
ON THE STABILITY OF EQUI NEIGHBOR POLYNOMIAL OF GRAPHS

Abstract

Let $\Gamma(V, E)$ be a simple graph of order n with vertex set V and edge set E . For an unordered pair of distinct vertices of Γ , we denote (x, y) . For a vertex $x \in \Gamma$, let $\mathcal{N}(x)$ be the set of all vertices of Γ that are adjacent to x . Then, for $0 \leq j \leq n - 1$, the j -equi neighbour set of Γ is defined as $\mathcal{N}_e(\Gamma, j) = \{(x, y) : x, y \in V, x \neq y \text{ and } |\mathcal{N}(x)| = |\mathcal{N}(y)| = j\}$. The equi-neighbour polynomial of Γ , denoted by $\mathcal{N}_e[\Gamma; y]$, is given by $\mathcal{N}_e[\Gamma; y] = \sum_{j=0}^{(n-1)} |\mathcal{N}_e(\Gamma, j)| y^j$. A root of the polynomial $\mathcal{N}_e[\Gamma; y]$ is defined as the equi neighbour root of the graph Γ . In this work, we study the distribution of zeros of the equi-neighbour polynomial associated with graphs and examine the stability of the common neighbour polynomial relative to the closed right half-plane. We further characterize the conditions under which the common neighbour polynomial for specific classes of graphs satisfies the Hurwitz stability criterion.

Keywords: j - equi neighbour set; equi neighbour polynomial, Hurwitz stability criterion, right half-plane

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1 Introduction

Let $\Gamma(V, E)$ be a simple graph of order n . Let (x, y) denotes an unordered vertex pair of distant vertices of Γ . The j -equi neighbour set of Γ is defined as $\mathcal{N}_e(\Gamma, j) = \{(x, y) : u, x \in V, x \neq y \text{ and } |\mathcal{N}(x)| = |\mathcal{N}(y)| = j\}$, for $0 \leq j \leq n - 1$. The polynomial $\mathcal{N}_e[\Gamma; y] = \sum_{j=0}^{(n-1)} |\mathcal{N}_e(\Gamma, j)| y^j$ is defined as the equi neighbour polynomial of Γ . The present authors derived the equi neighbour polynomial of some well known graphs. The equi neighbour polynomial of some graph operations were discussed. We say that two graphs Γ and H are said to be ENP-equivalent if and only if

$\mathcal{N}_e[\Gamma; y] = \mathcal{N}_e[H; y]$. A zero of the polynomial $\mathcal{N}_e[\Gamma; y]$ is defined as the equi neighbour root of the graph Γ .

The equi-neighbour polynomial for graphs formed through certain binary graph operations is investigated in (9). In (10) MN Husin et al examined neighbourhood polynomial of some interested standard graph networks. The detailed data on neighbourhood polynomials are discussed in (11), (12), and (13).

A polynomial $g(y_1, \dots, y_n)$ is said to be stable (7) with respect to a region $\Omega \in \mathbb{C}^n$ if none of its zeros lie inside Ω . In particular, a polynomial that is stable in the closed right half of the complex plane is known as the Hurwitz polynomial, while one that is stable in the open unit disk is termed a Schur polynomial. Hurwitz polynomials play a crucial role in control systems theory because they correspond to the characteristic equations of dynamically stable linear systems (2). The study of graph polynomials becomes meaningful when these polynomials are capable of modeling stability properties arising in physical systems. In this paper, we study the stability of the common neighbour polynomial of graphs relative to the closed right half-plane, and we characterize the circumstances in which this polynomial is Hurwitz for certain families of graphs.

2 Main results

Definition 2.1. A polynomial $g(y_1, y_2, \dots, y_n)$ is called stable with respect to the closed right half plane precisely when every one of its nonzero zeros lies strictly within the open left half plane.

Lemma 2.1. For a cycle graph C_k with k vertices, we have

$$\mathcal{N}_e[C_k; y] = \binom{k}{2} y^2, k \geq 3.$$

Theorem 2.2. If C_k is a cycle graph, then $\mathcal{N}_e[C_k; y]$ is stable; $k \geq 3$.

Proof. From Lemma, we have

$$\mathcal{N}_e[C_k; y] = \binom{k}{2} y^2, k \geq 3.$$

Since zero is the only root of $\mathcal{N}_e[C_k; y]$, it is stable for $k \geq 3$. □

Lemma 2.3. For $k \geq 2$, we have $\mathcal{N}_e[F_k; y] = k(2k - 1)y^2$.

Theorem 2.4. If F_k is a friendship graph, then $\mathcal{N}_e[F_k; y]$ is stable; $k \geq 2$.

Proof. From Lemma, we have $\mathcal{N}_e[F_k; y] = k(2k - 1)y^2; k \geq 2$.

Since zero is the only root of $\mathcal{N}_e[F_k; y]$, it is stable for $k \geq 2$. □

Lemma 2.5. For a bipartite cocktail party graph B_k , we have $\mathcal{N}_e[B_k; y] = k(2k - 1)y^{k-1}; k \geq 1$.

Theorem 2.6. For a bipartite cocktail party graph B_k , we have $\mathcal{N}_e[B_k; y]$ is stable; $k \geq 1$.

Proof. From Lemma, we have $\mathcal{N}_e[B_k; y] = k(2k - 1)y^{k-1}; k \geq 1$.

Since zero is the only root of $\mathcal{N}_e[B_k; y]$, it is stable for $k \geq 1$. □

Lemma 2.7. For a windmill graph $W_n^{(m)}$, $m \geq 2; n \geq 2$ we have

$$\mathcal{N}_e[W_n^{(m)}; y] = \binom{m(n-1)}{2} y^{n-1}.$$

Theorem 2.8. If $W_n^{(m)}$, is a windmill graph, then $\mathcal{N}_e[W_n^{(m)}; y]$ is stable; $m \geq 2; n \geq 2$.

Proof. From Lemma, we have $\mathcal{N}_e[W_n^{(m)}; y] = \binom{m(n-1)}{2} y^{n-1}; m \geq 2; n \geq 2$.

Since zero is the only root of $\mathcal{N}_e[W_n^{(m)}; y]$, it is stable for $m \geq 2; n \geq 2$. \square

Lemma 2.9. For a dutch wind mill graph $\mathcal{N}_e[D_n^{(m)}; y] = m(2m-1)y^2; m \geq 2$.

Theorem 2.10. If $D_n^{(m)}$, is a dutch wind mill graph, then $\mathcal{N}_e[D_n^{(m)}; y]$ is stable; $m \geq 2$.

Proof. From Lemma, we have $\mathcal{N}_e[D_n^{(m)}; y] = m(2m-1)y^2; m \geq 2$.

Since zero is the only root of $\mathcal{N}_e[D_n^{(m)}; y]$, it is stable for $m \geq 2$. \square

Lemma 2.11. For n - barbell graph, $\mathcal{N}_e[B_{n,1}; y] = y^n + (n-1)(2n-3)y^{n-1}; n \geq 1$.

Theorem 2.12. If $B_{n,1}$ is n - barbell graph, then $\mathcal{N}_e[B_{n,1}; y]$ is stable; $n \geq 1$.

Proof. From Lemma, we have

$$\begin{aligned}\mathcal{N}_e[B_{n,1}; y] &= y^n + (n-1)(2n-3)y^{n-1} \\ &= y^{n-1}[y + (n-1)(2n-3)]\end{aligned}$$

When $n = 1$, $\mathcal{N}_e[B_{1,1}; y] = y$, which has only one root namely zero. When $n > 1$, the equi neighbour root of $B_{n,1}$ is , $y = -(n-1)(2n-3)$; which lies in the open left half plane. Hence the result follows. \square

Theorem 2.13. Let Γ be a graph with common neighbour polynomial $\mathcal{N}_e[\Gamma; y]$ of degree 2. Then the following hold:

1. If $\mathcal{N}_e(\Gamma, 0) = \phi$ and $\mathcal{N}_e(\Gamma, 1) \neq \phi$, then $\mathcal{N}_e[\Gamma; y]$ is a stable polynomial.
2. If $\mathcal{N}_e(\Gamma, 0) \neq \phi$ and $\mathcal{N}_e(\Gamma, 1) = \phi$, then $\mathcal{N}_e[\Gamma; y]$ is not a stable polynomial.

Proof. Since $\mathcal{N}_e[\Gamma; y]$ is of degree 2, $|\mathcal{N}_e(\Gamma, 2)| \neq 0$. We consider the two cases:

1. Let $\mathcal{N}_e(\Gamma, 0) = \phi$ and $\mathcal{N}_e(\Gamma, 1) \neq \phi$. In this case, the roots of $\mathcal{N}_e[\Gamma; y]$ are given by $y = 0$ and $y = -\frac{|\mathcal{N}_e(\Gamma, 1)|}{|\mathcal{N}_e(\Gamma, 2)|}$. It follows that $\mathcal{N}_e[\Gamma; y]$ is stable.

2. Let $\mathcal{N}_e(\Gamma, 0) \neq \phi$ and $\mathcal{N}_e(\Gamma, 1) = \phi$. Then the roots of $\mathcal{N}_e[\Gamma; y]$ are given by $y = \pm \sqrt{\frac{|\mathcal{N}_e(\Gamma, 0)|}{|\mathcal{N}_e(\Gamma, 2)|}} i$.

Since $\mathcal{N}_e[\Gamma; y]$ has non zero roots in the closed right half plane, $\mathcal{N}_e[\Gamma; y]$ is not stable.

This completes the proof. \square

Lemma 2.14. For a path graph P_k , we have

$$\mathcal{N}_e[P_k; y] = y + \binom{k-2}{2} y^2; k \geq 2.$$

Theorem 2.15. If P_n , is a path graph, then $\mathcal{N}_e[P_n; y]$ is stable; $n \geq 2$.

Proof. From Lemma, we have $\mathcal{N}_e[P_n; y] = y + \binom{n-2}{2} y^2; n \geq 2$.

$\mathcal{N}_e[P_n; y]$ is of degree 2 with $|\mathcal{N}_e(P_n, 0)| = 0$ and $|\mathcal{N}_e(P_n, 1)| \neq 0$. So the result follows from Theorem 2.13 \square

Theorem 2.16. (8) (Routh-Hurwitz Criteria) Given a polynomial, $F(y) = y^n + a_1y^{n-1} + \dots + a_{n-1}y + a_0$, where the coefficients a_i are real constants, $i = 1, 2, \dots, n$ define the n Hurwitz matrices using the coefficients a_i of the above polynomial as

$$H_1 = [a_1] \quad H_2 = \begin{bmatrix} a_1 & 1 \\ a_3 & a_2 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} a_1 & 1 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{bmatrix} \quad \dots \quad H_n = \begin{bmatrix} a_1 & 1 & 0 & 0 & \dots & 0 \\ a_3 & a_2 & a_1 & 1 & \dots & 0 \\ a_5 & a_4 & a_3 & a_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ a_{2n-1} & a_{2n-2} & a_{2n-3} & a_{2n-4} & \dots & a_n \end{bmatrix}$$

where $a_j = 0$ if $j > n$. All the roots of the polynomial $F(y)$ are negative or have negative real part if and only if the determinants of all Hurwitz matrices are positive: $\det H_j > 0$, $j = 1, 2, \dots, n$.

Lemma 2.17. For a helm graph H_n , $n > 3$, we have

$$\mathcal{N}_e[H_n; y] = \begin{cases} \binom{n-1}{2}(y^4 + y), & \text{if } n \neq 5 \\ 10y^4 + 6y, & \text{if } n = 5. \end{cases}$$

Theorem 2.18. If H_k , $k > 3$ is a helm graph with $2k - 1$ vertices, then $\mathcal{N}_e[H_k; y]$ is not stable.

Proof. We consider two cases.

Case(i) Let $k \neq 5$. From Lemma, we have $\mathcal{N}_e[H_k; y] = \binom{k-1}{2}(y^4 + y)$.

So considering the polynomial $y^3 + 1$ the determinant of the first Hurwitz matrix is given by $|H_1| = 0$. Hence the result follows from Theorem 2.16.

Case(ii) Let $k = 5$. From Lemma, we have $\mathcal{N}_e[H_k; y] = 10y^4 + 6y$. So considering the polynomial $10y^3 + 6$ the determinant of the first Hurwitz matrix is given by $|H_1| = 0$. Hence the result follows from Theorem 2.16. □

Lemma 2.19. Let $L_{m,n}$ a lollipop graph. Then

$$\mathcal{N}_e[L_{m,n}; y] = \begin{cases} \binom{m-1}{2}y^{m-1} + \binom{n-1}{2}y^2, & \text{if } m \geq 4 \\ \binom{n-1}{2}y^2 + y, & \text{if } m = 1 \\ \binom{n}{2}y^2 + y, & \text{if } m = 2 \\ \binom{n+1}{2}y^2, & \text{if } m = 3. \end{cases}$$

Theorem 2.20. If $L_{m,n}$ is a lollipop graph, then the following results hold.

1. $\mathcal{N}_e[L_{m,n}; y]$ is not stable for $m \geq 4$.
2. $\mathcal{N}_e[L_{m,n}; y]$ is stable for $m = 1$ and $n > 2$.
3. $\mathcal{N}_e[L_{m,n}; y]$ is stable for $m = 2$ and $n > 1$.
4. $\mathcal{N}_e[L_{m,n}; y]$ is stable for $m = 3$.

Proof. Here consider the following cases.

Case(i) Let $m \geq 4$. From Lemma, we have

$$\mathcal{N}_e[L_{m,n}; y] = \binom{m-1}{2}y^{m-1} + \binom{n-1}{2}y^2.$$

Then the determinant of the first Hurwitz matrix itself is zero. Hence $\mathcal{N}_e[L_{m,n}; y]$ is not stable under this case, by Theorem 2.16.

Case(ii) Let $m = 1, n > 2$. From Lemma, we have

$\mathcal{N}_e[L_{m,n}; y] = \binom{n-1}{2}y^2 + y$. So considering the polynomial $y + \frac{1}{\binom{n-1}{2}}$; the determinant of the first Hurwitz matrix is given by $|H_1| = \frac{1}{\binom{n-1}{2}}$. Hence the result follows by Theorem 2.16.

Case(iii) Let $m = 2, n > 1$. From Lemma, we have

$\mathcal{N}_e[L_{m,n}; y] = \binom{n}{2}y^2 + y$. So considering the polynomial $y + \frac{1}{\binom{n}{2}}$; the determinant of the first Hurwitz matrix is given by $|H_1| = \frac{1}{\binom{n}{2}}$. Hence the result follows by Theorem 2.16.

Case(iv) Let $m = 2$. From Lemma, we have $\mathcal{N}_e[L_{m,n}; y] = \binom{n+1}{2}y^2$. Since zero is the only root of $\mathcal{N}_e[L_{m,n}; y]$, it is stable for $m = 2$.

This completes the proof. □

Lemma 2.21. For a web graph WB_n where $n > 3$, we have

$$\mathcal{N}_e[WB_n; y] = \begin{cases} (n-1)(2n-3)y^4 + \binom{n-1}{2}y, & \text{if } n \neq 5 \\ 36y^4 + 6y, & \text{if } n = 5. \end{cases}$$

Theorem 2.22. If $WB_n; n > 3$ is a web graph with $3(n-1)$ vertices, then $\mathcal{N}_e[WB_n; y]$ is not stable.

Proof. We consider two cases.

Case(i) Let $n \neq 5$. From Lemma, we have

$$\mathcal{N}_e[WB_n; y] = (n-1)(2n-3)y^4 + \binom{n-1}{2}y.$$

So considering the polynomial $(4n-6)y^3 + (n-2)$, the determinant of the first Hurwitz matrix is given by $|H_1| = 0$. Hence the result follows from Theorem 2.16.

Case(ii) Let $n = 5$. From Lemma, we have $\mathcal{N}_e[WB_n; y] = 36y^4 + 6y$. So considering the polynomial $6y^3 + 1$, the determinant of the first Hurwitz matrix is given by $|H_1| = 0$. Hence the result follows from Theorem 2.16. □

Lemma 2.23. For a shell graph $S_n; n \geq 3$, we have

$$\mathcal{N}_e[S_n; y] = \begin{cases} \binom{n-3}{2}y^3 + y^2, & \text{if } n \neq 3, 4 \\ 3y^2, & \text{if } n = 3 \\ y^3 + y^2, & \text{if } n = 4. \end{cases}$$

Theorem 2.24. If S_n is a shell graph with $n \geq 3$ vertices, then $\mathcal{N}_e[S_n; y]$ is stable.

Proof. We consider three cases.

Case(i) Let $n \neq 3, 4$. From Lemma, we have $\mathcal{N}_e[S_n; y] = \binom{n-3}{2}y^3 + y^2$.

So considering the polynomial $\binom{n-3}{2}y + 1$, the determinant of the first Hurwitz matrix is given by $|H_1| = 1$. Hence the result follows from Theorem 2.16.

Case(ii) Let $n = 3$. From Lemma, we have $\mathcal{N}_e[S_3; y] = 3y^2$. Since zero is the only root of $\mathcal{N}_e[S_n; y]$, it is stable for $n = 3$.

Case(iii) Let $n = 4$. From Lemma, we have $\mathcal{N}_e[S_4; y] = y^3 + y^2$. So considering the polynomial $y + 1$, the determinant of the first Hurwitz matrix is given by $|H_1| = 1$. Hence the result follows from Theorem 2.16.

□

Lemma 2.25. *If BF is a butterfly graph with $N \geq 7$ vertices, then*

$$\mathcal{N}_e[BF; y] = \binom{N-7}{2} y^3 + 6y^2 + y.$$

Theorem 2.26. *If BF is a butterfly graph with $N \geq 7$ vertices, then $\mathcal{N}_e[BF; y]$ is stable.*

Proof. We consider two cases.

Case(i) Let $N > 8$. From Lemma, we have $\mathcal{N}_e[BF; y] = \binom{N-7}{2} y^3 + 6y^2 + y$. So considering the polynomial $y^2 + \frac{6}{k}y + \frac{1}{k}$; where $k = \binom{N-7}{2}$, the determinants of the Hurwitz matrices are given by $|H_1| = \frac{6}{k}$. and $|H_2| = \begin{vmatrix} \frac{6}{k} & 1 \\ 0 & \frac{1}{k} \end{vmatrix} = \frac{6}{k^2}$. Since both the determinants are positive, by Theorem 2.16, $\mathcal{N}_e[BF; y]$ is stable for $N > 8$.

Case(ii) Let $N = 7, 8$. From lemma we have $\mathcal{N}_e[BF; y] = 6y^2 + y$.

So considering the polynomial $y + \frac{1}{6}$; the determinant of the first Hurwitz matrix is given by $|H_1| = \frac{1}{6}$. Hence the result follows by Theorem 2.16.

□

Lemma 2.27. *For a complete bipartite graph $K_{p,q}$; where $p, q \geq 2$, we have the following.*

$$\mathcal{N}_e[K_{p,q}; y] = \begin{cases} \binom{p}{2}y^q + \binom{q}{2}y^p, & p \neq q, \\ q(2q-1)y^q, & p = q. \end{cases}$$

Theorem 2.28. *If $K_{p,q}$; where $p > q; p, q \geq 2$ is a complete bipartite graph with $p+q$ vertices, then $\mathcal{N}_e[K_{p,q}; y]$ is stable if and only if $p = q$ or $p - q = 1$.*

Proof. Assume that $p = q$. From Lemma, we have

$\mathcal{N}_e[K_{p,q}; y] = q(2q-1)y^q$. Since zero is the only root of $\mathcal{N}_e[K_{p,q}; y]$, it is stable for $p = q$.

Now assume that $q - p = 1$. From Lemma, we have

$\mathcal{N}_e[K_{p,q}; y] = y^p \left[\binom{p}{2}y + \binom{q}{2} \right]$. So considering the polynomial $y + \frac{\binom{q}{2}}{\binom{p}{2}}$; the determinant of the first

Hurwitz matrix is given by $|H_1| = \frac{\binom{q}{2}}{\binom{p}{2}}$. Since the determinant of the Hurwitz matrix is positive, by

Theorem 2.16 $\mathcal{N}_e[K_{p,q}; y]$ is stable.

Assume that $q - p > 1$. Then the determinant of the first Hurwitz matrix itself is zero. Hence $\mathcal{N}_e[K_{p,q}; y]$ is not stable, by Theorem 2.16.

This completes the proof.

□

Theorem 2.29. *Let Γ be a graph with common neighbour polynomial $\mathcal{N}_e[\Gamma; y]$ of degree 2 where $|\mathcal{N}_e(\Gamma, i)| > 0$ for $i = 0, 1, 2$. Then $\mathcal{N}_e[\Gamma; y]$ is stable. Moreover, $\mathcal{N}_e[\Gamma; y]$ has two negative real roots if $|\mathcal{N}_e(\Gamma, 1)|^2 \geq 4|\mathcal{N}_e(\Gamma, 2)||\mathcal{N}_e(\Gamma, 0)|$ and $\mathcal{N}_e[\Gamma; y]$ has two complex roots with negative real parts otherwise.*

Proof. Since $\mathcal{N}_e[\Gamma; y]$ is a polynomial of degree 2, $\mathcal{N}_e[\Gamma; y]$ can be represented in the form $\mathcal{N}_e[\Gamma; y] = |\mathcal{N}_e(\Gamma, 2)|y^2 + |\mathcal{N}_e(\Gamma, 1)|y + |\mathcal{N}_e(\Gamma, 0)|$. The Hurwitz matrices of $\mathcal{N}_e[\Gamma; y]$ are given by $H_1 = \begin{bmatrix} |\mathcal{N}_e(\Gamma, 1)| \\ |\mathcal{N}_e(\Gamma, 2)| \end{bmatrix}$ and $H_2 = \begin{bmatrix} \frac{|\mathcal{N}_e(\Gamma, 1)|}{|\mathcal{N}_e(\Gamma, 2)|} & 1 \\ 0 & \frac{|\mathcal{N}_e(\Gamma, 0)|}{|\mathcal{N}_e(\Gamma, 2)|} \end{bmatrix}$. Since $|\mathcal{N}_e(\Gamma, i)| > 0$ for $i = 0, 1, 2$; it follows that $\det H_1 > 0$ and $\det H_2 > 0$. Hence by Theorem 2.16, $\mathcal{N}_e[\Gamma; y]$ is stable so that all the roots of $\mathcal{N}_e[\Gamma; y]$ lie in the open left half plane. Moreover, the discriminant of $\mathcal{N}_e[\Gamma; y]$ is given by $\Delta = |\mathcal{N}_e(\Gamma, 1)|^2 - 4|\mathcal{N}_e(\Gamma, 2)||\mathcal{N}_e(\Gamma, 0)|$. It follows that $\mathcal{N}_e[\Gamma; y]$ has 2 real roots if $\Delta \geq 0$ and has two complex roots if $\Delta < 0$. This completes the proof. \square

3 Conclusions

In this article, we investigate the distribution of zeros of the equi-neighbour polynomial associated with graphs and examine its stability properties for several classes of graphs. We also determine sufficient conditions under which the common-neighbour polynomial of specific graph families qualifies as a Hurwitz polynomial.

Some questions remains open, such as identify equi neighbour polynomial of cardinal unique graph classes. Another open problem is to characterize the properties of equi neighbour polynomial equivalent and equi neighbour polynomial cardinal unique graphs.

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