SOMBOR INDICES OF SOME GRAPHS

ABSTRACT. Let G be a graph with no loops and parallel edges. The Sombor index $SO(G)$ of graph G was introduced recently by I. Gutman and defined as $SO = SO(G) = \sum_{ij \in E(G)} \sqrt{d_i^2 + d_j^2}$, where the degree of the vertex i in G is denoted by d_i. In this paper, we find the Sombor index of Subdivisions of Graph S(G), Triangle Parallel Graph R(G),Semi Total graph Q(G) and the Total graph T(G) of a few classes of graph G

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1. Introduction

Topological index of many classes of graphs have been investigated deeply in the fields of Graph Theory. Topological indices help in predicting the properties of chemical compounds. Its importance is highly acknowledged in chemical mathematics as well. Though Sombor Index is a recently developed degree based topological index by I.Gutman (see [1]), it acquired the attention of scholars in a short span of time. As a result, many research papers were brought out on this topic. Sombor Index of the graph G is defined by

$$SO(G) = \sum_{ij \in E(G)} \sqrt{d_i^2 + d_j^2}$$

Over the years, many operations on graph and variants of corona have been intensively studied in literature. Subdivisions of Graph S(G), Triangle Parallel Graph R(G),Semi Total graph Q(G) and the Total graph T(G) are Four graph operations among them. These four graph operations and the work of I.Gutman have become the source of motivation to study and form general formulae for some modified graph of G.

Definition 1.1. "The subdivision graph $S(G)$ of a graph G is the graph obtained by inserting a new vertex into every edge of G". (see [2, 11])

Definition 1.2. "Triangle parallel graph (see [5, 11]) R(G) is obtained from G by adding a new vertex corresponding to each edge of G and then joining each new vertex to the end vertices of the corresponding edge".

Definition 1.3. "The semi total graph (see [5, 11]) Q(G) is obtained from G by adding a new vertex corresponding into every edge of G and then joining with edges those pairs of new vertices on adjacent edges of G".

Definition 1.4. "The total graph (see [5, 11]) R(G) and then joining with edges those pairs of new vertices on adjacent edges of G".

Lemma 1.5. [12] *For a graph G*

i $|V(S(G))| = |V(R(G))| = |V(Q(G))| = |V(T(G))| = |V(G)| + |E(G)|$.

ii $|E(S(G))| = 2|E(G)|$

iii $|E(R(G))| = 3|E(G)|$

iv $|E(Q(G))| = 2|E(G)| + |E(L(G))|$

v $|E(T(G))| = 23|E(G)| + |E(L(G))|$

If $v \in V(G)$*, then*

vi $d_{S(G)}(v) = d_{Q(G)}(v) = d_G(v)$ *and* $d_{R(G)}(v) = d_{T(G)}(v) = 2d_G(v)$

If $v \notin V(G)$ *but* $v \in V(S(G))/V(R(G))/V(Q(G))/V(T(G))$*, then*

vii $d_{S(G)}(v) = d_{R(G)}(v) = 2$ *and* $d_{Q(G)}(v) = d_{T(G)}(v) = 2d_{L(G)}(v) + 2$

2. Sombor Index of Subdivision of graphs

Here Sombor index of subdivision of some graphs are computed.

Theorem 2.1. *Sombor Index of subdivision (see [5]) of some graphs are given by the following results.*

(1) *If* $G = (n, m)$ *be a regular graph with regularity* r*, then*

$$SO(S(G)) = 2m\sqrt{2^2 + r^2}$$

.

(2) $SO(S(P_n)) = SO(P_{2n-1}) = 2\sqrt{5} + 4(n-2)\sqrt{2}$ *for* $n \geq 2$.

(3) $SO(S(F_n)) = SO(F_n) + 6n\sqrt{2}$.

(4) $SO(S(S_n)) = (n-1)(\sqrt{(n-1)^2 + 4} + \sqrt{5})$.

Proof. (1) For a regular graph $G = (n, m)$ with regularity r, $S(G)$ has $n+m$ vertex and $2m$ edges. Each edge of $S(G)$ has end degree $(r, 2)$. Therefore, $SO(S(G)) = 2m\sqrt{r^2 + 4}$

(2) Consider the path P_n, then by introducing a vertex to each edge of P_n, we get $S(P_n)$, which is again a path and has $2n - 1$ edges.
(See Figure 1)
Therefore, $SO(S(P_n)) = SO(P_{2n-1})$. Hence the result.(refer [2])

(3) In the case of friendship graph F_n, each cycle C_3 will change to C_6. Thus the graph $S(F_n)$ has 2n edges with end vertices of degree 2n and 2 and 4n edges with both end vertices of degree 2. (See Figure 2)

$$SO(S(F_n)) = 2n\sqrt{2n^2 + 2^2} + 4n\sqrt{2^2 + 2^2}$$

$$SO(S(F_n)) = 4n\sqrt{n^2 + 1} + 8n\sqrt{2}$$

$$SO(S(F_n)) = SO(F_n) + 6n\sqrt{2}.$$

(refer [2])

(4) The subdivision of star graph $S(S_n)$ has n-1 edges with end vertices of degree n-1 and 2 and n-1 edges with end vertices of degree 2 and 1.

$$SO(S(S_n)) = (n-1)\sqrt{(n-1)^2 + 2^2} + (n-1)\sqrt{2^2 + 1^2}$$
$$SO(S(S_n)) = (n-1)\sqrt{(n-1)^2 + 4} + (n-1)\sqrt{5}.$$

□

3. Sombor Index of Triangle parallel graphs

Theorem 3.1. *Sombor Index of Triangle parallel graph of some graphs are given by the following results.*

(1) *If $G = (n, m)$ be a regular graph with regularity r, then*
$$SO(R(G)) = 2rm\sqrt{2} + 4m\sqrt{r^2 + 1}$$
(2) $SO(R(P_n)) = 4(n-1)\sqrt{5} + 4(n-2)\sqrt{2}$ *for $n > 2$.*
(3) $SO(R(F_n)) = 2SO(F_n) + 4n(\sqrt{4n^2 + 1} + 2\sqrt{5})$.
(4) $SO(R(S_n)) = SO(F_{n-1}) = 2(n-1)\sqrt{2} + 4(n-1)\sqrt{(n-1)^2 + 1}$

Proof. (1) For a regular graph $G = (n, m)$ with regularity r, $S(G)$ has $n + m$ vertex and $3m$ edges.Also $R(G)$ has $2m$ edges of end degree $(2r, 2)$ and m edges of end degree $(2r, 2r)$.
Therefore, $SO(R(G)) = 2rm\sqrt{2} + 4m\sqrt{r^2 + 1}$

(2) Consider the path P_n. By introducing a vertex to each edge and joining to the adjacent vertices, we get $R(P_n)$ which has $2n - 1$ vertices and $3(n - 1)$ edges. $R(P_n)$ has 2 edges with end vertices of degree 2, $n - 3$ edges with end vertices of degree 4, and the remaining $2n - 2$ edges have end vertices of degree 2 and 4.
Therefore, $SO(R(P_n)) = 2\sqrt{2^2 + 2^2} + (n-3)\sqrt{4^2 + 4^2} + (2n-2)\sqrt{2^2 + 4^2}$
$SO(R(P_n)) = 4(n-1)\sqrt{5} + 4(n-2)\sqrt{2}$.

(3) The graph $R(F_n)$ has $9n$ edges in which 2n edges have end vertices of degree 4n and 4, 2n edges have end vertices of degree 4n and 2, 4n edges have end vertices of degree 2 and 4 and n edges with both end vertices of degree 4.
$$SO(R(F_n)) = 2n\sqrt{4n^2 + 4^2} + 2n\sqrt{4n^2 + 2^2} + 4n\sqrt{2^2 + 4^2} + +n\sqrt{4^2 + 4^2}$$
$$SO(R(F_n)) = 8n\sqrt{n^2 + 1} + 4n\sqrt{4n^2 + 1} + 8n\sqrt{5} + 4n\sqrt{2}$$
(refer [2])
$$SO(R(F_n)) = 2SO(F_n) + 4n(\sqrt{4n^2 + 1} + 2\sqrt{5})$$
.

(4) The triangle parallel graph of star graph $R(S_n)$ will be the friendship graph F_{n-1}.
$$SO(R(S_n)) = SO(F_{n-1})$$

(refer [2])

$$SO(R(S_n)) = 2(n-1)\sqrt{2} + 4(n-1)\sqrt{(n-1)^2+1}$$

□

4. Sombor Index of semi Total graphs

Theorem 4.1. *Sombor Index of semi total graph of some graphs are given by the following results.*

(1) *If $G = (n, m)$ be a regular graph with regularity r, then*

$$SO(Q(G)) = 2\sqrt{5}mr + r\sqrt{2}(nr^2 - 2m)$$

(2) $SO(Q(P_n)) = 10 + 2\sqrt{10} + 2\sqrt{13} + 4(n-3)\sqrt{5} + 4(n-4)\sqrt{2}$ *for* $n > 2$.

(3) $SO(Q(F_n)) = 4n\sqrt{4n^2 + (n+1)^2} + 4n\sqrt{(n+1)^2+1} + 4n\sqrt{5} + 16\sqrt{2} + 4n(n+1)\sqrt{2}$.

(4) $SO(Q(S_n)) = (n-1)\sqrt{n^2 + (n-1)^2} + (n-1)\sqrt{n^2+1} + \frac{n(n-1)(n-2)}{2}\sqrt{2}$

Proof. (1) For a regular graph $G = (n, m)$ with regularity r, $S(G)$ has $n + m$ vertex and $2m + m_l$ edges. $Q(G)$ has $2m$ edges of end degree $(r, 2r)$ and $m_l = \frac{nr^2-2m}{2}$ edges of end degree $(2r, 2r)$.

Therefore,

$$SO(Q(G)) = 2rm\sqrt{r^2 + (2r)^2} + \frac{nr^2 - 2m}{2}\sqrt{(2r)^2 + (2r)^2}$$
$$= 2\sqrt{5}mr + r\sqrt{2}(nr^2 - 2m)$$

(2) Consider the path P_n. Then $Q(P_n)$ has $2n - 12$ vertices and $2(n-1) + n - 2$ edges with vertices of degree 1,2,3 and 4. The edge partition of $Q(P_n)$ is given by

Degree of end vertices	Number of edges
$(1, 3)$	2
$(2, 3)$	2
$(3, 4)$	2
$(2, 4)$	$2(n-3)$
$(4, 4)$	n-4

Table 1. Edge Partition of $Q(P_n)$

Therefore,

$$SO(Q(P_n)) = 2\sqrt{1^2+3^2} + 2\sqrt{2^2+3^2} + 2\sqrt{3^2+4^2} + 2(n-3)\sqrt{2^2+4^2} + (n-4)\sqrt{4^2+4^2}.$$
$$= 10 + 2\sqrt{10} + 2\sqrt{13} + 4(n-3)\sqrt{5} + 4(n-4)\sqrt{2}$$

(3) The graph $Q(F_n)$ has $5n + 1$ vertices and $6n + m_1$ edges Where $m_l = n(2n + 1)$.The edge partition of $Q(F_n)$ is given by

Degree of end vertices	Number of edges
$(4n, 2(n+1))$	2n
$(2(n+1), 2)$	2n
$(2, 4)$	2n
$(4, 4)$	4
$(2(n+1), 2(n+1))$	2n

Table 2. Edge Partition of $Q(F_n)$

Therefore,

$$\begin{aligned} SO(Q(F_n)) =& 2n\sqrt{(4n)^2 + (2(n+1))^2} + 2n\sqrt{(2(n+1))^2 + 2^2} + 2n\sqrt{2^2 + 4^2} \\ &+ 4\sqrt{4^2 + 4^2} + 2n\sqrt{(2(n+1))^2 + (2(n+1))^2} \\ &= 4n\sqrt{4n^2 + (n+1)^2} + 4n\sqrt{(n+1)^2 + 1} + 4n\sqrt{5} + 16\sqrt{2} + 4n(n+1)\sqrt{2} \end{aligned}$$

(4) $Q(S_n)$ will have $n + n - 1 = 2n - 1$ vertices and $2(n-1) + m_l$ edges, where m_l is the number of edges in line graph of S_n equal to $\frac{(n-1)(n-2)}{2}$. The edge partition is as follows.

Degree of end vertices	Number of edges
$((n-1), n)$	n-1
$(n, 1)$	n-1
(n, n)	$\frac{(n-1)(n-2)}{2}$

Table 3. Edge Partition of $T(S_n)$

$$\begin{aligned} SO(Q(S_n)) =& (n-1)\sqrt{n^2 + (n-1)^2} + (n-1)\sqrt{n^2 + 1} + \frac{(n-1)(n-2)}{2}\sqrt{n^2 + n^2} \\ =& (n-1)\sqrt{n^2 + (n-1)^2} + (n-1)\sqrt{n^2 + 1} + \frac{n(n-1)(n-2)}{2}\sqrt{2} \end{aligned}$$

□

5. Sombor Index of Total graphs

Theorem 5.1. *Sombor Index of total graph of some graphs are given by the following results.*

(1) *If $G = (n, m)$ be a regular graph with regularity r, then*

$$SO(T(G)) = (3m + m_l)2r\sqrt{2}$$

where m_l is the number of edges line graph of G.

(2) $SO(T(P_n)) = 4\sqrt{5} + 4\sqrt{13} + 20 + (4n - 13)4\sqrt{2}$.

(3) $SO(T(F_n)) = 12n\sqrt{2} + 4n\sqrt{n^2 + 1} + 4n\sqrt{4n^2 + (n+1)^2} + 8n\sqrt{(n+1)^2 + 4} + 2n(n+1)(2n-1)\sqrt{2}$.

(4)
$$\begin{aligned} SO(T(S_n)) =& 2(n-1)\sqrt{(n-1)^2 + 1} + (n-1)\sqrt{4(n-1)^2 + n^2} \\ &+ (n-1)\sqrt{4 + n^2} + \frac{n(n-1)(n-2)}{6}\sqrt{2} \end{aligned}$$

Proof. (1) For a regular graph $G = (n, m)$ with regularity r, $S(G)$ has $n + m$ vertex and $3m + m_l$ edges where $m_l = \frac{1}{2}nr^2 - m$ is the number of edges in line graph of G . All the edges of Total graph $T(G)$ if G is regular has edges of end degree $(2r, 2r)$.

Therefore, $SO(T(G)) = (3m + m_l)2r\sqrt{2}$

(2) For total graph of the path P_n, ($T(P_n)$) has $2n-1$ vertices and $3(n-1)+n-2 = 4n-5$ edges. The edge partition of $T(P_n)$ is given by

Degree of end vertices	Number of edges
$(2, 4)$	2
$(2, 3)$	2
$(3, 4)$	4
$(4, 4)$	4n-13

Table 4. Edge Partition of $T(P_n)$

Therefore,

$$SO(T(P_n)) = 2\sqrt{2^2+4^2} + 2\sqrt{2^2+3^2} + 4\sqrt{3^2+4^2} + (4n-13)\sqrt{4^2+4^2}$$
$$= 4\sqrt{5} + 4\sqrt{13} + 20 + (4n-13)4\sqrt{2}.$$

(3) The graph $T(F_n)$ has $2n+1+3n = 5n+1$ vertices and $9n+m_l$ edges with the following edge partition.

Degree of end vertices	Number of edges
$(4, 4)$	3n
$(4n, 4)$	2n
$(4n, 2(n+1))$	2n
$(4, 2(n+1))$	4n
$(2(n+1), 2(n+1))$	n(2n-1)

Table 5. Edge Partition of $T(F_n)$

$$SO(T(F_n)) = 3n\sqrt{4^2+4^2} + 2n\sqrt{(4n)^2+4^2} + 2n\sqrt{(4n)^2+4(n+1)^2}$$
$$+4n\sqrt{4(n+1)^2+4^2} + n(2n-1)\sqrt{4(n+1)^2+4(n+1)^2}$$
$$= 12n\sqrt{2} + 4n\sqrt{n^2+1} + 4n\sqrt{4n^2+(n+1)^2}$$
$$+ 8n\sqrt{(n+1)^2+4} + 2n(n+1)(2n-1)\sqrt{2}$$

(refer [2])

(4) The Total graph of star graph $T(S_n)$ will have $n + n - 1 = 2n - 1$ vertices and $(n-1) + 2(n-1) + m_l = 3(n-1) + m_l$ edges, where m_l is the number of edges in line graph of S_n equal to $\frac{(n-1)(n-2)}{2}$. The edge partition is as follows.

Degree of end vertices	Number of edges
$(2(n-1), 2)$	n-1
$((n-1), n)$	n-1
$(2, n)$	n-1
(n, n)	$\frac{(n-1)(n-2)}{2}$

Table 6. Edge Partition of $T(S_n)$

$$\begin{aligned}SO(T(S_n)) =&(n-1)\sqrt{4(n-1)^2+2^2}+(n-1)\sqrt{4(n-1)^2+n^2}\\&+(n-1)\sqrt{2^2+n^2}+\frac{(n-1)(n-2)}{2}\sqrt{n^2+n^2}\\=&2(n-1)\sqrt{(n-1)^2+1}+(n-1)\sqrt{4(n-1)^2+n^2}\\&+(n-1)\sqrt{4+n^2}+\frac{n(n-1)(n-2)}{2}\sqrt{2}\end{aligned}$$

□

6. Conclusion

Topological index is the quantity which is invariant under graph isomorphism. It has several applications in chemical mathematics. Degree based index has great importance in molecular chemical properties. Sombor index is a recent developed vertex degree based topological index. Here Sombor index of a few modified graphs are calculated. Sombor index of many more classes of graphs can be calculated in future.

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