Inference for Stress-Strength Reliability Using the Exponential-Gamma Model: Theory and Applications

Abstract

The stress-strength reliability measure, defined as $R = P(Y < X)$, is a widely used index in reliability analysis, representing the probability that a system's strength exceeds the applied stress. This paper investigates the Exponential-Gamma stress-strength model, assuming the stress variable follows an exponential distribution while the strength variable follows a gamma distribution. Analytical expressions for the reliability function are derived and generalized to the case of standby redundant systems. Estimation of reliability is developed using maximum likelihood and uniformly minimum variance unbiased approaches, and both exact and asymptotic confidence intervals are obtained. A detailed Monte Carlo simulation study evaluates the finite-sample properties of the proposed estimators, highlighting the superior small-sample performance of the UMVUE and the asymptotic efficiency of the MLE. The practical usefulness of the model is demonstrated through real data applications, showing that the Exponential-Gamma framework provides an effective and tractable tool for modeling system reliability in applied settings.

Keywords: Stress-strength reliability; Standby redundancy; Maximum likelihood estimation; UMVUE; Confidence intervals

1 Introduction

Stress-strength reliability, defined as $R = P(Y < X)$ where X denotes component strength and Y the stress, is a central concept in reliability theory. Early work by Huang (2012) analyzed inference for R when strength follows a gamma distribution and stress follows an exponential distribution, deriving both the maximum likelihood estimator (MLE) and the uniformly minimum variance unbiased estimator (UMVUE) and comparing their mean squared errors in simulation.

Extensions to more flexible lifetime distributions have maintained vibrant momentum. Esfahani (2024) studied the stress-strength reliability parameter under the generalized exponential distribution, deriving MLE, UMVUE, and exact confidence intervals. Similarly, Temraz (2023) developed inference procedures in the exponentiated generalized Marshall-Olkin G family of distributions, bringing in asymptotic, bootstrap, and Bayesian estimation with real data illustrations.

Systems with multiple stress or strength components have also been investigated. Pal and Tiensuwan (2020) considered strength subject to two independent stresses and derived UMVUEs under generalized uniform distributions. Jha et al. (2022) addressed multicomponent systems using both frequentist and Bayesian approaches where stress and strength variables follow general distributions. The challenge of censoring under complex models has been tackled by Kohansal et al. (2025), who used a progressive first-failure censoring scheme for multi-component stress-strength parameters under a Lomax distribution.

Other authors have examined specialized distributions. (Khan and Khatoon, 2019) investigated classical and Bayesian estimation of R from the generalized inverted exponential distribution based on record values. (Pandit and Kavitha, 2024) studied stress-strength reliability under the Lomax-exponential distribution, combining MLE and Bayesian estimators with real data. (James et al., 2023) introduced the Type-1 Pathway Generated Exponential (PGE-1) distribution, providing both point and interval estimation and demonstrating applicability using AIDS incubation data.

Theoretical treatments of gamma-exponential combinations have also appeared. Wang (2011) analyzed inference about R when stress is gamma and strength is exponential- deriving UMVUEs, MLEs, and pivotal-based confidence intervals. More recently, Kotb (2025) provided both Bayesian and non-Bayesian illustrations in his analysis of generalized exponential stress-strength reliability, while Brownstein (2007) highlighted the role of the generalized gamma distribution in encompassing many reliability models. Finally, Thomas and Chacko (2022) analyzed the Exponential-Gamma stress-strength model, deriving analytical forms of R, maximum likelihood estimates, asymptotic confidence bounds, simulations, and a real data example.

Despite this rich literature, the Exponential-Gamma (EG) hybrid model remains underexplored. This paper addresses the gap by proposing a full treatment of the EG stress-strength setup: deriving both MLE and UMVUE estimators for R, extending to systems with redundancy, rigorously comparing estimator performance through simulation, and validating the approach with real data. The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 develops estimation methodologies. Section 4 presents a simulation study. Section 5 provides a real data application, and Section 6 concludes with insights and directions for future research.

2 Model Formulation and Reliability

The probability density function(pdf) and cumulative distribution function(cdf) of gamma distribution are respectively given by

$$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}; 0 < x < \infty; \alpha, \beta > 0$$

$$F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x); 0 < x < \infty; \alpha, \beta > 0$$

where, α is the shape parameter and β is the rate parameter.

Again, exponential distribution which is a special case of the gamma distribution when the shape parameter $\alpha = 1$ is a popular classical probability distribution used for analysing real life data.

Let X be a random variable from exponential distribution with respective pdf and cdf as,

$$f(x) = \lambda e^{-\lambda x}; x \geq 0, \lambda > 0$$

$$F(x) = e^{-\lambda x}; x \geq 0, \lambda > 0$$

Suppose $X \sim$ Gamma(α, β) be the strength variable and $Y \sim$ Exponential(λ) be the stress variable. X and Y are independently distributed, then the reliability of the system with stress variable Y and strength variable X is

$$R = \int_0^\infty (1 - e^{-\lambda x}) \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} dx$$

On simplification, we can get the reliability of the system as

$$R = 1 - \left(\frac{\beta}{\beta + \lambda}\right)^\alpha \tag{2.1}$$

Here, $R = 1 - c^{\alpha}$, where $c = \frac{\beta}{\beta+\lambda} \in (0,1)$. Taking the partial derivative of R with respect to α, we get

$$\frac{\partial R}{\partial \alpha} = -c^{\alpha} \log c < 0$$

Since $c \in (0,1) \Rightarrow \log c < 0$, so the whole derivative is negative. Therefore, R is a strictly decreasing function of α for fixed values of β and λ. That is, for fixed stress and scale parameters, the reliability decreases as the shape parameter α increases.

Similarly, taking the partial derivative of $R$with respect to α, we get

$$\frac{\partial R}{\partial \lambda} = \alpha c^{\alpha} \frac{1}{\beta + \lambda} > 0$$

which shows that R increases with λ, when α and β are fixed. Hence, the reliability R is an increasing function of the stress parameter λ, and a decreasing function of the strength shape parameter α.

2.1 Reliability with Redundancy

Standby redundancy is a common design strategy in reliability engineering where additional units remain inactive until the primary component fails. This mechanism is particularly relevant in safety-critical systems such as power grids, aircraft engines, and medical devices, where uninterrupted operation is essential.

For the EG stress–strength model, suppose each component is exposed to stress $Y \sim$ Exponential(λ_i) and has strength $X \sim$ Gamma(α_i, β_i). For a single standby unit$(n = 1)$, the reliability is obtained as,

$$R(1) = \left(\frac{\alpha_1}{\lambda_1 + \alpha_1}\right)^{\lambda_1} \tag{2.2}$$

which represents the probability that the first active component withstands the applied stress.

When two standby units are available $(n = 2)$, the system survives if the first unit fails but the second unit resists the stress. The resulting reliability is,

$$R(2) = \left[1 - \left(\frac{\alpha_1}{\alpha_1 + \lambda_1}\right)^{\lambda_1}\right] \left(\frac{\alpha_2}{\alpha_2 + \lambda_2}\right)^{\lambda_2} \tag{2.3}$$

For three standby components $(n = 3)$, the expression naturally extends to

$$R(3) = \left[1 - \left(\frac{\alpha_1}{\alpha_1 + \lambda_1}\right)^{\lambda_1}\right] \left[1 - \left(\frac{\alpha_2}{\alpha_2 + \lambda_2}\right)^{\lambda_2}\right] \left(\frac{\alpha_3}{\alpha_3 + \lambda_3}\right)^{\lambda_3} \tag{2.4}$$

The general form of reliability for an n-component standby redundant system is therefore given by,

$$R(n) = \left[\prod_{i=1}^{n-1} \left(1 - \left(\frac{\alpha_i}{\alpha_i + \lambda_i}\right)^{\lambda_i}\right)\right] \left(\frac{\alpha_n}{\alpha_n + \lambda_n}\right)^{\lambda_n} \tag{2.5}$$

This expression shows that each standby unit is sequentially activated, and overall system reliability improves with redundancy, though the gain diminishes with higher n. Such formulations for cold/standby redundancy have been widely discussed in reliability literature, including the work of Liu et al. (2018) on multicomponent standby systems and Cüran and Kızılaslan (2021) on parallel systems with cold standby.

The analytical form of $R(n)$ under the EG framework is valuable because it allows direct computation of system reliability without extensive simulation. Moreover, it highlights the influence of the Gamma shape parameter α_i and the Exponential rate parameter λ_i on the overall system performance.

Specifically, larger values of α_i reduce the survival probability, while larger λ_i improve system reliability. These insights help guide the design of redundant systems in practice, providing a balance between redundancy, cost, and performance.

3 Estimation of Model Parameters

3.1 Maximum Likelihood Estimation (MLE) Method

To accomplish the MLE of the stress-strength reliability R, we first need to obtain the MLEs of the involved parameters. Let $X_1, X_2, ..., X_n$ be a random sample from gamma distribution with shape parameter α and scale parameter β, representing the strength variable and let $Y_1, Y_2, ..., Y_m$ be a random sample from exponential distribution with rate parameter λ, representing the stress variable. Assuming that the samples are independent, the joint log-likelihood function for the observed data is given by,

$$L(\alpha, \beta, \lambda) = \prod_{i=1}^{n} f(x_i) \prod_{j=1}^{m} f(y_j)$$

$$\ln L = n\alpha \log \beta - n \log \Gamma(\alpha) + (\alpha - 1) \sum_{i=1}^{n} \log x_i - \beta \sum_{i=1}^{n} x_i + m \log \lambda - \lambda \sum_{j=1}^{m} y_j \tag{3.1}$$

The MLEs of α, β, λ are obtained by maximizing this log-likelihood function with respect to the respective parameters. Therefore, the MLE of R becomes

$$\hat{R}_{MLE} = 1 - \left(\frac{\hat{\beta}}{\hat{\beta} + \hat{\lambda}} \right)^{\hat{\alpha}}$$

3.2 Uniformly Minimum Variance Unbiased Estimation (UMVUE) Method

The UMVUE of the stress-strength reliability parameter $R = P(Y < X)$, where $X \sim$ Gamma(α, β) and $Y \sim$ Exponential(λ) under the assumption that the scale parameters β and λ are known and only the shape parameter α is unknown.

Let $X_1, X_2, ..., X_n$ be a random sample from gamma distribution with the same parameters. It is well known that the sum of independent gamma variables with common scale forms another gamma variable. Specifically, the statistic $T = \sum_{i=1}^{n} X_i$ follows a gamma distribution with shape parameter $n\alpha$ and scale parameter β. Since T is both complete and sufficient for α, it provides a strong foundation for constructing the UMVUE of the function R. Using established results for functions of exponential family parameters, such as those presented by Tong (2009), we can derive the unbiased estimator of c^{α}, where $c = \frac{\beta}{\beta+\lambda} \in (0, 1)$, and hence for $R = 1 - c^{\alpha}$. Therefore, the UMVUE of R is given by

$$\hat{R}_{UMVUE} = 1 - \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{(-\log c)^k}{k!} \left(\frac{T}{\beta} \right)^k \left(1 + \frac{T}{\beta} \right)^{-(n+k)} \tag{3.2}$$

This estimator is based solely on the sufficient statistic T ensuring its unbiasedness and optimality in terms of variance among all unbiased estimators.

3.3 Simulation Study

To assess the performance of the proposed estimators of the stress-strength reliability under EG stress-strength model, a detailed Monte Carlo simulation study was carried out. The shape parameter α was varied over the values 0.5,1.0,1.5,2.0 and 2.5, while β and $\lambda$was fixed at 2.5 and 1.5 respectively. The sample sizes were taken as $n = 5, 10, 15, 20$ for the strength variable and $m = 5, 10, 15$ for the stress variable. For each combination of parameters and sample sizes, 5000 independent replications were simulated. The reliability measure R was estimated using both MLE and UMVUE. **Table 1** presents the average bias and MSE of these estimators across all configurations.

Table 1: Bias and Mean Squared Error of Estimated Reliability R Using MLE and UMVUE

(n,m)	α				
	0.5	1.0	1.5	2.0	2.5
(5,5)	-0.00875(0.01968)	-0.01055(0.02226)	-0.00963 (0.01726)	-0.01117 (0.01203)	-0.01126 (0.00774)
	0.17357(0.07859)	0.20052 (0.06120)	0.16654 (0.03446)	0.12210 (0.01702)	0.08462 0.00779
(5,10)	-0.02200(0.01653)	-0.01587 (0.01724)	-0.01859(0.01378)	-0.01533(0.00899)	-0.01344(0.00559)
	0.17067 (0.07811)	0.20329(0.06204)	0.16621(0.03425)	0.12176(0.01689)	0.08443(0.00778)
(5,15)	-0.02549(0.01580)	-0.02030(0.01653)	-0.01865(0.01193)	-0.01670(0.00787)	-0.01448(0.00490)
	0.16757(0.07707)	0.20122(0.06201)	0.16634(0.03422)	0.12160(0.01687)	0.08399(0.00773)
(10,5)	0.00557(0.01408)	0.00869(0.01625)	-0.00016(0.01338)	-0.00248(0.00953)	-0.00690(0.00640)
	0.58135(0.34619)	0.41278(0.17046)	0.26858(0.07214)	0.17357(0.03013)	0.11206(0.01256)
(10,10)	-0.00657(0.01032)	0.00002(0.01126)	-0.00536(0.00910)	-0.00748(0.00605)	-0.00608(0.00390)
	0.58205(0.34673)	0.41292(0.17057)	0.26859(0.07214)	0.17356(0.03012)	0.11206(0.01256)
(10,15)	-0.00991(0.00894)	-0.00753(0.00951)	-0.00700(0.00713)	-0.00581(0.00468)	-0.00619(0.00297)
	0.58150(0.34641)	0.41276(0.17043)	0.26859(0.07214)	0.17357(0.03012)	0.11206(0.01256)
(15,5)	0.00797(0.01187)	0.01104(0.01470)	0.00533(0.01181)	-0.00190(0.00830)	-0.00584(0.00565)
	0.64234(0.41276)	0.41666(0.17360)	0.26896(0.07234)	0.17361(0.03014)	0.11207(0.01256)
(15,10)	-0.00089 0.00799	0.00020(0.00933)	-0.00526(0.00765)	-0.00411(0.00526)	-0.00529(0.00329)
	0.64246(0.41289)	0.41666(0.17360)	0.26896(0.07234)	0.17361(0.03014)	0.11207(0.01256)
(15,15)	-0.00562(0.00674)	-0.00310(0.00784)	-0.00380(0.00586)	-0.00399(0.00405)	-0.00459(0.00261)
	0.64266(0.41311)	0.41665(0.17360)	0.26896(0.07234)	0.17361(0.03014)	0.11207(0.01256)
(20,5)	0.01266(0.01072)	0.01032(0.01376)	0.00814(0.01158)	-0.00052(0.00794)	-0.00389(0.00536)
	0.64542(0.41657)	0.41667(0.17361)	0.26896(0.07234)	0.17361(0.03014)	0.11207(0.01256)
(20,10)	0.00082(0.00697)	-0.00009(0.00843)	-0.00054(0.00709)	-0.00074(0.00487)	-0.00247(0.00314)
	0.64544(0.41659)	0.41667(0.17361)	0.26896(0.07234)	0.17361(0.03014)	0.11207(0.01256)
(20,15)	-0.00053(0.00588)	0.00109(0.00664)	-0.00143(0.00537)	-0.00396(0.00356)	-0.00442(0.00231)
	0.64542(0.41657)	0.41667(0.17361)	0.26896(0.07234)	0.17361(0.03014)	0.11207(0.01256)

The average bias and the corresponding MSE are reported within brackets using MLE in first row and using UMVUE in the second row.

Now, we present the graphical representations of the bias and mean squared error (MSE) of the estimators with respect to different values of α.

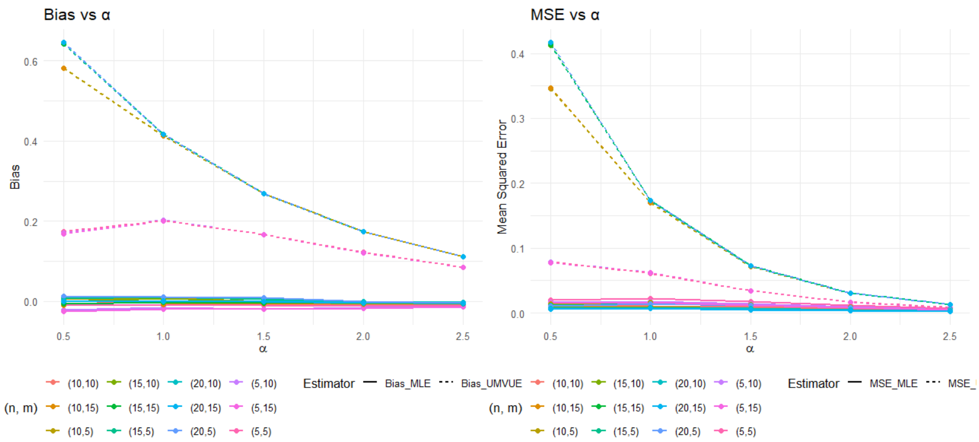

Figure 1: Plot of Bias and MSE for Reliability Estimators

4 Estimation of Confidence Interval

4.1 Exact Confidence Interval

Let $X_1, X_2, \dots, X_n$ be a random sample from the gamma distribution with unknown shape parameter α and known rate β, and let the stress variable $Y \sim$ Exponential(λ), with known λ. The stress-strength reliability is given by,

$$R = P(Y < X) = 1 - \left(\frac{\beta}{\beta+\lambda}\right)^{\alpha} = 1 - \nu^{\alpha},$$

where $\nu = \beta/(\beta+\lambda)$.

Since $T = \sum_{i=1}^{n} X_i \sim$ Gamma($n\alpha, \beta$), it follows that the transformation $Z = 2\beta T$ follows a chi-square distribution with $2n\alpha$ degrees of freedom, i.e., $Z \sim \chi^2_{2n\alpha}$. This pivotal quantity is used to construct an exact confidence interval for α, which is then transformed into an exact confidence interval for R.

Let $\hat{\alpha}$ be an estimate of α. Then the $100(1-\gamma)\%$ exact confidence interval for R is given by,

$$\left(1 - \nu^{\chi^2_{2n\hat{\alpha},1-\gamma/2}/(2n)}, \quad 1 - \nu^{\chi^2_{2n\hat{\alpha},\gamma/2}/(2n)}\right) \tag{4.1}$$

where, $\chi^2_{v,p}$ denotes the p-th percentile of the chi-square distribution with v degrees of freedom.

4.2 Asymptotic Confidence Interval

In this section, we derive the asymptotic distribution of the estimator of R and construct the corresponding large-sample confidence interval. Let $X_1, X_2, \dots, X_n$ be a random sample from Gamma(α, β), where β is known and assume $Y \sim$ Exponential(λ) with known λ. The reliability function is given by,

$$R = 1 - \left(\frac{\beta}{\beta+\lambda}\right)^{\alpha} = 1 - \nu^{\alpha}$$

Let $\hat{\alpha}$ be the maximum likelihood estimator of α. It is well known that under regularity conditions, $\hat{\alpha}$ is asymptotically normal with mean α and variance $1/(n\psi'(\alpha))$, where $\psi'(\alpha)$ is the trigamma function. Using the delta method, the asymptotic variance of the estimator $\hat{R} = 1 - \nu^{\hat{\alpha}}$ is given by,

$$\text{Var}(\hat{R}) = (\nu^{\alpha} \ln \nu)^2 \frac{1}{n\psi'(\alpha)} \tag{4.2}$$

Replacing α by $\hat{\alpha}$, the approximate $100(1-\gamma)\%$ confidence interval for R becomes

$$\hat{R} \pm Z_{1-\gamma/2} \cdot \nu^{\hat{\alpha}} |\ln \nu| \sqrt{\frac{1}{n\psi'(\hat{\alpha})}} \tag{4.3}$$

where, $Z_{1-\gamma/2}$ is the upper $(1-\gamma/2)$-th quantile of the standard normal distribution.

4.3 Simulation Study

To evaluate the accuracy of the proposed interval estimators under the EG stress-strength model, we computed the average lengths and corresponding coverage probabilities of the confidence intervals for various parameter configurations. α was varied over the values 0.5,1.0,1.5,2.0 and 2.5, while β and $\lambda$was fixed at 2.5 and 1.5 respectively throughout the study. The strength sample size was chosen as $n = 15, 20, 25, 30$ and the stress sample size as $m = 20, 25, 30$. For each configuration, the simulation was repeated 5000 times. **Table 2** presents the simulation results, comparing the performance of exact and asymptotic confidence intervals in terms of average interval length and coverage probability.

Table 2:Average confidence interval length and coverage probability of R using Asymptotic and Exact methods

(n,m)	α				
	0.5	1.0	1.5	2.0	2.5
(5,5)	0.51552 (0.67040)	0.56313 (0.84880)	0.58085 (0.70360)	0.59048 (0.44920)	0.59546 (0.19580)
	0.50240 (0.74580)	0.53788 (0.91980)	0.55034 (0.71520)	0.55693 (0.29640)	0.56030 (0.03640)
(5,10)	0.46452 (0.68160)	0.49868 (0.84520)	0.51059 (0.60060)	0.51748 (0.24820)	0.52103 (0.04260)
	0.45191 (0.76960)	0.48002 (0.87000)	0.48917 (0.49200)	0.49454 (0.07500)	0.49707 (0.00080)
(5,15)	0.44366 (0.68000)	0.47463 (0.84220)	0.48357 (0.55660)	0.48975 (0.17180)	0.49255 (0.01400)
	0.43229 (0.77900)	0.45831 (0.83860)	0.46563 (0.40980)	0.47054 (0.02940)	0.47259 (0.00000)
(10,5)	0.46420 (0.64580)	0.49834 (0.86440)	0.51121 (0.64200)	0.51726 (0.27140)	0.52107 (0.05900)
	0.45191 (0.64940)	0.47971 (0.93640)	0.48974 (0.70740)	0.49442 (0.20940)	0.49712 (0.01400)
(10,10)	0.39899 (0.64940)	0.41725 (0.85620)	0.42417 (0.44680)	0.42761 (0.06860)	0.42950 (0.00100)
	0.39027 (0.67300)	0.40567 (0.89220)	0.41136 (0.39320)	0.41417 (0.02040)	0.41571 (0.00000)
(10,15)	0.36980 (0.63860)	0.38435 (0.84180)	0.38908 (0.33860)	0.39199 (0.01540)	0.39359 (0.00020)
	0.36255 (0.67060)	0.37514 (0.85220)	0.37917 (0.25020)	0.38162 (0.00220)	0.38301 (0.00000)
(15,5)	0.44643 (0.63900)	0.47317 (0.87320)	0.48368 (0.60980)	0.48942 (0.21260)	0.49277 (0.03480)
	0.43472 (0.61220)	0.45721 (0.94180)	0.46549 (0.70660)	0.47021 (0.18740)	0.47286 (0.01020)
(15,10)	0.36967 (0.60520)	0.38425 (0.85600)	0.38932 (0.36260)	0.39200 (0.02420)	0.39360 (0.00020)
	0.36239 (0.59660)	0.37506 (0.89660)	0.37939 (0.34080)	0.38164 (0.00820)	0.38299 (0.00000)
(15,15)	0.33578 (0.58800)	0.34600 (0.82340)	0.35001 (0.23560)	0.35180 (0.00480)	0.35302 (0.00020)
	0.33007 (0.59320)	0.33910 (0.84580)	0.34261 (0.18300)	0.34416 (0.00160)	0.34522 (0.00000)
(20,5)	0.43520 (0.64460)	0.46049 (0.87460)	0.46953 (0.58540)	0.47536 (0.17660)	0.47839 (0.01820)
	0.42431 (0.60500)	0.44564 (0.94800)	0.45313 (0.70620)	0.45800 (0.16400)	0.46052 (0.00580)
(20,10)	0.35330 (0.59620)	0.36555 (0.84120)	0.37036 (0.31680)	0.37257 (0.01520)	0.37415 (0.00000)
	0.34701 (0.57240)	0.35778 (0.89540)	0.36189 (0.31480)	0.36388 (0.00720)	0.36522 (0.00000)
(20,15)	0.31583 (0.55920)	0.32525 (0.82840)	0.32845 (0.17180)	0.32979 (0.00080)	0.33076 (0.00000)
	0.31105 (0.54720)	0.31947 (0.86160)	0.32232 (0.14420)	0.32350 (0.00020)	0.32436 (0.00000)

The average confidence length and coverage probability are reported within brackets for confidence intervals using asymptotic in first row, using exact in the second row.

Now, we present graphical representation of average confidence interval lengths and coverage probabilities for the reliability estimators across varying values of α.

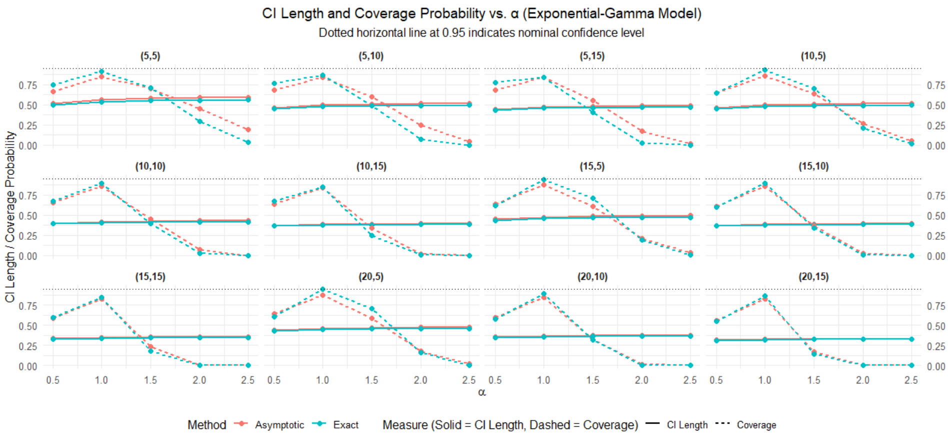

Figure 2: Plot of Confidence Interval Length and Coverage Probability for Reliability Estimators

5 Real Data Application

In this section, we analyze two independent datasets. The first, taken from Folks and Chhikara (1978), records runoff amounts at Jug Bridge, Maryland.

Data set I: 0.17, 0.23, 0.33, 0.39, 0.39, 0.40, 0.45, 0.52, 0.56, 0.59, 0.64, 0.66, 0.70, 0.76, 0.77, 0.78, 0.95, 0.97, 1.02, 1.12, 1.19, 1.24, 1.59, 1.74, 2.92.

The second dataset gives active repair times (hours) for an airborne communication transceiver. It was originally reported by Chhikara and Folks (1989) and later compiled by Balakrishnan et al. (2009).

Data set II: 0.2, 0.3, 0.5, 0.5, 0.5, 0.5, 0.6, 0.6, 0.7, 0.7, 0.7, 0.8, 0.8, 1.0, 1.0, 1.0, 1.0, 1.1, 1.3, 1.5, 1.5, 1.5, 1.5, 2.0, 2.0, 2.2, 2.5, 2.7, 3.0, 3.0, 3.3, 3.3, 4.0, 4.0, 4.5, 4.7, 5.0, 5.4, 5.4, 7.0, 7.5, 8.8, 9.0, 10.3, 22.0, 24.5.

Dataset I (stress) with 25 observations modeled by $Exp(\lambda)$, and Dataset II (strength) with 46 observations modeled by $Gamma(\alpha, \beta)$.
Parameters were estimated by maximum likelihood from the full joint log-likelihood. The fitted values are $\hat{\lambda} = 0.27728, \hat{\alpha} = 2.69623$ and $\hat{\beta} = 3.19761$.
Model adequacy was examined with Kolmogorov–Smirnov (K-S) test computed against the fitted CDFs. For Dataset I, the K-S distance is 0.15974 with a p-value of 0.19106. For Dataset II, the K-S distance is 0.10835 with a p-value of 0.93087. At the 5% level neither test rejects, so the exponential and gamma forms are acceptable for these datasets.

Using the fitted parameters, $\hat{R} = 0.20086$. This indicates that there is 20.086% probability that strength exceeds stress under the observed conditions. Two interval estimates were obtained- the asymptotic 95% CI based on the delta-method variance is $(0.16193, 0.23978)$ and the exact CI is $(0.15962, 0.24483)$. The two intervals are close; although the exact CI is slightly wider, as expected.
For each dataset we plot the empirical and fitted survival functions which is given in **Figure 3**. In Dataset I, the exponential curve follows the empirical steps across the support with no systematic departures. In Dataset II, the fitted gamma survival tracks the empirical curve well, including the mid-range where most observations lie. These visuals are consistent with the K-S outcomes.

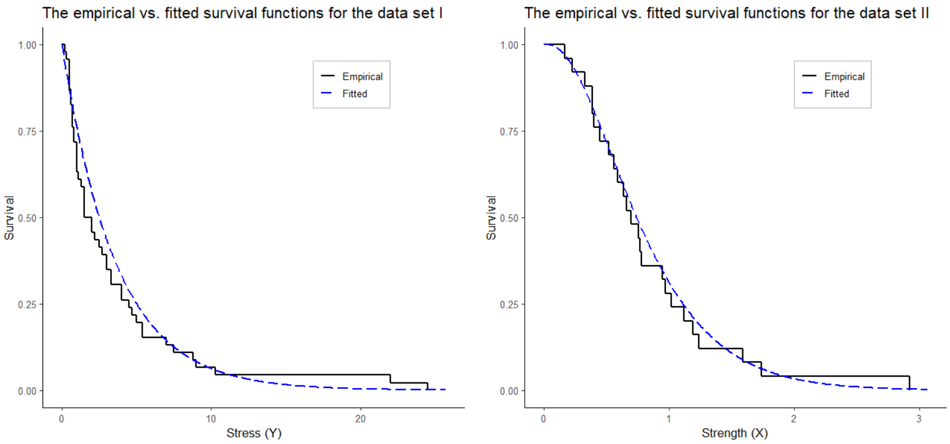

Figure 3: Empirical vs. fitted survival functions for the two datasets

6 Conclusion

The present work has examined the EG stress-strength model in detail, focusing on both theoretical properties and inferential aspects. Reliability expressions were obtained in closed form, and the framework was further generalized to incorporate standby redundancy. The redundancy results clearly show that system performance improves with additional standby units, though the incremental gain becomes smaller as the level of redundancy increases.

For parameter estimation, both MLE and UMVUE methods were derived and studied. Simulation results demonstrated that the UMVUE performs better for small sample sizes, whereas the MLE is preferable when larger samples are available due to its asymptotic efficiency. Two approaches to confidence interval construction were also considered- exact intervals provided stable coverage, while asymptotic intervals were more efficient but less reliable in smaller samples.

Application to real data illustrated the practical suitability of the EG model, confirming that it can capture the reliability behavior of real systems under stress-strength configurations. Taken together, the findings underline the usefulness of this model as a flexible and tractable tool in reliability analysis. Potential extensions include Bayesian methods, more general censoring schemes, and applications to dependent stress-strength structures.

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