Forcasting Stock Prices In Nigeria Using Bayesian Vector Autoregression

Abstract

Forecasting stock market is one of the challenges facing investors and portfolio managers today. There are several forecasting techniques in the literature, forecasting stock prices for investment decision making. However, only few researchers have employed Bayesian Techniques in the forecasts. This study aimed at forecasting the stock prices of five leading banks and the banking sector index of Nigeria using Bayesian Vector Autoregression(BVAR). This research adopted Minnesota priors, Stochastic search variable selection(SSVS), Steady state with inverse wishart prior and steady state prior with diffuse priors. The data were divided into two sets: One set containing 400 datasets for training while the other containing 100 datasets were used for evaluation. Covariance matrices were obtained for these priors as well as the coefficients of the BVAR models. Forecasts for the five priors were obtained as well as theirStandardVectorAutoregressions (SVAR). The forecast performancesfor the priors and SVAR were examined using Root Means Square Error(RMSE). The resultsshowed that the minnesota prior of BVAR model out-performed the other priors and Ordinary Least Square method of SVAR in predicting stock prices.

Keywords: Minnesota, Stochastic searchvariableselection, Steady state with inverse wishart, Root mean square error

1.0 Introduction

The stock market is a segment of the financial market where long-term funds packaged in the form of securities, such as shares, stocks bonds, debentures, loan stocks, and derivatives, are traded. Ezeoha, Ogamba, and Onyiuke, (2009) and Ogunmuyiwa, (2010) both agreed in their research that Nigeria's stock market spurs economic growth. The challenge of an investor is how to identify viable stocks and guide them towards making a profit as well as discovering a way to predict the stock market[Akanbi O. B. (2022), Akanbi O. B. et al (2018), Yaya O. S. et al (2019)]. In that connection, stock price forecasting has always been a subject of interest particular interest to investors, speculators, economists, and governments. This paper employs Bayesian Vector Autoregression (BVAR) that are essential to capture prior distributions and also improve out-of-sample performances. In BVAR, models are treated as random variables and prior probability is assigned to those variables, unlike the standard VAR. Since VAR is flexible enough to allow many free parameters, and VAR coefficients are not constant, Time-Varying coefficient-VAR (TVC-VAR) models were introduced to allow for time variation in the model.

The generality of the VAR model brings along a large number of parameters even for systems of moderate size. (Joris de Wind (2015)). Over-parametrization results in the problem of lack of precision and reliability of the forecast. The large number of parameters and limited temporal availability of macroeconomic datasets often lead to over-parameterization problems (Koop and Korobilis 2010) that can be mitigated by introducing prior information within a Bayesian approach. To solve the problem of over-parameterization in VAR, Litterman (1986) proposed the concept of Bayesian VAR. The Bayesian VAR involves a combination of prior (information available to the researcher before seeing the data) and the likelihood (data information) to arrive at the posterior. Bayesian estimation provides a convenient framework for incorporating prior information with as much weight as the analyst feels it merits (Hamilton, 1994). Bayesian inference treats VAR parameters as random variables and provides a framework for updating the distribution of those parameters. (Silvia and Giovanni 2018).

Silva and Giannone (2018) stated that when there is no pre-sample information, Bayesian VAR inference can be thought of as adopting 'non-informative' (or 'diffuse' or 'flat') priors, which expresses complete ignorance about the model parameters, in the light of the sample evidence summarized by the likelihood function (i.e. the probability density function of the data as a function of the parameters).

Koop (2013) observed that informative priors are used to impose additional structure on the model and shrink it towards proven benchmarks. The results are models with reduced parameter uncertainty and significantly enhanced out-of-sample forecasting performance.

The choice of these priors and their informativeness poses a challenge and remains the fulcrum of discussion and criticism. A Bayesian approach to VAR estimation was originally advocated by Litterman (1980) as a solution to the overfitting problem. They express the belief that an independent random-walk model for each variable in the system is a reasonable 'center' for the beliefs about their time series behavior. While not motivated by economic theory, they are computationally convenient priors, meant to capture commonly held beliefs about how economic time series behave. Minnesota priors can be cast in the form of a Normal Inverse-Wishart (NIW) prior, which is the conjugate prior for the likelihood of a VAR with normally distributed disturbances (Kadiyala and Karlsson, 1997). Giannone, Lenza, and Primiceri (2015) provide a data-based, theoretically grounded approach to setting prior informativeness in the spirit of hierarchical modeling. They alleviate the subjectivity of setting hyperparameters and demonstrate remarkable performance in common analyses. The Bayesian VAR involves a combination of prior (information available to the researcher before seeing the data) and the likelihood (data information) to arrive at the posterior. (George, Olubusoye and Nwabueze, 2018). Frank Schorfheidea&Dongho Song, 2014 in their joint paper develops a vector autoregression (VAR) for time series which are observed at mixed frequencies – quarterly and monthly. The model is cast in state-space form and estimated with Bayesian methods under a Minnesota-style prior. In their paper evaluation of the marginal data density was demonstrated to implement a data-driven hyperparameter selection. Mauro Bernardi, Daniele Bianchib, and Nicolas Bianco (2024) propose a novel variational Bayes approach to estimate high-dimensional Vector Autoregressive (VAR)models with hierarchical shrinkage priors. Sugita, K. (2022), adopts Bayesian VAR models with three different priors – independent Normal-Wishart prior, the Minnesota prior, and the stochastic search variable selection (SSVS). His results show that iterated forecasts tend to outperform direct forecasts, particularly with longer lag models and with longer forecast horizons. He believes that Implementing SSVS prior generally improves forecasting performance over unrestricted VAR model for either nonstationary or stationary data. Amidst the rise of Markov chain Monte Carlo (MCMC) methods, Bayesian statistical software has evolved rapidly. Established software provides flexible and extensible tools for Bayesian inference, which are available cross-platform. BVAR is the first R package implementing these hierarchical Bayesian VAR models and provides a complete and easy-to-use toolkit for estimation and analysis. (Nikolas and Lucas (2019)).

2.0 Methodology

2.1 Bayesian Inference

For a given model:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \varepsilon_t \quad \varepsilon_t \sim IID\ N(0, \Sigma) \qquad (1)$$

Where ε_t is an independent and identically distributed random variable for each t. The distribution from which is ε_t drawn determines the distribution of y_t, conditional on its past. The standard assumption in the macro-econometric literature is that errors are Gaussian. The

Bayesian approach to VAR treats time series data $y = \{y_1, \cdots, y_t\}$ as known stochastic data and A and Σ as unknown parameters. The inference about the unknown parameters A and Σ is made conditioned to the set of known data y as $f\ (y|A, \Sigma)$. The previous information about $A\ and\ \Sigma$ is known as prior of $(A, \Sigma), \pi(A, \Sigma)$ is defined in the form of the probability density function. By applying Baye's theorem to represent the posterior distribution of VAR(p), we have;

$$P(A, \Sigma|y) = \frac{P(A,\Sigma)P(y|A,\Sigma)}{P(y)} \quad (2)$$

$$P(A, \Sigma|y) \propto P(A, \Sigma)P(y|A, \Sigma) \quad (3)$$

$P(A, \Sigma|y)$ is the posterior distribution of $(A, \Sigma)\ given\ y$. The posterior distribution of VAR(p) is the combination of prior distribution $P(A, \Sigma)$ and the information given by the time series data through $P(y|A, \Sigma)$. $P(y)$ is the sample density that is independent of A and Σ. It is the normalizing constant for a given sample y.

$$P(y|A, \Sigma) \propto P(A, \Sigma)P(y|A, \Sigma) = L(A, \Sigma|y)P(A, \Sigma) \quad (4)$$

Bayesian inference on the model in Eq. (1) amounts to updating prior beliefs about the VAR parameters, that are seen as stochastic variables, after having observed a sample.

VAR model of equation 1 can be written as

$$Y = \alpha X + \epsilon \quad (5)$$

where Y is a $T \times n$ matrix of regressands, X is a $T \times k$ matrix of regressors, ϵ is a $T \times n$ matrix of shocks and α is a $k \times n$matrix of regression parameters, with $k = 1 + pn$is the number of regression parameters per VAR equation.

If we consider the vectorised form of equation (1),

$$y = (I \otimes X)\alpha + u \qquad u \backsim N(0, \Sigma \otimes I) \quad (6)$$

$$p\ (Y|A, \Sigma) = \frac{1}{(2\pi)^{\frac{Tm}{2}}} |\Sigma|^{\frac{T}{2}} \exp\left\{\frac{1}{2}\ tr\left[\Sigma^{-1}\hat{S}\right]\right\} \times \exp\left\{-\frac{1}{2}\ (\alpha - \hat{\alpha})(\Sigma \otimes (X'X)^{-1})^{-1}(\alpha - \hat{\alpha})\right\} (7)$$

The posterior of y will now depend on the prior of $\alpha\ and\ \Sigma$.

2.2 Prior Selection

2.2.1 Non-Informative Prior

In the absence of pre-sample information, Bayesian VAR inference can be thought of as adopting 'non-informative' (or 'diffuse' or 'flat') priors, that express complete ignorance about the model parameters. Non-informative or flat priors are designed to extract the maximum amount of expected information from the data. In this case, the prior distribution has minimal influence on the posterior distribution, and the estimates are primarily driven by the likelihood function, which summarizes the sample evidence. When using non-informative priors, we assumed that α and Σ are independent.

$$p(\alpha, \Sigma) = p(\alpha).p(\Sigma) \tag{8}$$

$$p(\alpha) = constant \tag{9}$$

$$p(\Sigma) = |\Sigma|^{-(n+1)} \tag{10}$$

Using the condition (8), (9) and (10), we have:

$$p(A, \Sigma|y) \propto |\Sigma|^{-\frac{(T+n+1)}{2}} \exp\left\{\frac{1}{2}\, tr\left[\Sigma^{-1}\hat{S}\right]\right\} \times \exp\left\{-\frac{1}{2}\,(\alpha - \hat{\alpha})(\Sigma \otimes (X'X)^{-1})^{-1}(\alpha - \hat{\alpha})\right\} \tag{11}$$

Note that:

$$p(A, \Sigma|y) = p(\alpha|\Sigma, y).p(\Sigma|y) \tag{12}$$

By ignoring the constant of the proportionality we have,

$$\boldsymbol{\alpha}|\Sigma, y \sim N(\hat{\alpha}, \Sigma \otimes (X'X)^{-1}) \tag{13}$$

$$\Sigma|y \sim IW\left((y - \hat{A}x)'(y - \hat{A}x),\ T - k\right) \tag{14}$$

2.2.2 Minnesota Prior

The Minnesota prior is based on the assumption that Σ is known, so in practice it should be pre-estimated. The major idea behind the Minnesota prior is to shrink the model towards the random walk, with stronger shrinkage for coefficients on longer lags and across variables. The Minnesota prior is a shrinkage method that is used to set most or all the elements of αtowards zero. This is achieved by shrinking the diagonal elements of the coefficients of regression to 1 and others to zero. This will lead to each variable of the VAR model following a simple Random Walk with a drift. The prior for the variance-covariance matrix, Σ is assumed to be fixed and diagonal. Specifically, Litterman suggested setting the prior standard deviations to:

$\alpha \sim N(\hat{a}, \hat{\Sigma})$,

$$\hat{\Sigma} = \begin{cases} \frac{\pi_1}{l^{\pi_3}} & for\ coefficient\ of\ its\ own\ lags, l = 1, 2, \dots, p \\ \frac{\pi_1 \pi_2 s_j}{l^{\pi_3} s_r} & for\ coefficient\ of\ its\ own\ lags, l = 1, 2, \dots, p \\ \infty & for\ coefficient\ of\ constant \end{cases} \quad (15)$$

Where π_1 is referred to as the ”overall tightness, π_2 the ”relative tightness of other variables” and π_3 the ”lag decay rate” and the variance of the prior is proportional to $\frac{s_r}{s_j}$ which is a scale factor accounting for the different variances of the dependent and explanatory variable.

The error covariance matrix $\Sigma = diag(s_1^2, \dots, s_p^2)$.

Estimating the BVAR model requires the predictor to determine the value of the above hyperparameters. Because the main purpose of the BVAR model is prediction, so, unlike other models, the value standard of hyperparameters is to obtain the optimal prediction effect, rather than relying on various model settings and tests. The determination of hyperparameters is a process similar to raster search, searching for the value that can obtain the best prediction effect within the range of hyperparameters. For this reason, the total sample T obtained is usually divided into two periods T and $T - T_0$. The data of period T is used to estimate the BVAR model and forecast, and the data of $T - T_0$ is used to calculate and compare the forecast error and determine the final hyperparameter value.

2.2.3 Steady State VARwith Variable Selection Prior.

A reduced-form VAR is written as:

$$B(L)y_t = cd_t + \varepsilon_t \quad (16)$$

where $y_t$is $m \times 1$ vectors of time series with time $t = 1, \dots, T$ *observations*.

$$B(L) = I_m - B_1 L - \dots - B_p L^p \quad (17)$$

with

$$L y_t = y_{t-1}, \quad (18)$$

ε_t are the errors distributed as $N(0, \Sigma)$ with Σ being the $m \times m$ covariance matrix and d_t is a q-dimensional vector of exogenous deterministic variables such as constants, dummies or time trends. Suppose $y_t$is stationary, that is the expected mean of y_t exists and is stationary the unconditional mean or steady-state of the VAR process in Eq. (16) is defined as

$$E(y_t) = \mu_t = B(L)^{-1} c d_t. \quad (19)$$

By setting $\theta = B(L)^1 c$, then

$$E(y_t) = \mu_t = \theta d_t. \quad (20)$$

The deviation from the mean parametrization can be represented by:

$$B(L)(y_t - \theta d_t) = \varepsilon_t \quad (21)$$

Equation 21 above is called the steady-state representation of VAR with $\mu_t = \theta d_t$ being the long run mean. The prior belief about μ_t can be incorporated by specifying the prior of θ. $\underline{y}_t = y_t - \theta d_t$ (22)

is the mean-adjusted time series of y_t.

Therefore,(16) can be written in a normal form as

$$\underline{y}_t = \alpha X + \varepsilon_t \quad (23)$$

(23) is an unrestricted steady state VAR since no restriction is incorporated in the $(\alpha)_{j=1}^{m}$elements of α. Applying the Bayesian variable selection method proposed by Korobilis (2013) restricts some of the α_j coefficients to be zero as follows:

$$\begin{cases} \alpha_j = 0, if\ \gamma_j = 0 \\ \alpha_j \neq 0\ \ if\ \gamma_j = 1 \end{cases} \quad (24)$$

where γ_j is an indicator variable and the j element of the vector $\{\gamma_1 \cdots \gamma_n\}'$. We can then define steady-state VAR with variable selection as

$$\underline{y}_t = \tilde{\theta} X + \varepsilon_t \quad (25)$$

2.2.4 Stochastics Search Variable Selection Prior

The basic idea of SSVS is to assign commonly used prior variances to parameters, which should be included in a model, and prior variances close to zero to irrelevant parameters. By that, relevant parameters are estimated in the usual way and posterior draws of irrelevant variables are close to zero so that they have no significant effect on forecasts and impulse responses. This is achieved by adding a hierarchial prior to the model, where the relevance of a variable is assessed in each step of the sampling algorithm. The main innovations of the method were: (i) the introduction of a vector of binary parameters, denoted by $\boldsymbol{\gamma}$, which was used to indicate if a variable should be included or excluded from the model (active or inactive) and (ii) that each regression coefficient was not set exactly equal to zero when a covariate was assumed to be inactive, but it was a posteriori restricted to a small neighborhood around zero via very informative zero-centered priors.

$$\alpha_j|\gamma_j \sim N(\underline{\alpha}, D) \quad (26)$$

Where $\alpha$is the prior mean, $D = diag\{\Gamma_1, \dots, \Gamma_n\}\ and\ \gamma = \{\gamma_1, \dots, \gamma_n\}$which takes the values of 0 and 1.

$$\Gamma = \begin{cases} \Gamma_{0j} & \gamma = 0 \\ \Gamma_{1j} & \gamma = 1 \end{cases}$$

Given the latent inclusion of $\gamma_j$using the hierarchical form:

$$\alpha_j|\gamma_j \sim (1 - \gamma_j)N(a, \Gamma_{0j}^2) + \gamma j N(a, \Gamma_{1j}^2) \quad (27)$$

$$P(\gamma_j = 1) = 1 - P(\gamma_j = 0) = P_{1j} \quad (28)$$

3.0 Data Analysis And Results

3.1Bayesian Macroeconomics in R(BMR)

Bayesian Macroeconometrics in **R** (‘BMR’) is a collection of **R** and C + + routines for estimating Bayesian Vector Autoregressive (BVAR) and Dynamic Stochastic General Equilibrium (DSGE) models in the **R** statistical environment.

3.2Data Collection

The data used in this study is the daily stock price of five leading banks and the banking sector index. These banks are GTBank, Zenith Bank, UBA, First Bank, and Eco Bank. The data collected was from January 2022 to December 2023. A total of 500 data points were splitted into two parts with 400 data for training and the remaining for evaluating the forecast performance. The data was stationary after the first difference as shown in Figures I and II below:

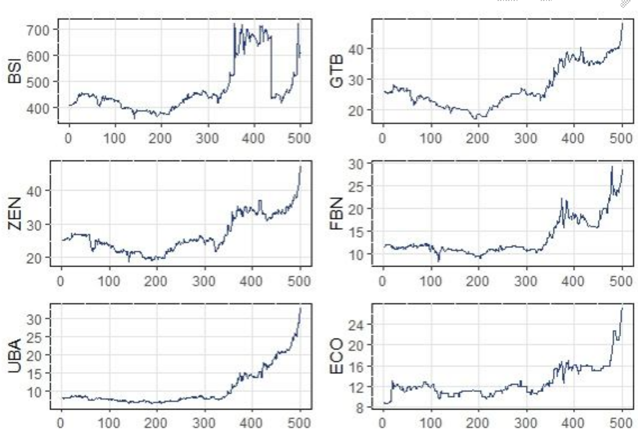

Figure I:Data Unstationary before differing

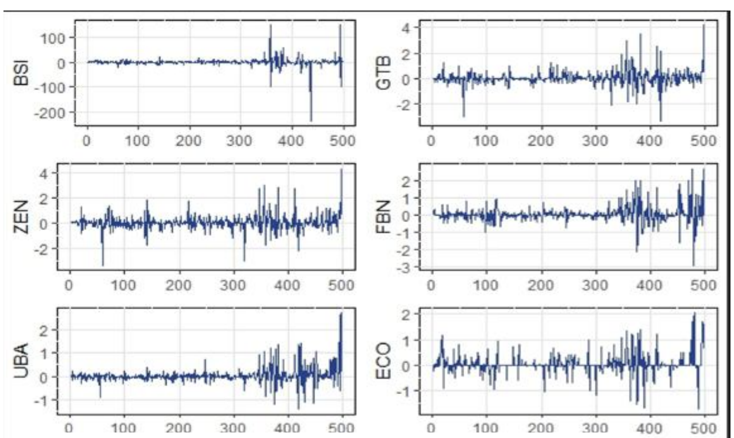

Figure II: Data stationary after first diference

The coefficients of the Bayesian vector autoregressions are computed using Minnesota, OLS_VAR, stochastic search variable selection, Minnesota prior with diffuse prior for variance and steady state with inverse Wishart as shown below:

Table I: Coefficient of BVAR using OLS

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI.01	-0.339	-0.018	-0.011	-0.003	-0.005	-0.003
GTB.01	5.507	0.166	0.183	0.192	0.104	0.079
ZEN.01	3.311	0.213	-0.002	-0.03	0.007	0.031
FBN.01	0.599	0.098	0.048	0.198	0.092	0.106
UBA.01	7.718	0.32	0.293	0.221	-0.067	0.144
ECO.01	1.265	0.068	-0.093	0.049	-0.01	0.009
BSI.02	-0.435	-0.003	-0.006	0.001	0.001	0.005
GTB.02	1.253	-0.106	0.076	0.048	-0.023	0.058
ZEN.02	2.668	0.041	0.057	0.025	-0.007	-0.055
FBN.02	5.099	0.164	0.258	-0.09	0.03	0.056
UBA.02	1.022	0.174	-0.319	-0.226	-0.091	-0.259
ECO.02	1.248	0.019	-0.004	-0.08	-0.037	-0.136
BSI.03	0.018	0	0.001	0.007	0.003	0.001
GTB.03	0.396	-0.022	-0.014	0.147	0.034	0.038
ZEN.03	-2.826	-0.142	-0.131	-0.166	-0.057	-0.062
FBN.03	-4.334	-0.232	-0.033	-0.188	-0.07	-0.033
UBA.03	1.451	0.456	0.238	-0.281	-0.094	-0.119
ECO.03	-1.719	-0.011	0.008	0.065	0.023	-0.001
BSI.04	-0.232	-0.001	-0.004	0	0	0.001
GTB.04	0.776	-0.105	-0.053	-0.012	0.01	0.048
ZEN.04	2.709	0.177	0.114	0.034	0.025	0.008
FBN.04	0.827	0.094	0.025	-0.016	0.099	-0.016
UBA.04	4.284	0.01	0.006	-0.246	-0.096	-0.068
ECO.04	-0.564	-0.043	0.09	-0.041	-0.055	-0.15
const	0.699	0.023	0.021	0.014	0.017	0.019

Table II: Coefficient of BVAR using Minnesota Prior

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI-lag1	-0.329	-0.018	-0.012	-0.003	-0.006	-0.004
BSI-lag2	5.421	0.201	0.173	0.171	0.088	0.07

BSI-lag3	3.218	0.188	0.024	-0.023	0.008	0.035
BSI-lag4	0.561	0.077	0.037	0.232	0.073	0.103
GTB-lag1	7.112	0.291	0.284	0.214	0.026	0.142
GTB-lag2	1.314	0.062	-0.086	0.057	-0.008	0.044
GTB-lag3	-0.401	-0.003	-0.006	0	0	0.004
GTB-lag4	1.153	-0.081	0.067	0.028	-0.023	0.043
ZEN-lag1	2.33	0.032	0.054	0.037	0.003	-0.043
ZEN-lag2	4.489	0.16	0.227	-0.092	0.02	0.042
ZEN-lag3	0.755	0.116	-0.244	-0.16	-0.052	-0.189
ZEN-lag4	0.845	0.013	-0.007	-0.073	-0.028	-0.114
FBN-lag1	0.011	0.001	0.001	0.005	0.002	0
FBN-lag2	0.303	-0.003	-0.009	0.105	0.026	0.024
FBN-lag3	-2.24	-0.112	-0.103	-0.126	-0.042	-0.041
FBN-lag4	-3.191	-0.162	-0.014	-0.167	-0.055	-0.037
UBA-lag1	0.145	0.228	0.144	-0.151	-0.048	-0.056
UBA-lag2	-1.407	-0.003	0.015	0.051	0.021	0.005
UBA-lag3	-0.174	0	-0.003	-0.001	0	0
UBA-lag4	0.664	-0.063	-0.031	-0.016	0.005	0.033
ECO-lag1	1.978	0.114	0.073	0.028	0.017	0.014
ECO-lag2	0.475	0.055	0.014	-0.024	0.056	-0.023
ECO-lag3	1.864	-0.015	-0.003	-0.101	-0.03	-0.021
ECO-lag4	-0.473	-0.035	0.055	-0.031	-0.038	-0.102
constant	0.817	0.019	0.025	0.014	0.016	0.016

Table III: Coefficient of BVAR using SSVS

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI.01	-0.002	-0.002	0	0	0	0
GTB.01	0.76	0.003	0.001	0.083	0.002	0.001
ZEN.01	1.156	0.068	-0.027	-0.003	-0.001	0.004
FBN.01	-0.059	0.004	-0.001	0.183	0.049	0.089
UBA.01	0.37	0.017	0.005	0.111	-0.151	0.034
ECO.01	0.17	0.004	-0.047	0.02	-0.002	0.001
BSI.02	-0.324	0	0	0	0	0
GTB.02	0.08	-0.093	0.006	0.001	-0.037	0.004
ZEN.02	0.875	-0.002	-0.002	0.003	-0.002	-0.004
FBN.02	0.516	0.016	0.025	-0.121	-0.008	0

UBA.02	5.091	0.187	-0.159	-0.026	-0.01	-0.021
ECO.02	0.518	0.003	-0.005	-0.01	-0.002	-0.037
BSI.03	-0.003	0	0	0.001	0	0
GTB.03	-0.035	-0.003	-0.002	0.089	0.003	0
ZEN.03	-0.043	-0.006	-0.003	-0.055	0.002	0
FBN.03	-0.147	-0.013	0.109	-0.144	-0.003	0
UBA.03	-0.132	0.054	0.022	-0.029	-0.006	-0.004
ECO.03	-1.724	-0.001	0.006	0.011	0.004	-0.001
BSI.04	-0.08	0	0	0	0	0
GTB.04	0.194	-0.014	-0.004	-0.003	0.002	0.002
ZEN.04	0.053	0.033	0	0	0	0
FBN.04	-0.045	0.005	-0.002	-0.023	0.047	-0.01
UBA.04	2.819	0.003	-0.004	-0.064	-0.002	0.001
ECO.04	0.009	0	0.021	-0.001	-0.004	-0.045
const	0.731	0.027	0.023	0.012	0.016	0.017

Table IV: Coefficient of BVAR using Steady State Inverse Wishart

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI.l1	-0.113	-0.003	-0.005	-0.032	0.001	-0.001
GTB.l1	6.064	0.085	-0.012	0.156	0.006	-0.006
ZEN.l1	1.494	-0.006	0.022	-0.808	0.002	0.005
FBN.l1	1.367	0.146	0.315	0.394	0.005	0.03
UBA.l1	-1.281	-0.115	-0.205	1.337	-0.019	0.119
ECO.l1	0.561	-0.051	0.072	-0.926	0.032	0.04
BSI.l2	-0.015	-0.002	-0.002	-0.003	0.002	-0.001
GTB.l2	0.105	0.01	-0.019	0.017	0.014	-0.062
ZEN.l2	0.181	0.064	0.051	-0.034	-0.048	-0.009
FBN.l2	0.126	0.133	-0.081	0.015	-0.019	0.013
UBA.l2	0.19	0.053	-0.004	0.184	-0.067	-0.106
ECO.l2	0.049	0.002	-0.05	0.019	0.036	0.023
BSI.l3	-0.007	-0.21	-0.001	-0.004	-0.025	0.001
GTB.l3	0.19	-0.795	-0.041	-0.021	0.924	0.008
ZEN.l3	-0.096	1.365	0.02	0.028	-1.056	-0.006
FBN.l3	0.085	4.118	0.004	0.054	0.192	0.044
UBA.l3	0.057	-0.813	-0.032	0.165	1.097	-0.03
ECO.l3	-0.079	1.736	-0.007	-0.064	0.601	-0.001

BSI.l4	-0.002	-0.001	0.001	0.002	0.002	0.001
GTB.l4	0.154	-0.13	0.047	0.066	0.023	-0.003
ZEN.l4	-0.017	0.006	-0.033	-0.051	0.002	-0.006
FBN.l4	0.208	0.182	0.014	-0.13	0.021	-0.008
UBA.l4	0.202	0.125	-0.074	-0.039	-0.053	-0.032
ECO.l4	0.077	-0.031	-0.087	0.097	0.034	-0.046

Table V: Coefficient of BVAR using Steady State Diffuse Prior

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI.l1	-0.29	0.099	0.12	-2.537	-0.057	0.081
GTB.l1	4.387	0.009	0.086	-0.871	-0.059	0.001
ZEN.l1	2.044	0.066	-0.23	0.034	0	-0.019
FBN.l1	-2.63	0.045	0.168	0.001	0.021	0.028
UBA.l1	15.67	0.023	-0	0.05	-0.044	-0.036
ECO.l1	0.046	-0.004	0.003	-0.073	-0.067	-0.056
BSI.l2	-0.02	0.13	0.081	-0.165	0.013	0.064
GTB.l2	0.123	0.007	-0.15	0.135	-0.093	0
ZEN.l2	0.23	0.061	-0.11	0.047	-0.042	0.013
FBN.l2	0.072	0.11	0.059	-0.001	0.832	0.019
UBA.l2	0.311	4.519	0	0.012	0.148	0.032
ECO.l2	0.034	-0.226	-0.03	0.007	0.348	-0.028
BSI.l3	-0.01	0.159	0.025	-0.036	-0.559	0.027
GTB.l3	0.182	2.462	-0.04	0.064	0.082	0.002
ZEN.l3	0.083	1.32	-0.01	-0.127	0.001	0.026
FBN.l3	-0.06	-1.171	0.052	0	0.005	-0.026
UBA.l3	0.418	0.223	0.001	0.049	0.048	0.006
ECO.l3	0.029	-0.002	0.016	-0.024	0.013	-0.09
BSI.l4	-0	-0.088	0.011	-0.163	-0.009	-0.249
GTB.l4	0.195	0.113	0.024	-0.031	0.104	-0.049
ZEN.l4	-0.07	0.006	-0.11	0.001	0	0.031
FBN.l4	0.207	0.272	0.437	0.001	0.017	-0.038
UBA.l4	0.252	0.028	0.015	0.027	-0.049	-0.012
ECO.l4	0.013	-0.002	-0.44	-0.025	0.049	-0.124
Const	1.008	-0.006	0.004	-1.213	-0.058	-0.116

Table VI: Coefficient of BVAR using Diffuse Prior

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI.l1	-0.05	-0.008	0.002	0.001	-0.002	0.002
GTB.l1	2.423	0.059	0.068	0.037	0.029	0.001
ZEN.l1	-0.19	-0.013	-0.2	-0.002	0.01	-0.004
FBN.l1	-1.93	-0.08	-0.16	0.156	0.035	0.093
UBA.l1	4.106	0.279	0.229	0.228	-0.053	0.001
ECO.l1	1.299	0.065	-0.11	-0.026	-0.001	0.088
BSI.l2	-0.24	-0.001	0.001	-0.001	0	0
GTB.l2	1.129	-0.014	0.086	0.01	-0.012	0.036
ZEN.l2	0.659	0.003	-0.1	-0.011	-0.024	-0.036
FBN.l2	-0.22	-0.019	0.041	-0.094	-0.027	0.029
UBA.l2	0.353	-0.021	-0.08	-0.047	0.008	-0.048
ECO.l2	0.178	0.033	-0.05	-0.072	-0.057	-0.002
BSI.l3	0.006	0.002	0.001	0.001	0	0
GTB.l3	-1.01	-0.034	-0.03	0.025	0.012	0
ZEN.l3	0.738	-0.015	0.032	-0.033	-0.003	0
FBN.l3	-0.88	-0.009	0.088	-0.122	-0.037	-0.008
UBA.l3	-2.84	0.107	-0.04	-0.07	0.048	-0.038
ECO.l3	0.329	-0.006	0	0.022	-0.003	-0.001
BSI.l4	-0.07	0.001	0	0.001	0	-0.001
GTB.l4	1.342	-0.035	-0	-0.04	-0.003	-0.012
ZEN.l4	0.208	0.033	0.023	-0.008	-0.002	0.009
FBN.l4	-0.08	-0.042	-0.1	0.017	0.017	-0.049
UBA.l4	-1.97	-0.03	-0.02	-0.064	0.012	0.088
ECO.l4	-1.01	0.015	0.067	-0.021	-0.006	-0.073
const	1.052	0.035	0.023	0.022	0.014	0.04

The coefficients comparison among the priors reveals a similarity between the posterior distributions of the Minnesota prior and those of the OLS_VAR for stock market predictions. The table also illustrates the interrelationship among the variables. Additionally, it can be observed that the SSVS prior predominantly shrinks most variables to zero, while OLS_VAR and Minnesota priors only allow the variables to shrink toward the vicinity of zero. Across the model, SSVS have the lowest coefficient and Minnesota have the highest coefficient.

Table VII: Posterior Mean of all the coefficient of the six priors.

	OLS	DIFFUSE	MINN	SSVS	STEADY
BSI.l1	-0.339	-0.05	-0.329	-0.002	-0.113
GTB.l1	5.507	2.423	5.421	0.76	6.064

ZEN.l1	3.311	-0.19	3.218	1.156	1.494
FBN.l1	0.599	-1.93	0.561	-0.059	1.367
UBA.l1	7.718	4.106	7.112	0.37	-1.281
ECO.l1	1.265	1.299	1.314	0.17	0.561
BSI.l2	-0.435	-0.24	-0.401	-0.324	-0.015
GTB.l2	1.253	1.129	1.153	0.08	0.105
ZEN.l2	2.668	0.659	2.33	0.875	0.181
FBN.l2	5.099	-0.22	4.489	0.516	0.126
UBA.l2	1.022	0.353	0.755	5.091	0.19
ECO.l2	1.248	0.178	0.845	0.518	0.049
BSI.l3	0.018	0.006	0.011	-0.003	-0.007
GTB.l3	0.396	-1.01	0.303	-0.035	0.19
ZEN.l3	-2.826	0.738	-2.24	-0.043	-0.096
FBN.l3	-4.334	-0.88	-3.191	-0.147	0.085
UBA.l3	1.451	-2.84	0.145	-0.132	0.057
ECO.l3	-1.719	0.329	-1.407	-1.724	-0.079
BSI.l4	-0.232	-0.07	-0.174	-0.08	-0.002
GTB.l4	0.776	1.342	0.664	0.194	0.154
ZEN.l4	2.709	0.208	1.978	0.053	-0.017
FBN.l4	0.827	-0.08	0.475	-0.045	0.208
UBA.l4	4.284	-1.97	1.864	2.819	0.202
ECO.l4	-0.564	-1.01	-0.473	0.009	0.077
const	0.699	1.052	0.817	0.731	-0.113

TablesVIII, IX,X, & XI show the variance-covariance matrix of variables BSI, GTB, ZEN, FBN, UBA and ECO. The observed variance-covariance matrix confirms a strong relationship between the Banking Sector Index (BSI) and the individual stock prices of the five banks (GTB, ZEN, FBN, UBA, and ECO). This suggests that movements in the overall banking sector index are closely related to fluctuations in the stock prices of individual banks. The results highlights that GTBank (GTB) and Zenith Bank (ZEN) stock prices exhibit a particularly strong relationship compared to other bank stocks. This is supported by the variance-covariance values indicating higher covariance between GTB and ZEN compared to other banks.

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI	135.568	3.605	4.572	1.832	1.867	1.939
GTB	3.605	0.252	0.159	0.054	0.061	0.059
ZEN	4.572	0.159	0.301	0.058	0.071	0.049
FBN	1.832	0.054	0.058	0.117	0.039	0.039
UBA	1.867	0.061	0.071	0.039	0.050	0.031
ECO	1.939	0.059	0.049	0.039	0.031	0.122

Table VIII: Variance_Covariance of OLS

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI	147.929	4.117	5.005	2.025	2.021	2.072
GTB	4.117	0.279	0.180	0.060	0.068	0.063
ZEN	5.005	0.180	0.319	0.065	0.076	0.053
FBN	2.025	0.060	0.065	0.126	0.043	0.043
UBA	2.021	0.068	0.076	0.043	0.053	0.035
ECO	2.072	0.063	0.053	0.043	0.035	0.125

Table X: Variance_Covariance of SSVS

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI	126.331	3.337	4.216	1.700	1.709	1.779
GTB	3.337	0.241	0.147	0.049	0.056	0.054
ZEN	4.216	0.147	0.284	0.054	0.064	0.045
FBN	1.700	0.049	0.054	0.114	0.036	0.036
UBA	1.709	0.056	0.064	0.036	0.048	0.029
ECO	1.779	0.054	0.045	0.036	0.029	0.116

Table IX: Variance_Covariance of MINN

	BSI	GTB	ZEN	FBN	UBA	ECO
BSI	132.713	3.244	4.422	1.767	1.780	1.520
GTB	3.244	0.236	0.159	0.056	0.056	0.046
ZEN	4.422	0.159	0.289	0.060	0.070	0.038
FBN	1.767	0.056	0.060	0.123	0.039	0.034
UBA	1.780	0.056	0.070	0.039	0.048	0.023
ECO	1.520	0.046	0.038	0.034	0.023	0.105

Table XI: Variance_Covariance of Steady Steady Inverse Wishart

Variance-covariance matrix above reinforces the idea of interdependence and significant relationships between the stock prices of individual banks and the overall banking sector index, with GTBank and Zenith Bank playing particularly influential roles.

It can be deduced from Table I and II that frequentist and Bayesian estimators agreed on estimate of the coefficients of vector autoregression. The stochastic search variable selection prior differs from the result of Minnesota and Ordinary Least Square estimates. Table XII shows the forecast performance of the priors:

3.3Forecasting Performance of OLS, Minnesota, Ssvs And Steady State

Forecast for 10 steps ahead was obtained using the training data and the estimate obtained in Table V above.

Table XII: Forcasting performances of the priors for 10 steps ahead.

	Actual	**OLS_VARS**	**MINNESOTA**	**MINNESOTA IW**	**SSVS**	**DIFFUSE**	**STEADY STATE IW**
	-2.98	-2.562	-3.47	0.2213	1.623	-5.721	-0.096

	-3.18	2.487	3.07	0.0847	-1.579	14.962	-4.632
	0.42	-0.498	0.60	0.2598	0.080	-4.21	-10.543
	-2.36	2.150	2.35	0.2742	-0.315	6.683	-3.127
	-6.02	0.494	-1.43	-0.1946	-0.079	-7.03	-10.414
	-4.49	0.678	1.77	-0.1029	0.145	-12.44	-12.956
	-11.86	-0.530	1.58	-0.1302	0.041	-13.823	-3.592
	0.89	0.877	0.06	0.0273	-0.233	-14.182	19.339
	-5.79	1.288	1.71	-0.1047	0.043	-5.676	-8.286
	10.40	0.758	3.41	-0.1352	-0.078	32.638	17.127

Table XII also show that there are similarities among OLS_VAR and Minnesota prior forecast. Minnesota inverse Wishart prior and stochastic search variable selection prior has the lowest and the diffuse prior has the highest.

The forecast as shown in table XIII was evaluated using Mean average, Mean Average Error and Root Means Square Error.

Table XIII: Forcasts evaluations for the priors

	ME	RMSE	MAE	MPE	MAPE
Minnesota	-0.092	0.35085	0.252	66	98
Minnesota(NIW)	2.34526	7.663893	6.360506	296.4384	353.9403
SSVS	-2.461787	6.095331	4.850008	100.2396	100.2396
STEADY STATE	-0.0296	0.4449004	0.3726	-98.63846	271.4615
Diffuse	-1.6171	11.08892	8.2903	323.5318	426.771
OLS	-2.608714	6.174951	4.781576	60.49834	94.45113

Using Table XIII above, Minnesota prior and Steady state prior has the lowest RMSE of 0.35085 and 0.4449004 respectively,indicating better forecast accuracy. SSVS and OLS has approximately the same RMSE. Thus,minnesota and steady state priors out-performed all other priors.

4.0 Conclusion

In this paper, Bayesian Vector Autoregressions(BVAR) was adopted in forcasting the stock prices of some banks in Nigeria using different priors. The VAR model was formed using six variables of BSI, GTB, ZEN, FBN, UBA, and ECO stock prices. The three major priors proposed by earlier researchers: Minnesota byLitterman (1980, 1986), stochastic search variable selection (SSVS) by George et al. (2008), and Steady State by Villain(2009) were compared, andwas observed that the forecasting performance of Minnesota prior of Bayesian vector autoregression out-performed other priors using the criteria: RMSE, MAPE, MAE and ME. The result was supported by the Random Walk Theory which was proposed by Louise Bachelier(1900) and Burton Malkiel(1973). This study will be essential to many researchers especially thefinancial analysts and stock market professionals, to predict the prices of stock using the inter-relationship among the prices of other prices and the contribution of the sectoral index. This will enable them to come up with strategic plans for inflation targeting. It is recommended that other classes of stochastic search variable selection prior for Bayesian VAR be considered for further studies.

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