, incompressible fluid flow over an oscillating infinite vertical plate in the presence of radiation has been attempted to be investigated analytically. The fluid is seen as a gray, non-scattering medium that emits and absorbs radiation. The dimensionless governing equations have been solved using the Laplace transform approach. With the aid of various graphs, the expressions for the velocity, temperature, and concentration profiles, as well as for the skin friction, Nusselt number, and Sherwood number are obtained and examined for various physical parameters, such as the thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, radiation parameter, and time.

Keywords: Mass transfer, radiation, free convection, oscillating plate.

AMS subject classification: 76D

1. Introduction

In nature, heat and mass transfer processes occur frequently. It occurs due to variations in temperature, concentration, or both, as well as in different geophysical situations and so on. Heat and mass transfer rates are known to be higher in oscillatory flows. Numerous investigations have been conducted to comprehend its properties in various systems, including chemical reactors, reciprocating engines, and pulse combustors. The Navier-Stokes equation, which deals with the flow of viscous incompressible fluid past a horizontal plate oscillating in its own plate, was first solved exactly by Stokes. The boundary layer near a solid in oscillatory flow of a viscous fluid is referred to in fluid dynamics as the Stokes boundary layer, also known as the oscillatory boundary layer. Alternatively, it describes the analogous situation of a plate oscillating in a viscous fluid at rest, with the oscillation direction (s) parallel to the plate. The effects of small-amplitude and finite-amplitude free-stream oscillations on boundary layer flow past a semi-infinite body were investigated separately by Lin [1] and Lighthill [2]. Lighthill used the integral method to solve the problem, while Lin used the approximation method. An experiment conducted by Hill and Stenning validated Lighthill's expectations [3]. Soundalgekar [4] addressed the flow of a viscous, incompressible fluid past an infinite isothermal vertical plate oscillating in its own plane. Revankar [5] examined the same issue for an oscillating or impulsively initiated plate. Soundalgekar et al. [6] investigated the impact of mass transfer on the flow past an infinite vertical oscillating plate in the presence of a constant heat flux. Fluid mechanics of oscillatory and modulated flows and related applications in heat and mass transfer have been thoroughly studied by Copper et al. [7]. The impact of heat and mass transfer on flow over an oscillating vertical plate with varying temperature has been investigated by Muthucumaraswamy [8]. Ahmed and Das [9] used the Laplace Transform approach to investigate the impact of natural convection on the unsteady, viscous, incompressible fluid flow across an oscillating plate. Using the Laplace transform, Ghosh [10] examined how an induced magnetic field affected MHD Stokes' flow past an oscillating flat plate. Rajput and Shareef [11] have addressed MHD free convective flow with oscillations of an infinite nonconducting vertical flat surface through a porous material with Hall current in a rotating system. They discovered that fluid motion is significantly impacted by variations in radiation, Hall current, porous media, and plate oscillation. Heat transmission on MHD oscillatory flow in a vertical double-passage channel was investigated by Disu et al. [12]. The oscillation frequency significantly affects both the flow velocity and the thermal field, according to the closed-form results.

The combined impact of heat and mass transfer in the presence of thermal radiation has been investigated for use in remote sensing for astronomy and space exploration, nuclear power plants, gas turbines, and different aircraft propulsion mechanisms. In space technology applications, including cosmic flight aerodynamics, rocket propulsion systems, plasma physics, and spacecraft reentry aerothermodynamics, radiation effects become significant when the surrounding fluid temperature is rather high. The radiation effects in free convective flows must be taken into account in these situations. Cess [13] studied how radiation interacts with laminar free convection heat transfer from a vertical plate for an emitting, absorbing fluid in the optically thick area. The radiation effects on flow past an impulsively initiated infinite vertical plate were examined by Das et al. [14], and the Laplace transform method was used to solve the governing equations. Mansour [15] investigated how oscillatory flow via a vertical plate is affected by radiative and free convection. The impact of radiation on the free convective flow past an impulsively initiated vertical plate in the presence of mass transfer was examined by Loganathan and Ganesan [16]. The effects of thermal radiation on unsteady free convective flow across a moving vertical plate in the presence of uniform mass flux and variable temperature were investigated by Muthucumaraswamy and Vijayalakshmi [17]. The MHD oscillatory flow of a viscous fluid in an asymmetric permeable channel with a heat source was analysed by Sasikumar et al. [18], who also visually examined the combined effects of thermal radiation and heat source on the magnetohydrodynamic flow features. The heat and mass transfer of MHD for an unstable viscous oscillatory flow in the presence of radiation and a chemical reaction was examined by Agaie et al. [19]. The radiation effect on MHD free convective flow through a semi-infinite porous vertical plate embedded in a porous material was recently studied by Sharma et al. [20]. They revealed that as the radiation parameter increased, the temperature and velocity dropped, as well as the skin friction at the plate. N. Ahmed et al. [21] have studied the effects of thermal radiation, chemical reaction, heat sink, magnetic field, and thermal diffusion on a steady two-dimensional MHD convection-free heat and mass transfer flow over a semi-infinite porous plate.

Analysing the unsteady free convective heat and mass transfer flow along an oscillating infinite vertical plate while taking thermal radiation into account is the primary purpose of the present work. The investigation is carried out using the Laplace transform technique.

2. Mathematical Formulation of the Problem

In this problem, an oscillating infinite vertical plate is passed by an unsteady natural convective flow of a viscous, incompressible, and radiating fluid. The x' is taken along the plate in the vertically upward direction, the y' axis is normal to the plate and the z' axis is taken along the width of the plate in the rectangular Cartesian coordinate system. The plate and the surrounding fluid had been assumed to be at the same constant temperature T'_∞ and concentration level C'_∞. At time $t' > 0$, the plate is oscillated in its own plane with a velocity $u' = u_o \cos \omega' t'$. At the same time, the plate temperature is also raised to T'_w and the mass is diffused from the plate to the fluid at a uniform speed as the plate temperature is simultaneously raised to T'_w at the same time. The physical model of the problem is shown in Figure 1.

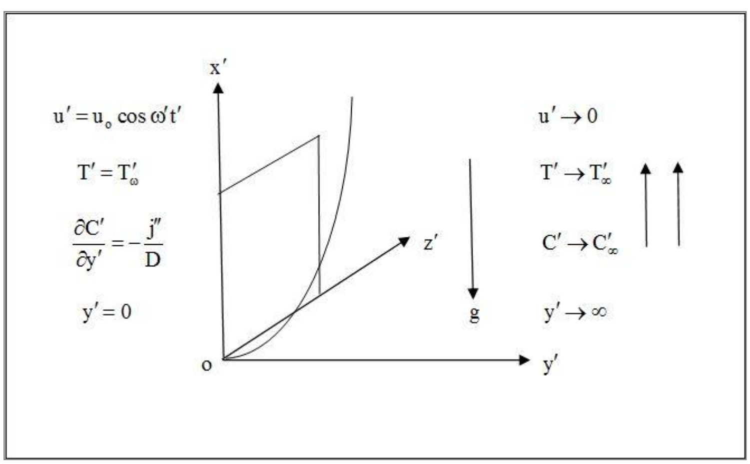

Figure 1: Flow configuration

We now make the following assumptions

1. The plate is not subjected to an intense magnetic field.
2. All the physical properties of fluid are regarded as constant, with the exception of the body force term's influence.
3. The fluid is considered as gray, non-scattering medium that absorbs and emits radiation.
4. The plate is infinite along the x' and z' direction, therefore, all the physical quantities depend on t' and y' only.

Under the above assumptions and according to Boussinesq's approximation

$$\rho = \rho_\infty \left[1 - \beta\left(T' - T'_\infty\right) - \beta^*\left(C' - C'_\infty\right)\right]$$

The set of governing equations of the unsteady flow in dimensional form is given by

Equation of continuity:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum Equation**:**

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \tag{3}$$

Mass Diffusion Equation:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

The initial and boundary conditions are specified as

$$t' \le 0: \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \qquad \text{for all } y' \tag{5}$$

$$t' > 0: \quad \begin{cases} u' = u_o \cos\omega' t', \quad T' = T'_\omega, \quad \dfrac{\partial C'}{\partial y'} = -\dfrac{j''}{D} & \text{at } y' = 0 \\ u' \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty & \text{as } y' \to \infty \end{cases} \tag{6}$$

The local gradient for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right) \tag{7}$$

We assumed that the temperature differences within the flow are sufficiently small that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order of terms, we get

$$T'^4 \cong 4T'^3_\infty - 3T'^4_\infty \tag{8}$$

By using equations (7) and (8). Equation (3) becomes

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\left[16a^*\sigma T_\infty'^3\left(T' - T_\infty'\right)\right] \tag{9}$$

We introduce the following non-dimensional variables and parameters:

$$u = \frac{u'}{u_o},\quad t = \frac{t'u_o^2}{\upsilon},\quad y = \frac{y'u_0}{\upsilon},\quad \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty},\quad C = \frac{C' - C'_\infty}{\left(\dfrac{j''\upsilon}{Du_o}\right)},\quad \omega = \frac{\upsilon\omega'}{u_o^2},\ Sc = \frac{\upsilon}{D}$$

$$\Pr = \frac{\mu C_p}{k},\ R = \frac{16a^*\upsilon^2\sigma T_\infty'^3}{ku_o^2}\ \ Gr = \frac{g\beta\upsilon\left(T'_w - T'_\infty\right)}{u_o^3},\ \ Gm = \frac{g\beta^*\upsilon\left(\dfrac{j''\upsilon}{Du_o}\right)}{u_o^3} \tag{10}$$

With the aid of (10), the equations (2), (9), and (4) with appropriate boundary conditions reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{\Pr}\theta \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{13}$$

Subject to initial and boundary conditions:

$$t \le 0:\ u = 0,\ \theta = 0,\ C = 0 \quad \text{for all } y \tag{14}$$

$$t > 0:\ \begin{Bmatrix} u = \cos\omega t, & \theta = 1, & \dfrac{\partial C}{\partial y} = -1 & \text{at } y = 0 \\ u \to 0, & \theta \to 0, & C \to 0 & \text{at } y \to \infty \end{Bmatrix} \tag{15}$$

All the physical quantities are defined in the Nomenclature

3. Method of Solution

Taking the Laplace transform of the equations (11), (12), and (13), we derive the following ordinary differential equations:

$$\frac{d^2\bar{u}}{dy^2} - s\bar{u} = -Gr\bar{\theta} - Gm\bar{C} \tag{16}$$

$$\frac{d^2\bar{\theta}}{dy^2} - \Pr(s + \alpha)\bar{\theta} = 0 \tag{17}$$

$$\frac{d^2\bar{C}}{dy^2} - sSc\bar{C} = 0 \tag{18}$$

Subject to the boundary conditions

$$\bar{u} = \frac{s}{s^2+\omega^2}, \quad \bar{\theta} = \frac{1}{s}, \quad \frac{d\bar{C}}{dy} = -\frac{1}{s} \quad \text{at } y=0$$
$$\bar{u} = 0, \quad \bar{\theta} = 0, \quad \bar{C} = 0 \quad \text{at } y \to \infty \tag{19}$$

The solutions of the equations (16), (17) and (18) under the conditions (19) are as follows

$$\bar{u} = \left[\frac{1}{2(s+i\omega)} + \frac{1}{2(s-i\omega)} + \frac{a}{a_1}\left(\frac{1}{s-a_1} - \frac{1}{s}\right) + \frac{b}{s^2\sqrt{s}}\right] e^{-y\sqrt{s}}$$
$$- \frac{a}{a_1}\left(\frac{1}{s-a_1} - \frac{1}{s}\right)\frac{e^{-y\sqrt{s+\alpha}\sqrt{Pr}}}{s^2} - b\frac{e^{-y\sqrt{s}\sqrt{Sc}}}{s^2\sqrt{s}} \tag{20}$$

$$\bar{\theta} = \frac{1}{s} e^{-y\sqrt{Pr}\sqrt{s+\alpha}} \tag{21}$$

$$\bar{C} = \frac{1}{s\sqrt{s}\sqrt{Sc}} e^{-y\sqrt{sSc}} \tag{22}$$

On taking the inverse Laplace transform of equations (20), (21), and (22), we get the expressions for velocity, temperature and concentration fields as follows:

$$u = \frac{1}{2}\left[e^{-i\omega t}\phi_1 + e^{i\omega t}\phi_2\right] + \frac{a}{a_1} e^{a_1 t}(\phi_3 - \phi_4) + b(\psi_1 - \psi_2) + \frac{a}{a_1}\left[\phi_5 - \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right)\right] \tag{23}$$

$$\theta = \phi_5 \tag{24}$$

$$C = \frac{1}{\sqrt{Sc}}\left[2\sqrt{\frac{t}{\pi}}\, e^{-\frac{y^2 Sc}{4t}} - y\sqrt{Sc}\ \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)\right] \tag{25}$$

3.1 Skin Friction

The expression for skin friction at the plate, which is in the dimensionless form, is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\left[\frac{1}{2}\left[e^{-i\omega t}\varphi_1 + e^{i\omega t}\varphi_2\right] + \frac{a}{a_1} e^{a_1 t}(\varphi_3 - \varphi_4) + bt\left(1 - \sqrt{Sc}\right) + \frac{a}{a_1}\left(\varphi_5 - \frac{1}{\sqrt{\pi t}}\right)\right] \tag{26}$$

3.2 Nusselt Number

The rate of heat transfer at the plate in terms of the non-dimensional Nusselt number is given by

$$Nu = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0} = \varphi_5 \tag{27}$$

3.3 Sherwood Number

The rate of mass transfer at the plate in terms of the non-dimensional Sherwood number is given by

$$Sh = -\left[\frac{\partial C}{\partial y}\right]_{y=0} = \frac{2}{\sqrt{\pi}} \tag{28}$$

where,

$$a = \frac{Gr}{Pr-1}, \quad b = \frac{Gm}{\sqrt{Sc}\,(Sc-1)}, \quad a_1 = \frac{R}{1-Pr}, \quad \alpha = \frac{R}{Pr}$$

$$\phi_1 = \phi(-i\omega, y, 1, t), \quad \phi_2 = \phi(i\omega, y, 1, t), \quad \phi_3 = (a_1, y, 1, t), \quad \phi_4 = \phi(\alpha + a_1, y, Pr, t)$$

$$\phi_5 = \phi(\alpha, y, Pr, t), \quad \psi_1 = \psi(x, y, t), \quad \psi_2 = \psi(Sc, y, t), \quad \varphi_1 = \varphi(-i\omega, 1, t)$$

$$\varphi_2 = \varphi(i\omega, 1, t), \quad \varphi_3 = \varphi(a_1, 1, t), \quad \varphi_4 = \varphi(\alpha + a_1, Pr, t), \quad \varphi_5 = \varphi(\alpha, Pr, t)$$

$$\phi(x, y, z, t) = \frac{1}{2}\left[e^{y\sqrt{z}\sqrt{x}}\,\mathrm{erfc}\left(\frac{y\sqrt{z}}{2\sqrt{t}} + \sqrt{xt}\right) + e^{-y\sqrt{z}\sqrt{x}}\,\mathrm{erfc}\left(\frac{y\sqrt{z}}{2\sqrt{t}} - \sqrt{xt}\right)\right]$$

$$\psi(x, y, t) = \frac{\sqrt{t}\, e^{-\left(\frac{y\sqrt{x}}{2\sqrt{t}}\right)^2}\left(4t + \left(y\sqrt{x}\right)^2\right)}{3\sqrt{\pi}} - \frac{1}{6}y\sqrt{x}\left[6t + \left(y\sqrt{x}\right)^2\right]\mathrm{erfc}\left(\frac{y\sqrt{x}}{2\sqrt{t}}\right)$$

$$\varphi(x, z, t) = \sqrt{z}\sqrt{x}\ \mathrm{erf}\sqrt{xt} + \frac{\sqrt{z}}{\sqrt{\pi t}}e^{-xt}$$

4. Results and discussion

In order to get physical insight into the problem numerical computations are carried out for the different values of the parameters viz thermal Grashof number, solutal Grashof number, radiation parameter, Prandtl number, Schmidt number, frequency of oscillation and time which are involved in the governing equations on the velocity, temperature and concentration fields, skin friction and the Nusselt number at the plate. Numerical computations for the above fields are performed by assigning some specific arbitrary values to the thermal Grashof number Gr, mass Grashof number Gm, frequency of oscillation ω and time t. The values of the Prandtl number Pr are selected as Pr = 0.71 and Pr =7, which correspond to air and water, respectively at 20ºC, and the values of the Schmidt number Sc are considered as 0.3, 0.78 and 2.01, which stand for Helium, Ammonia and Ethylbenzene, respectively. The numerical results are demonstrated through different graphs.

4.1 Velocity Profile

The velocity profiles against the normal coordinate y for different values of the radiation parameter R, Schmidt number Sc, frequency of oscillation ω, thermal Grashof number Gr, and mass Grashof number Gm are exhibited in Figures 2 to 6 . These figures demonstrate that the velocity grows consistently close to the plate until it reaches a specific point, at which point it starts to decrease.

It is clear from Figure 2 that when radiation is present, the momentum boundary layer deepens and the velocity decreases as the radiation parameter R increases. Higher Schmidt number Sc values slow down fluid flow, as shown in Figure 3, suggesting that velocity increases with mass diffusivity.

Figure 4 illustrates how the fluid velocity decreases with increasing oscillation frequency ω. In the meantime, Figures 5 and 6 show that the fluid flow is accelerated by increases in the mass Grashof number Gm and thermal Grashof number Gr. This is explained by the increased fluid velocity caused by the increasing buoyancy force.

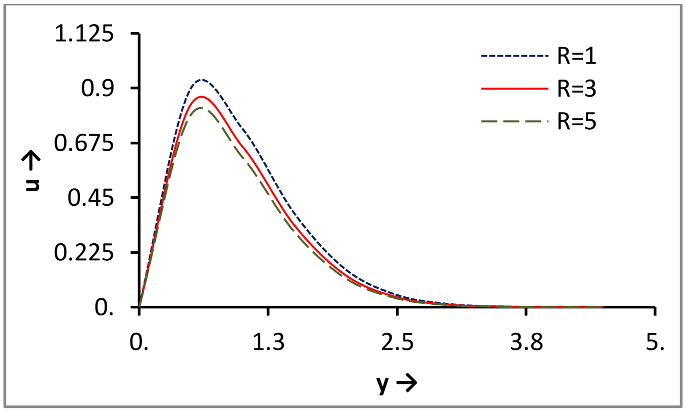

Figure 2: Velocity profiles for different R and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.78, t = 0.5, ω = π

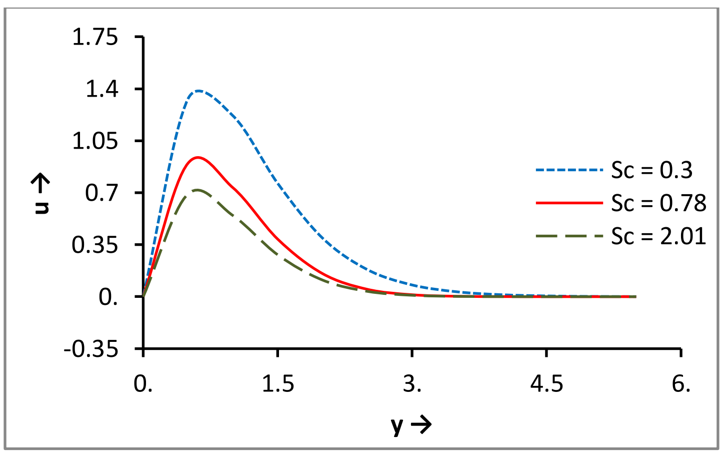

Figure 3: Velocity profiles for different Sc and Pr = 0.71, Gr = 3, Gm = 5, R = 1, t = 0.5, ω = π

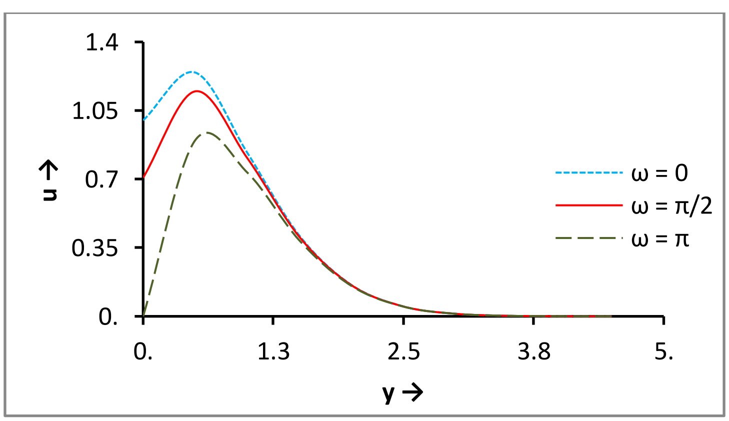

Figure 4: Velocity profiles for different ω and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.78, R = 1, t = 0.5

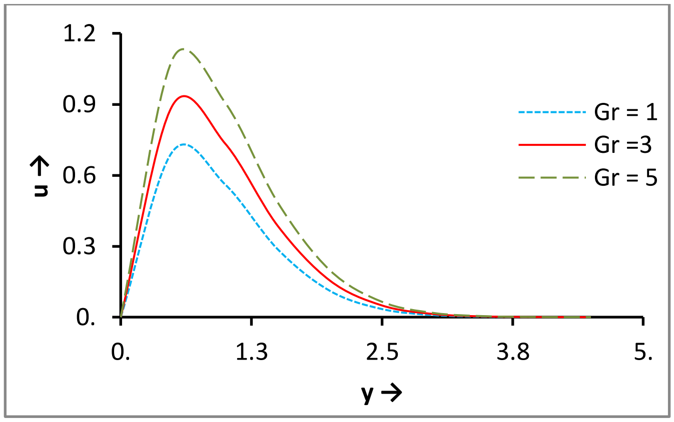

Figure 5: Velocity profiles for different Gr and Pr = 0.71, Gm = 5, Sc = 0.78, R =1, t = 0.5, ω = π

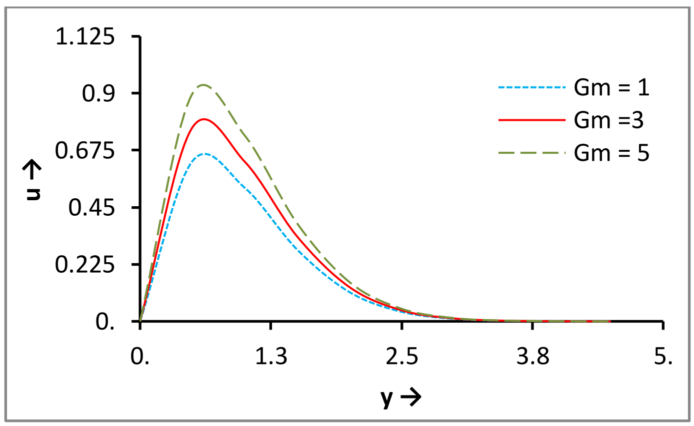

Figure 6: Velocity profiles for different Gm and Pr = 0.71, Gr = 3, Sc = 0.78, R =1, t = 0.5, ω = π

4.2 Temperature Profile

Figures 7, 8, and 9 show how radiation, Prandtl number, and time affect the temperature field over the boundary layer, respectively. Figures 7 and 8 show that when R and Pr are increased, the fluid's temperature level decreases. In other words, it is seen that the thermal boundary layer thickens and shrinks when radiation is present. Once more, a decrease in thermal diffusivity is physically correlated with an increase in Pr. In other words, the thermal boundary layer grows as a result of thermal diffusivity.

On the other hand, Figure 9 shows that the fluid temperature rises with time. Furthermore, the temperature rises close to the plate and then progressively drops, asymptotically approaching zero in the free-stream area.

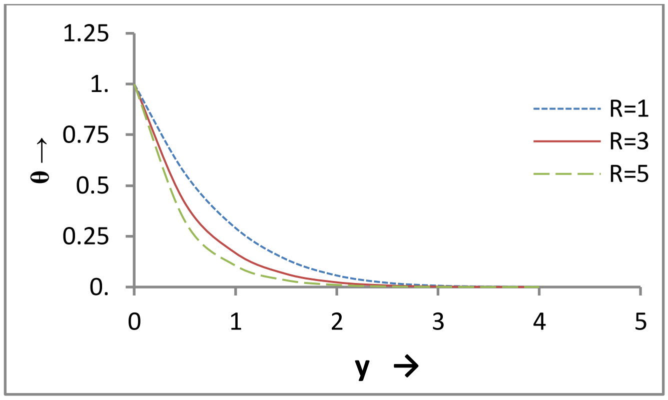

Figure 7: Temperature profiles for different R and Pr = 0.71, t = 0.5

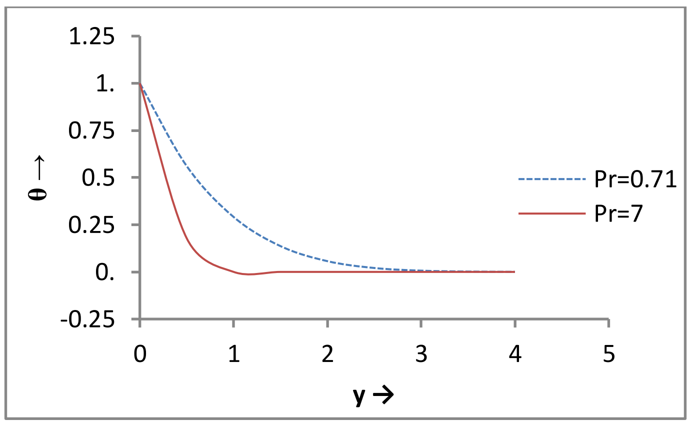

Figure 8: Temperature profiles for different Pr and R = 1, t = 0.5

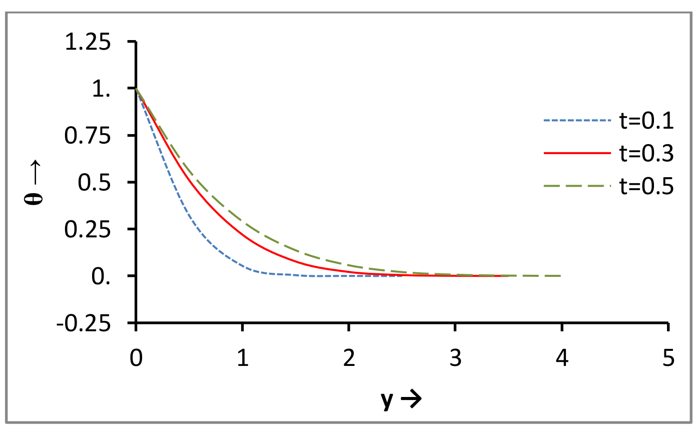

Figure 9: Temperature profiles for different t and Pr = 0.71, R = 1

4.3 Concentration Profile

Figures 10 and 11 show how the concentration profile's behaviour changes for various values of the Schmidt number Sc and time t. It has been seen that the concentration profile decreases as the Schmidt number rises. It is physically true since molecular mass diffusivity decreases with increasing Schmidt number, which results in a thinner concentration boundary layer.

Furthermore, as time (t) increases, the fluid concentration rises. The concentration field asymptotically drops to zero in the free-stream area, just like the temperature field.

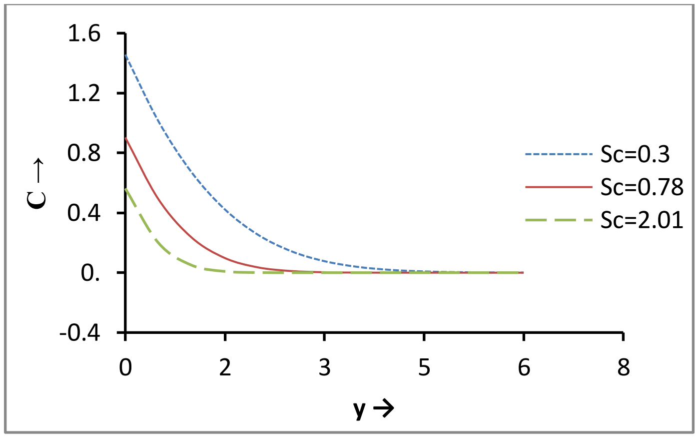

Figure 10: Concentration profiles for different Sc and t = 0.5

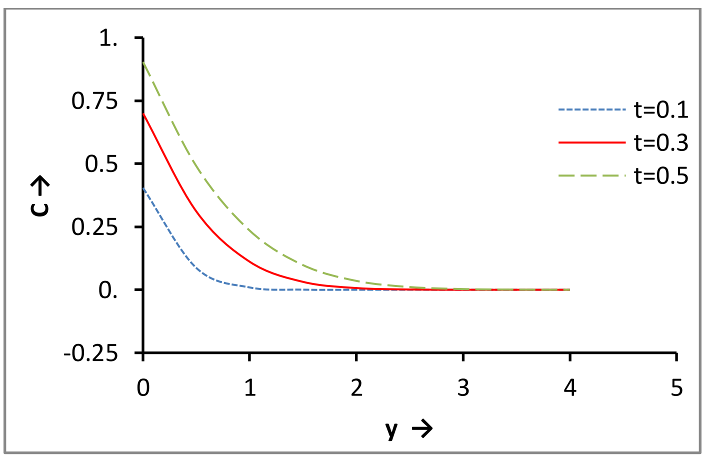

Figure 11: Concentration profiles for different t and Sc = 0.78

4.4 Skin Friction

For various values of the radiation parameter (R), thermal Grashof number (Gr), mass Grashof number (Gm), and oscillation frequency (ω), Figures 12 to 15 show how skin friction varies with respect to time (t). The results show that the skin friction coefficient rises with greater Gr, Gm, and ω values but falls with increasing R. To put it another way, the buoyancy forces Gr and Gm, as well as oscillatory motion ω, increase the viscous drag on the plate while it decreases with increasing radiation.

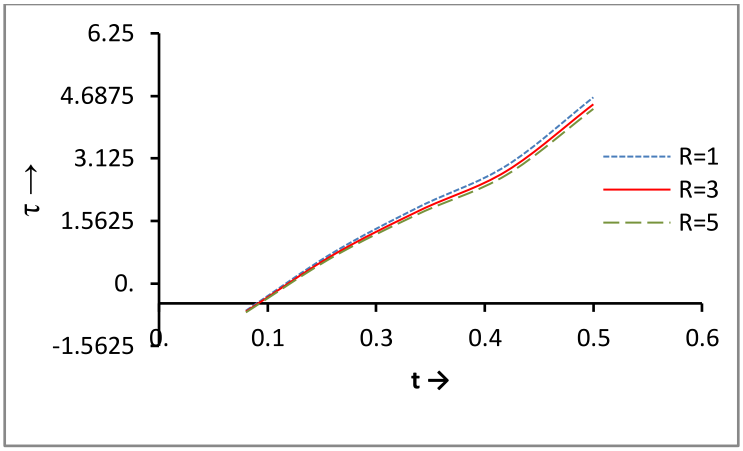

Figure 12: Skin friction for different R and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.3, ω = π

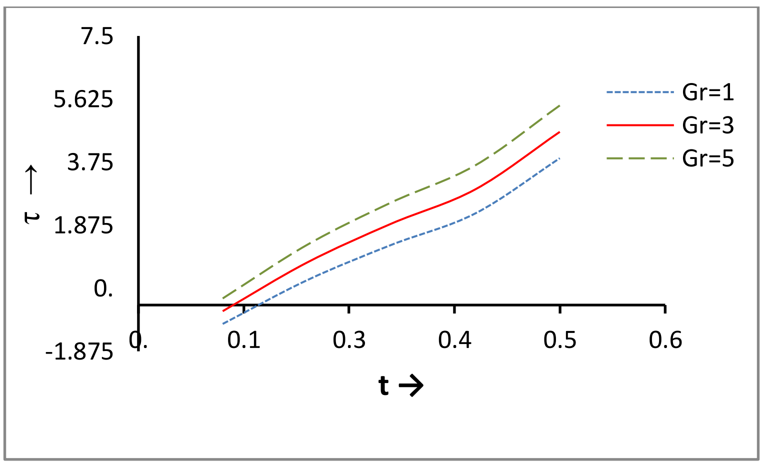

Figure 13: Skin friction for different Gr and Pr = 0.71, Gm = 5, Sc = 0.3, R=1, ω = π

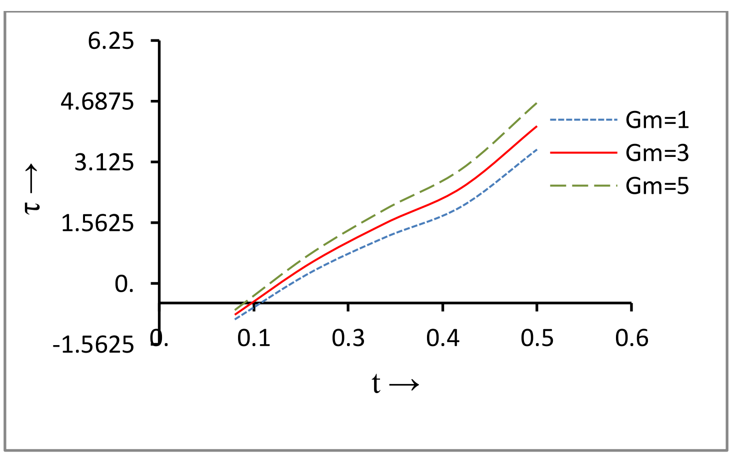

Figure 14: Skin friction for different Gm and Pr = 0.71, Gr = 3, Sc = 0.3, R =1, ω = π

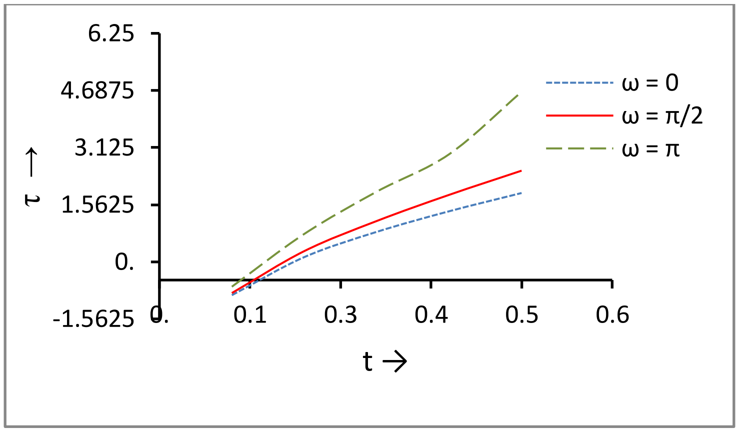

Figure 15: Skin friction for different ω and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.3, R=1

4.5 Nusselt number

Figures 16 and 17 show the coefficient of heat transfer expressed in terms of Nusselt numbers. It is evident that as the values of the radiation parameter and the Prandtl number rise, so does the Nusselt number. This suggests that the rate of heat transfer at the plate is increased by fluids with higher radiative properties.

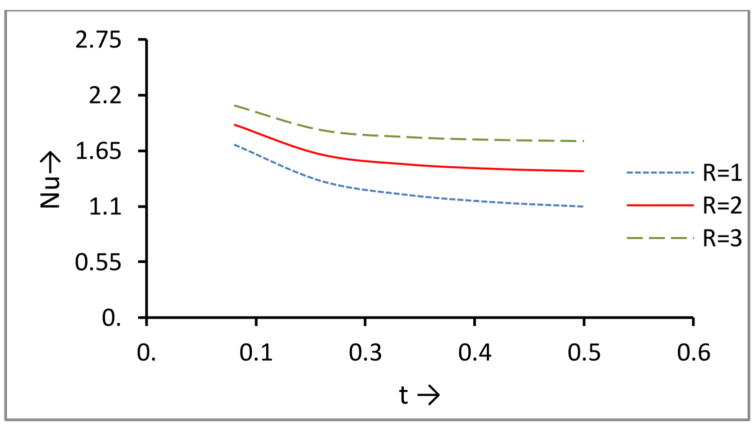

Figure 16: Nusselt number for different R and Pr = 0.71

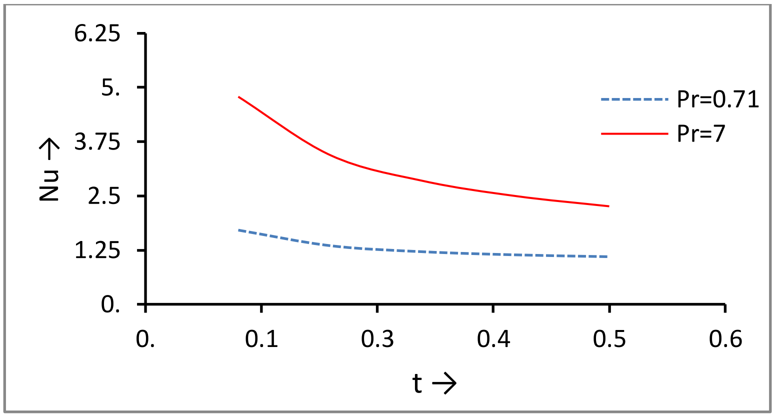

Figure 17: Nusselt number for different Pr and R = 1

4.6 Sherwood Number

It is observed from equation (28) that the Sherwood number at the plate is twice the reciprocal of the square root of *pi,* which is constant. Consequently, it can be said that the rate of solutal concentration at the plate stays constant.

5. CONCLUSIONS

The Laplace Transform Technique is used to examine the analytical solutions of unsteady free convective flow across an oscillating plate in the presence of thermal radiation. The following findings are reached.

1. As various values of radiation R increase, velocity decreases.
2. The thermal boundary layer thickens and decreases when radiation is present.
3. The concentration profile decreases when the Schmidt number rises.
4. As R increases, the coefficient of skin friction declines.
5. The Nusselt number rises as the radiation parameter R increases.

Nomenclature

a^* absorption coefficient

C' species concentration in the fluid

C'_w concentration of the plate

C'_∞ concentration in the fluid far away from the plate

C dimensionless concentration

C_p specific heat at constant pressure

D mass diffusion coefficient

g acceleration due to gravity

Gr thermal Grashof number

Gm mass Grashof number

j'' mass flux per unit area at the plate

k thermal conductivity of the fluid

Pr Prandtl number

q_r radiation heat flux

R Radiation parameter

Sc Schmidt number

T' temperature of the fluid near the plate

T'_w temperature of the plate

T'_∞ temperature of the fluid far away from the plate

t' time

t dimensionless time

u' velocity of the fluid in x' direction

u_o velocity of the plate

u dimensionless velocity

y' coordinate axis normal to the plate

y dimensionless coordinate axis normal to the plate

β volumetric coefficient of thermal expansion

β^* volumetric coefficient of solutal expansion

μ coefficient of viscosity

ρ density

ρ_∞ reference density

υ kinematic viscosity

σ^* Stefean Boltzment constant

τ dimensionless skin friction

ω' dimensional frequency of oscillation

ω dimensionless frequency of oscillation

θ dimensionless temperature

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