

Estimation of Stress-Strength Reliability based on the Exponential-Gamma Model: Theory and Applications

Original Research Article

Abstract

The stress-strength reliability measure, defined as $R = P(Y < X)$, is a widely used index in reliability analysis, representing the probability that a system's strength exceeds the applied stress. This paper investigates the Exponential-Gamma stress-strength model, assuming the stress variable follows an exponential distribution while the strength variable follows a gamma distribution. Analytical expressions for the reliability function are derived and generalized to the case of standby redundant systems. Estimation of reliability is developed using maximum likelihood and uniformly minimum variance unbiased approaches, and both exact and asymptotic confidence intervals are obtained. A detailed Monte Carlo simulation study evaluates the finite-sample properties of the proposed estimators, highlighting the superior small-sample performance of the UMVUE and the asymptotic efficiency of the MLE. The practical usefulness of the model is demonstrated through real data applications, showing that the Exponential-Gamma framework provides an effective and tractable tool for modeling system reliability in applied settings.

Keywords: Stress-strength reliability; Standby redundancy; Maximum likelihood estimation; UMVUE; Confidence intervals

1 Introduction

Stress-strength reliability, defined as $R = P(Y < X)$ where X denotes component strength and Y the stress, is a central concept in reliability theory. Early work by Huang (2012) analyzed inference for R when strength follows a gamma distribution and stress follows an exponential distribution, deriving both the maximum likelihood estimator (MLE) and the uniformly minimum variance unbiased estimator (UMVUE) and comparing their mean squared errors in simulation.

Extensions to more flexible lifetime distributions have maintained vibrant momentum. Esfahani (2024) studied the stress-strength reliability parameter under the generalized exponential distribution, deriving MLE, UMVUE, and exact confidence intervals. Similarly, Temraz (2023) developed inference procedures in the exponentiated generalized Marshall-Olkin G family of distributions, bringing in asymptotic, bootstrap, and Bayesian estimation with real data illustrations.

Systems with multiple stress or strength components have also been investigated. Pal and Tiensuwan (2020) considered strength subject to two independent stresses and derived UMVUEs under generalized uniform distributions. Jha et al. (2022) addressed multicomponent systems using both frequentist and Bayesian approaches where stress and strength variables follow general distributions. The challenge of censoring under complex models has been tackled by Kohansal et al. (2025), who used a progressive first-failure censoring scheme for multi-component stress-strength parameters under a Lomax distribution.

Other authors have examined specialized distributions. Khan and Khatoon (2019) investigated classical and Bayesian estimation of R from the generalized inverted exponential distribution based on record values. Pandit and Kavitha (2024) studied stress-strength reliability under the Lomax-exponential

distribution, combining MLE and Bayesian estimators with real data. James et al. (2023) introduced the Type-1 Pathway Generated Exponential (PGE-1) distribution, providing both point and interval estimation and demonstrating applicability using AIDS incubation data.

Theoretical treatments of gamma-exponential combinations have also appeared. Wang (2011) analyzed inference about R when stress is gamma and strength is exponential- deriving UMVUEs, MLEs, and pivotal-based confidence intervals. More recently, Kotb (2025) provided both Bayesian and non-Bayesian illustrations in his analysis of generalized exponential stress-strength reliability, while Brownstein (2007) highlighted the role of the generalized gamma distribution in encompassing many reliability models. Finally, Thomas and Chacko (2022) analyzed the Exponential-Gamma stress-strength model, deriving analytical forms of R , maximum likelihood estimates, asymptotic confidence bounds, simulations, and a real data example.

Despite this rich literature, the Exponential-Gamma (EG) hybrid model remains underexplored. This paper addresses the gap by proposing a full treatment of the EG stress-strength setup: deriving both MLE and UMVUE estimators for R , extending to systems with redundancy, rigorously comparing estimator performance through simulation, and validating the approach with real data. The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 develops estimation methodologies. Section 4 presents a simulation study. Section 5 provides a real data application, and Section 6 concludes with insights and directions for future research.

2 Model Formulation and Reliability

The probability density function(pdf) and cumulative distribution function(cdf) of gamma distribution are respectively given by

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}; 0 < x < \infty; \alpha, \beta > 0$$

$$F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x); 0 < x < \infty; \alpha, \beta > 0$$

where, α is the shape parameter, β is the rate parameter and $\gamma(\alpha, \beta x) = \int_0^{\beta x} t^{\alpha-1} e^{-t} dt$ denotes the lower incomplete gamma function.

Again, exponential distribution which is a special case of the gamma distribution when the shape parameter $\alpha = 1$ is a popular classical probability distribution used for analysing real life data.

Let X be a random variable from exponential distribution with respective pdf and cdf as,

$$f(x) = \lambda e^{-\lambda x}; x \geq 0, \lambda > 0$$

$$F(x) = e^{-\lambda x}; x \geq 0, \lambda > 0$$

Suppose $X \sim \text{Gamma}(\alpha, \beta)$ be the strength variable and $Y \sim \text{Exponential}(\lambda)$ be the stress variable. X and Y are independently distributed, then the reliability of the system with stress variable Y and strength variable X is

$$R = \int_0^\infty (1 - e^{-\lambda x}) \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} dx$$

On simplification, we can get the reliability of the system as

$$R = 1 - \left(\frac{\beta}{\beta + \lambda} \right)^\alpha \quad (2.1)$$

Here, $R = 1 - \nu^\alpha$, where $\nu = \frac{\beta}{\beta + \lambda} \in (0, 1)$. Taking the partial derivative of R with respect to α , we obtain

$$\frac{\partial R}{\partial \alpha} = -\nu^\alpha \log \nu$$

Since $\log \nu < 0$ for all $\nu \in (0, 1)$, it follows that $-\log \nu > 0$, and $\nu^\alpha > 0$. Therefore,

$$\frac{\partial R}{\partial \alpha} = \nu^\alpha (-\log \nu) > 0$$

which shows that R is a strictly increasing function of α when β and λ are fixed. This behaviour is consistent with the fact that the mean of a $\text{Gamma}(\alpha, \beta)$ distribution, α/β , increases with α . Hence, as the strength shape parameter increases, the expected strength becomes larger and the reliability of the system rises accordingly.

Similarly, taking the partial derivative of R with respect to the stress parameter λ , we get

$$\frac{\partial R}{\partial \lambda} = \alpha \nu^\alpha \frac{1}{\beta + \lambda} > 0,$$

showing that R also increases with λ for fixed α and β .

2.1 Reliability with Redundancy

In many safety-critical applications the reliability of a single component is not sufficient to guarantee uninterrupted operation. To mitigate the impact of a failure, designers often incorporate standby redundancy by providing one or more identical components that remain idle until they are needed. Under this scheme only one unit is active at any given time; the remaining units are kept in reserve and are activated sequentially when the active component fails. This arrangement differs from parallel (active) redundancy, where all units operate simultaneously.

Consider the EG stress–strength model in which the strength of the i^{th} component is modelled by a Gamma distribution with shape parameter $\alpha_i > 0$ and rate parameter $\beta_i > 0$, and the stress applied to that component follows an Exponential distribution with rate $\lambda_i > 0$. The reliability of a single component, the probability that it withstands the stress, is given by equation (2.1).

Now suppose a system comprises n such components connected in a cold standby arrangement. Let A_i denote the probability that the i^{th} component fails under stress, and let $B_i = 1 - A_i$ denote the probability that it survives. From the single-component reliability above,

$$A_i = \left(\frac{\beta_i}{\beta_i + \lambda_i} \right)^{\alpha_i}, \quad B_i = 1 - \left(\frac{\beta_i}{\beta_i + \lambda_i} \right)^{\alpha_i} \quad (2.2)$$

Because only one component is active at a time, the system survives if the r^{th} component is the first to survive the stress after each of the preceding $(r - 1)$ components has failed. Assuming independence between components, the contribution of the r -th component to the system reliability is,

$$R(r) = A_1 A_2 \cdots A_{r-1} \cdot B_r, \quad r = 1, 2, \dots, n \quad (2.3)$$

The overall reliability of an n component cold standby system is therefore, $R = \sum_{r=1}^n R(r)$.

For illustration, the first three terms are

$$R(1) = B_1 = 1 - \left(\frac{\beta_1}{\beta_1 + \lambda_1} \right)^{\alpha_1} \quad (2.4)$$

$$R(2) = A_1 B_2 = \left(\frac{\beta_1}{\beta_1 + \lambda_1} \right)^{\alpha_1} \left[1 - \left(\frac{\beta_2}{\beta_2 + \lambda_2} \right)^{\alpha_2} \right] \quad (2.5)$$

$$R(3) = A_1 A_2 B_3 = \left(\frac{\beta_1}{\beta_1 + \lambda_1} \right)^{\alpha_1} \left(\frac{\beta_2}{\beta_2 + \lambda_2} \right)^{\alpha_2} \left[1 - \left(\frac{\beta_3}{\beta_3 + \lambda_3} \right)^{\alpha_3} \right] \quad (2.6)$$

In general,

$$R(r) = \prod_{i=1}^{r-1} \left(\frac{\beta_i}{\beta_i + \lambda_i} \right)^{\alpha_i} \cdot \left[1 - \left(\frac{\beta_r}{\beta_r + \lambda_r} \right)^{\alpha_r} \right] \quad (2.7)$$

and the system reliability is the sum of these sequential contributions. Such formulations for cold/standby redundancy have been widely discussed in reliability literature, including the work of Liu et al. (2018) on multicomponent standby systems and Cüran and Kızılaslan (2021) on parallel systems with cold standby. This closed-form expression demonstrates how standby redundancy improves reliability, each additional standby provides another opportunity for the system to survive, although the marginal benefit diminishes because later standbys are only used if all earlier components fail.

3 Estimation of Model Parameters

3.1 Maximum Likelihood Estimation (MLE) Method

To accomplish the MLE of the stress-strength reliability R , we first need to obtain the MLEs of the involved parameters. Let X_1, X_2, \dots, X_n be a random sample from gamma distribution with shape parameter α and scale parameter β , representing the strength variable and let Y_1, Y_2, \dots, Y_m be a random sample from exponential distribution with rate parameter λ , representing the stress variable. Assuming that the samples are independent, the joint log-likelihood function for the observed data is given by,

$$L(\alpha, \beta, \lambda) = \prod_{i=1}^n f(x_i) \prod_{j=1}^m f(y_j)$$

$$\ln L = n\alpha \log \beta - n \log \Gamma(\alpha) + (\alpha - 1) \sum_{i=1}^n \log x_i - \beta \sum_{i=1}^n x_i + m \log \lambda - \lambda \sum_{j=1}^m y_j \quad (3.1)$$

The MLEs of α, β, λ are obtained by maximizing this log-likelihood function with respect to the respective parameters. Therefore, the MLE of R becomes

$$\hat{R}_{MLE} = 1 - \left(\frac{\hat{\beta}}{\hat{\beta} + \hat{\lambda}} \right)^{\hat{\alpha}}$$

3.2 Uniformly Minimum Variance Unbiased Estimation (UMVUE) Method

The UMVUE of the stress-strength reliability parameter $R = P(Y < X)$, where $X \sim \text{Gamma}(\alpha, \beta)$ and $Y \sim \text{Exponential}(\lambda)$ under the assumption that the scale parameters β and λ are known and only the shape parameter α is unknown.

Let X_1, X_2, \dots, X_n be a random sample from gamma distribution with the same parameters. It is well known that the sum of independent gamma variables with common scale forms another gamma variable. Specifically, the statistic $T = \sum_{i=1}^n X_i$ follows a gamma distribution with shape parameter $n\alpha$ and scale parameter β . Since T is both complete and sufficient for α , it provides a strong foundation for constructing the UMVUE of the function R . Using established results for functions of exponential family parameters, such as those presented by Tong (2009), we can derive the unbiased estimator of c^α , where $c = \frac{\beta}{\beta + \lambda} \in (0, 1)$, and hence for $R = 1 - c^\alpha$. Therefore, the UMVUE of R is given by

$$\hat{R}_{UMVUE} = 1 - \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{(-\log c)^k}{k!} \left(\frac{T}{\beta} \right)^k \left(1 + \frac{T}{\beta} \right)^{-(n+k)} \quad (3.2)$$

This estimator is based solely on the sufficient statistic T ensuring its unbiasedness and optimality in terms of variance among all unbiased estimators.

3.3 Simulation Study

To assess the performance of the proposed estimators of the stress-strength reliability under EG stress-strength model, a detailed Monte Carlo simulation study was carried out. The shape parameter α was varied over the values 0.5, 1.0, 1.5, 2.0 and 2.5, while β and λ was fixed at 2.5 and 1.5 respectively. The sample sizes were taken as $n = 5, 10, 15, 20$ for the strength variable and $m = 5, 10, 15$ for the

stress variable. For each combination of parameters and sample sizes, 5000 independent replications were simulated. The reliability measure R was estimated using both MLE and UMVUE. **Table 1** presents the average bias and MSE of these estimators across all configurations.

Table 1: Bias and Mean Squared Error of Estimated Reliability R Using MLE and UMVUE

(n,m)	α				
	0.5	1.0	1.5	2.0	2.5
(5,5)	-0.00875(0.01968) 0.17357(0.07859)	-0.01055(0.02226) 0.20052(0.06120)	-0.00963(0.01726) 0.16654(0.03446)	-0.01117(0.01203) 0.12210(0.01702)	-0.01126(0.00774) 0.08462(0.00779)
(5,10)	-0.02200(0.01653) 0.17067(0.07811)	-0.01587(0.01724) 0.20329(0.06204)	-0.01859(0.01378) 0.16621(0.03425)	-0.01533(0.00899) 0.12176(0.01689)	-0.01344(0.00559) 0.08443(0.00778)
(5,15)	-0.02549(0.01580) 0.16757(0.07707)	-0.02030(0.01653) 0.20122(0.06201)	-0.01865(0.01193) 0.16634(0.03422)	-0.01670(0.00787) 0.12160(0.01687)	-0.01448(0.00490) 0.08399(0.00773)
(10,5)	0.00557(0.01408) 0.58135(0.34619)	0.00869(0.01625) 0.41278(0.17046)	-0.00016(0.01338) 0.26858(0.07214)	-0.00248(0.00953) 0.17357(0.03013)	-0.00690(0.00640) 0.11206(0.01256)
(10,10)	-0.00657(0.01032) 0.58205(0.34673)	0.00002(0.01126) 0.41292(0.17057)	-0.00536(0.00910) 0.26859(0.07214)	-0.00748(0.00605) 0.17356(0.03012)	-0.00608(0.00390) 0.11206(0.01256)
(10,15)	-0.00991(0.00894) 0.58150(0.34641)	-0.00753(0.00951) 0.41276(0.17043)	-0.00700(0.00713) 0.26859(0.07214)	-0.00581(0.00468) 0.17357(0.03012)	-0.00619(0.00297) 0.11206(0.01256)
(15,5)	0.00797(0.01187) 0.64234(0.41276)	0.01104(0.01470) 0.41666(0.17360)	0.00533(0.01181) 0.26896(0.07234)	-0.00190(0.00830) 0.17361(0.03014)	-0.00584(0.00565) 0.11207(0.01256)
(15,10)	-0.00089(0.00799) 0.64246(0.41289)	0.00020(0.00933) 0.41666(0.17360)	-0.00526(0.00765) 0.26896(0.07234)	-0.00411(0.00526) 0.17361(0.03014)	-0.00529(0.00329) 0.11207(0.01256)
(15,15)	-0.00562(0.00674) 0.64266(0.41311)	-0.00310(0.00784) 0.41665(0.17360)	-0.00380(0.00586) 0.26896(0.07234)	-0.00399(0.00405) 0.17361(0.03014)	-0.00459(0.00261) 0.11207(0.01256)
(20,5)	0.01266(0.01072) 0.64542(0.41657)	0.01032(0.01376) 0.41667(0.17361)	0.00814(0.01158) 0.26896(0.07234)	-0.00052(0.00794) 0.17361(0.03014)	-0.00389(0.00536) 0.11207(0.01256)
(20,10)	0.00082(0.00697) 0.64544(0.41659)	-0.00009(0.00843) 0.41667(0.17361)	-0.00054(0.00709) 0.26896(0.07234)	-0.00074(0.00487) 0.17361(0.03014)	-0.00247(0.00314) 0.11207(0.01256)
(20,15)	-0.00053(0.00588) 0.64542(0.41657)	0.00109(0.00664) 0.41667(0.17361)	-0.00143(0.00537) 0.26896(0.07234)	-0.00396(0.00356) 0.17361(0.03014)	-0.00442(0.00231) 0.11207(0.01256)

The average bias and the corresponding MSE are reported within brackets using MLE in first row and using UMVUE in the second row.

Now, we present the graphical representations of the bias and mean squared error (MSE) of the estimators with respect to different values of α .

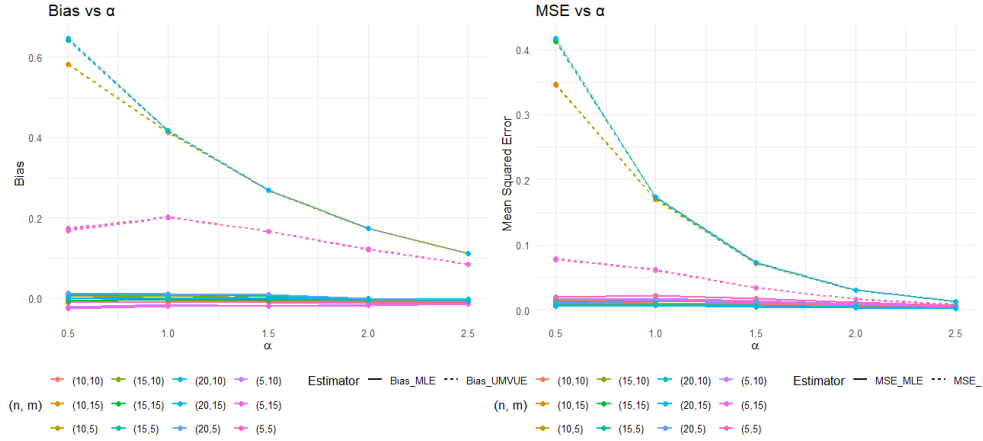


Figure 1: Plot of Bias and MSE for Reliability Estimators

Figure 1 displays the average bias and MSE of the MLE and UMVUE of the reliability parameter R for a range of values of the Gamma shape parameter α and different sample sizes. The underlying numerical results are given in **Table 1**; the first row in each cell corresponds to the MLE and the second row corresponds to the UMVUE. The graphical representation helps to visualise how estimator performance changes with α .

For small α values, the MLE exhibits a slight negative bias, while the UMVUE remains essentially unbiased by construction. As α increases, the biases of both estimators diminish towards zero. This decreasing trend arises because larger shape parameters provide more information about the strength distribution, yielding more accurate reliability estimates. The MSE curves show that the UMVUE has a smaller MSE than the MLE when α is small; however, the MSE values due to the MLE decreases rapidly as α grows, and for larger α values the MLE and UMVUE perform comparably.

4 Estimation of Confidence Interval

4.1 Exact Confidence Interval

Let X_1, X_2, \dots, X_n be a random sample from the gamma distribution with unknown shape parameter α and known rate β , and let the stress variable $Y \sim \text{Exponential}(\lambda)$, with known λ .

Since $T = \sum_{i=1}^n X_i \sim \text{Gamma}(n\alpha, \beta)$, it follows that the transformation $Z = 2\beta T$ follows a chi-square distribution with $2n\alpha$ degrees of freedom, i.e., $Z \sim \chi_{2n\alpha}^2$. This pivotal quantity is used to construct an exact confidence interval for α , which is then transformed into an exact confidence interval for R .

Let $\hat{\alpha}$ be an estimate of α . Then the $100(1 - \gamma)\%$ exact confidence interval for R is given by,

$$\left(1 - \nu^{\chi_{2n\hat{\alpha}, 1-\gamma/2}^2/(2n)}, \quad 1 - \nu^{\chi_{2n\hat{\alpha}, \gamma/2}^2/(2n)} \right) \quad (4.1)$$

where, $\chi_{v,p}^2$ denotes the p -th percentile of the chi-square distribution with v degrees of freedom.

4.2 Asymptotic Confidence Interval

In this section, we derive the asymptotic distribution of the estimator of R and construct the corresponding large-sample confidence interval. Let X_1, X_2, \dots, X_n be a random sample from $\text{Gamma}(\alpha, \beta)$, where β is known and assume $Y \sim \text{Exponential}(\lambda)$ with known λ .

Let $\hat{\alpha}$ be the maximum likelihood estimator of α . It is well known that under regularity conditions, $\hat{\alpha}$ is asymptotically normal with mean α and variance $1/(n\psi'(\alpha))$, where $\psi'(\alpha)$ is the trigamma function. Using the delta method, the asymptotic variance of the estimator $\hat{R} = 1 - \nu^{\hat{\alpha}}$ is given by,

$$\text{Var}(\hat{R}) = (\nu^\alpha \ln \nu)^2 \frac{1}{n\psi'(\alpha)} \quad (4.2)$$

Replacing α by $\hat{\alpha}$, the approximate $100(1 - \gamma)\%$ confidence interval for R becomes

$$\hat{R} \pm Z_{1-\gamma/2} \cdot \nu^{\hat{\alpha}} |\ln \nu| \sqrt{\frac{1}{n\psi'(\hat{\alpha})}} \quad (4.3)$$

where, $Z_{1-\gamma/2}$ is the upper $(1 - \gamma/2)$ -th quantile of the standard normal distribution.

4.3 Simulation Study

To evaluate the accuracy of the proposed interval estimators under the EG stress-strength model, we computed the average lengths and corresponding coverage probabilities of the confidence intervals for various parameter configurations. α was varied over the values 0.5, 1.0, 1.5, 2.0 and 2.5, while β and λ was fixed at 2.5 and 1.5 respectively throughout the study. The strength sample size was chosen as $n = 15, 20, 25, 30$ and the stress sample size as $m = 20, 25, 30$. For each configuration, the simulation was repeated 5000 times. **Table 2** presents the simulation results, comparing the performance of exact and asymptotic confidence intervals in terms of average interval length and coverage probability.

Table 2: Average confidence interval length and coverage probability of R using Asymptotic and Exact methods

(n,m)	α				
	0.5	1.0	1.5	2.0	2.5
(5,5)	0.51552 (0.67040)	0.56313 (0.84880)	0.58085 (0.70360)	0.59048 (0.44920)	0.59546 (0.19580)
	0.50240 (0.74580)	0.53788 (0.91980)	0.55034 (0.71520)	0.55693 (0.29640)	0.56030 (0.03640)
(5,10)	0.46452 (0.68160)	0.49868 (0.84520)	0.51059 (0.60060)	0.51748 (0.24820)	0.52103 (0.04260)
	0.45191 (0.76960)	0.48002 (0.87000)	0.48917 (0.49200)	0.49454 (0.07500)	0.49707 (0.00080)
(5,15)	0.44366 (0.68000)	0.47463 (0.84220)	0.48357 (0.55660)	0.48975 (0.17180)	0.49255 (0.01400)
	0.43229 (0.77900)	0.45831 (0.83860)	0.46563 (0.40980)	0.47054 (0.02940)	0.47259 (0.00000)
(10,5)	0.46420 (0.64580)	0.49834 (0.86440)	0.51121 (0.64200)	0.51726 (0.27140)	0.52107 (0.05900)
	0.45191 (0.64940)	0.47971 (0.93640)	0.48974 (0.70740)	0.49442 (0.20940)	0.49712 (0.01400)
(10,10)	0.39899 (0.64940)	0.41725 (0.85620)	0.42417 (0.44680)	0.42761 (0.06860)	0.42950 (0.00100)
	0.39027 (0.67300)	0.40567 (0.89220)	0.41136 (0.39320)	0.41417 (0.02040)	0.41571 (0.00000)
(10,15)	0.36980 (0.63860)	0.38435 (0.84180)	0.38908 (0.33860)	0.39199 (0.01540)	0.39359 (0.00020)
	0.36255 (0.67060)	0.37514 (0.85220)	0.37917 (0.25020)	0.38162 (0.00220)	0.38301 (0.00000)
(15,5)	0.44643 (0.63900)	0.47317 (0.87320)	0.48368 (0.60980)	0.48942 (0.21260)	0.49277 (0.03480)
	0.43472 (0.61220)	0.45721 (0.94180)	0.46549 (0.70660)	0.47021 (0.18740)	0.47286 (0.01020)
(15,10)	0.36967 (0.60520)	0.38425 (0.85600)	0.38932 (0.36260)	0.39200 (0.02420)	0.39360 (0.00020)
	0.36239 (0.59660)	0.37506 (0.89660)	0.37939 (0.34080)	0.38164 (0.00820)	0.38299 (0.00000)
(15,15)	0.33578 (0.58800)	0.34600 (0.82340)	0.35001 (0.23560)	0.35180 (0.00480)	0.35302 (0.00020)
	0.33007 (0.59320)	0.33910 (0.84580)	0.34261 (0.18300)	0.34416 (0.00160)	0.34522 (0.00000)
(20,5)	0.43520 (0.64460)	0.46049 (0.87460)	0.46953 (0.58540)	0.47536 (0.17660)	0.47839 (0.01820)
	0.42431 (0.60500)	0.44564 (0.94800)	0.45313 (0.70620)	0.45800 (0.16400)	0.46052 (0.00580)
(20,10)	0.35330 (0.59620)	0.36555 (0.84120)	0.37036 (0.31680)	0.37257 (0.01520)	0.37415 (0.00000)
	0.34701 (0.57240)	0.35778 (0.89540)	0.36189 (0.31480)	0.36388 (0.00720)	0.36522 (0.00000)
(20,15)	0.31583 (0.55920)	0.32525 (0.82840)	0.32845 (0.17180)	0.32979 (0.00080)	0.33076 (0.00000)
	0.31105 (0.54720)	0.31947 (0.86160)	0.32232 (0.14420)	0.32350 (0.00020)	0.32436 (0.00000)

The average confidence length and coverage probability are reported within brackets for confidence intervals using asymptotic in first row, using exact in the second row.

Now, we present graphical representation of average confidence interval lengths and coverage probabilities for the reliability estimators across varying values of α .

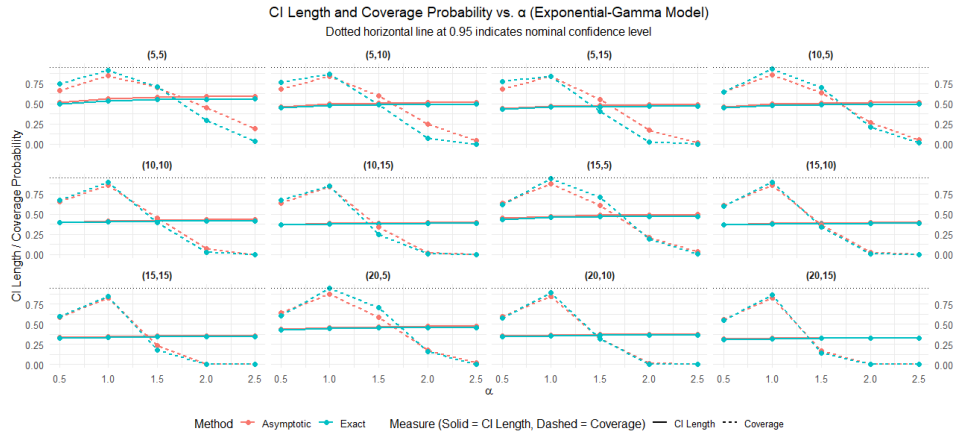


Figure 2: Plot of Confidence Interval Length and Coverage Probability for Reliability Estimators

Figure 2 contains two panels summarising the behaviour of the exact and asymptotic confidence intervals for R across varying values of α and sample sizes. The first panel shows the average length of the confidence intervals and the second panel shows the empirical coverage probabilities. The average lengths and coverage probabilities tabulated in **Table 2** correspond to these plots.

The first panel illustrates that average interval lengths decline as α increases. The exact confidence intervals are consistently longer than the asymptotic intervals for the same (n, m) configuration, reflecting the conservativeness of the exact method. This difference is most pronounced when α and the sample sizes are small; as α and the sample sizes increase, the lengths of the two types of intervals converge. The second panel demonstrates that exact intervals achieve coverage close to the nominal confidence level across all scenarios. Asymptotic intervals tend to underestimate coverage when α and sample sizes are small, but their coverage improves as α grows and samples become larger. Collectively, **Figure 2** highlights that exact intervals provide reliable coverage at the cost of wider intervals, whereas asymptotic intervals are shorter but only perform satisfactorily in larger-sample settings.

5 Real Data Application

In this section, we analyze two independent datasets. The first, taken from Folks and Chhikara (1978), records runoff amounts at Jug Bridge, Maryland.

Data set I: 0.17, 0.23, 0.33, 0.39, 0.39, 0.40, 0.45, 0.52, 0.56, 0.59, 0.64, 0.66, 0.70, 0.76, 0.77, 0.78, 0.95, 0.97, 1.02, 1.12, 1.19, 1.24, 1.59, 1.74, 2.92.

The second dataset gives active repair times (hours) for an airborne communication transceiver. It was originally reported by Chhikara and Folks (1989) and later compiled by Balakrishnan et al. (2009).

Data set II: 0.2, 0.3, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.7, 0.7, 0.7, 0.8, 0.8, 1.0, 1.0, 1.0, 1.0, 1.1, 1.3, 1.5, 1.5, 1.5, 2.0, 2.0, 2.2, 2.5, 2.7, 3.0, 3.0, 3.3, 3.3, 4.0, 4.0, 4.5, 4.7, 5.0, 5.4, 5.4, 7.0, 7.5, 8.8, 9.0, 10.3, 22.0, 24.5.

Dataset I (stress) with 25 observations modeled by $Exp(\lambda)$, and Dataset II (strength) with 46 observations modeled by $Gamma(\alpha, \beta)$.

Parameters were estimated by maximum likelihood from the full joint log-likelihood. The fitted values are $\hat{\lambda} = 0.27728$, $\hat{\alpha} = 2.69623$ and $\hat{\beta} = 3.19761$.

Model adequacy was examined with Kolmogorov–Smirnov (K-S) test computed against the fitted CDFs. For Dataset I, the K-S distance is 0.15974 with a p-value of 0.19106. For Dataset II, the K-S distance is 0.10835 with a p-value of 0.93087. At the 5% level neither test rejects, so the exponential and gamma forms are acceptable for these datasets.

Using the fitted parameters, $\hat{R} = 0.20086$. This indicates that there is 20.086% probability that strength exceeds stress under the observed conditions. Two interval estimates were obtained- the asymptotic 95% CI based on the delta-method variance is (0.16193, 0.23978) and the exact CI is (0.15962, 0.24483). The two intervals are close; although the exact CI is slightly wider, as expected.

For each dataset we plot the empirical and fitted survival functions which is given in **Figure 3**. In Dataset I, the exponential curve follows the empirical steps across the support with no systematic departures. In Dataset II, the fitted gamma survival tracks the empirical curve well, including the mid-range where most observations lie. These visuals are consistent with the K-S outcomes.

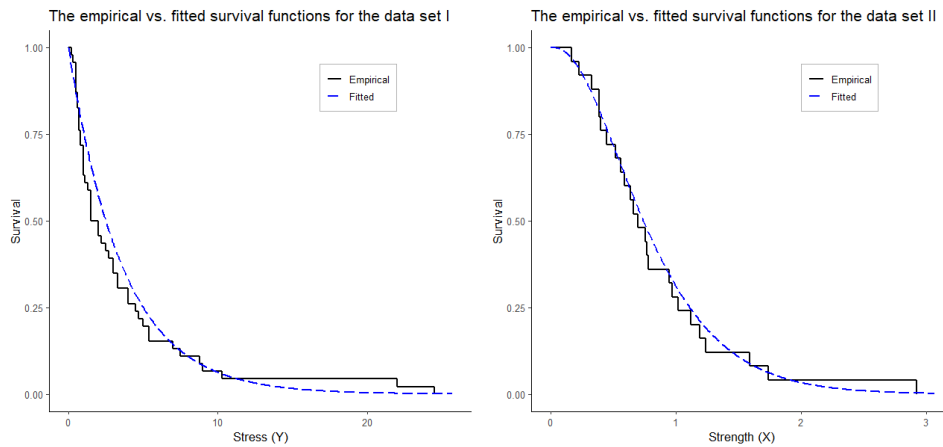


Figure 3: Empirical vs. fitted survival functions for the two datasets

6 Conclusion

The present work has examined the EG stress-strength model in detail, focusing on both theoretical properties and inferential aspects. Reliability expressions were obtained in closed form, and the framework was further generalized to incorporate standby redundancy. The redundancy results clearly show that system performance improves with additional standby units, though the incremental gain becomes smaller as the level of redundancy increases.

For parameter estimation, both MLE and UMVUE methods were derived and studied. Simulation results demonstrated that the UMVUE performs better for small sample sizes, whereas the MLE is preferable when larger samples are available due to its asymptotic efficiency. Two approaches to confidence interval construction were also considered- exact intervals provided stable coverage, while asymptotic intervals were more efficient but less reliable in smaller samples.

Application to real data illustrated the practical suitability of the EG model, confirming that it can capture the reliability behavior of real systems under stress-strength configurations. Taken together, the findings underline the usefulness of this model as a flexible and tractable tool in reliability analysis.

Potential extensions include Bayesian methods, more general censoring schemes, and applications to dependent stress-strength structures.

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