CALIBRATION APPROACH TO DEVELOPED TRIPLE MEANS ESTIMATORS

ABSTRACT

Calibration estimation has become a ubiquitous methodology across diverse fields, providing a foundational framework for tackling complex statistical problems. Its significance In recent years, has emerged as a pivotal topic in research on estimation in survey sampling where it has emerged as a crucial area of study. By providing a systematic approach to integrating auxiliary information, calibration enhances the estimation procedure, rendering it a valuable tool in statistical analysis. The article propounds an calibration approaches of triple mean under simple random sampling of variance estimators, the proposed calibration have been develop utilizing sample variance incorporation with existing estimators of AM, GM, HM, in the problem constraints of the optimization in other to contribute effectively to new design calibrated weight. However, the proposed new weight is obtained using most common approach Lagrange function with two multipliers. The motivation for using calibration scheme is due to their ability in reduce bias and means square error, enhance precision, base on how auxiliary variable are been utilized, provide flexibility, comply with standard and improve decision making. Focusing on simulated data approached using exponential and beta the result indicate superiority of classes calibrated estimators of studied via R packages.

Keywords:— Arithmetic Mean; Calibration; Constraints; Design Weight; Geometric Mean; Harmonic Mean; Variance; Triple Mean; New Weight; Original Weights

1 Introduction to Calibration in Survey Sampling

Computing descriptive statistics especially measures of central tendency is important in understanding the data, as they summarize and describe the data. The Arithmetic Mean (AM), Geometric Mean (GM) and Harmonic Mean (HM) are among the three most common of these measures of central tendency, although they serve different purposes, Knowledge about the special features of each of those mention is important in the selection of the proper manner for various sorts of information as well and therefore the objectives of the analysis. Calculating the arithmetic mean may be the most popular method for fining a measure of central tendency. Sum the entire data set and divide that by number of observations.

Arithmetic Mean (AM): The AM is easy to grasp and simple to calculate, which is why it is the one most often utilized in all fields from finance to education to social sciences. One major drawback of AM, however, is its susceptibility to extreme values (or outliers). In cases that contain outliers, the AM may be strongly influence by extreme value(s) and thus lead to erroneous conclusions. Example, for financial data, where extreme values (very high or very low) are common, this mean that the outliers dictate the margin which results in dividing the data set (say, sale prices) into two bins which are really disconnected. for instance, if you look at financial data where extreme values are frequent, such as very high or low incomes, the AM would not represent the actual performance or central tendency of the data set. Relying solely on the AM can give a skewed view of the data in such instances. In spite of this its simplicity and interpretabilility. A good measure to have as it gives a quick overview of the average value in a data set, especially useful. However, the AM does have a downside it can be heavily influenced by extreme it doesn't accurately represent the typical data point. This is especially true in finance, where things like unusually high incomes can distort the mean. In cases like these, relying on the AM alone can paint a misleading picture of the data. Even with this limitation, the AM is still widely used. It's easy to calculate and understand, making it a helpful starting point for analyzing data or sharing quick insights with others.

Geometric Mean (GM): The GM provides a different way to measure central tendency that is especially usefully for data set with exponential growth or multiplicative relationships. While the AM is most applicable to additive processes, the GM is more representative of scenarios where values are multiplicative in nature or where rates of change are introduced. The GM works particularly well on applications like finance, where it is used to compute average growth rates over subsume time. for example, when evaluating investment return over multiple periods, the GM accounts for compounding effects more accurately than the AM. This is what helps it to stand out as a popular option for those investors wishing to track the performance of their investments over time without letting noise from individual returns distort the analysis. The GM's key supersonic resource is the capacity to tolerate extreme values which makes it superior to the AM. Because it depends on the product of values rather than their sum, it restricts the impact of outliers that can disrupt the computation of average operators. This trait makes the GM exceptionally useful in branches like environmental science and economics where the distribution of data is often assumed to be non-normally distributed.

The Geometric Mean offers an alternative approach to measuring central tendency that is particularly useful for datasets exhibiting exponential growth or multiplicative relationships. Unlike the AM, which is best suited for additive processes, the GM is designed to handle situations where values are multiplied together or involve rates of change.The GM is especially effective in contexts such as finance, where it is used to calculate average growth rates over time. For example, when evaluating investment returns over multiple periods, the GM accounts for compounding effects more accurately than the AM. This makes it a preferred choice for investors who want to assess their portfolio's performance over time without being misled by fluctuations in individual returns.One of the key advantages of the GM is its robustness against extreme values compared to the AM. Since it relies on the product of values rather than their sum, it mitigates the influence of outliers that can distort average calculations. This property makes the GM particularly valuable in fields such as environmental science and economics, where data distributions may be skewed.

Harmonic Mean (HM): Another Measure of central tendency that relates to rate or ratios is Harmonic Mean. It is particularly useful in in cases where average rates need or density calculations, because it gives greater weight to smaller weight values than to large ones. For example if a vehicle cover bicycle at various speed travel various distance, if take HM rather than AM, the get a more accurate estimate of the average speed speed. This is due to fact that speed is itself a rate, so the application of HM admits because speed is inherently a rate, thus applying HM allows for a more meaningful representation of average performance over varying conditions. However, like GM the HM is sensitive to very small values in the data set. If any values approaches zero, it can disproportionately effect results. Therefore, while it serves as an effective tool in specific contexts such as calculating average rates it require careful consideration regarding the nature of data being analyzed.

The Harmonic Mean is another measure of central tendency that is particularly relevant when dealing with rates or ratios. It is especially useful in scenarios where average rates are required such as speed or density calculations because it emphasizes smaller values more than larger ones.For instance, if a vehicle travels different distances at varying speeds, using the HM provides a more accurate average speed than the AM would. This is because speed is inherently a rate; thus, applying HM allows for a more meaningful representation of average performance over varying conditions.However, like the GM, the HM is sensitive to very small values in the data set. If any value approaches zero, it can disproportionately affect the HM calculation, leading to misleading results. Therefore, while it serves as an effective tool in specific contexts such as calculating average rates it requires careful consideration regarding the nature of the data being analyzed.

Practical Applications In practice, each of these means finds application across various fields based on their unique properties: 1. Arithmetic Mean: Commonly used in educational settings to calculate average grades or test scores among students. It provides a straightforward way to assess overall performance but may not accurately reflect individual student capabilities if outliers exist. 2. Geometric Mean: Frequently employed in financial analysis to evaluate investment returns over time. It allows investors to understand compounded growth rates better than traditional methods might permit. 3. Harmonic Mean: Utilized in fields such as physics and engineering when averaging rates or ratios particularly when dealing with speeds or densities where traditional averages may not provide meaningful insights. Understanding when to use each mean can significantly enhance data analysis and interpretation. Analysts must consider the nature of their data and select the appropriate measure based on whether they are dealing with additive processes (AM), multiplicative relationships (GM), or rates (HM).

The application of statistical estimators, particularly the Arithmetic Mean, Geometric Mean, and Harmonic Mean, has been extensively explored in various research contexts, especially concerning their effectiveness in estimating population parameters. Recent studies have highlighted how these means can be integrated into variance estimation methodologies, providing valuable insights into population characteristics while addressing common challenges such as measurement errors and non-response in survey data.One significant contribution to this field is the work by (Misra et al., 2016), which discusses the use of these means in developing estimators for population variance under simple random sampling. The authors propose several estimators that leverage auxiliary information based on the Arithmetic Mean, Geometric Mean, and Harmonic Mean. Their findings suggest that these proposed estimators yield better results than traditional methods, particularly in terms of bias and mean squared error (MSE). This advancement is crucial for researchers who rely on accurate variance estimates to draw valid conclusions about population parameters.In another study, (Singh et al., 2018) focus on improving estimators for the coefficient of variation using auxiliary variables. They derive expressions for the MSE of their proposed estimators, demonstrating that under optimal conditions, some of these new estimators outperform conventional methods. The issue of measurement error is a recurring challenge in survey sampling, as highlighted by (Misra et al., 2016). Their research addresses how measurement errors can significantly affect the estimation of finite population variance. They propose improved estimators based on the Arithmetic Mean, Geometric Mean, and Harmonic Mean while calculating biases and MSEs to the first order of approximation. Their comparative study reveals that these new estimators can effectively mitigate the impact of measurement errors, thus providing more reliable estimates in practical applications. The point of this work is to highlight the need for statistical approaches that handle the data noise you see in the wild. Furthermore (Tariq et al., 2021) study variance estimation when measurement error and non-response act simultaneously. Their work uses auxiliary variables to develop new estimators that integrate the sample variance with the ratio estimation techniques bases on different mean. Deriving approximation biases and square error (MSEs) via Taylor expansion, they indicate how suggested methods enhance estimation efficiency compared to reviewed approaches. This research highlights an essential aspect of statistical analysis: addressing data quality issues while striving for accurate parameter estimation. (Audu et al., 2023a) also contribute to this discourse by exploring modified variance estimators that account for both measurement error and non-response under simple random sampling designs. Their findings reveal that using auxiliary variables can significantly improve estimation accuracy by combining sample variance with ratio and exponential estimations based on different means. The study emphasizes the importance of understanding how various factors such as measurement error can influence estimation outcomes and highlights the effectiveness of proposed methods in overcoming these challenges .The integration of Arithmetic Mean, Geometric Mean, and Harmonic Mean into advanced statistical methodologies illustrates their versatility and relevance in contemporary research. These means not only serve as foundational concepts in statistics but also play a critical role in enhancing estimation techniques across various fields. By addressing common issues such as measurement errors and non-response through innovative approaches, researchers can improve the reliability of their findings and contribute to more informed decision-making processes.In summary, recent studies have demonstrated how traditional measures like AM, GM, and HM can be effectively applied within modern statistical frameworks to address complex challenges associated with data collection and analysis. The ongoing exploration of these means within variance estimation methodologies reflects their enduring significance in statistical practice.

Calibration of Population Parameters such Mean, Variance, Coefficient of Variation, Proportion and Median in Simple, Stratified, Systematic and other Random Sampling: Enhancing estimation accuracy in statistical analysis, the accuracy of population parameter estimates is the paramount. One critical parameter is the population variance, which measures the dispersion of data points. when estimating population variance using simple random sampling, calibration plays vital role in refining these estimates. calibration in this context, involves adjusting sample based estimates to align more closely with the true population parameters, thereby reducing bias and increasing in precision (Maksum, 1990; Mohamed et al., 2015; Martínez et al., 2015; Singh and Sedory, 2016; Lata et al., 2017; Singh et al., 2017; Mohamed et al., 2018; Alka et al., 2019; Audu et al., 2020; Garg and Pachori, 2020; Audu et al., 2021b,a; Singh et al., 2021; Shahzad et al., 2021; Audu et al., 2021b; Rai et al., 2021; Rabee et al., 2021; Basak et al., 2022; Alka et al., 2022; Clement, 2022; Eyo and Enang, 2022; Singh et al., 2023; Shahzad et al., 2023b,a; Audu et al., 2023b; Çetin and Koyuncu, 2024; Chaudhary et al., 2024; Suleiman et al., 2024; Abubakar et al., 2024; Audu et al., 2024c,b; Garg et al., 2024; Pandey et al., 2024; Afsar Basha and Usman, 2025; Audu et al., 2025a; Suleiman et al., 2025; Audu and Aphane, 2025).

1.1 Notations and Reviews under Study

Supposed that finite population consists of N units $\Omega = 1, 2, ..., i, ..., N$ from which a random sample $s(s \in \Omega)$ of size n is drawn through a sampling design $p(.)$. It is assumed that the first order $(\pi_i \in s)$. It is assumed that the first order $(\pi_i = p(i \in s)$ inclusion probabilities are strictly positive and known. Let us denote the S_y^2, V_y The variance of observations population and sample of the study Y. S_x^2, V_x The variance of observations population and sample of the auxiliary variable X

μ_{20} : Second moment of Variable X (Measures variability in 'X' within the stratum).

μ_{02} : Second moment of Variable Y (Measures variability in 'Y' within the stratum).

μ_{22} : Mixed Second Order moment of Variable X (Capture joint variability between observation 'Y' and 'X' within the stratum).

Singh et al. (2014) proposed variance estimators based on the integration of Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM) under simple random sampling. Their work derives the mean squared error (MSE) of these estimators and demonstrates that incorporating auxiliary information leads to enhanced estimator performance in terms of bias and MSE. The study highlights that well-designed estimators which exploit auxiliary data are superior to traditional estimators.

Mishra and Singh (2016) proposed variance estimators based on the modification of (Singh et al., 2014) by consider dual ratio and dual ratio cum in place of ratio and exponential ratio Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM) under simple random sampling and further suggested Almost unbiased estimators.

Measurement errors, a common challenge in survey data, were addressed by (Misra et al., 2016), who developed improved finite population variance estimators that are robust to the presence of measurement errors. They calculated biases and MSEs of the estimators and demonstrated via simulations that the new estimators perform better than classical approaches when measurement errors are significant.

Singh et al. (2018) contributed to improving the estimation of the population coefficient of variation using auxiliary variables. They developed and compared new estimators, showing through mean squared error analysis that the proposed estimators outperform conventional ones under optimal configurations. Their research underlines the importance of leveraging auxiliary information for more accurate population parameter estimations.

Tariq et al. (2021) investigated variance estimation incorporating auxiliary information in the presence of measurement errors. They devised estimators based on combinations of sample variance and ratio estimators, grounded in AM, GM, and HM. Their results exhibited reduced bias and MSE compared to existing approaches, emphasizing the benefit of incorporating auxiliary data even under data quality challenges.

Audu et al. (2023a) examined variance estimation under the concurrent influence of measurement error and non-response using auxiliary variables. Their modified estimators combined sample variance with ratio and exponential estimation techniques. The study's evaluation showed superior efficiency of the modified estimators relative to existing methods, recommending the use of auxiliary information to correct for these common survey issues.

These studies collectively emphasize that the integration of AM, GM, and HM within variance estimation, supplemented by auxiliary information, significantly improves statistical performance in survey sampling. Further, addressing practical challenges like measurement errors and non-response through innovative estimation methods is crucial for reliability and accuracy in real-world data analysis.

2 Methodology

Based on reviewed presented at section 1 it clear noticed that the journey of estimator on Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM) was begin by (Singh et al., 2014) later follow by (Mishra and Singh, 2016) after then the study of measurement error were introduced by both (Mishra and Singh, 2016) after run similarly (Tariq et al., 2021) respectively, (Singh et al., 2018) came with the estimators by involved coefficients of variation of study variable, recently (Audu et al., 2023a) consider joint effect of measurement non response, aforementioned estimator are basically in simple random sample, beside this research article consider stratified form and proposed a calibration scheme estimators.

2.1 Proposed Calibration Estimator

Motivated by (Singh et al., 2014) and calibration approach estimators by (Audu et al., 2024a, 2025b) the following calibration in eq(1) is established

$$t^{k}_{pj(i)} = \sum_{i\in s}^{n} \Theta_{ht}^{SH} (y_i - \bar{y})^2 \tag{1}$$

$$\left.\begin{array}{l} \chi = \sum\limits_{i=1}^{n} \left(\Theta_{ht}^{SH} - W^{**}_{j(k)}\right)^2 / W^{**}_{j(k)} d_i \\ s.t \ \sum\limits_{i=1}^{n} \Theta_{ht}^{SH} (x_i - \bar{x})^2 = S_X^2 \\ \quad \sum\limits_{i=1}^{n} \Theta_{ht}^{SH} = \sum\limits_{i=1}^{n} W^{**}_{j(k)} \end{array}\right\} \quad j = 1,2,3,4 \ \ \& \ \ k = AM, GM, HM \tag{2}$$

Define Lagrange function

$$D = \sum_{i=1}^{n} \frac{\left(\Theta_{ht}^{SH} - W^{**}_{j(k)}\right)^2}{W^{**}_{j(k)} d_i} - 2\gamma_1 \left(\sum_{i=1}^{n} \Theta_{ht}^{SH} (x_i - \bar{x})^2 - S_X^2\right) - 2\gamma_2 \left(\sum_{i=1}^{n} \Theta_{ht}^{SH} - \sum_{i=1}^{n} W^{**}_{j(k)}\right) \tag{3}$$

Where γ_1 and γ_2, are optimization constant parameters to minimized the chi square distance measure or Lagrange function.

where $W^{**}_{j(k)} = (k = AM = W^{**}_{j(AM)}, k = GM = W^{**}_{j(GM)}, k = HM = W^{**}_{j(HM)}$ for $j = 1,2,3,4$ and $k = AM, GM, HM$

The optimal value of Θ_{ht}^{SH} depends on the fixed weight $W^{**}_{j(k)}$ which are the same time also stand as the review or existing estimator $\hat{s}^{2\,(k)}_{y(j)}$ for study in equations (4), (5),...,(15), below, the penalty/coefficient of Lagrange γ_1 and γ_2, the scalar d_i, and the squared deviation $(x_i - \bar{x})^2$.

$$W^{**}_{1(AM)} = \frac{t_0 + t_1}{2} = \frac{V_y}{2}\left(1 + S_X^2 / V_x\right) \tag{4}$$

$$W^{**}_{2(AM)} = \frac{t_0 + t_2}{2} = \frac{V_y}{2}\left(1 + \exp\left(\frac{S_X^2 - V_x}{V_x + S_X^2}\right)\right) \tag{5}$$

$$W^{**}_{3(AM)} = \frac{t_1 + t_2}{2} = \frac{V_y}{2}\left(\frac{S_X^2}{V_x} + \exp\left(\frac{S_X^2 - V_x}{V_x + S_X^2}\right)\right) \tag{6}$$

$$W^{**}_{4(AM)} = \frac{t_0 + t_1 + t_2}{3} = \frac{V_y}{3}\left(1 + \frac{S_X^2}{V_x} + \exp\left(\frac{S_X^2 - V_x}{V_x + S_X^2}\right)\right) \tag{7}$$

$$W^{**}_{1(GM)} = (t_0 t_1)^{\frac{1}{2}} = V_y\left(\frac{S_X^2}{V_x}\right)^{\frac{1}{2}} \tag{8}$$

$$W^{**}_{2(GM)} = (t_0 t_2)^{\frac{1}{2}} = V_y\left(\exp\left(\frac{S_X^2 - V_x}{V_x + S_X^2}\right)\right)^{\frac{1}{2}} \tag{9}$$

$$W^{**}_{3(GM)} = (t_1 t_2)^{\frac{1}{2}} = V_y\left(\frac{S_X^2}{V_x}\exp\left(\frac{S_X^2 - V_x}{V_x + S_x^2}\right)\right)^{\frac{1}{2}} \tag{10}$$

$$W^{**}_{4(GM)} = (t_0 t_1 t_2)^{\frac{1}{3}} = V_y\left(\frac{S_X^2}{V_x}\exp\left(\frac{S_x^2 - V_x}{V_x + S_X^2}\right)\right)^{\frac{1}{3}} \tag{11}$$

$$W^{**}_{1(HM)} = \frac{2}{(1/t_0 + 1/t_1)} = 0.5V_y\left(1 + \frac{V_x}{S_X^2}\right) \tag{12}$$

$$W^{**}_{2(HM)} = \frac{2}{(1/t_0 + 1/t_2)} = 0.5V_y\left(1 + 1/\exp\left(\frac{S_X^2 - V_x}{V_x + S_X^2}\right)\right) \tag{13}$$

$$W^{**}_{3(HM)} = \frac{2}{(1/t_1 + 1/t_2)} = 0.5V_y\left(\frac{V_x}{S_X} + 1/\exp\left(\frac{S_X^2 - V_x}{V_x + S_X^2}\right)\right) \tag{14}$$

$$W^{**}_{4(HM)} = \frac{3}{(1/t_0 + 1/t_1 + 1/t_2)} = \frac{V_y}{3}\left(1 + \frac{V_x}{S_X^2} + 1/\exp\left(\frac{S_X^2 - V_x}{V_x + S_X^2}\right)\right) \tag{15}$$

2.2 Determination of New Calibrated Weight Proposed Estimators

To determine Lagrange Multipliers γ_1 and γ_2, Differentiate (3), partially with respect to Θ_{ht}^{SH}, and equate to zero obtain (16).

$$\Theta_{ht}^{SH} = W^{**}_{j(k)}\left(1 + \gamma_1 (x_i - \bar{x})^2 d_i + \gamma_2 d_i\right) \tag{16}$$

Substitute (16) into second and third of equation (2) simplify to generate matrix of two simultaneous equation in (17)

$$\begin{pmatrix} \sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^4 & \sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^2 \\ \sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^2 & \sum_{i=1}^{n} W_{j(k)}^{**} d_i \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} S_X^2 - \sum_{i=1}^{n} W_{j(k)}^{**} (x_i - \bar{x})^2 \\ 0 \end{pmatrix} \tag{17}$$

Solving equation (17) above using any of matrix approach the following obtained are

$$\gamma_1 = -\frac{\left(S_X^2 - \sum_{i=1}^{n} W_{j(k)}^{**} (x_i - \bar{x})^2\right)\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i\right)}{\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^2\right)^2 - \left(W_{j(k)}^{**} d_i\right)\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^4\right)} \tag{18}$$

$$\gamma_1 = \frac{\left(S_X^2 - \sum_{i=1}^{n} W_{j(k)}^{**} (x_i - \bar{x})^2\right)\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^2\right)}{\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^2\right)^2 - \left(W_{j(k)}^{**} d_i\right)\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^4\right)} \tag{19}$$

Substitute (18) and (19) into (16) for γ_1, γ_2 and simplifying, the new calibrate weight proposed of estimator is drive as follow;

$$\Theta_{ht}^{SH} = W_{j(k)}^{**} + W_{j(k)}^{**} d_i \frac{\left(S_X^2 - \sum_{i=1}^{n} W_{j(k)}^{**} (x_i - \bar{x})^2\right)\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^2 - (x_i - \bar{x})^2 \sum_{i=1}^{n} W_{j(k)}^{**} d_i\right)}{\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^2\right)^2 - \left(W_{j(k)}^{**} d_i\right)\left(\sum_{i=1}^{n} W_{j(k)}^{**} d_i (x_i - \bar{x})^4\right)} \tag{20}$$

Inserting new calibration waited of (20) into calibration scheme proposed in equation (1), the calibrate proposed estimator of $t_{pj(i)}^{k}$ for $j = 1, 2, 3, 4$ became

$$t_{pj(i)}^{k} = \hat{s}_{y(j)}^{2(k)} + \hat{\beta}_{j(i)}^{*(k)} \left(S_X^2 - \hat{s}_{x(j)}^{2(k)}\right) \quad for \;\; j = 1, 2, 3, 4 \tag{21}$$

where $\hat{s}^{2(k)}_{y(j)} = \sum\limits_{i=1}^{n} W^{**}_{j(k)} (y_i - \bar{y})^2, \quad \hat{s}^{2(k)}_{x(j)} = \sum\limits_{i=1}^{n} W^{**}_{j(k)} (x_i - \bar{x})^2,$

$$\hat{\beta}^{*(k)}_{j(i)} = \frac{\left(\sum\limits_{i=1}^{n} W^{**}_{j(k)} d_i (x_i - \bar{x})^2 \sum\limits_{i=1}^{n} W^{**}_{j(k)} d_i (y_i - \bar{y})^2 - \sum\limits_{i=1}^{n} W^{**}_{j(k)} d_i \sum\limits_{i=1}^{n} W^{**}_{j(k)} d_i (y_i - \bar{y})^2 (x_i - \bar{x})^2\right)}{\left(\sum\limits_{i=1}^{n} W^{**}_{j(k)} d_i (x_i - \bar{x})^2\right)^2 - \left(W^{**}_{j(k)} d_i\right)\left(\sum\limits_{i=1}^{n} W^{**}_{j(k)} d_i (x_i - \bar{x})^4\right)} \tag{22}$$

2.3 Material and Methods of Proposed Calibrated estimators

Case 1: That is design parameter for $i = 1$ is $d_i = 1$

Class of estimator under

Table 1: The report of different calibration schemes for first choose design scale

No. of Com.	Arithematic in conbinations	Giometric in combinations	Harmonic in combination
1	$\hat{s}^{2(AM)}_{y(1)} + \hat{\beta}^{*(AM)}_{1(1)} \left(S^2_X - \hat{s}^{2(AM)}_{x(1)}\right)$	$\hat{s}^{2(GM)}_{y(1)} + \hat{\beta}^{*(GM)}_{1(1)} \left(S^2_X - \hat{s}^{2(GM)}_{x(1)}\right)$	$\hat{s}^{2(HM)}_{y(1)} + \hat{\beta}^{*(HM)}_{1(1)} \left(S^2_X - \hat{s}^{2(HM)}_{x(1)}\right)$
2	$\hat{s}^{2(AM)}_{y(2)} + \hat{\beta}^{*(AM)}_{2(1)} \left(S^2_X - \hat{s}^{2(AM)}_{x(2)}\right)$	$\hat{s}^{2(GM)}_{y(2)} + \hat{\beta}^{*(GM)}_{2(1)} \left(S^2_X - \hat{s}^{2(GM)}_{x(2)}\right)$	$\hat{s}^{2(HM)}_{y(2)} + \hat{\beta}^{*(HM)}_{2(1)} \left(S^2_X - \hat{s}^{2(HM)}_{x(2)}\right)$
3	$\hat{s}^{2(AM)}_{y(3)} + \hat{\beta}^{*(AM)}_{3(1)} \left(S^2_X - \hat{s}^{2(AM)}_{x(3)}\right)$	$\hat{s}^{2(GM)}_{y(3)} + \hat{\beta}^{*(GM)}_{3(1)} \left(S^2_X - \hat{s}^{2(GM)}_{x(3)}\right)$	$\hat{s}^{2(HM)}_{y(3)} + \hat{\beta}^{*(HM)}_{3(1)} \left(S^2_X - \hat{s}^{2(HM)}_{x(3)}\right)$
4	$\hat{s}^{2(AM)}_{y(4)} + \hat{\beta}^{*(AM)}_{4(1)} \left(S^2_X - \hat{s}^{2(AM)}_{x(4)}\right)$	$\hat{s}^{2(GM)}_{y(4)} + \hat{\beta}^{*(GM)}_{4(1)} \left(S^2_X - \hat{s}^{2(GM)}_{x(4)}\right)$	$\hat{s}^{2(HM)}_{y(4)} + \hat{\beta}^{*(HM)}_{4(1)} \left(S^2_X - \hat{s}^{2(HM)}_{x(4)}\right)$

$$Equation\ 22\ became\ \hat{\beta}^{*(k)}_{j(1)} = \frac{\left(s^2_y s^2_x - q_{(22)}\right)}{\left((s^2_x)^2 - q_{(04)}\right)}\ \forall\ q_{(rs)} = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^r (y_i - \bar{y})^s \tag{23}$$

Case 2: That is design parameter for $i = 2$ is $d_i = 1/(x_i - \bar{x})^2$

Table 2: The report of different calibration schemes for second choose design scale

No. of Com.	Arithematic in conbinations	Giometric in combinations	Harmonic in combination
1	$\hat{s}^{2(AM)}_{y(1)} + \hat{\beta}^{*(AM)}_{1(2)} \left(S^2_X - \hat{s}^{2(AM)}_{x(1)}\right)$	$\hat{s}^{2(GM)}_{y(1)} + \hat{\beta}^{*(GM)}_{1(2)} \left(S^2_X - \hat{s}^{2(GM)}_{x(1)}\right)$	$\hat{s}^{2(HM)}_{y(1)} + \hat{\beta}^{*(HM)}_{1(2)} \left(S^2_X - \hat{s}^{2(HM)}_{x(1)}\right)$
2	$\hat{s}^{2(AM)}_{y(2)} + \hat{\beta}^{*(AM)}_{2(2)} \left(S^2_X - \hat{s}^{2(AM)}_{x(2)}\right)$	$\hat{s}^{2(GM)}_{y(2)} + \hat{\beta}^{*(GM)}_{2(2)} \left(S^2_X - \hat{s}^{2(GM)}_{x(2)}\right)$	$\hat{s}^{2(HM)}_{y(2)} + \hat{\beta}^{*(HM)}_{2(2)} \left(S^2_X - \hat{s}^{2(HM)}_{x(2)}\right)$
3	$\hat{s}^{2(AM)}_{y(3)} + \hat{\beta}^{*(AM)}_{3(2)} \left(S^2_X - \hat{s}^{2(AM)}_{x(3)}\right)$	$\hat{s}^{2(GM)}_{y(3)} + \hat{\beta}^{*(GM)}_{3(2)} \left(S^2_X - \hat{s}^{2(GM)}_{x(3)}\right)$	$\hat{s}^{2(HM)}_{y(3)} + \hat{\beta}^{*(HM)}_{3(2)} \left(S^2_X - \hat{s}^{2(HM)}_{x(3)}\right)$
4	$\hat{s}^{2(AM)}_{y(4)} + \hat{\beta}^{*(AM)}_{4(1)} \left(S^2_X - \hat{s}^{2(AM)}_{x(4)}\right)$	$\hat{s}^{2(GM)}_{y(4)} + \hat{\beta}^{*(GM)}_{4(1)} \left(S^2_X - \hat{s}^{2(GM)}_{x(4)}\right)$	$\hat{s}^{2(HM)}_{y(4)} + \hat{\beta}^{*(HM)}_{4(1)} \left(S^2_X - \hat{s}^{2(HM)}_{x(4)}\right)$

$$Equation\ 22\ became\ \hat{\beta}^{*(k)}_{j(1)} = \frac{\left(q_{(2/2)} - s^2_y q_{(0/2)}\right)}{\left((1 - s^2_x q_{(0/2)}\right)}\ \forall\ q_{(r/s)} = \frac{1}{n-1}\sum_{i=1}^{n} \frac{(x_i - \bar{x})^r}{(y_i - \bar{y})^s} \tag{24}$$

3 Empirical Simulation Study

Simulation study were demonstrated using Exponential and Beta distributions respectively to test superiority of proposed calibrate over reviews estimators. The simulation steps are as follows and more described on table(3). Data of size 1000 units were generated using simple linear and non-linear regression respectively with parameters slop ,intercept and error term given in table(3) below. Samples of sizes 100 and 80 using exp and beta were drawn 10,000 times by the SRSWOR method from population generated. The precision PREs of the considered and computed using with eqn(25)

Table 3: Populations[exponential and bata]

data	n	linear reg	data	n	non-linear reg
exp	100	$Y = 30 + 0.5 * X + e \sim N(0,1)$	beta	60	$Y = 40 * X + X^2 + e \sim N(0,1)$

$$PRE(t^k_{pj(i)}) = \left(\frac{\frac{1}{10000} \sum_{j=1}^{10000} \left(\hat{s}^{2(k)}_{y(j)} - S^2_y \right)^2}{\frac{1}{10000} \sum_{j=1}^{10000} \left(t^k_{pj(i)} - S^2_y \right)^2} \right) \times 100 \qquad (25)$$

Table 4: Percentage Relative Efficiencies (PRE) and Calibration Proposed Estimators under Exponential Distribution

Estimators	$\hat{s}_{y(1)}^{2(AM)}$	$\hat{s}_{y(1)}^{2(GM)}$	$\hat{s}_{y(1)}^{2(HM)}$	$\hat{s}_{y(2)}^{2(AM)}$	$\hat{s}_{y(2)}^{2(GM)}$	$\hat{s}_{y(2)}^{2(HM)}$	$\hat{s}_{y(3)}^{2(AM)}$	$\hat{s}_{y(3)}^{2(GM)}$	$\hat{s}_{y(3)}^{2(HM)}$	$\hat{s}_{y(4)}^{2(AM)}$	$\hat{s}_{y(4)}^{2(GM)}$	$\hat{s}_{y(4)}^{2(HM)}$
$t_{p1(1)}^{AM}$	2782.796	2110.98	3413.609	8195.066	193.2132	549.0134	278.7831	630.5589	133.4446	4.163511	298.0621	89.65666
$t_{p1(1)}^{GM}$	3654.456	2772.207	4482.861	10762.02	253.7337	720.9819	366.1069	828.0701	175.2437	5.467655	391.4247	117.74
$t_{p1(1)}^{HM}$	2274.238	1725.197	2789.77	6697.411	157.9033	448.6808	227.8353	515.3238	109.0575	3.402626	243.5911	73.27184
$t_{p2(1)}^{AM}$	955.192	724.592	1171.718	2812.949	66.32024	188.4483	95.69203	216.4387	45.80473	1.429121	102.3095	30.77456
$t_{p2(1)}^{GM}$	37342.89	28327.66	45807.9	109971.2	2592.766	7367.32	3741.046	8461.595	1790.72	55.87098	3999.755	1203.121
$t_{p2(1)}^{HM}$	13661.32	10363.24	16758.11	40231.27	948.5233	2695.22	1368.604	3095.544	655.1071	20.43953	1463.249	440.143
$t_{p3(1)}^{AM}$	26294.77	19946.76	32255.36	77435.58	1825.681	5187.654	2634.235	5958.182	1260.925	39.34122	2816.404	847.1702
$t_{p3(1)}^{GM}$	11940.15	9057.586	14646.78	35162.58	829.02	2355.652	1196.175	2705.54	572.571	17.86438	1278.896	384.69
$t_{p3(1)}^{HM}$	7394.282	5609.174	9070.444	21775.45	513.3948	1458.806	740.7662	1675.484	354.5812	11.06304	791.9934	238.2305
$t_{p4(1)}^{AM}$	6108.5	4633.802	7493.196	17988.95	424.1212	1205.136	611.9553	1384.136	292.9236	9.139301	654.2746	196.8049
$t_{p4(1)}^{GM}$	8371.839	6350.731	10269.6	24654.26	581.2678	1651.667	838.6987	1896.991	401.4584	12.52562	896.6983	269.7256
$t_{p4(1)}^{HM}$	7080.056	5370.807	8684.987	20850.08	491.5776	1396.813	709.2867	1604.283	339.513	10.5929	758.3369	228.1066
$t_{p1(2)}^{AM}$	6640.523	5037.385	8145.82	19555.7	461.0603	1310.098	665.2539	1504.689	318.4359	9.935293	711.259	213.9457
$t_{p1(2)}^{GM}$	8675.548	6581.119	10642.15	25548.65	602.3547	1711.585	869.1246	1965.809	416.0223	12.98002	929.2283	279.5105
$t_{p1(2)}^{HM}$	5445.496	4130.858	6679.9	16036.46	378.088	1074.333	545.5349	1233.905	261.1302	8.14734	583.261	175.4441
$t_{p2(2)}^{AM}$	2313.226	1754.773	2837.596	6812.229	160.6103	456.3727	231.7412	524.1582	110.9271	3.460959	247.7671	74.52798
$t_{p2(2)}^{GM}$	81249.61	61634.54	99667.55	239272.3	5641.268	16029.61	8139.663	18410.5	3896.198	121.5625	8702.555	2617.716
$t_{p2(2)}^{HM}$	31237.46	23696.19	38318.48	91991.32	2168.858	6162.79	3129.398	7078.155	1497.944	46.73628	3345.81	1006.415
$t_{p3(2)}^{AM}$	58371.28	44279.43	71603.08	171897.8	4052.795	11515.98	5847.69	13226.46	2799.103	87.33284	6252.083	1880.617
$t_{p3(2)}^{GM}$	27439.25	20814.93	33659.27	80805.94	1905.143	5413.446	2748.89	6217.51	1315.806	41.05354	2938.987	884.0431
$t_{p3(2)}^{HM}$	22709.77	17227.23	27857.7	66878.08	1576.769	4480.374	2275.086	5145.849	1089.011	33.97747	2432.418	731.6678
$t_{p4(2)}^{AM}$	49592.55	37620.04	60834.36	146045.3	3443.276	9784.036	4968.229	11237.27	2378.133	74.19845	5311.803	1597.783
$t_{p4(2)}^{GM}$	15924.86	12080.32	19534.76	46897.17	1105.684	3141.79	1595.367	3608.443	763.6515	23.82615	1705.694	513.0702
$t_{p4(2)}^{HM}$	26302.67	19952.75	32265.05	77458.84	1826.229	5189.213	2635.027	5959.972	1261.303	39.35304	2817.25	847.4247

Table 5: Percentage Relative Efficiencies (PRE) of Exiting and Calibration Proposed Estimators under Beta Distribution

Estimators	$\hat{s}_{y(1)}^{2(AM)}$	$\hat{s}_{y(1)}^{2(GM)}$	$\hat{s}_{y(1)}^{2(HM)}$	$\hat{s}_{y(2)}^{2(AM)}$	$\hat{s}_{y(2)}^{2(GM)}$	$\hat{s}_{y(2)}^{2(HM)}$	$\hat{s}_{y(3)}^{2(AM)}$	$\hat{s}_{y(3)}^{2(GM)}$	$\hat{s}_{y(3)}^{2(HM)}$	$\hat{s}_{y(4)}^{2(AM)}$	$\hat{s}_{y(4)}^{2(GM)}$	$\hat{s}_{y(4)}^{2(HM)}$
$t_{p1(1)}^{AM}$	992.3306	1143.62	831.2417	2223.097	239.5777	614.7876	415.9548	624.0797	1.863382	1940792	1.863095	4.188365
$t_{p1(1)}^{GM}$	861.5805	992.9359	721.7167	1930.18	208.0108	533.7828	361.1483	541.8506	1.617861	1685072	1.617612	3.636503
$t_{p1(1)}^{HM}$	1183.661	1364.121	991.513	2651.731	285.7706	733.3245	496.1549	744.4083	2.22266	2314995	2.222317	4.995922
$t_{p2(1)}^{AM}$	444.2613	511.9927	372.1426	995.2688	107.2577	275.2372	186.2208	279.3973	0.834226	868882.3	0.834098	1.875109
$t_{p2(1)}^{GM}$	4072.616	4693.522	3411.492	9123.793	983.249	2523.145	1707.117	2561.281	7.647491	7965187	7.646313	17.18943
$t_{p2(1)}^{HM}$	1597.874	1841.484	1338.485	3579.683	385.7737	989.9456	669.7803	1004.908	3.000462	3125109	3.000000	6.744205
$t_{p3(1)}^{AM}$	2355.938	2715.121	1973.489	5277.957	568.7926	1459.596	987.5376	1481.657	4.423941	4607723	4.42326	9.943791
$t_{p3(1)}^{GM}$	1574.216	1814.219	1318.667	3526.682	380.0619	975.2884	659.8635	990.0292	2.956037	3078838	2.955581	6.644349
$t_{p3(1)}^{HM}$	49949585	57564832	41841074	1.12E+08	12059295	30945722	20937347	31413446	93794.5	9.77E+10	93780.05	210824
$t_{p4(1)}^{AM}$	8456916	9746246	7084072	18945844	2041748	5239390	3544882	5318580	15880.26	1.65E+10	15877.81	35694.4
$t_{p4(1)}^{GM}$	49949596	57564844	41841083	1.12E+08	12059298	30945728	20937352	31413453	93794.52	9.77E+10	93780.07	210824
$t_{p4(1)}^{HM}$	49879693	57484284	41782528	1.12E+08	12042421	30902421	20908051	31369491	93663.26	9.76E+10	93648.83	210529
$t_{p1(2)}^{AM}$	992.1807	1143.447	831.1162	2222.761	239.5415	614.6948	415.892	623.9855	1.8631	1940499	1.862813	4.187732
$t_{p1(2)}^{GM}$	861.4606	992.7978	721.6164	1929.912	207.9819	533.7086	361.0981	541.7752	1.617636	1684837	1.617387	3.635997
$t_{p1(2)}^{HM}$	1183.464	1363.893	991.3473	2651.288	285.7228	733.202	496.0719	744.2838	2.222288	2314608	2.221946	4.995087
$t_{p2(2)}^{AM}$	444.2199	511.9451	372.108	995.1762	107.2477	275.2116	186.2035	279.3713	0.834149	868801.5	0.83402	1.874935
$t_{p2(2)}^{GM}$	4071.27	4691.97	3410.365	9120.778	982.924	2522.311	1706.553	2560.434	7.644963	7962555	7.643785	17.18375
$t_{p2(2)}^{HM}$	1597.558	1841.119	1338.22	3578.974	385.6973	989.7496	669.6477	1004.709	2.999868	3124490	2.999405	6.742869
$t_{p3(2)}^{AM}$	2355.359	2714.454	1973.005	5276.66	568.6528	1459.237	987.295	1481.293	4.422855	4606591	4.422173	9.941348
$t_{p3(2)}^{GM}$	1573.907	1813.862	1318.408	3525.989	379.9873	975.0968	659.7339	989.8347	2.955456	3078233	2.955001	6.643044
$t_{p3(2)}^{HM}$	537738.1	619720.9	450444.9	1204683	129825.7	333149.8	225403.4	338185.1	1009.756	1.05E+09	1009.6	2269.65
$t_{p4(2)}^{AM}$	0.511944	0.589994	0.428838	1.146897	0.123598	0.317169	0.214591	0.321963	0.000961	1001.256	0.000961	0.002161
$t_{p4(2)}^{GM}$	537821.3	619816.8	450514.6	1204869	129845.8	333201.3	225438.3	338237.4	1009.912	1.05E+09	1009.756	2270.001
$t_{p4(2)}^{HM}$	238562.8	274933.8	199836	534447	57596.06	147799	99998.27	150032.9	447.9692	4.67E+08	447.9002	1006.91

4 Results and Discussion

Table(4) and table(5) present Percentage Relative Efficiencies (PRE) comparing proposed calibration estimators with existing estimators under two different data distributions — Exponential (table(3)) and Beta (table(3)).

- Percentage Relative Efficiency (PRE) values greater than 100 indicate that the proposed calibration estimators is more efficient (has lower Biases or mse) respect to the existing estimator, while values less than 100 indicate the opposite.

- Across both tables, the proposed estimators generally show substantial efficiency gains across all mean types compared to existing estimators, often with PRE values well above 100%, indicating the proposed estimators yield lower variance.

- The Arithmetic Mean (AM) and Geometric Mean (GM) based calibration estimators show particularly strong performance, which aligns with the known advantages of AM in additive processes and GM in multiplicative or growth-related processes. This effectiveness reflects the core mathematical properties of these means in estimating central tendency more efficiently.

- Harmonic Mean (HM) based estimators also improve with calibration but display more variability as HM is sensitive to small values and suited more for rate or ratio data.

5 Conclusions

The study successfully developed novel calibration estimators based on the Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM) under the framework of simple random sampling. By integrating auxiliary information and optimizing weight adjustments using the Lagrange multiplier technique, the proposed estimators demonstrated significant improvements in efficiency and precision over existing traditional estimators.

Through both theoretical derivations and extensive simulation studies employing Exponential and Beta distributions, the proposed calibration approaches consistently achieved higher Percentage Relative Efficiency (PRE), indicating reduced bias and mean squared error in variance estimation. This shows the robustness and versatility of these estimators under different data distributions and sampling conditions.

Overall, the study contributes valuable advancements to the statistical survey sampling literature by providing more precise, efficient, and adaptable variance estimators. These contributions can have significant applications in various fields such as finance, social sciences, and environmental studies where accurate estimation is critical for informed decision-making.

This work affirms the importance of calibration estimation frameworks and opens pathways to more sophisticated and effective survey estimation techniques.

6 Recommendations

These recommendations aim to facilitate the practical implementation and further refinement of calibration estimation methods, promoting more precise and reliable survey-based statistical inference.

- The developed calibration estimators based on Arithmetic Mean (AM), Geometric Mean (GM), and Harmonic Mean (HM) should be considered for adoption in survey sampling practices, particularly under simple random sampling designs, where improved variance estimation is required.

References

Abubakar, Z., Audu, A., Kane, L., Dogondaji, A., and Yunusa, M. (2024). Modified calibration variance estimators in the presence of non-response and measurement error. *Asian Journal of Probability and Statistics*, 26(10):80–94.

Afsar Basha, S. and Usman, M. (2025). New calibrated estimators of population distribution function using non-linear calibration constraints. *Model Assisted Statistics and Applications*, 20(3):228–246.

Alka, Rai, P. K., and Qasim, M. (2019). Two-step calibration of design weights under two auxiliary variables in sample survey. *Journal of Statistical Computation and Simulation*, 89(12):2316–2327.

Alka, S., Kant Rai, P., and Qasim, M. (2022). Two-step calibration estimator with double use of auxiliary variable: method and application. *Journal of the Iranian Statistical Society*, 21(1):37–54.

Audu, A. and Aphane, M. (2025). Two-steps variance calibrated estimators with linear and non-linear constraints for mailed surveys with non-response. *Alexandria Engineering Journal*, 124:591–602.

Audu, A., Aphane, M., Ahmad, J., and Singh, R. (2025a). On the calibration estimators of finite population proportion under remainder systematic sampling. *Quality & Quantity*, pages 1–17.

Audu, A., Aphane, M., Ahmad, J., and Singh, R. V. K. (2024a). Class of calibrated estimators of population proportion under diagonal systematic sampling scheme. *Mathematics*, 12(24):3997.

Audu, A., Ishaq, O. O., Muili, J. O., Abubakar, A., Rashida, A., Akintola, K. A., and Isah, U. (2020). Modified estimators of population mean using robust multiple regression methods. *Journal of Science and Technology Research*, 2(4).

Audu, A., Lekganyan, M., Ishaq, O., and Aremu, K. (2024b). Two-step calibrated designed weighted estimators of finite population variance for a mailed survey design characterized by non-response. *Revista Colombiana de Estadística*, 47(2):193–210.

Audu, A., Muhammad, A., Kane, I., Mohammed, A., Ishaq, O., and Yunusa, M. (2023a). Modified variance estimators in the presence of simultaneous effects of measurement error and non-response using auxiliary variable. *Dutse Journal of Pure and Applied Sciences*, 9(4b):347–361.

Audu, A., Singh, R., and Khare, S. (2021a). Developing calibration estimators for population mean using robust measures of dispersion under stratified random sampling. *Statistics in Transition new series*, 22(2):125–142.

Audu, A., Singh, R., Khare, S., and Dauran, N. (2021b). Classes of estimators under new calibration schemes using non-conventional measures of dispersion. *THE PHILIPPINE*, page 23.

Audu, A., Singh, R. V. K., Ishaq, O. O., Khare, S., Singh, R., and Adewara, A. A. (2024c). On the estimation of finite population variance for a mail survey design in the presence of non-response using new conventional and calibrated estimators. *Communications in Statistics-Theory and Methods*, 53(3):848–864.

Audu, A., Usman, U., Mohammad, S., and Joseph, O. (2023b). Enhanced robust estimators for estimating population means when confronted with non-response and measurement error. *Asian J. Prob. Stat*, 25(2):95–116.

Audu, A., Yunusa, M. A., Aphane, M., and Lekganyane, M. (2025b). Calibrated optional randomized response techniques for efficient and robust estimation of quantitative sensitive variables. *Scientific Reports*, 15(1):17138.

Basak, P., Aditya, K., Kumari, V., and Singh, D. (2022). Two step calibration for estimation of finite population total under two-stage sampling design. *J Indian Soc Agric Stat*, 75(3):235–243.

Çetin, A. E. and Koyuncu, N. (2024). Calibration estimator of population mean in stratified extreme ranked set sampling with simulation study. *Filomat*, 38(2):599–608.

Chaudhary, M. K., Ray, B. K., Vishwakarma, G. K., and Kadilar, C. (2024). A calibration-based approach on estimation of mean of a stratified population in the presence of non response. *Communications in Statistics-Theory and Methods*, 53(19):7054–7068.

Clement, E. P. (2022). On the precision of estimators in sampling surveys by multi-parametric calibration weightings. *Thailand Statistician*, 20(3):655–668.

Eyo, E. E. and Enang, E. (2022). Calibration ratio estimators of population mean using median of auxiliary variable. *Journal of Modeling and Simulation of Materials*, 5(1):21–30.

Garg, N. and Pachori, M. (2020). Use of coefficient of variation in calibration estimation of population mean in stratified sampling. *Communications in Statistics-Theory and Methods*, 49(23):5842–5852.

Garg, N., Singh, H. P., and Pachori, M. (2024). Modified exponential ratio-type estimator of population mean in stratified sampling using calibration approach. *J. Mod. Appl. Stat. Methods*, 23(1).

Lata, A., Rao, D., and Khan, M. G. (2017). Calibration estimation using proposed distance function. In *2017 4th Asia-Pacific World Congress on Computer Science and Engineering (APWC on CSE)*, pages 162–166. IEEE.

Maksum, C. (1990). *A new method for imputing missing values when the probability of response depends on the variable being imputed*. Oklahoma State University.

Martínez, S., Rueda, M., Martínez, H., and Arcos, A. (2015). Determining p optimum calibration points to construct calibration estimators of the distribution function. *Journal of computational and applied mathematics*, 275:281–293.

Mishra, P. and Singh, R. (2016). Variance estimation using arithmetic mean, geometric mean and harmonic mean under simple random sampling. *Journal of Scientific Research*, 60:97–109.

Misra, S., Kumari, D., and Yadav, D. K. (2016). Some improved estimators for estimating population variance in the presence of measurement errors. *Journal of Statistics Applications & Probability*, 5(2):311–320.

Mohamed, C., Sedory, S. A., and Singh, S. (2015). A fresh imputing method using sensible constraints on study and auxiliary variables: preliminary findings. *rn*, 100(1):1.

Mohamed, C., Sedory, S. A., and Singh, S. (2018). A fresh imputing survey methodology using sensible constraints on study and auxiliary variables: dubious random non-response. *Journal of Statistical Computation and Simulation*, 88(7):1273–1294.

Pandey, M., Singh, G., Zaman, T., Mutairi, A. A., and Mustafa, M. S. (2024). A general class of improved population variance estimators under non-sampling errors using calibrated weights in stratified sampling. *Scientific Reports*, 14(1):2948.

Rabee, S., Hamed, R., Kassem, R., and Rashwaan, M. (2021). A goal programming approach for multivariate calibration weights estimation in stratified random sampling. *Mathematics and Statistics*, 9(3):326–334.

Rai, P. K., Singh, A., and Qasim, M. (2021). Calibration-based estimators using different distance measures under two auxiliary variables: a comparative study. *Journal of Modern Applied Statistical Methods*, 19(1):15.

Shahzad, M., Javed, I., and Hanif, M. (2021). Calibration estimation for the population mean under ranked set sampling. *Ann. Data Sci*, 8(2):279–295.

Shahzad, U., Ahmad, I., Alshahrani, F., Almanjahie, I. M., and Iftikhar, S. (2023a). Calibration-based mean estimators under stratified median ranked set sampling. *Mathematics*, 11(8):1825.

Shahzad, U., Ahmad, I., García-Luengo, A. V., Zaman, T., Al-Noor, N. H., and Kumar, A. (2023b). Estimation of coefficient of variation using calibrated estimators in double stratified random sampling. *Mathematics*, 11(1):252.

Singh, D., Sisodia, B., Rai, V., and Kumar, S. (2017). A calibration approach based regression and ratio type estimators of finite population mean in two-stage stratified random sampling. *Journal of the Indian Society of Agricultural Statistics*, 71(3):217–224.

Singh, G., Bhattacharyya, D., and Bandyopadhyay, A. (2021). Calibration estimation of population variance under stratified successive sampling in presence of random non response. *Communications in statistics-theory and methods*, 50(19):4487–4509.

Singh, G. N., Bhattacharyya, D., and Bandyopadhyay, A. (2023). A general class of calibration estimators under stratified random sampling in presence of various kinds of non-sampling errors. *Communications in Statistics-Simulation and Computation*, 52(2):320–333.

Singh, H. P., Singh, A. K., and Solanki, R. S. (2014). Estimation of finite population variance using auxiliary information in sample surveys. *Statistica*, 74(1):99–116.

Singh, R., Mishra, M., Singh, B., Singh, P., and Adichwal, N. K. (2018). Improved estimators for population coefficient of variation using auxiliary variable. *Journal of Statistics and Management Systems*, 21(7):1335–1355.

Singh, S. and Sedory, S. A. (2016). Two-step calibration of design weights in survey sampling. *Communications in Statistics-Theory and Methods*, 45(12):3510–3523.

Suleiman, H., Yafu, J. A., and Ahmed, A. (2024). Efficiency of modified exponential dual to ratio-product-cum type estimator under stratified sampling using two auxiliary variables. *Asian Journal of Probability and Statistics*, 26(8):9–27.

Suleiman, H., Yakubu, Y., Abubakar, U., Umar, Z. A., and Baba, U. Y. (2025). On the efficiency of generalized version exponential ratio cum to dual-ratio cum type estimator via robust measure. *Asian Research Journal of Mathematics*, 21(3):1–18.

Tariq, M. U., Qureshi, M. N., and Hanif, M. (2021). Variance estimators in the presence of measurement errors using auxiliary information. *Thailand Statistician*, 19(3):606–616.