# Detecting Mean Shifts in a Class of Time Series CHARN models with Application to Financial Data.

***ORIGINAL ARTICLE***


## Abstract

In this paper, we propose a fully automated method for detecting changes in the mean of piecewise Conditional Heteroskedastic Autoregressive Nonlinear (CHARN) models. Detecting weak changes, those of small magnitude, is crucial in financial and economic applications, where they may signal important structural breaks. Our approach combines an adaptive model selection algorithm with a robust break detection procedure based on local power estimation. By dynamically selecting the most appropriate model for each stationary segment, the method reduces false alarms and improves sensitivity to subtle transitions. Applied to financial datasets such as the S&P 500 and FTSE 100 indices, the algorithm operates automatically and not only reproduces known breakpoints documented in the literature, but also uncovers previously undetected structural changes. These additional findings correspond to meaningful real-world events, highlighting the effectiveness and reliability of the proposed framework for analyzing financial time series and its potential value for financial stability, trading strategies, and risk management.



*Keywords: CHARN model; changepoint; Algorithm; S&P 500; FTSE 100*

2020 Mathematics Subject Classification: 62M10; 62G10; 62F05; 62P20; 91B84.


## 1 Introduction

Detecting changepoints in time series is a crucial problem in many disciplines, particularly in economics and finance, where sudden shifts in market behavior, volatility, or underlying structural regimes can have significant impacts. A changepoint refers to a moment when the statistical properties of a series such as its mean, variance, or dependence structure experience a fundamental alteration. Understanding these shifts is essential not only for accurate modeling and forecasting but also for identifying the underlying causes driving such changes, whether they stem from policy interventions, macroeconomic shocks, or structural market transformations.

The study of structural breaks was initiated by Page (1954) in the context of quality control and has since evolved into a major field of research spanning economics Perron et al. (2006), finance Andreou and Ghysels (2009), climatology Beaulieu et al. (2012); Reeves et al. (2007), and engineering Stoumbos et al. (2000). Techniques such as the CUSUM test Page (1954), and its least-squares adaptation $\text{CUSUM}^{\text{ols}}$ by Brown et al. (1975), have laid the foundation for change detection in various time series settings. Subsequent advancements, including p-value corrections Zeileis (2001, 2004) and robust methods for dependent data Aue and Horváth (2013), have further enhanced detection capabilities.

In economic and financial data, detecting changepoints is particularly challenging due to features such as serial dependence, heteroskedasticity, and short-lived or weak changes. Offline methods optimizing information criteria Horváth (1993); Yao (1987), efficient algorithms like PELT Killick et al. (2012), and segmentation approaches Fryzlewicz (2014); Vostrikova (1981) have been proposed to address these issues. Moreover, Bayesian methods Hadj-Amar et al. (2020) and likelihood ratio scanning Yau and Zhao (2016) provide flexible frameworks for identifying multiple changes in complex financial environments.

More recent contributions have focused on the detection of subtle or localized changes in financial time series, addressing the significant challenges posed by nonstationarity and weak signals Diop and Kengne (2023); Hallgren et al. (2022); Truong et al. (2020). More recent surveys and methodological advances also emphasize robustness and high-dimensional contexts. For instance, Truong et al. (2020) provide a selective review of modern offline changepoint detection techniques, while Fearnhead and Rigaill (2019) develop approaches robust to outliers, a problem often encountered in financial data. In parallel to these statistical advances, machine-learning and deep-learning approaches have also been developed for changepoint detection. For example, Li et al. (2024) propose a deep-learning framework for automatic detection that performs well under dependence and heavy-tailed noise, while Gupta et al. (2022) develop a neural network approach for real-time detection. These contributions highlight the growing role of neural and hybrid models in finance and econometrics and complement the classical statistical literature.

In particular, detecting weak structural changes, such as small shifts in the mean under conditional heteroskedasticity, has attracted growing attention Ltaifa (2021); Ngatchou-Wandji and Ltaifa (2025); Salman et al. (2024b). In financial contexts, it is crucial not only to determine *when* a change occurs, but also to understand *why*, whether it results from monetary policy interventions, geopolitical events, or endogenous market dynamics. Such understanding is essential for effective risk management and informed decision-making.

Despite these advances, the problem of detecting weak changes where shifts are of small magnitude and potentially transient remains relatively under-explored. Ltaifa (2021) and Ngatchou-Wandji and Ltaifa (2025) specifically addressed this issue within the framework of Conditional Heteroskedastic Autoregressive Nonlinear (CHARN) models, with further generalizations proposed by Salman et al. (2024b). When changes are brief and atypical, distinguishing between false alarms and genuine structural breaks becomes particularly challenging. Developing robust methodologies capable of reliably detecting weak and transient changes is therefore crucial for improving the analysis of piecewise stationary financial data.
To address this issue, Salman et al. (2024a) proposed a new automatic algorithm for the detection of weak changes in the mean, incorporating operations designed to distinguish between *true changepoints* and false alarms. A *true changepoint* is defined as a structural shift that results in a piecewise stationary process, whereas a *false alarm* refers to a transient change that affects only a small number of observations without altering the overall stationarity of the series.

In this paper, we first revisit the method and theoretical results developed in Salman et al. (2024b), along with the algorithm proposed in Salman et al. (2024a). Building on this foundation, we propose a complementary algorithm designed to automatically select the appropriate time series model for the real data under investigation. The combination of both algorithms forms a unified framework capable of handling a wide variety of data across different fields. Furthermore, we demonstrate the enhanced performance of the complete method through applications to real-world financial data, with comparisons drawn against results from other studies in the literature.

The structure of the paper is organized as follows. In Section 2, we recall the essential theoretical results from Salman et al. (2024b) and review the algorithm introduced in Salman et al. (2024a). In Section 3, we present our complementary algorithm and highlight its significance. Section 4 illustrates the application of the full method to a real financial dataset. Finally, Section 5 concludes the paper.

# 2 Approach and Algorithm

In this section, we first recall the general principle of changepoint detection based on likelihood ratio tests. Such tests compare parameter estimates across different segments of the series in order to determine whether a structural change has occurred. This framework has been widely applied to changes in mean, variance, and dependence structures, and it serves as the theoretical foundation of our study.

Building on this idea, we provide a concise overview of the method developed in Salman et al. (2024b), which generalizes the approaches introduced in Ngatchou-Wandji and Ltaifa (2025). These methods aim to detect weak changes in the mean by leveraging the theoretical power properties of a likelihood ratio test. In addition, we review the algorithm proposed in Salman et al. (2024a) and detail its underlying mechanisms.

The statistical model used in Salman et al. (2024b) belongs to the class of Conditional Heteroskedastic Autoregressive Nonlinear (CHARN) models (see, e.g., Härdle et al. (1998)). More specifically, let $d, p, k, n \in \mathbb{N}$, with $k \ll n$. Assume that the observations $X_1, \ldots, X_n$ are generated from the following piecewise stationary CHARN model

$$X_t = T(\rho_{\mathbf{0}} + \gamma \odot \omega(t); \mathbf{X}_{t-1}) + V(\mathbf{X}_{t-1})\varepsilon_t, \quad t \in \mathbb{Z}, \tag{2.1}$$

with

$$X_t = Y_{t,j} = T(\rho_{\mathbf{0}} + \boldsymbol{\gamma}_j \omega_j(t); \mathbf{X}_{t-1,j}) + V(\mathbf{X}_{t-1,j})\varepsilon_t, \quad \begin{array}{l} \tau_{j-1} \leq t < \tau_j, \\ j = 1, \ldots, k+1, \end{array} \tag{2.2}$$

where for $j = 1, \ldots, k$, $(Y_{t,j})_{t\in\mathbb{Z}}$ is a stationary and ergodic process; $\boldsymbol{\rho}_0 \in \mathbb{R}^p$, $T(\boldsymbol{\rho}_0, .)$ and $V(.)$ are real-valued functions with $\inf_{x\in\mathbb{R}^d} V(x) > 0$; the $\tau_j$, $j = 0, \ldots, k+1$, are potential instants of changes with $\tau_0 = 1$ and $\tau_{k+1} = n+1$; for $j = 1, \ldots, k$, $\mathbf{X}_{t,j} = (Y_{t,j}, \ldots, Y_{t-d+1,j})^\top$, $\mathbf{X}_{\tau_{j-1}+\ell} = \mathbf{X}_{\tau_{j-1}+\ell,j}$, $\ell = 0, \ldots, d-1$ and for $t \in [\tau_{j-1} + d - 1, \tau_j)$, $\mathbf{X}_t = (X_t, \ldots, X_{t-d+1})^\top$; for $j, \ell = 1, \ldots, k$, $j \neq \ell$, the process $(Y_{t,j})_{t\in\mathbb{Z}}$ and $(Y_{t,\ell})_{t\in\mathbb{Z}}$ are mutually independent (Yau and Zhao (2016) noted that this assumption can be extended to some weak dependence assumption); $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a standard white noise with density $f$. $\boldsymbol{\gamma} = \left(\boldsymbol{\gamma}_1^\top, \ldots, \boldsymbol{\gamma}_{k+1}^\top\right)^\top$, $\boldsymbol{\gamma}_j \in \mathbb{R}^p$, $j = 1, \ldots, k+1$; $\omega(t) = (\mathbb{1}_{[\tau_0,\tau_1)}(t), \mathbb{1}_{[\tau_1,\tau_2)}(t), ..., \mathbb{1}_{[\tau_{k-1},\tau_k)}(t), \mathbb{1}_{[\tau_k,\tau_{k+1})}(t))^\top = (\omega_1(t), \ldots, \omega_{k+1}(t)) \in \{0,1\}^{k+1}$; for $\boldsymbol{\gamma} = (\boldsymbol{\gamma}_1^\top, \ldots, \boldsymbol{\gamma}_{k+1}^\top)^\top$ and $\omega(t) = (\omega_1(t), \ldots, \omega_{k+1}(t))^\top$, $\boldsymbol{\gamma} \odot \omega(t)$ stands for $\boldsymbol{\gamma} \odot \omega(t) = \boldsymbol{\gamma}_1\omega_1(t) + \cdots + \boldsymbol{\gamma}_{k+1}\omega_{k+1}(t) \in \mathbb{R}^p$, and $\boldsymbol{\gamma}_i\omega_i = (\gamma_{i,1}\omega_i, \ldots, \gamma_{i,p}\omega_i) \in \mathbb{R}^p$.

This class of models is broad and includes various structures such as AR($p$), ARCH($p$), EXPAR($p$), and GEXPAR($p$) models. Their statistical and probabilistic properties have been extensively studied in the literature (see, for example, Chen et al. (2018) for an analysis of the ergodicity of GEXPAR models).

For $\boldsymbol{\gamma}_0 \in \mathbb{R}^{p(k+1)}$ and $\boldsymbol{\beta} \in \mathbb{R}^{p(k+1)}$ depending on the $\tau_j$'s, Salman et al. (2024b) construct a likelihood ratio test for testing

$$H_0 : \boldsymbol{\gamma} = \boldsymbol{\gamma}_0 \quad \text{against} \quad H_{\boldsymbol{\beta}}^{(n)} : \boldsymbol{\gamma} = \boldsymbol{\gamma}_n = \boldsymbol{\gamma}_0 + \frac{\boldsymbol{\beta}}{\sqrt{n}}. \tag{2.3}$$

The validity of our procedure relies on the standard assumptions of the CHARN framework, namely piecewise stationarity within each regime, ergodicity of the component processes, and the existence of finite second moments. These assumptions are consistent with prior studies (e.g., Härdle et al. (1998); Ngatchou-Wandji and Ltaifa (2025); Salman et al. (2024b)) and ensure that the likelihood-ratio test retains its asymptotic optimality.

Note also that the norm of $\boldsymbol{\beta}$ is small relative to $n$, and thus the two hypotheses under consideration become increasingly close as the sample size $n$ grows.

First, the authors establish that the constructed test satisfies the locally asymptotically normal (LAN) property, and that the hypotheses are contiguous in the sense of Le Cam (see Le Cam (1986) and Droesbeke and Fine (1996)). These properties enable the study of the theoretical power of the proposed test and lead to an explicit expression for it. Specifically, under certain technical assumptions, they show that the constructed likelihood ratio test is asymptotically optimal and that its asymptotic power is given by

$$\mathcal{P}_{k,\tau^k} = 1 - \Phi(z_\alpha - \vartheta(\boldsymbol{\rho}_0, \boldsymbol{\gamma}_0, \boldsymbol{\beta})) \tag{2.4}$$

where $\boldsymbol{\rho}_0$ represents the true nuisance parameter, $\alpha \in (0,1)$ denotes the level of significance, $z_\alpha$ is the $(1-\alpha)$-quantile of the standard Gaussian distribution with cumulative distribution function $\Phi$, $\vartheta$ is a real-valued function defined on $\mathbb{R}^{p(k+1)\times p(k+1)}$, whose explicit form is provided in Salman et al. (2024b).

In practice, the model parameters are unknown and must be estimated. Salman et al. (2024a) summarized and clearly explained the methodology for estimating the power of the test under unknown parameters. It is worth noting that parameter estimation has been extensively studied in the literature; for instance, Chen et al. (2018) discusses the estimation of both linear and nonlinear components in GExpAR models, a particular case of the CHARN model considered in Salman et al. (2024b), while Brockwell et al. (1990) addresses parameter estimation in linear models such as ARMA. In Salman et al. (2024b), the decision procedure for the testing problem is based on the estimated power $\widehat{\mathcal{P}}_{k,\tau^k}$, obtained by substituting the true parameters with their estimators in the expression of $\mathcal{P}_{k,\tau^k}$.

To describe the estimation procedure, let $1 \le j \le k+1$ and $1 \le h \le p$. Denote by $\widehat{\rho}_{j,h}$ a consistent estimator (e.g., the maximum likelihood estimator) of $\rho_{0,h} + \beta_{j,h}/\sqrt{n}$, based on the observations within the interval $[\tau_{j-1}, \tau_j)$. The estimator of $\beta_{j,h}$ is then defined as $\widehat{\beta}_{j,h} = \sqrt{n}(\widehat{\rho}_{j,h} - \widehat{\rho}_{0,h})$, where $\widehat{\rho}_{0,h}$ denotes the estimator of the stationary parameter $\rho_{0,h}$ based on the first segment of observations $[1, \tau_1)$. By replacing the true parameters with their estimates, it is shown that the constructed test retains its asymptotic optimality, and the corresponding estimated power is explicitly expressed as $\widehat{\mathcal{P}}_{k,\tau^k}$.

To determine whether a given observation $X_t$ corresponds to a changepoint, we rely on the estimated local power of the test, denoted by $\mathcal{P}_{k,t}$, as previously described. Estimating this quantity requires computing all its components, which motivates the need for a dedicated algorithm. One of the most challenging components to estimate is the vector $\boldsymbol{\beta}$, which, by construction, reflects the contrast between two sets of estimated parameters: one based on a segment of stationary observations, and the other on a segment that includes the candidate changepoint. This comparison enables the local power $\mathcal{P}_{k,t}$ to serve as a statistical decision criterion for identifying potential structural changes at time $t$.

To implement this idea, Salman et al. (2024a) assume that the first $m$ observations, $X_1, \ldots, X_m$, are stationary. Based on this initial segment, the underlying statistical model is estimated/assumed to be a particular CHARN model providing a reference for comparison.

To detect the first potential changepoint at a time $t > m$, the algorithm defines two intervals: $I_1 = \{X_1, \ldots, X_{t-1}\}$, and $I_2 = \{X_1, \ldots, X_t\}$, where $X_t$ is the observation under investigation. Parameter estimation is carried out for both intervals, and the contrast between them yields an estimate of $\boldsymbol{\beta}$, from which the local power $\widehat{\mathcal{P}}_{k,t}$ is computed.

If this estimated power exceeds a predefined threshold, the observation $X_t$ is flagged as a critical point. To distinguish between a true changepoint and a false alarm, the algorithm removes

$X_t$ from $I_2$ and sequentially replaces it with future observations $X_{t+\iota}$, for $\iota = 1, \ldots, \ell$, where $1 \leq \ell < m$, repeating the same local power calculation. If the power remains above the threshold across replacements, a changepoint is confirmed. The algorithm then assumes that the following $m$ observations are stationary and restarts the process for subsequent detection.

Conversely, if the local power does not exceed the threshold, $X_t$ is not considered a changepoint. In that case, the algorithm updates both intervals $I_1$ and $I_2$ by appending the next observation, and continues automatically.

# 3 Proposed complementary algorithm

The approach proposed by Salman et al. (2024b) assumes piecewise stationarity. The algorithm of Salman et al. (2024a) fits a CHARN model to the initial $m$ stationary observations and then applies that model across the full dataset. However, once changepoints are detected, the initially chosen model may no longer be appropriate for the newly identified stationary segments. Fitting a single model to all observations is also problematic, as it ignores the nonstationarities that the detection procedure is designed to uncover. These considerations motivate an adaptive procedure that updates model selection dynamically as new stationary regimes are identified.

## 3.1 Algorithm

Consider a set of $n$ observations, denoted as $X_1, X_2, \ldots, X_n$. We use $m$ as the minimum number of observations assumed to be stationary for reliable parameter estimation. Our complementary algorithm is applied before that of Salman et al. (2024a) and proceeds as follows:

1. Consider $h$ independent Uniform random variables $u \in U[1, n-m]$, $1 \leq h < n-m$, where $m$ denotes the minimum number of observations assumed to be stationary as per the algorithm mentioned above:
   (a) Select a subset $S_u$, which contains the observations $X_{u+1}, X_{u+2}, \ldots, X_{u+m}$.
   (b) Fit the CHARN model to the time series subset $S_u$.
   (c) Based on different selection criteria (such as AIC, BIC, etc.), extract the best-fitting time series model that explains the behavior of the observations in $S_u$, denoted as $M_u$.
2. The optimal model to be used for applying the algorithm described in Salman et al. (2024a) is:

$$M = \text{ Most frequent model } M_u \text{ among } h \text{ models}$$

# 4 Application to real-world financial data

In this section, we apply the approach proposed by Salman et al. (2024b) to detect structural changes in real-world financial data, specifically focusing on the Standard & Poor's 500 and the Financial Times Stock Exchange 100 indices. To achieve this, we employ the technique presented in Subsection 3.1, combined with the algorithm proposed by Salman et al. (2024a) and summarized in Section 2.

## 4.1 Standard & Poor's 500 Index (S&P 500)

Here, we apply our new approach to financial data, specifically the daily values of the S&P 500 index. We utilize daily data from January 1992 to December 2000, the same dataset used in Salman et al. (2024b), to determine whether the new algorithm detects subtle changes that may have gone undetected by the previously proposed algorithm.

Since the data exhibits a trend indicating that the S&P 500 index is non-stationary, we work with the transformed series $X_t$, defined as:

$$X_t = \log\left(\frac{P_t}{P_{t-1}}\right),$$

where $P_t$ represents the S&P 500 price index at time $t$. This transformation ensures that any potential breaks in the series $P_t$ are preserved in $X_t$, due to the continuity property of the logarithmic function.

We begin by assuming stationarity over $m$ observations, where we set $m = 25$ based on the approximate number of trading days in a month. Additionally, we consider a vector of 200 different random variables $u$ $(h = 200)$, $u \in U[1, n - m = 2020 - 25]$, representing randomly selected sub-samples from the dataset.

Next, to identify the most suitable particular CHARN model, we implement our algorithm presented in Section 3.1. The dominant repetitive model is summarized in Table 1.

| Model | Frequency |
|---|---|
| ARIMA(0,0,0) | 166 |
| ARIMA(1,0,0) | 5 |
| ARIMA(2,0,0) | 2 |

Table 1: The most frequently selected particular CHARN models for S&P 500 Index.

Finally, using our methodology, we adopt the same model proposed in Salman et al. (2024b), which is defined as follows:

$$X_t = \frac{\beta_j}{\sqrt{n}} + \theta_j \varepsilon_t, \quad t \in [\tau_j, \tau_{j+1}[, \quad \varepsilon_t \sim N(0,1),$$

where $\beta_j$ and $\theta_j$ represent the model parameters, and $\varepsilon_t$ is a standard normally distributed error term. Then, applying the algorithm for changes detection using this model, we detect the changes in this series, as shown in Figure 1.

While the algorithm of Salman et al. (2024b) successfully identified several significant changes in the S&P 500 index, the new algorithm detects additional important breaks that correspond to events not captured by the previous technique.

For instance, the new algorithm identified **1993-02-12**, a changepoint which could be linked to the inauguration of President Bill Clinton and the anticipation surrounding his economic policies aimed at reducing the federal deficit. This period marked the beginning of shifts in fiscal policy, which could explain the market's response. However, this subtle change was not captured by the old algorithm.

Similarly, the date **1994-09-19**, identified by the new algorithm, corresponds to the bond market crisis, also known as the "Bond Market Massacre" of 1994. The old algorithm identified a related point in **1994-03-03**, which we linked to the U.S. lifting of the trade embargo on Vietnam, but failed to capture the subsequent market turbulence later in the year.

Moreover, the detected changepoint at **1997-04-10**, which likely reflects the market's reaction to early concerns about inflation and potential interest rate hikes. This date is not detected by the previous algorithm, which only identified **1997-07-14**, likely driven by the deepening Asian financial crisis.

Another example is the detection of **1996-03-07**, a subtle change linked to the Federal Reserve's decision to hold interest rates steady after prior hikes. This change was missed by the old algorithm, which only captured a later date in July 1996 (**1996-07-11**), possibly linked to strong corporate earnings at the time.

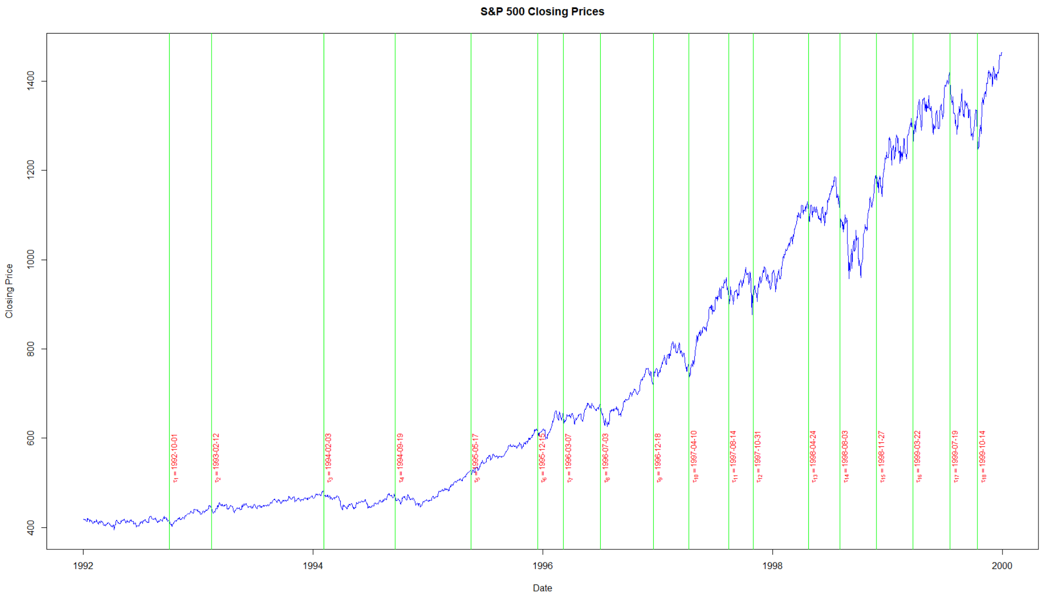


Figure 1: Estimated breaks detected in S&P 500 indices.

Additionally, the detected changepoint on **1998-08-03** coincides with the onset of the Russian financial crisis, whereas the algorithm of Salman et al. (2024b) identified 1998-06-22 and 1998-11-02, which we link to market reactions surrounding the LTCM crisis and the Federal Reserve's intervention.

An important remark must be made regarding the dates 1997-04-10 and 1997-07-19, when the estimated local power crossed the threshold, signaling the presence of a critical point. It subsequently fell below the threshold after 4 and 5 observations, respectively. We can therefore conclude that, in real-world data, the events occurring at these times had a distinct, short-term impact, unlike those that caused more persistent changes at other dates.

These results confirm that our algorithm is consistent with existing methods, as it successfully detects well-established breakpoints also found in previous studies (e.g., Fryzlewicz and Subba Rao (2014)). At the same time, it identifies additional changepoints, demonstrating improved sensitivity to subtle structural shifts while maintaining a low rate of false alarms.

In conclusion, the new combined algorithm not only reproduces several changepoints identified by earlier approaches but also detects additional, earlier, or subtler changes linked to significant market events. This enhancement highlights the improved sensitivity of the new algorithm in capturing weak changes in financial markets that previous algorithms overlooked. The gain in sensitivity is primarily attributed to the dynamic selection of an appropriate model from a large number of subsets and its integration with the automatic detection algorithm proposed in Salman et al. (2024a).

## 4.2 Financial Times Stock Exchange 100 Index (FTSE 100)

In this section, we apply our approach to detect changes in the FTSE 100 index. We use daily data from July 27, 2005 to July 13, 2009 and analyze log-returns as defined in Subsection 4.1. As we explain in the previous section, we begin by assuming that the first $m = 25$ observations are stationary, corresponding approximately to one month of trading days. Using the same settings as in

Subsection 4.1, the dominant model among 300 subsets is summarized in Table 2.

| Model | Frequency |
|---|---|
| ARIMA(0,0,0) | 226 |
| ARIMA(0,1,0) | 11 |
| ARIMA(1,0,0) | 16 |
| ARIMA(3,0,0) | 13 |

Table 2: The most frequently selected particular CHARN models for a subset of FTSE 100 Index.

Out of 300 subsets derived from the original dataset of 1004 observations, 16 subsets suggest an Autoregressive AR(1) model, indicating that the daily price depends on the previous day's price value. Additionally, 13 subsets suggest that the price depends on the price values from the past three days. However, the dominant model identified is the shifted model, which consists of a mean plus an error term. This shifted model will be the one utilized for further analysis. Using these results, we adjust the following particular CHARN model

$$X_t = \frac{\beta_j}{\sqrt{n}} + \theta_j \varepsilon_t, \quad t \in [\tau_j, \tau_{j+1}[, \quad \varepsilon_t \sim N(0,1),$$

where $\beta_j$ and $\theta_j$ represent the model parameters, and $\varepsilon_t$ is a standard normally distributed error term. By applying our change detection algorithm using this model, we successfully identified the structural changes present in the series, as illustrated in Figure 2. Fryzlewicz and Subba Rao (2014)


Figure 2: Estimated breaks detected in FTSE 100 Index.

proposed a technique for detecting changes, referred to as the BASTA-res technique, which the authors applied to the FTSE 100 index over the same period analyzed here. The corresponding dates detected by their method are notably June 5, 2007, August 18, 2008, and December 4, 2008. These dates align with significant financial events, such as the onset of the subprime mortgage crisis and the collapse of Lehman Brothers. While their method successfully identifies these key turning points, our approach provides additional insights by detecting earlier and intermediate structural changes, such as on February 26, 2007, and July 25, 2007, indicating shifts in market dynamics preceding the financial crisis.

For instance, the changepoint on May 11, 2006, detected by our algorithm, likely reflects global market concerns about inflation and interest rates, which triggered a broader sell-off. Similarly, the changepoint on July 25, 2007, reflects early signs of market volatility linked to the subprime crisis, a significant moment that went undetected by Fryzlewicz and Subba Rao (2014). Furthermore, our algorithm detected important changes on January 18, 2008, March 20, 2008, and September 18, 2008, providing a more detailed representation of market turbulence during the 2008 financial crisis.

These additional changepoints underscore the increased sensitivity and granularity of our approach. By identifying earlier and more frequent structural changes, it offers a deeper understanding of the market's evolving behavior, which could be crucial for more proactive risk management. The results demonstrate that our approach not only captures major financial disruptions, as found by Fryzlewicz and Subba Rao (2014), but also identifies critical early warning signals and minor shifts, providing a more comprehensive view of the market's volatility.

# 5 Conclusion

In this study, we combined our proposed algorithmic component with that introduced by Salman et al. (2024a) to apply, in a fully automated manner, the approach developed by Salman et al. (2024b) for selecting and detecting breaks in the mean of piecewise CHARN models. Applied to financial datasets, this integrated framework effectively identifies structural changes driven by major market events. Our method not only reproduces known breakpoints found in previous studies but also uncovers previously undetected shifts that can be linked to specific historical events, underscoring the robustness and enhanced sensitivity of our approach in detecting nuanced transitions in time series data.

Compared with existing approaches such as the BASTA-res procedure (Fryzlewicz and Subba Rao (2014)) or wild binary segmentation (Fryzlewicz (2014)), our method provides a fully automated framework that integrates adaptive model selection with robust changepoint detection. This integration ensures consistency with established results while revealing meaningful structural breaks that prior methods often overlooked. By reducing false alarms and improving sensitivity to weak changes, the proposed approach offers a practical advancement for the analysis of financial time series and complements the existing literature on structural break detection.

A key contribution of this work is the development of an automated procedure for selecting the optimal threshold tailored to the domain under investigation. This innovation addresses a long-standing challenge in changepoint detection and will be a central focus of our future research efforts.

# Notes on contributor(s)

Conceptualization, Y.S.; methodology, Y.S., A.H. and M.B.-H.; software, Y.S.; validation, Y.S., A.H. and M.B.-H.; formal analysis, Y.S. and A.H.; data collection and pretreatment, Y.S. and A.H.; writing—original draft preparation, Y.S.; writing—review and editing, Y.S., A.H. and M.B.-H. All authors have read and agreed to the published version of the manuscript.

## References


Andreou, E. and Ghysels, E. (2009). Structural breaks in financial time series. *Handbook of financial time series*, pages 839–870.

Aue, A. and Horváth, L. (2013). Structural breaks in time series. *Journal of Time Series Analysis*, 34(1):1–16.

Beaulieu, C., Chen, J., and Sarmiento, J. L. (2012). Change-point analysis as a tool to detect abrupt climate variations. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 370(1962):1228–1249.

Brockwell, P., Davis, R., and Salehi, H. (1990). A state-space approach to transfer-function modeling. *Statistical Inference in Stochastic Processes*, 6:233.

Brown, R. L., Durbin, J., and Evans, J. M. (1975). Techniques for testing the constancy of regression relationships over time. *Journal of the Royal Statistical Society: Series B (Methodological)*, 37(2):149–163.

Chen, G.-y., Gan, M., and Chen, G.-l. (2018). Generalized exponential autoregressive models for nonlinear time series: stationarity, estimation and applications. *Information Sciences*, 438:46–57.

Diop, M. L. and Kengne, W. (2023). A general procedure for change-point detection in multivariate time series. *TEST*, 32(1):1–33.

Droesbeke and Fine, J.-J. (1996). *Inférence non paramétrique: Les statistiques de rangs*. Ed. de l'Université de Bruxelles; Ed. Ellipses.

Fearnhead, P. and Rigaill, G. (2019). Changepoint detection in the presence of outliers. *Journal of the American Statistical Association*, 114(525):169–183.

Fryzlewicz, P. (2014). Wild binary segmentation for multiple change-point detection. *The Annals of Statistics*, 42(6):2243–2281.

Fryzlewicz, P. and Subba Rao, S. (2014). Multiple-change-point detection for auto-regressive conditional heteroscedastic processes. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 76(5):903–924.

Gupta, M., Wadhvani, R., and Rasool, A. (2022). Real-time change-point detection: A deep neural network-based adaptive approach for detecting changes in multivariate time series data. *Expert Systems with Applications*, 209:118260.

Hadj-Amar, B., Rand, B. F., Fiecas, M., Lévi, F., and Huckstepp, R. (2020). Bayesian model search for nonstationary periodic time series. *Journal of the American Statistical Association*, 115(531):1320–1335.

Hallgren, K. L., Heard, N. A., and Adams, N. M. (2022). Changepoint detection in non-exchangeable data. *Statistics and Computing*, 32(6):110.

Härdle, W., Tsybakov, A., and Yang, L. (1998). Nonparametric vector autoregression. *Journal of Statistical Planning and Inference*, 68(2):221–245.

Horváth, L. (1993). The maximum likelihood method for testing changes in the parameters of normal observations. *The Annals of statistics*, pages 671–680.

Killick, R., Fearnhead, P., and Eckley, I. A. (2012). Optimal detection of changepoints with a linear computational cost. *Journal of the American Statistical Association*, 107(500):1590–1598.

Le Cam, L. (1986). The central limit theorem around 1935. *Statistical science*, pages 78–91.

Li, J., Fearnhead, P., Fryzlewicz, P., and Wang, T. (2024). Automatic change-point detection in time series via deep learning. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 86(2):273–285.

Ltaifa, M. (2021). *Tests optimaux pour détecter les signaux faibles dans les séries chronologiques*. Theses, Université de Lorraine ; Université de Sousse (Tunisie).

Ngatchou-Wandji, J. and Ltaifa, M. (2025). Detecting weak changes in the mean of a class of nonlinear heteroscedastic models. *Communications in Statistics-Simulation and Computation*, 54(1):160–192.

Page, E. S. (1954). Continuous inspection schemes. *Biometrika*, 41(1/2):100–115.

Perron, P. et al. (2006). Dealing with structural breaks. *Palgrave handbook of econometrics*, 1(2):278–352.

Reeves, J., Chen, J., Wang, X. L., Lund, R., and Lu, Q. Q. (2007). A review and comparison of changepoint detection techniques for climate data. *Journal of applied meteorology and climatology*, 46(6):900–915.

Salman, Y., Hoayek, A., and Batton-Hubert, M. (2024a). New algorithm for detecting weak changes in the mean in a class of charn models with application to welding electrical signals. *Engineering Proceedings*, 68(1):42.

Salman, Y., Ngatchou-Wandji, J., and Khraibani, Z. (2024b). Testing a class of piece-wise charn models with application to change-point study. *Mathematics*, 12(13):2092.

Stoumbos, Z. G., Reynolds Jr, M. R., Ryan, T. P., and Woodall, W. H. (2000). The state of statistical process control as we proceed into the 21st century. *Journal of the American Statistical Association*, 95(451):992–998.

Truong, C., Oudre, L., and Vayatis, N. (2020). Selective review of offline change point detection methods. *Signal Processing*, 167:107299.

Vostrikova, L. Y. (1981). Detecting "disorder" in multidimensional random processes. In *Doklady akademii nauk*, volume 259, pages 270–274. Russian Academy of Sciences.

Yao, Y.-C. (1987). Approximating the distribution of the maximum likelihood estimate of the change-point in a sequence of independent random variables. *The Annals of Statistics*, pages 1321–1328.

Yau, C. Y. and Zhao, Z. (2016). Inference for multiple change points in time series via likelihood ratio scan statistics. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 78(4):895–916.

Zeileis, A. (2001). p values and alternative boundaries for cusum tests. Technical report, Technical Report.

Zeileis, A. (2004). Alternative boundaries for cusum tests. *Statistical Papers*, 45(1):123–131.