Radiation Effect on Natural Convection Flow Past an Oscillatory Moving Infinite Vertical Plate

Abstract

An analytical investigation has been conducted into the collaboration of free convection with mass transfer in an unsteady, viscous, incompressible fluid flow past an oscillatory, endless, vertical plate in the presence of radiation. The fluid is considered gray, absorbing/emitting radiation, yet a non-scattering medium. The Laplace transform scheme has been adopted to solve the dimensionless governing equations. The expressions for the velocity, temperature and concentration profiles and in addition for the skin friction, Nusselt number and Sherwood number are obtained and the effects of various physical parameters viz, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, radiation parameter and time are discussed with the assistance of various graphs.

Keywords: Mass transfer, radiation, free convection, oscillating plate.

AMS subject classification: 76D

1. Introduction

Processes including coupled heat and mass transfer often happen in nature. It happens because of temperature differences, concentration differences, or the amalgamation of these two in various geophysical cases and so forth. Oscillatory flows are known to result in higher rates of heat and mass transfer. Many studies have been done to understand its characteristics in different systems, such as reciprocating engines, pulse combustors, chemical reactors, etc. Stokes first found the exact solution of the Navier–Stokes equation, which is concerned with the flow of viscous incompressible fluid past a horizontal plate oscillating in its own plate. In fluid dynamics, the Stokes boundary layer, or oscillatory boundary, refers to the boundary layer close to a solid in the oscillatory flow of a viscous fluid. Or, it refers to the similar case of an oscillating plate in a viscous fluid at rest, with the oscillation direction (s) parallel to the plate. Lin [1] and Lighthill [2] studied the effects of finite–amplitude and small amplitude free–stream oscillations on boundary layer flow past a semi-infinite body, respectively and independently. Lin [1] solved the problem by the approximation method, whereas Lighthill [2] solved it by the integral method. Lighthill's predictions were confirmed by an experiment performed by Hill and Stenning [3]. The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was discussed by Soundalgekar [4]. The same problem was considered by Revankar [5] for an impulsively started or oscillating plate. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et. al. [6], and Copper et. al. [7] have made a detailed study of the fluid mechanics of oscillatory and modulated flows and associated applications in heat and mass transfer. Muthucumaraswamy [8] has studied the effect of heat and mass transfer on flow past an oscillatory vertical plate with variable temperature. Ahmed and Das [9] studied the effect of natural convection on the unsteady, viscous, incompressible fluid flow over an oscillating plate by using the Laplace Transform technique. Ghosh [10] investigated the effect of induced magnetic field on MHD Stokes' flow past an oscillating flat plate by applying the Laplace transform. MHD free convective flow with oscillations of an infinite nonconducting vertical flat surface through a porous medium with Hall current in a rotating system has been discussed by Rajput and Shareef [11]. They found that changes in plate oscillation, porous medium, radiation and Hall current have significant effects on fluid motion. Disu et al. [12] carried out an investigation on heat transfer on MHD oscillatory flow in a vertical double-passage channel. The closed-form results revealed that the oscillatory frequency has a significant impact on both the flow velocity and the thermal field.

The combined effect of heat and mass transfer in the presence of thermal radiation has been studied for its application in nuclear power plants, gas turbines and various propulsion devices for aircraft, remote sensing for astronomy and space exploration. When the temperature of the surrounding fluid is rather high, radiation effects play an important role, and this situation exists in space technology applications such as cosmic flight aerodynamics, rocket propulsion systems, plasma physics and spacecraft reentry aerothermodynamics. In these cases, it is necessary to consider the radiation effects in free convective flows. The interaction of radiation with laminar free convection heat transfer from a vertical plate was investigated by Cess [13] for an absorbing, emitting fluid in the optically thick region. Das *et al* [14] analysed the radiation effects on flow past an impulsively started infinite vertical plate, and the governing equations were solved by the Laplace transform technique. Mansour [15] studied the radiative and free convection effects on the oscillatory flow past a vertical plate. Loganathan and Ganesan [16] considered the effect of radiation on the free convective flow past an impulsively started vertical plate in the presence of mass transfer. Muthucumaraswamy and Vijayalakshmi [17] studied the thermal radiation effects on unsteady free convective flow over a moving vertical plate in the presence of variable temperature and uniform mass flux. Sasikumar et al. [2019] analysed the MHD oscillatory flow of a viscous fluid in an asymmetric permeable channel with a heat source, and graphically investigated the combined effects of thermal radiation and heat source on the magnetohydrodynamic flow characteristics. Agaie et al. [2021] studied the heat and mass transfer of MHD for an unsteady viscous oscillatory flow in the presence of radiation and chemical reaction. In recent work, Sharma et al. [2024] investigated the radiation effect on MHD free convective flow past a semi-infinite porous vertical plate through a porous medium. They found that increasing the radiation parameter decreased the velocity and temperature, and the skin friction at the plate decreased with increasing values of the radiation parameter.

The primary objective of this study is to analyse the unsteady free convective heat and mass transfer flow along an oscillating infinite vertical plate, considering the effects of thermal radiation. The investigation is carried out using the Laplace Transform Technique.

2. Mathematical Formulation of the Problem

In this problem, we consider an unsteady natural convective flow of a viscous, incompressible and radiating fluid past an oscillatory infinite vertical plate. The x' is taken along the plate in the vertically upward direction, y' axis is normal to the plate and z' axis is taken along the width of the plate in the rectangular Cartesian coordinate system. Initially, it is assumed that the plate and the surrounding fluid were at the same constant temperature T'_∞ with a concentration level C'_∞. At time $t' > 0,$ the plate is oscillated with a velocity $u' = u_o \cos\omega' t'$ in its own plane. At the same time, the plate temperature is also raised to T'_w and the mass is diffused from the plate to the fluid at a uniform speed. The physical model of the problem is shown in Figure 1.

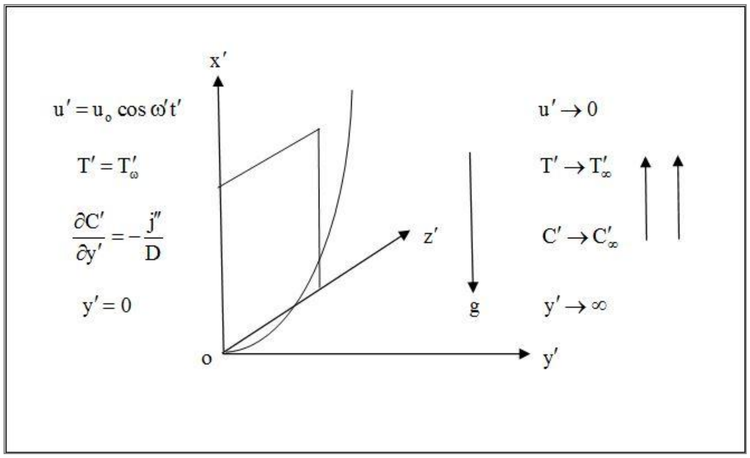

Figure 1: Flow configuration

We now make the following assumptions

1. A magnetic field of intensity is not applied to the plate.
2. All the physical properties of fluid are considered to be constant except for the influence of the body force term.
3. The fluid is considered as gray, absorbing/emitting radiation, but a non – scattering medium.
4. The plate is infinite along the x' and z' direction, therefore, all the physical quantities depend on t' and y' only.

Under the above assumptions and according to Boussinesq's approximation

$$\rho = \rho_\infty\left[1-\beta\left(T'-T'_\infty\right)-\beta^*\left(C'-C'_\infty\right)\right]$$

The set of governing equations of the unsteady flow in dimensional form is given by

Equation of continuity:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum Equation**:**

$$\frac{\partial u'}{\partial t'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T'-T'_\infty\right) + g\beta^*\left(C'-C'_\infty\right) \tag{2}$$

Energy Equation:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'} \tag{3}$$

Mass Diffusion Equation:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

The initial and boundary conditions are specified as

$$t' \le 0: \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \qquad \text{for all } y' \tag{5}$$

$$t' > 0: \quad \left\{\begin{array}{l} u' = u_o\cos\omega' t', \quad T' = T'_\omega, \quad \dfrac{\partial C'}{\partial y'} = -\dfrac{j''}{D} \qquad \text{at } y' = 0 \\ u' \to 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y' \to \infty \end{array}\right\} \tag{6}$$

The local gradient for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right) \tag{7}$$

We assumed that the temperature differences within the flow are sufficiently small that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order of terms, we get

$$T'^4 \cong 4T'^3_\infty - 3T'^4_\infty \tag{8}$$

By using equations (7) and (8). Equation (3) becomes

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\left[16a^*\sigma T_\infty'^3\left(T' - T_\infty'\right)\right] \tag{9}$$

We introduce the following non-dimensional variables and parameters:

$$u = \frac{u'}{u_o},\quad t = \frac{t'u_o^2}{\upsilon},\quad y = \frac{y'u_0}{\upsilon},\quad \theta = \frac{T' - T_\infty'}{T_\omega' - T_\infty'},\quad C = \frac{C' - C_\infty'}{\left(\dfrac{j''\upsilon}{Du_o}\right)},\quad \omega = \frac{\upsilon\omega'}{u_o^2},\quad Sc = \frac{\upsilon}{D}$$

$$Pr = \frac{\mu C_p}{k}\ ,R = \frac{16a^*\upsilon^2\sigma T_\infty'^3}{ku_o^2}\quad Gr = \frac{g\beta\upsilon\left(T_w' - T_\infty'\right)}{u_o^3},\quad Gm = \frac{g\beta^*\upsilon\left(\dfrac{j''\upsilon}{Du_o}\right)}{u_o^3} \tag{10}$$

With the aid of (10), the equations (2), (9), and (4) with appropriate boundary conditions reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{Pr}\theta \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{13}$$

Subject to initial and boundary conditions:

$$t \le 0:\ u = 0,\ \theta = 0,\ C = 0 \quad \text{for all } y \tag{14}$$

$$t > 0:\ \begin{cases} u = \cos\omega t,\quad \theta = 1,\quad \dfrac{\partial C}{\partial y} = -1 \quad \text{at } y = 0 \\ u \to 0,\quad \theta \to 0,\quad C \to 0 \quad \text{at } y \to \infty \end{cases} \tag{15}$$

All the physical quantities are defined in the Nomenclature

3. Method of Solution

Taking the Laplace transform of the equations (11), (12), and (13), we derive the following ordinary differential equations:

$$\frac{d^2\bar{u}}{dy^2} - s\bar{u} = -Gr\bar{\theta} - Gm\bar{C} \tag{16}$$

$$\frac{d^2\bar{\theta}}{dy^2} - Pr(s + \alpha)\bar{\theta} = 0 \tag{17}$$

$$\frac{d^2\bar{C}}{dy^2} - sSc\bar{C} = 0 \tag{18}$$

Subject to the boundary conditions

$$\bar{u} = \frac{s}{s^2 + \omega^2}, \quad \bar{\theta} = \frac{1}{s}, \quad \frac{d\bar{C}}{dy} = -\frac{1}{s} \quad \text{at } y = 0$$
$$\bar{u} = 0, \quad \bar{\theta} = 0, \quad \bar{C} = 0 \quad \text{at } y \to \infty \tag{19}$$

The solutions of the equations (16), (17) and (18) under the conditions (19) are as follows

$$\bar{u} = \left[\frac{1}{2(s+i\omega)} + \frac{1}{2(s-i\omega)} + \frac{a}{a_1}\left(\frac{1}{s-a_1} - \frac{1}{s}\right) + \frac{b}{s^2\sqrt{s}}\right] e^{-y\sqrt{s}}$$
$$- \frac{a}{a_1}\left(\frac{1}{s-a_1} - \frac{1}{s}\right)\frac{e^{-y\sqrt{s+\alpha}\sqrt{Pr}}}{s^2} - b\frac{e^{-y\sqrt{s}\sqrt{Sc}}}{s^2\sqrt{s}} \tag{20}$$

$$\bar{\theta} = \frac{1}{s} e^{-y\sqrt{Pr}\sqrt{s+\alpha}} \tag{21}$$

$$\bar{C} = \frac{1}{s\sqrt{s}\sqrt{Sc}} e^{-y\sqrt{sSc}} \tag{22}$$

On taking the inverse Laplace transform of equations (20), (21), and (22), we get the expressions for velocity, temperature and concentration fields as follows:

$$u = \frac{1}{2}\left[e^{-i\omega t}\phi_1 + e^{i\omega t}\phi_2\right] + \frac{a}{a_1} e^{a_1 t}\left(\phi_3 - \phi_4\right) + b\left(\psi_1 - \psi_2\right) + \frac{a}{a_1}\left[\phi_5 - \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right)\right] \tag{23}$$

$$\theta = \phi_5 \tag{24}$$

$$C = \frac{1}{\sqrt{Sc}}\left[2\sqrt{\frac{t}{\pi}}\, e^{-\frac{y^2 Sc}{4t}} - y\sqrt{Sc}\ \operatorname{erfc}\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)\right] \tag{25}$$

3.1 Skin Friction

The expression for skin friction at the plate, which is in the dimensionless form, is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\left[\frac{1}{2}\left[e^{-i\omega t}\varphi_1 + e^{i\omega t}\varphi_2\right] + \frac{a}{a_1} e^{a_1 t}\left(\varphi_3 - \varphi_4\right) + bt\left(1 - \sqrt{Sc}\right) + \frac{a}{a_1}\left(\varphi_5 - \frac{1}{\sqrt{\pi t}}\right)\right] \tag{26}$$

3.2 Nusselt Number

The rate of heat transfer at the plate in terms of the non-dimensional Nusselt number is given by

$$Nu = -\left[\frac{\partial \theta}{\partial y}\right]_{y=0} = \varphi_5 \tag{27}$$

3.3 Sherwood Number

The rate of mass transfer at the plate in terms of the non-dimensional Sherwood number is given by

$$\mathrm{Sh}=-\left[\frac{\partial C}{\partial y}\right]_{y=0}=\frac{2}{\sqrt{\pi}} \tag{28}$$

where,

$$a=\frac{Gr}{Pr-1},\quad b=\frac{Gm}{\sqrt{Sc}\,(Sc-1)},\quad a_1=\frac{R}{1-Pr},\quad \alpha=\frac{R}{Pr}$$

$$\phi_1=\phi(-i\omega,y,1,t),\quad \phi_2=\phi(i\omega,y,1,t),\quad \phi_3=(a_1,y,1,t),\quad \phi_4=\phi(\alpha+a_1,y,Pr,t)$$

$$\phi_5=\phi(\alpha,y,Pr,t)\ ,\quad \psi_1=\psi(x,y,t)\ ,\quad \psi_2=\psi(Sc,y,t),\quad \varphi_1=\varphi(-i\omega,1,t)$$

$$\varphi_2=\varphi(i\omega,1,t)\ ,\quad \varphi_3=\varphi(a_1,1,t)\ ,\quad \varphi_4=\varphi(\alpha+a_1,Pr,t)\ ,\quad \varphi_5=\varphi(\alpha,Pr,t)$$

$$\phi(x,y,z,t)=\frac{1}{2}\left[e^{y\sqrt{z}\sqrt{x}}\operatorname{erfc}\left(\frac{y\sqrt{z}}{2\sqrt{t}}+\sqrt{xt}\right)+e^{-y\sqrt{z}\sqrt{x}}\operatorname{erfc}\left(\frac{y\sqrt{z}}{2\sqrt{t}}-\sqrt{xt}\right)\right]$$

$$\psi(x,\ y,\ t)=\frac{\sqrt{t}\ e^{-\left(\frac{y\sqrt{x}}{2\sqrt{t}}\right)^2}\left(4t+\left(y\sqrt{x}\right)^2\right)}{3\sqrt{\pi}}-\frac{1}{6}y\sqrt{x}\left[6t+\left(y\sqrt{x}\right)^2\right]\operatorname{erfc}\left(\frac{y\sqrt{x}}{2\sqrt{t}}\right)$$

$$\varphi(x,z,t)=\sqrt{z}\sqrt{x}\ \operatorname{erf}\sqrt{xt}+\frac{\sqrt{z}}{\sqrt{\pi t}}e^{-xt}$$

4. Results and discussion

In order to get physical insight into the problem numerical computations are carried out for the different values of the parameters viz thermal Grashof number, solutal Grashof number, radiation parameter, Prandtl number, Schmidt number, frequency of oscillation and time which are involved in the governing equations on the velocity, temperature and concentration fields, skin friction and the Nusselt number at the plate. Numerical computations for the above fields are performed by assigning some specific arbitrary values to the thermal Grashof number Gr, mass Grashof number Gm, frequency of oscillation ω and time t. The values of the Prandtl number Pr are chosen as Pr = 0.71 and Pr =7, which correspond to air and water, respectively at 20°C, and the values of the Schmidt number Sc are considered as 0.3, 0.78 and 2.01, which represent respectively Helium, Ammonia and Ethylbenzene. The numerical results are displayed through different graphs.

4.1 Velocity Profile

The velocity profiles against the normal coordinate y are presented in Figures 2 to 6 for varying values of the radiation parameter R, Schmidt number Sc, frequency of oscillation ω, thermal Grashof number Gr, and mass Grashof number Gm. These figures reveal that the velocity near the plate increases steadily up to a certain point before decreasing farther away from the plate.

From Figure 2, it is evident that an increase in the radiation parameter R reduces the velocity, and the momentum boundary layer thickens in the presence of radiation. Figure 3 indicates that higher values of the Schmidt number Sc decelerate the fluid flow, implying that the velocity rises with increased mass diffusivity.

Figure 4 shows that a higher frequency of oscillation ω diminishes the fluid velocity. Meanwhile, Figures 5 and 6 demonstrate that increases in the thermal Grashof number Gr and mass Grashof number Gm accelerate the fluid flow. This can be attributed to the enhanced buoyancy force, which increases the fluid velocity.

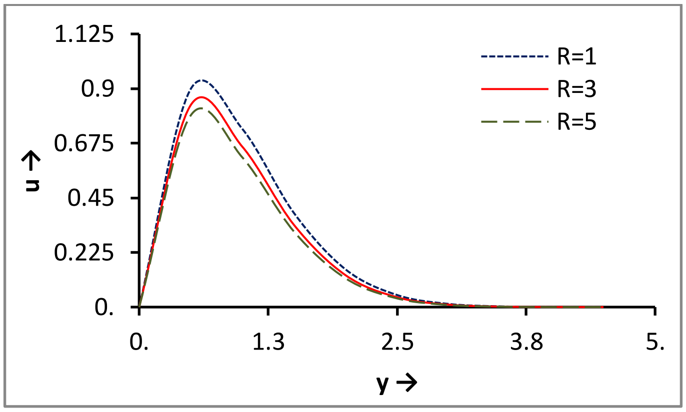

Figure 2: Velocity profiles for different R and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.78, t = 0.5, ω = π

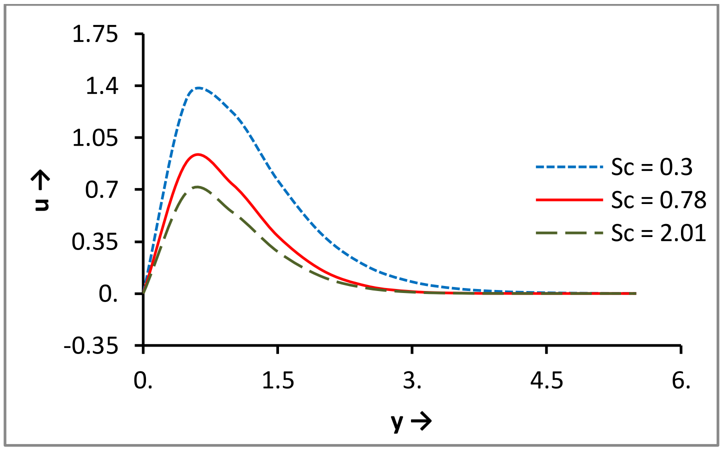

Figure 3: Velocity profiles for different Sc and Pr = 0.71, Gr = 3, Gm = 5, R = 1, t = 0.5, ω = π

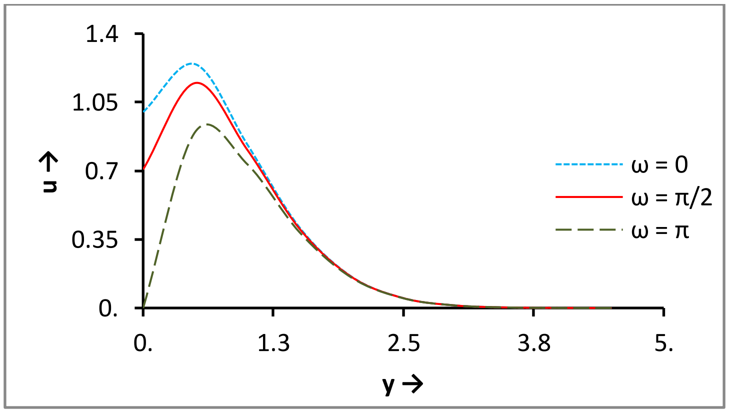

Figure 4: Velocity profiles for different ω and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.78, R = 1, t = 0.5

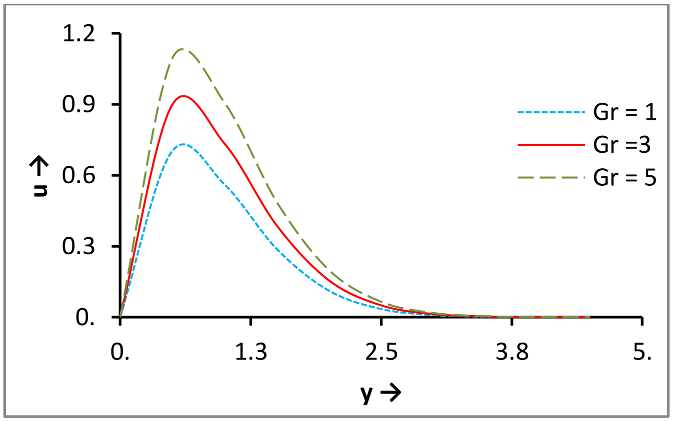

Figure 5: Velocity profiles for different Gr and Pr = 0.71, Gm = 5, Sc = 0.78, R =1, t = 0.5, ω = π

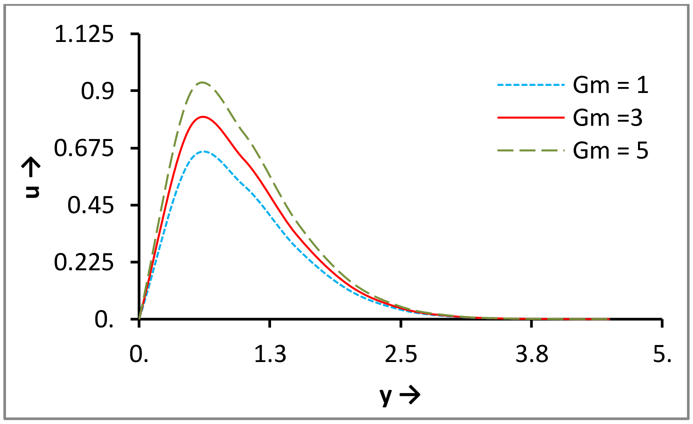

Figure 6: Velocity profiles for different Gm and Pr = 0.71, Gr = 3, Sc = 0.78, R =1, t = 0.5, ω = π

4.2 Temperature Profile

The effects of radiation, Prandtl number, and time on the temperature distribution across the boundary layer are depicted in Figures 7, 8, and 9, respectively. Figures 7 and 8 show that increasing R (radiation parameter) and Pr (Prandtl number) leads to a reduction in the fluid temperature, indicating a thinner thermal boundary layer in the presence of radiation. Physically, a higher Pr corresponds to lower thermal diffusivity, which causes a decrease in the thermal boundary layer thickness.

Conversely, Figure 9 illustrates that the fluid temperature increases over time. Additionally, the temperature is highest near the plate and gradually diminishes, approaching zero asymptotically in the free-stream region.

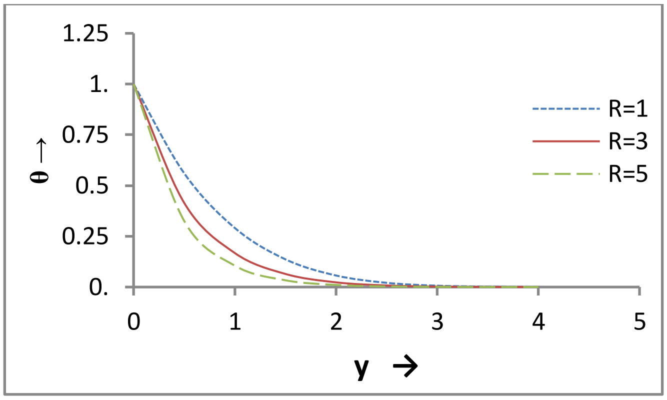

Figure 7: Temperature profiles for different R and Pr = 0.71, t = 0.5

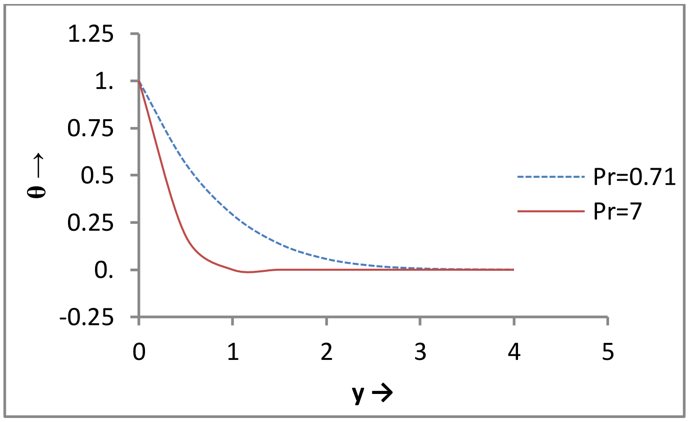

Figure 8: Temperature profiles for different Pr and R = 1, t = 0.5

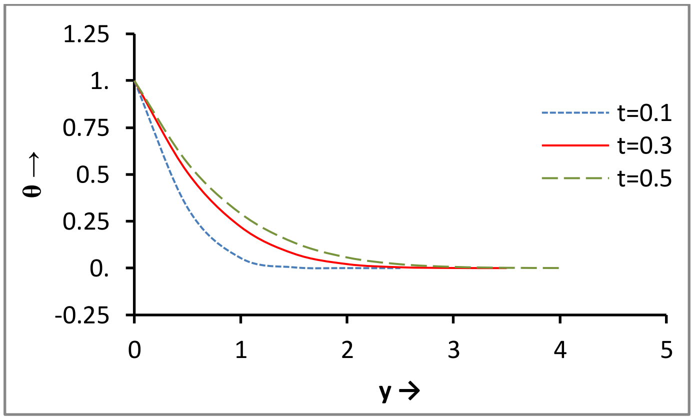

Figure 9: Temperature profiles for different t and Pr = 0.71, R = 1

4.3 Concentration Profile

The variation in concentration profiles for different values of the Schmidt number (Sc) and time (t) is presented in Figures 10 and 11, respectively. It is observed that an increase in Sc leads to a reduction in the concentration profile. This behaviour is consistent with physical expectations, as higher Schmidt numbers correspond to lower molecular diffusivity, resulting in a thinner concentration boundary layer.

Furthermore, the fluid concentration increases with advancing time (t). Similar to the temperature field, the concentration field asymptotically decreases to its minimum value of zero in the free-stream region.

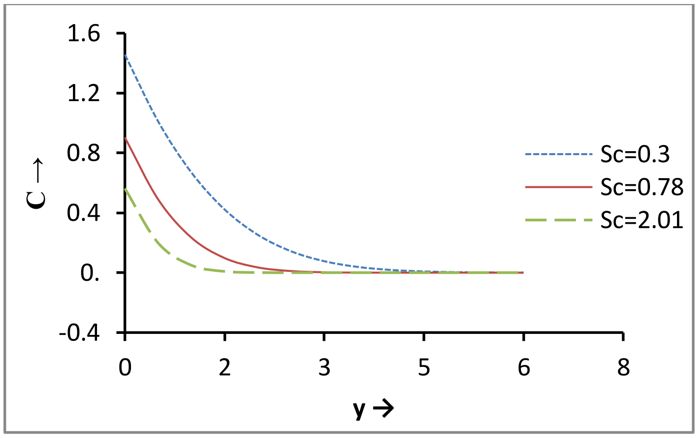

Figure 10: Concentration profiles for different Sc and t = 0.5

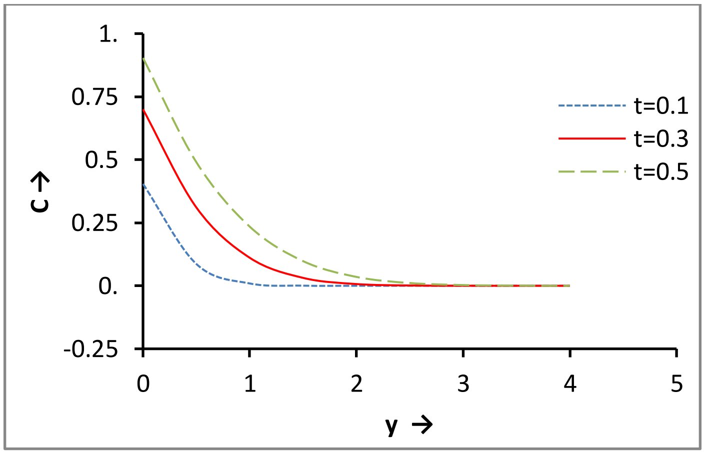

Figure 11: Concentration profiles for different t and Sc = 0.78

4.4 Skin Friction

Figures 12 to 15 illustrate the variation of skin friction with respect to time (t) for different values of the radiation parameter ®, thermal Grashof number (Gr), mass Grashof number (Gm), and oscillation frequency (ω). The results indicate that the skin friction coefficient decreases with increasing R but increases with higher values of Gr, Gm, and ω. In other words, the viscous drag on the plate diminishes with increasing radiation but is enhanced under the influence of buoyancy forces Gr and Gm and oscillatory motion ω.

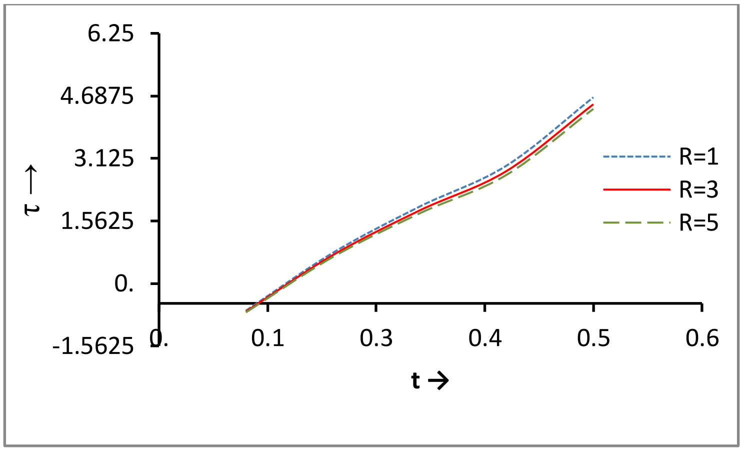

Figure 12: Skin friction for different R and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.3, ω = π

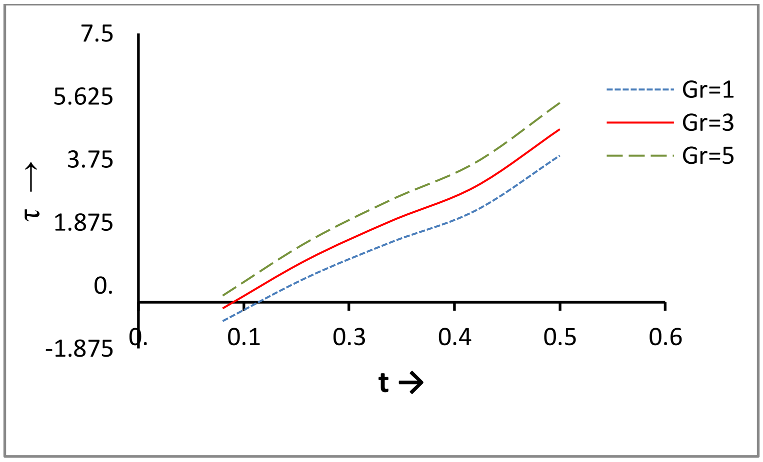

Figure 13: Skin friction for different Gr and Pr = 0.71, Gm = 5, Sc = 0.3, R=1, ω = π

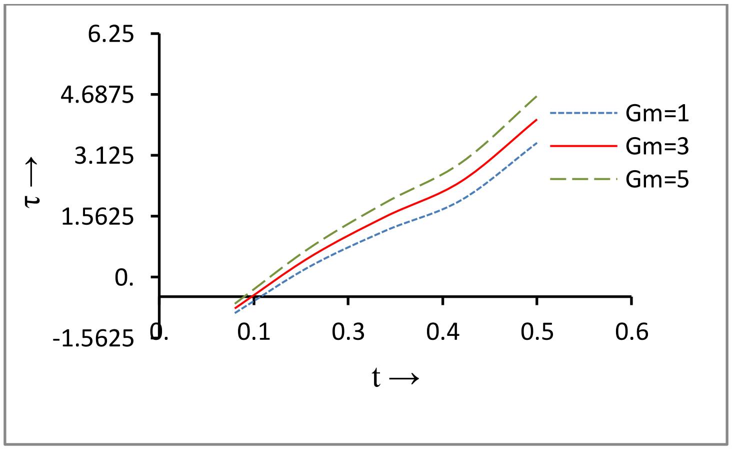

Figure 14: Skin friction for different Gm and Pr = 0.71, Gr = 3, Sc = 0.3, R =1, ω = π

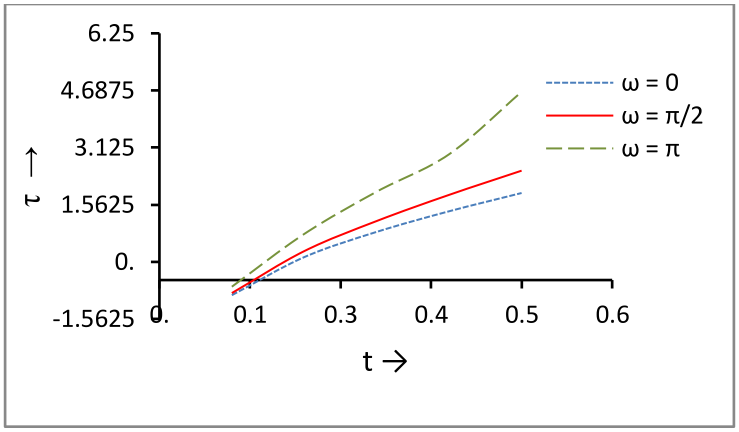

Figure 15: Skin friction for different ω and Pr = 0.71, Gr = 3, Gm = 5, Sc = 0.3, R=1

4.5 Nusselt number

The coefficient of heat transfer in terms of Nusselt numbers is displayed in Figures 16 and 17. It is observed that the Nusselt number increases with higher values of the radiation parameter ® and the Prandtl number (Pr). This implies that fluids with greater radiative properties enhance the heat transfer rate at the plate.

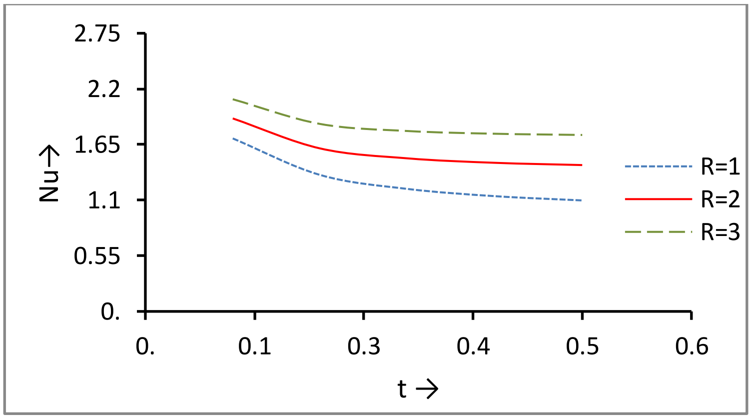

Figure 16: Nusselt number for different R and Pr = 0.71

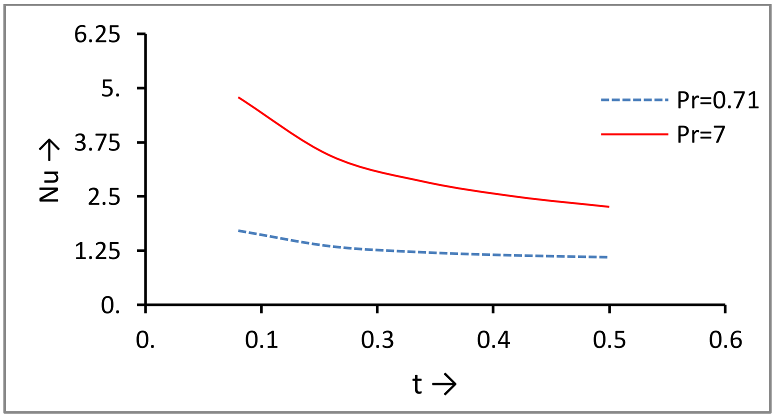

Figure 17: Nusselt number for different Pr and R = 1

4.6 Sherwood Number

It is observed from equation (28) that the Sherwood number at the plate is twice the reciprocal of the square root of *pi,* which is constant. Therefore, it can be concluded that the rate of solutal concentration at the plate remains uniform.

5. CONCLUSIONS

The analytical solutions of unsteady free convective flow over an oscillating plate in the presence of thermal radiation are investigated by using the Laplace Transform Technique, and the following conclusions are drawn

1. Velocity decreases with an increase in different values of radiation R.
2. The thermal boundary layer decreases in the presence of radiation and is found to thicken.
3. An increase in Schmidt number diminishes the concentration profile.
4. The coefficient of skin friction decreases with increasing values of R.
5. Nusselt number increases with the values of the radiation parameter R.

Nomenclature

a^*	absorption coefficient
C'	species concentration in the fluid
C'_w	concentration of the plate
C'_∞	concentration in the fluid far away from the plate
C	dimensionless concentration
C_p	specific heat at constant pressure
D	mass diffusion coefficient
g	acceleration due to gravity
Gr	thermal Grashof number
Gm	mass Grashof number
j''	mass flux per unit area at the plate
k	thermal conductivity of the fluid
Pr	Prandtl number
q_r	radiation heat flux
R	Radiation parameter
Sc	Schmidt number
T'	temperature of the fluid near the plate
T'_w	temperature of the plate
T'_∞	temperature of the fluid far away from the plate
t'	time
t	dimensionless time
u'	velocity of the fluid in x' direction
u_o	velocity of the plate
u	dimensionless velocity
y'	coordinate axis normal to the plate
y	dimensionless coordinate axis normal to the plate
β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of solutal expansion
μ	coefficient of viscosity
ρ	density
ρ_∞	reference density
υ	kinematic viscosity
σ^*	Stefean Boltzment constant
τ	dimensionless skin friction
ω'	dimensional frequency of oscillation
ω	dimensionless frequency of oscillation
θ	dimensionless temperature

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