NewAnalysis of the second-class current in the radiative scattering process of neutrinos and antineutrinos by atomic nuclei

Abstract

The study of the radiative neutrino(antineutrino)-nucleus scattering process on the lithium-6 nucleus (^{6}Li) allowed us to derive the general expression of the differential cross section for this type of process and to highlight several significant results. In particular, the analysis focused on the influence of the tensor form factor F_T, associated with the second-class currents (SCCs), on two key observables : the charge asymmetry coefficient and the degree of polarization of the emitted photon. The study of the charge asymmetry coefficient $A_{\nu\tilde{\nu}}$ reveals a strong dependence on the presence of the tensor form factor. In the absence of SCCs ($F_T = 0$), the behavior of $A_{\nu\tilde{\nu}}$ remains moderate and stable as a function of the neutrino energy. Conversely, when $F_T \neq 0$, the coefficient increases significantly and exhibits pronounced variations, including sign changes at high energy. The analysis of the relative contribution $\delta_{A_{\nu\tilde{\nu}}}$ confirms this trend, emphasizing an enhanced sensitivity to SCCs, particularly in the high-energy domain ($E_\nu \gtrsim 200$ MeV). The examination of the photon polarization degree P_{S_γ} also shows that it serves as a relevant indicator of the presence of SCCs. The introduction of a nonzero tensor form factor ($F_T = 5 \times 10^{-3}\,\mathrm{MeV}^{-1}$) leads to substantial modifications of the polarization degree. The analysis of the relative contribution $\delta_{P_{S_\gamma}}$ further confirms this strong sensitivity, with variations exceeding 200% at high energy.

Keywords : second-class current ; charge asymmetry coefficient ; degree of polarization ; neutrino scattering ; neutral current, radiative scattering.

1 Introduction

The study of the structure of weak neutral currents is one of the main areas of research in electroweak interaction physics. The Standard Model has successfully interpreted neutrino-nucleon interaction processes, as well as inelastic electron-nucleus scattering, particularly on the deuteron [1–6]. The experimental discovery of the intermediate bosons $W^\pm$ and Z^0 at CERN [7, 8] confirmed this theory. Introduced by Weinberg [9], SCCs are theoretical contributions to the weak current that do not transform like first-class currents under the G-parity symmetry operation. Their direct observation remains difficult in weak decays, due to the small amounts of momentum transferred in these processes [10,11]. On the other hand, electroweak scattering processes, and in particular radiative processes, are a promising avenue for probing the existence of SCCs. Significant experimental studies have been conducted in this direction. Sugimoto et al. [12] observed effects consistent with the existence of a second-class pseudo-tensor term by analyzing the angular distribution of electrons in β decays of polarized mirror nuclei. Based on their data analysis, Morita and Tanihata [13] quantified the pseudo-tensor coupling constant at the nucleon level to a value of $(3.5 \pm 1.3)/2M.f_A(0)$, where M denotes the nucleon mass and $f_A(0)$ the axial form factor. Subsequently, similar results were obtained by Calaprice et al. using a method analogous to that developed in [12], but based on a polarized ^{19}Ne beam [14]. Fatima et al. studied the influence of SCCs on the production of hyperons induced by antineutrinos, as well as on the differential cross sections associated with quasi-elastic scattering processes of neutrinos and antineutrinos [16–21]. Their results suggest that it is possible to explore the effects of SCCs in quasi-elastic interactions, in particular by analyzing the strangeness component of the hadronic current. More recently, a first systematic study [22] of relativistic distributions of the weak neutral axial vector current in a spin 1/2 hadron, including the second-class contribution associated with a pseudotensor form factor, was conducted. The authors demonstrate that SCCs affect the spatial distribution of the axial charge in the Breit reference frame, but do not affect the mean square radii, which limits their impact on global observables such as axial radii or spin radii. Thanks to recent advances in detection technology, it is now possible to explore the neutral scattering of neutrinos on atomic nuclei with radiative emission as a potential channel for detecting SCCs. In this work, we analyze precisely the influence of the form factor F_T associated with the second-class current in the neutral radiative scattering process :

$$\nu(\tilde{\nu}) + (A, Z) \longrightarrow \nu'(\tilde{\nu'}) + (A, Z)^* \longrightarrow (A, Z) + \gamma_{RL} \tag{1}$$

We implement a multipole decomposition of the matrix elements in order to obtain an explicit expression of the differential cross section for the radiative scattering process under consideration. This approach allows us to structure the contributions according to the multipole operators of the neutral hadronic current. Particular attention is paid to the sensitivity of the process to second-class currents, which could manifest themselves in this context.

2 Differential cross section for neutral radiative scattering

In this article, we study the process of neutral radiative scattering of a neutrino on an atomic nucleus. To do this, we use a method based on the multipole decomposition of the hadronic current, which allows us to calculate the expression for the differential cross section of the process. At the lowest order of perturbation theory, the square of the scattering amplitude of the process under consideration is given by :

$$\sum_{\mathcal{M}_i\mathcal{M}_f} |\mathcal{M}_{fi}|^2 = \frac{1}{2J_i+1} \sum_{\mathcal{M}_i\mathcal{M}_f} |\sum_{\mathcal{M}_n} \mathcal{M}_{fn}\mathcal{M}_{ni}|^2 \tag{2}$$

where $\mathcal{M}_{ni}$ and $\mathcal{M}_{fn}$ are respectively the weak nuclear transitions and the emission of the gamma photon matrix elements defined in terms of the lepton current ℓ^z_μ and the hadron current $J^z_\mu(q)$ as well as the electric multipole operators $\hat{T}^e_{J_1\sigma}$ and magnetic multipole operators $\hat{T}^m_{J_1\sigma}$.
The magnetic and electric multipole operators, $\hat{T}^m_{J_1\sigma}$ and $\hat{T}^e_{J_1\sigma}$, are initially defined in a coordinate system $X'Y'Z'$, whose z'-axis is oriented along the photon momentum vector $\vec{k}$. In order to express these operators in a coordinate system where the z-axis is aligned with the transferred

momentum $\vec{q}$, a rotation of the $X'Y'Z'$ frame by the Euler angles $(\varphi_\gamma, \theta_\gamma)$ is performed. This transformation leads to the following relation [23–25] :

$$\hat{T}^{m(e)}_{J_1\sigma}(k) = \sum_{M_1} \hat{T}^{m(e)}_{J_1 M_1} D^{J_1}_{M_1\sigma}(\phi_\gamma, \theta_\gamma, \phi_\gamma) \tag{3}$$

$D^{J_1}_{M_1\sigma}(\phi_\gamma, \theta_\gamma, \phi_\gamma)$ is the Wigner function, and the angles θ_γ and ϕ_γ determine the direction of flight of the emitted photon.
The matrix elements of weak nuclear transitions and gamma photon emission are given by :

$$\mathcal{M}_{ni} = -\frac{G_F}{\sqrt{2}} \ell^z_\mu J^z_\mu \tag{4}$$

$$\mathcal{M}_{fn} = -\frac{2\pi}{\sqrt{\omega\Omega}} \sum_{J_1 \geq 1} (-i)^{J_1} [J_1] D^{J_1}_{M_1\sigma}(\phi_\gamma, \theta_\gamma, \phi_\gamma) \begin{pmatrix} J_f & J_1 & J_n \\ -M_f & M_1 & M_n \end{pmatrix} \left\langle J_f || \sigma \hat{T}^m_{J_1\sigma} - \hat{T}^e_{J_1\sigma} || J_n \right\rangle \tag{5}$$

Here $[J_1] = (2J_1 + 1)^{1/2}$.
The constant G_F is called the Fermi constant of the weak interaction.
The weak lepton ℓ^z_μ and hadron $J^z_\mu(q)$ currents are defined respectively by :

$$\ell^z_\mu = \bar{u}_2 \gamma_\mu (1 + \gamma_5) u_1 \tag{6}$$

$$J^z_\mu(q) = \left\langle J_n M_n | \int d\vec{x} exp(-i\vec{q}_\mu \vec{x}) \hat{\mathcal{J}}^{ni}_\mu(\vec{x}) | J_i M_i \right\rangle \tag{7}$$

$\hat{\mathcal{J}}^{ni}_\mu(\vec{x})$ is the hadronic current density operator. The structure of the hadronic current takes into account its isospin component and is written as a combination of vector and axial-vector currents [24].

$$\hat{\mathcal{J}}^{ni}_\mu = \beta^{(\tau)}_V (J_\mu)_{\tau M_\tau} + \beta^{(\tau)}_A (J^5_\mu)_{\tau M_\tau} \tag{8}$$

The coupling constants $\beta^{(\tau)}_{V,A}$ depend on the Weinberg angle θ_W and are fixed according to the model. For example, in the Weinberg-Salam electroweak model we are considering here, these constants are given by [26] :

$$\beta^{(0)}_V = -2sin^2\theta_W \qquad \beta^{(0)}_A = 0 \tag{9}$$

$$\beta^{(1)}_V = 1 - 2sin^2\theta_W \qquad \beta^{(1)}_A = 1 \tag{10}$$

In the case of processes with weak neutral currents, $M_\tau = 0 (\tau = 0, 1)$, then the hadronic current takes the form :

$$J^Z = \beta^{(0)}_V (J_\mu)_{00} + \beta^{(0)}_A (J^5_\mu)_{00} + \beta^{(1)}_V (J_\mu)_{10} + \beta^{(1)}_A (J^5_\mu)_{10} \tag{11}$$

The expression for the differential cross section is obtained from the multipole decomposition method of the transition matrix elements [26–28] :

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\nu d\Omega_\gamma} = \frac{\Gamma_\gamma^{(n\to f)}}{\Gamma_{total}^{(n\to f)}} \sum_{\tau\tau'} \begin{pmatrix} \tau_n & \tau & \tau_i \\ -M_{\tau_n} & 0 & M_{\tau_i} \end{pmatrix} \begin{pmatrix} \tau_n & \tau' & \tau_i \\ -M_{\tau_n} & 0 & M_{\tau_i} \end{pmatrix} \\
\times \frac{d\sigma_0}{d\Omega_\nu} \{1 + \frac{1}{K_0^0} \sum_{L\geq 2} f_L^{(n\to f+\gamma)} (P_L(cos\theta_\gamma) K_L^0 \\
+ P_L^1(cos\theta_\gamma) cos\Phi_\gamma K_L^1 + P_L^2(cos\theta_\gamma) cos2\Phi_\gamma K_L^2) \\
+ \frac{s_\gamma}{K_0^0} \sum_{L\geq 1} f_L^{(n\to f+\gamma)} (P_L(cos\theta_\gamma) \widetilde{K}_L^0 + P_L^1(cos\theta_\gamma) cos\Phi_\gamma \widetilde{K}_L^1) \\
+ \frac{s_\gamma}{K_0^0} \sum_{L\geq 3} f_L^{(n\to f+\gamma)} (P_L^2(cos\theta_\gamma) 2cos\Phi_\gamma \widetilde{K}_L^2)\}
\end{aligned} \tag{12}$$

where $f_L^{(n\to f+\gamma)}$ represents the multipole dependence, $d\sigma_0/d\Omega_\nu$ is the cross section for unpolarized scattering, and the associated Legendre polynomials P_L^M describe the angular distribution of the emitted photon. The functions $f_L^{(n\to f+\gamma)}$ are calculated from the electric (F_{EJ}) and magnetic (F_{MJ}) multipole amplitudes, combining these amplitudes symmetrically or antisymmetrically depending on the parity of the multipoles. These expressions are given by :

$$f_L^{(n\to f+\gamma)} = \begin{cases} \dfrac{Y_L^T(q_0)}{Y_0^T(q_0)} \text{ pour } L \text{ paire} \\ \\ \dfrac{Y_L^{T'}(q_0)}{Y_0^T(q_0)} \text{ pour } L \text{ impaire} \end{cases} \tag{13}$$

In the case of neutral radiative scattering, which we are dealing with here, the functions K_L^m and $\widetilde{K}_L^m$, which appear in the general expression for the cross section, retain the same formal structure as those used in [23]. They are still written as linear combinations of the lepton functions $f_j (j = 1...10)$, multiplied by the hadron functions Y_i^L and $\bar{Y}_i^L (i = 1, 2......10)$ according to :

$$\begin{aligned}
K_L^0 &= f_1 Y_1^L + f_2 Y_2^L + f_3 Y_3^L + f_4 Y_4^L + f_5 Y_5^L \\
K_L^1 &= f_6 Y_6^L + f_7 Y_7^L + f_8 Y_8^L + f_9 Y_9^L \\
K_L^2 &= f_{10} Y_{10}^L \\
\bar{K}_L^0 &= f_1 \bar{Y}_1^L + f_2 \bar{Y}_2^L + f_3 \bar{Y}_3^L + f_4 \bar{Y}_4^L + f_5 \bar{Y}_5^L \\
\bar{K}_L^1 &= f_6 \bar{Y}_6^L + f_7 \bar{Y}_7^L + f_8 \bar{Y}_8^L + f_9 \bar{Y}_9^L \\
\bar{K}_L^2 &= f_{10} \bar{Y}_{10}^L \\
\bar{K}_1^2 &\equiv 0
\end{aligned} \tag{14}$$

The lepton functions f_j retain the same definition as in [23], but the hadron functions Y_i^L and $\bar{Y}_i^L$ are modified (see appendix) : they depend on the coupling constants of the Weinberg-Salam model through the factors $\beta^{(\tau)}_{V,A}$ and the multipole matrix elements F_{MJ}, F_{EJ}, F_{CJ} and F_{LJ} (F^5_{MJ}, F^5_{EJ}, F^5_{CJ} and F^5_{LJ}). These functions therefore describe the specific contribution of the internal structure of the nucleus in the context of neutral interaction.

3 Study of neutrino(antineutrino)-nucleus radiative transition $\nu(\tilde{\nu}) + {}^6\mathbf{Li} \longrightarrow \nu'(\tilde{\nu'}) + {}^6\mathbf{Li}^* \longrightarrow {}^6\mathbf{Li} + \gamma_{RL}$

As an illustration, consider the following neutrino(antineutrino)-nucleus radiative scattering processes :

$$\nu(\tilde{\nu}) + {}^6\mathrm{Li}(1^+, T=0) \longrightarrow \nu'(\tilde{\nu'}) + {}^6\mathrm{Li}^*(0^+, T=1) \longrightarrow {}^6\mathrm{Li}(1^+, 0) + \gamma_{RL} \tag{15}$$

This emission process corresponds to the excitation of the lithium-6 nucleus from its ground state ($J^\pi_i = 1^+$) to an excited state ($J^\pi_n = 0^+$), followed by a return to the ground state through the emission of a gamma photon (denoted γ_{RL}).
The quantum numbers associated with the orbital angular momentum L and the total angular momenta (J and J') are defined by the following relations : $0 \leq L \leq 2J_n \Rightarrow 0 \leq L \leq 2$; $|J_i - J_n| \leq J \leq J_i + J_n \Rightarrow J = 1$; $\quad |J_i - J_n| \leq J' \leq J_i + J_n \Rightarrow J' = 1$.

The differential cross section associated with this radiative process is obtained from the general expression (12), applied to the specific conditions of the transition described in (15) :

$$\begin{aligned}\frac{d\sigma}{d\Omega_\nu d\Omega_\gamma} = \frac{\Gamma^{(n\to f)}_\gamma}{\Gamma^{(n\to f)}_{total}}\Sigma_0\{K^0_0 + f^{(n\to f+\gamma)}_2(P_2(cos\theta_\gamma)K^0_2 + P^1_2(cos\theta_\gamma)cos\phi_\gamma K^1_2 \\ +P^2_2(cos\theta_\gamma)cos2\phi_\gamma K^2_2) + s_\gamma f^{(n\to f+\gamma)}_1(P_1(cos\theta_\gamma)\widetilde{K}^0_1 + P^1_1(cos\theta_\gamma)cos\phi_\gamma\widetilde{K}^1_1)\}\end{aligned} \tag{16}$$

with $\Sigma_0 = \dfrac{8\pi^3 G_F^2}{3\omega\Omega}$, $\quad f^{(n\to f+\gamma)}_1 = -\dfrac{\sqrt{3}}{\sqrt{2}}$, $\quad f^{(n\to f+\gamma)}_2 = \dfrac{1}{\sqrt{2}}$.
The functions K^m_L and $\widetilde{K}^m_L$ are given by the following relations :

$$\begin{aligned}&K^0_0 = \frac{2}{\sqrt{3}}\{v_1H_1 - v_2H_2 + v_3H_3 + v_4H_4 + v_5H_5\}, \quad K^0_2 = \frac{2}{\sqrt{6}}\{v_1H_1 - v_2H_2 - 2v_3H_3 - 2v_4H_4 - 2v_5H_5\} \\ &K^1_2 = -(2\sqrt{6}/3)\{v_6H_6 - v_7H_7 + v_8H_8 - v_9H_9\}, \quad K^2_2 = (\sqrt{6}/6)v_{10}H_{10}, \quad \widetilde{K}^0_1 = (-2/\sqrt{2})\{v_1H_2 - v_2H_1\} \\ &\widetilde{K}^1_1 = 2\sqrt{2}\{v_6H_7 - v_7H_6 + v_8H_9 - v_9H_8\}\end{aligned} \tag{17}$$

with

$$\begin{aligned}&H_1 = \beta^{(1)}_V\beta^{(1)}_V F^{(1)}_{M1}F^{(1)}_{M1} + \beta^{(1)}_A\beta^{(1)}_A F^{5(1)}_{E1}F^{5(1)}_{E1}, \quad H_2 = \beta^{(1)}_V\beta^{(1)}_A F^{(1)}_{M1}F^{5(1)}_{E1}, \quad H_3 = \beta^{(1)}_A\beta^{(1)}_A F^{5(1)}_{L1}F^{5(1)}_{L1} \\ &H_4 = \beta^{(1)}_A\beta^{(1)}_A F^{5(1)}_{L1}F^{5(1)}_{C1}, \quad H_5 = \beta^{(1)}_A\beta^{(1)}_A F^{5(1)}_{C1}F^{5(1)}_{C1}, \quad H_6 = \beta^{(1)}_A\beta^{(1)}_A F^{5(1)}_{L1}F^{5(1)}_{E1}, \quad H_7 = \beta^{(1)}_A\beta^{(1)}_V F^{5(1)}_{L1}F^{(1)}_{M1} \\ &H_8 = \beta^{(1)}_A\beta^{(1)}_A F^{5(1)}_{C1}F^{5(1)}_{E1}, \quad H_9 = \beta^{(1)}_A\beta^{(1)}_V F^{5(1)}_{C1}F^{(1)}_{M1}, \quad H_{10} = \beta^{(1)}_A\beta^{(1)}_A F^{5(1)}_{E1}F^{5(1)}_{E1} - \beta^{(1)}_V\beta^{(1)}_V F^{(1)}_{M1}F^{(1)}_{M1}\end{aligned} \tag{18}$$

The functions $v_j (j = 1...10)$ are given in the appendix.

4 Charge asymmetry coefficient

The charge asymmetry coefficient is a coefficient that allows the difference between the differential cross sections of neutrino and antineutrino scattering to be evaluated. It is defined by :

$$A_{\nu\tilde{\nu}} = \frac{d\sigma_\nu - d\sigma_{\tilde{\nu}}}{d\sigma_\nu + d\sigma_{\tilde{\nu}}} \tag{19}$$

This coefficient $A_{\nu\tilde{\nu}}$ is sensitive to the presence of interference between vector and axial currents, in particular to contributions from SCCs, via the tensor form factor F_T.

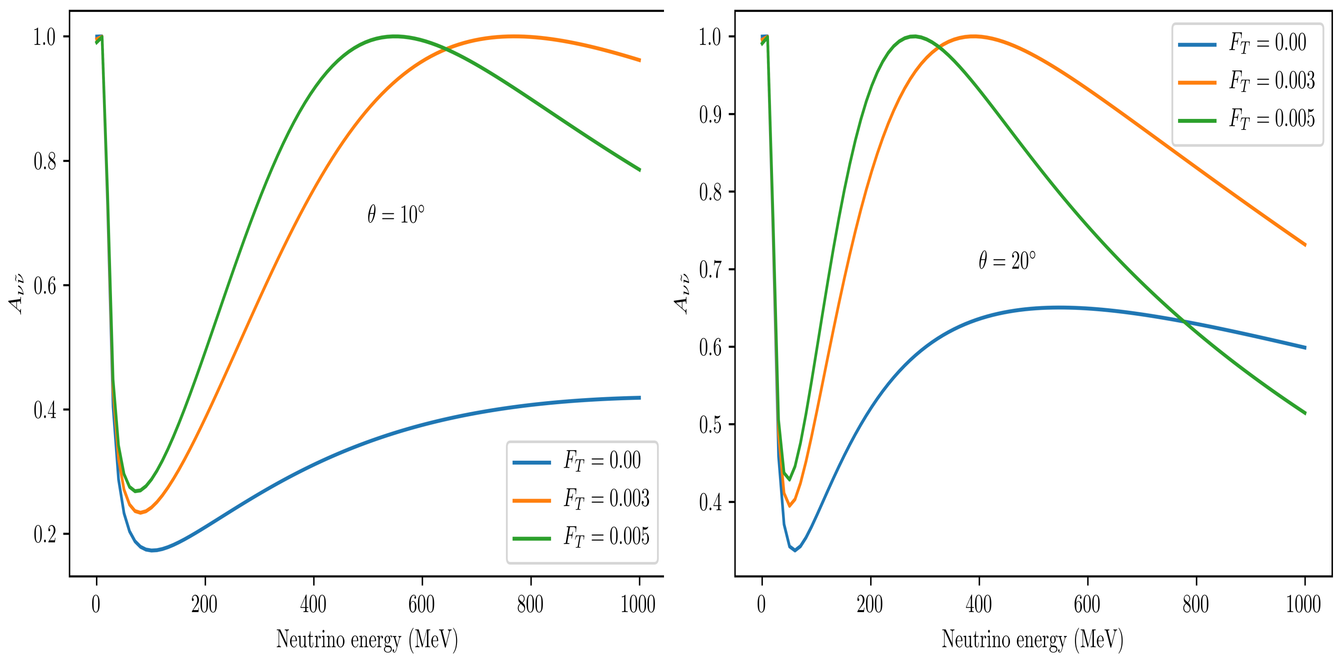

FIGURE 1 – Charge asymmetry coefficient for $\theta = 10°$ and $20°$

Figure 1 shows the evolution of the charge asymmetry coefficient $A_{\nu\tilde{\nu}}$ as a function of the incident neutrino energy for the ^{6}Li nucleus at two scattering angles ($\theta = 10°$ and $\theta = 20°$) and for three values of the tensor form factor $F_T = 0$, $F_T = 0.003MeV^{-1}$, $F_T = 0.005MeV^{-1}$. We observe that the charge asymmetry coefficient $A_{\nu\tilde{\nu}}$ depends strongly on the form factor F_T. For $F_T = 0$, the curves are relatively stable and remain positive over the entire energy range, reaching values around 750% ($\theta = 20°$) and 55% ($\theta = 10°$). When $F_T \neq 0$, the asymmetry evolves negatively after an initially positive region (up to approximately $50 - 100MeV$) and tends toward extreme values close to -100% at high energy. At $\theta = 20°$, the values of $A_{\nu\tilde{\nu}}$ are generally higher in absolute value than at $\theta = 10°$.
These analyses suggest that charge asymmetry is more sensitive to the tensor form factor at larger angles.
These results highlight the sensitivity of $A_{\nu\tilde{\nu}}$ to the form factor F_T.
In order to better quantify the sensitivity of the asymmetry $A_{\nu\tilde{\nu}}$ to SCCs, we introduce a relative contribution defined by the following ratio between $F_T = 0$ and $F_T = 5.10^{-3}MeV^{-1}$.

$$\delta_{A_{\nu\tilde{\nu}}} = \frac{A_{\nu\tilde{\nu}}(F_T = 5.10^{-3}MeV^{-1}) - A_{\nu\tilde{\nu}}(F_T = 0)}{A_{\nu\tilde{\nu}}(F_T = 5.10^{-3}MeV^{-1})} \tag{20}$$

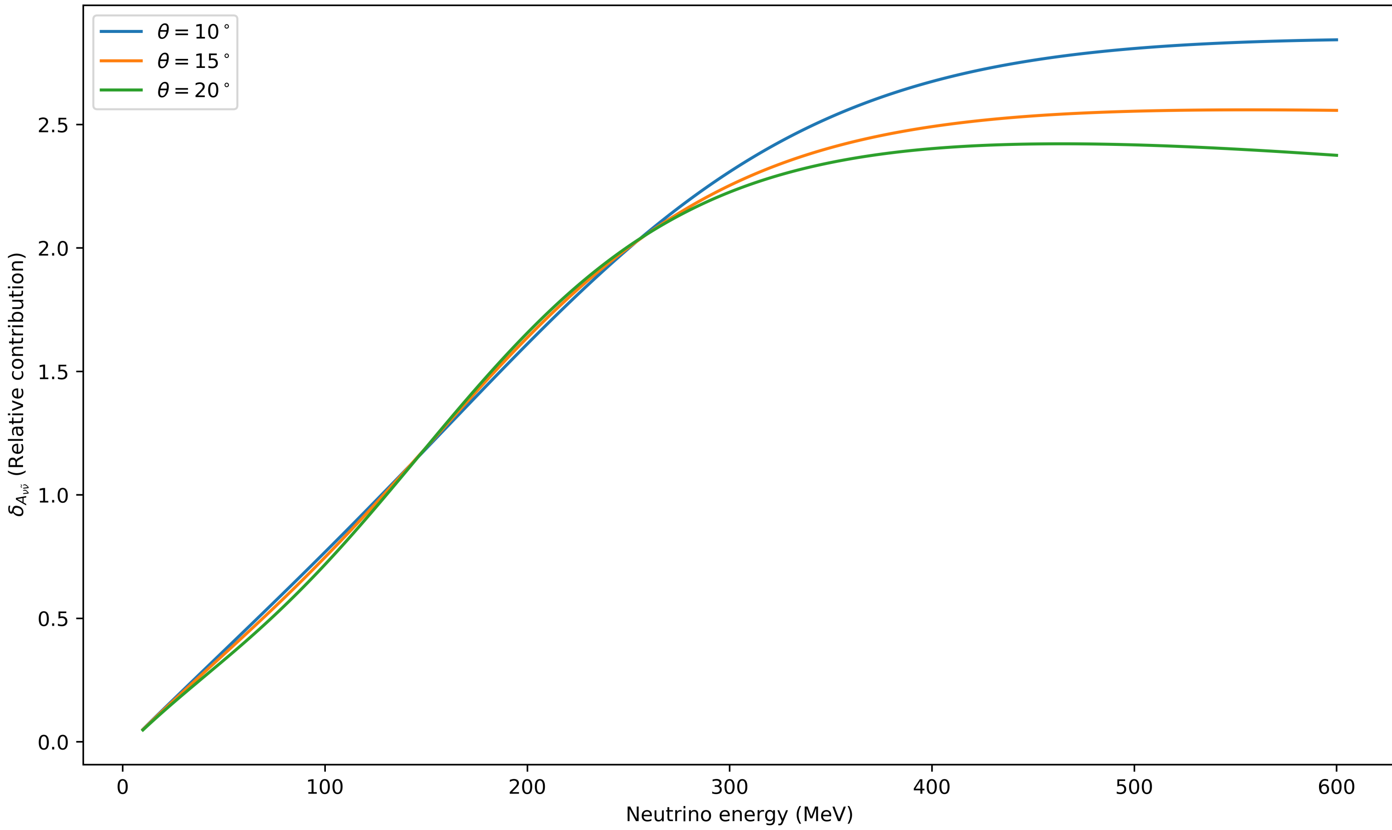

FIGURE 2 – Relative contribution of SCCs to the charge asymmetry coefficient

Figure 2 shows the evolution of $\delta_{A_{\nu\tilde{\nu}}}$ as a function of the incident neutrino energy for the ^{6}Li nucleus and for three values of the scattering angle. We observe that the relative contribution $\delta_{A_{\nu\tilde{\nu}}}$ increases sharply with the energy of the incident neutrino, and the behavior is similar for all three angles up to an energy of approximately $300MeV$. This shows a weak angular dependence in this region. For an energy of $E_\nu = 600MeV$, this contribution reaches a value of 280% for $\theta = 10°$, 260% for $\theta = 15°$ and 245% for $\theta = 20°$.
The relative contribution $\delta_{A_{\nu\tilde{\nu}}}$ shows much stronger sensitivity to high-energy SCCs ($E_\nu \gtrsim$ 200 MeV). These results confirm that studying the relative contribution $\delta_{A_{\nu\tilde{\nu}}}$ is a useful tool for highlighting the influence of SCCs in neutral radiative scattering on light nuclei.

5 Photon Polarization Coefficient

The photon polarization coefficient P_{S_γ} measures the asymmetry between two helicity states of the emitted photon ($S_\gamma = \pm 1$) :

$$P_{S_\gamma} = \frac{d\sigma(S_\gamma = 1) - d\sigma(S_\gamma = -1)}{d\sigma(S_\gamma = 1) + d\sigma(S_\gamma = -1)} \tag{21}$$

This coefficient makes it possible to probe for the existence of SCCs. Figure 3 shows the evolution

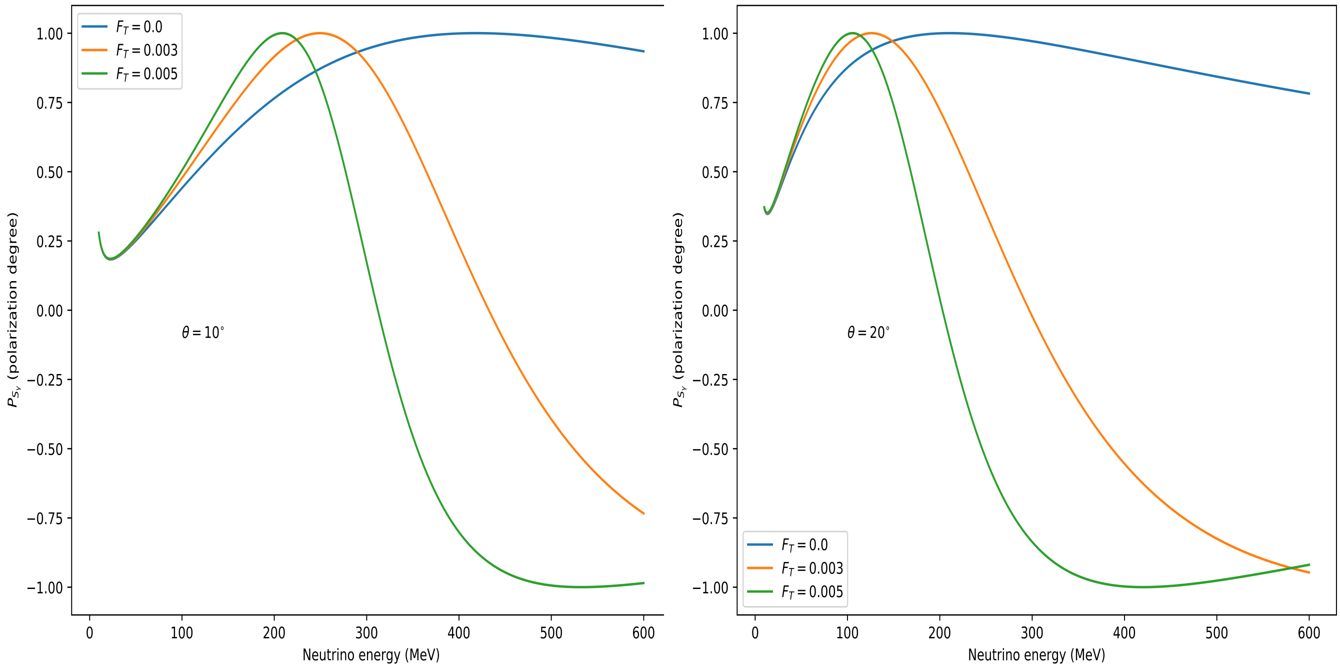

FIGURE 3 – Photon Polarization Coefficient for $F_T = 0$ and $5 \times 10^{-3} MeV^{-1}$

of the polarization coefficient P_{S_γ} for two scattering angles, $\theta = 10^\circ$ and $\theta = 20^\circ$, and for three values of the form factor F_T. Analysis of these curves shows that introducing a value $F_T \neq 0$ causes a significant variation in P_{S_γ}, especially at high energies. For example, at $\theta = 20^\circ$ and $E_\nu \simeq 200$ MeV, the degree of polarization changes from $P_{S_\gamma} \approx 99\%$ (for $F_T = 0$) to $P_{S_\gamma} \approx 12\%$ (for $F_T = 5 \times 10^{-3}$), i.e., a relative decrease of approximately 87%.
At $\theta = 10^\circ$ and $E_\nu \simeq 300$ MeV, the absolute difference between the curves with and without F_T for P_{S_γ} is approximately 74%. This confirms the sensitivity of the polarization degree P_{S_γ} to SCCs. Thus, the coefficient P_{S_γ} appears to be an effective indicator for experimentally testing the existence of SCCs. The optimal conditions for this research are a scattering angle ($\theta \leqslant 20^\circ$) and a neutrino energy $\leqslant 300$ MeV, particularly for light targets such as ^{6}Li.
In order to analyze more precisely the influence of the form factor F_T on the degree of polarization of the emitted photon, we introduce the following relative contribution :

$$\delta_{P_{S_\gamma}} = \frac{P_{S_\gamma}(F_T = 5.10^{-3} MeV^{-1}) - P_{S_\gamma}(F_T = 0)}{P_{S_\gamma}(F_T = 5.10^{-3} MeV^{-1})} \tag{22}$$

This quantity allows us to evaluate the relative contribution of the degree of polarization P_{S_γ} induced by the presence of SCCs. The behavior of this contribution is illustrated in the following figure :

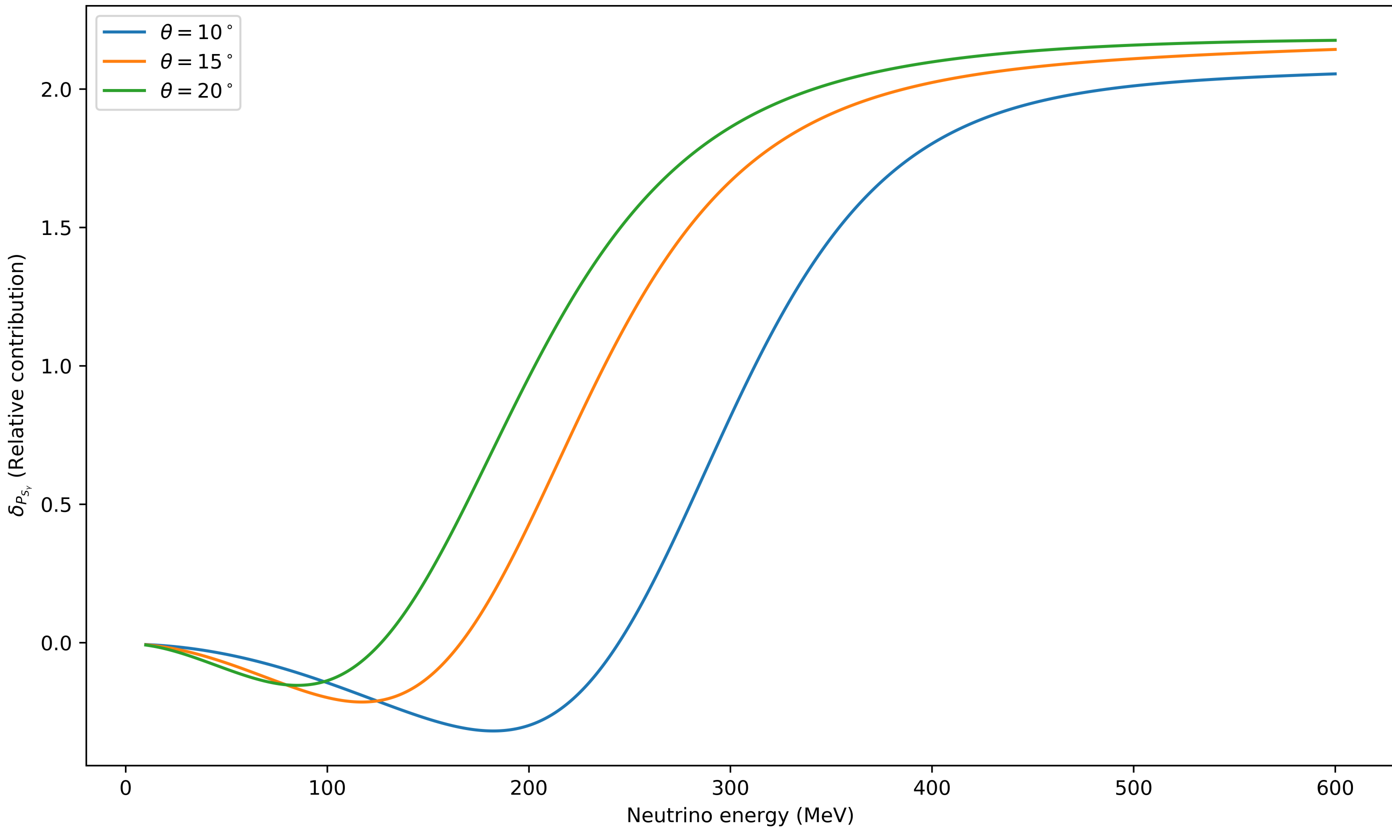

FIGURE 4 – Relative contribution of SCCs to the photon polarization degree

Figure 4 shows the energy dependence of the relative contribution $\delta_{P_{S_\gamma}}$ of the polarization degree of the emitted photon for three values of the scattering angle $\theta = 10°$, $\theta = 15°$, $\theta = 20°$.
Analysis of this curve shows the sensitivity of the relative contribution $\delta_{P_{S_\gamma}}$ to SCCs. First, for all values of θ, a negative evolution is observed at low energies. Above this threshold, $\delta_{P_{S_\gamma}}$ increases rapidly to reach values greater than 200% at high energy. These results suggest that the ^{6}Li nucleus is a prime candidate for the experimental detection of SCCs in neutral radiative scattering processes.

6 Conclusion

In this article, we have established the general expression for the differential cross section for the neutrino(antineutrino)-nucleus radiative scattering process by neutral current. A study was conducted on the charge asymmetry coefficient and the degree of polarization of the emitted photon for the ^{6}Li nucleus. The results obtained show that the observables $A_{\nu\tilde{\nu}}$ and P_{S_γ} are effective tools for experimentally probing SCCs. These results highlight that the analysis of the coefficients $A_{\nu\tilde{\nu}}$ and P_{S_γ} is a sensitive tool for the experimental search for SCCs in the neutral radiative scattering of neutrino(antineutrino) on atomic nuclei. The introduction of the relative contributions $\delta_{A_{\nu\tilde{\nu}}}$ and $\delta_{P_{S_\gamma}}$ has made these results more significant by quantifying the impact of SCCs. These quantities are particularly sensitive at high energies, where they reach approximately $\approx 260\%$ and $\approx 200\%$ when $\theta = 10°$ and $E_\nu = 600 MeV$.
Ultimately, these results show that the light nucleus of ^{6}Li is an ideal candidate for experimental research into SCCs in the neutral radiative scattering process. The energy and angular analysis of these observables provides favorable experimental conditions for experimental studies of SCCs, particularly for incident neutrino energies in the range of a few hundred MeV and scattering angles below 20°.
This work paves the way for future experimental research aimed at testing the existence of SCCs, the discovery of which would represent a major advance in the understanding of weak interactions.

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Appendix

The most general expression of the leptonic current in neutral scattering is given by :

$\ell_\mu = (\vec{\ell}, \ell_4 = i\ell_0) = \bar{u}_2\gamma_\mu(1+\gamma_5)u_1$

The leptonic tensor is defined by : $\ell_\mu \ell_\nu^* = \delta_\nu \bar{u}_2\gamma_\mu(1+\gamma_5)u_1\bar{u}_1(1-\gamma_5)\gamma_\nu u_2$,

$\delta_\nu = \begin{cases} -1, & \nu = 1,2,3 \\ +1, & \nu = 4 \end{cases}$

$\ell_\mu \ell_\nu^* = \delta_\nu Tr\left\{\gamma_\mu(1+\gamma_5)\Lambda_1(1-\gamma_5)\gamma_\nu\Lambda_2\right\}$,

$\Lambda_1 = \frac{1}{4\epsilon E_1}(m_1 - i\epsilon\hat{P}_1)(1 - i\hat{S}_1\gamma_5)$ et $\Lambda_2 = \frac{1}{4\epsilon E_2}(m_2 - i\epsilon\hat{P}_2)(1 - i\hat{S}_2\gamma_5)$

$\epsilon = +1$ for particles, $\epsilon = -1$ for anti-particles.

The evaluation of the tensor $\ell_\mu \ell_\nu^*$ requires computing traces of Dirac matrix products, which can be carried out by applying the standard algebraic relations of the gamma matrices :

$$\begin{aligned}
f_1 &= \frac{1}{2}(\ell_1\ell_1^* + \ell_2\ell_2^*) \qquad f_2 = Im(\ell_1\ell_2^*) \\
f_3 &= \ell_3\ell_3^* \qquad f_4 = -2R_e(\ell_3\ell_0^*) \\
f_5 &= \ell_0\ell_0^* \qquad f_6 = -\frac{1}{\sqrt{2}}Re(\ell_1\ell_3^*) \\
f_7 &= \frac{1}{\sqrt{2}}Im(\ell_2\ell_3^*) \qquad f_8 = \frac{1}{\sqrt{2}}Re(\ell_1\ell_0^*) \\
f_9 &= \frac{1}{\sqrt{2}}Im(\ell_2\ell_0^*) \qquad f_{10} = \frac{1}{2}(\ell_1\ell_1^* - \ell_2\ell_2^*) \\
\widetilde{f}_1 &= \frac{1}{\sqrt{2}}Im(\ell_3^*\ell_1) \qquad \widetilde{f}_2 = -\frac{1}{\sqrt{2}}Re(\ell_3^*\ell_2) \\
\widetilde{f}_3 &= -\frac{1}{\sqrt{2}}Im(\ell_1\ell_0^*) \qquad \widetilde{f}_4 = \frac{1}{\sqrt{2}}Re(\ell_2\ell_0^*) \\
\widetilde{f}_5 &= Re(\ell_1\ell_2^*) \qquad \widetilde{f}_6 = 2Im(\ell_0\ell_3^*)
\end{aligned} \tag{23}$$

In the case of massless neutrinos, the leptonic functions take the form :

$$\begin{aligned}
&v_1 = 2(1 - C_1C_2), \quad v_2 = 2\eta(C_2 - C_1), \quad v_3 = 2(1 + 2C_1C_2 - \cos\theta), \quad v_4 = -4(C_1 + C_2), \\
&v_5 = 2(1+\cos\theta), \quad v_6 = \frac{-2\sqrt{2}}{\sin\theta}(C_2^2 - C_1^2), \quad v_7 = \frac{2\eta\sqrt{2}}{\sin\theta}(C_1 + C_2)(\cos\theta - 1), \\
&v_8 = \frac{2\sqrt{2}}{\sin\theta}(C_2 - C_1)(1+\cos\theta), \quad v_9 = \frac{2\eta\sqrt{2}}{\sin\theta}(C_1^2 + C_2^2 - 2C_1C_2\cos\theta), \\
&v_{10} = -\frac{2}{\sin^2\theta}(C_2 - C_1\cos\theta)(C_2\cos\theta - C_1)
\end{aligned} \tag{24}$$

Here, θ is the angle between the neutrino(antineutrino) and electron(positron) pulses, η takes the value $+1$ for neutrino scattering and -1 for antineutrino and the coefficients C_1, C_2 are given by the relations : $C_1 = (E_\ell cos\theta - E_\nu)/q, \quad C_2 = (E_\ell - E_\nu cos\theta)/q$

The hadronic functions are given by the following formulas :

$$Y_1^L = -\sum_{J'J} A^L_{1,-1}\{P^+_{J'+J}\{\beta_V^{(\tau')}\beta_V^{(\tau)}(F^{(\tau')}_{MJ'}F^{(\tau)}_{MJ}+F^{(\tau')}_{EJ'}F^{(\tau)}_{EJ})+\beta_A^{(\tau')}\beta_A^{(\tau)}(F^{5(\tau')}_{MJ'}F^{5(\tau)}_{MJ}+F^{5(\tau')}_{EJ'}F^{5(\tau)}_{EJ})\}+ P^-_{J'+J}\{\beta_V^{(\tau')}\beta_V^{(\tau)}(F^{(\tau')}_{MJ'}F^{(\tau)}_{EJ}-F^{(\tau')}_{EJ'}F^{(\tau)}_{MJ})+\beta_A^{(\tau')}\beta_A^{(\tau)}(F^{5(\tau')}_{EJ'}F^{5(\tau)}_{MJ}-F^{5(\tau')}_{MJ'}F^{5(\tau)}_{EJ})\}\}$$

$$Y_2^L = -\sum_{J'J} A^L_{1,-1}\{P^+_{J'+J}\{(\beta_V^{(\tau')}\beta_A^{(\tau)}(F^{(\tau')}_{MJ'}F^{5(\tau)}_{EJ}+F^{(\tau')}_{EJ'}F^{5(\tau)}_{MJ})+\beta_A^{(\tau')}\beta_V^{(\tau)}(F^{5(\tau')}_{MJ'}F^{(\tau)}_{EJ}+F^{5(\tau')}_{EJ'}F^{(\tau)}_{MJ})\}+ P^-_{J'+J}\{(\beta_V^{(\tau')}\beta_A^{(\tau)}(F^{(\tau')}_{MJ'}F^{5(\tau)}_{MJ}-F^{(\tau')}_{EJ'}F^{5(\tau)}_{EJ})+\beta_A^{(\tau')}\beta_V^{(\tau)}(F^{5(\tau')}_{EJ'}F^{(\tau)}_{EJ}-F^{5(\tau')}_{MJ'}F^{(\tau)}_{MJ})\}\}$$

$$Y_3^L = \sum_{J'J} A^L_{0,0}P^+_{J'+J}\{\beta_V^{(\tau')}\beta_V^{(\tau)}F^{(\tau')}_{LJ'}F^{(\tau)}_{LJ}+\beta_A^{(\tau')}\beta_A^{(\tau)}F^{5(\tau')}_{LJ'}F^{5(\tau)}_{LJ}\}$$

$$\bar{Y}_3^L = \sum_{J'J} A^L_{0,0}P^-_{J'+J}\{\beta_A^{(\tau')}\beta_V^{(\tau)}F^{5(\tau')}_{LJ'}F^{(\tau)}_{LJ}-\beta_V^{(\tau')}\beta_A^{(\tau)}F^{(\tau')}_{LJ'}F^{5(\tau)}_{LJ}\}$$

$$Y_4^L = \sum_{J'J} A^L_{0,0}P^+_{J'+J}\{\beta_V^{(\tau')}\beta_V^{(\tau)}F^{(\tau')}_{LJ'}F^{(\tau)}_{CJ}+\beta_A^{(\tau')}\beta_A^{(\tau)}F^{5(\tau')}_{LJ'}F^{5(\tau)}_{CJ}\}$$

$$\bar{Y}_4^L = \sum_{J'J} A^L_{0,0}P^-_{J'+J}\{\beta_A^{(\tau')}\beta_V^{(\tau)}F^{5(\tau')}_{LJ'}F^{(\tau)}_{CJ}-\beta_V^{(\tau')}\beta_A^{(\tau)}F^{(\tau')}_{LJ'}F^{5(\tau)}_{CJ}\}$$

$$Y_5^L = \sum_{J'J} A^L_{0,0}P^+_{J'+J}\{\beta_V^{(\tau')}\beta_V^{(\tau)}F^{(\tau')}_{CJ'}F^{(\tau)}_{CJ}+\beta_A^{(\tau')}\beta_A^{(\tau)}F^{5(\tau')}_{CJ'}F^{5(\tau)}_{CJ}\}$$

$$\bar{Y}_5^L = \sum_{J'J} A^L_{0,0}P^-_{J'+J}\{\beta_A^{(\tau')}\beta_V^{(\tau)}F^{5(\tau')}_{CJ'}F^{(\tau)}_{CJ}-\beta_V^{(\tau')}\beta_A^{(\tau)}F^{(\tau')}_{CJ'}F^{5(\tau)}_{CJ}\}$$

$$Y_6^L = -2\sqrt{2}\sum_{J'J} A^L_{1,0}\{P^+_{J'+J}(\beta_V^{(\tau')}\beta_V^{(\tau)}F^{(\tau')}_{LJ'}F^{(\tau)}_{EJ}+\beta_A^{(\tau')}\beta_A^{(\tau)}F^{5(\tau')}_{LJ'}F^{5(\tau)}_{EJ})+P^-_{J'+J}(\beta_A^{(\tau')}\beta_A^{(\tau)}F^{5(\tau')}_{LJ'}F^{5(\tau)}_{MJ}- \beta_V^{(\tau')}\beta_V^{(\tau)}F^{(\tau')}_{LJ'}F^{(\tau)}_{MJ})\}$$

$$Y_7^L = -2\sqrt{2}\sum_{J'J} A^L_{1,0}\{P^+_{J'+J}(\beta_V^{(\tau')}\beta_A^{(\tau)}F^{(\tau')}_{LJ'}F^{5(\tau)}_{MJ}+\beta_A^{(\tau')}\beta_V^{(\tau)}F^{5(\tau')}_{LJ'}F^{(\tau)}_{MJ})+P^-_{J'+J}(\beta_A^{(\tau')}\beta_V^{(\tau)}F^{5(\tau')}_{LJ'}F^{(\tau)}_{EJ}- \beta_V^{(\tau')}\beta_A^{(\tau)}F^{(\tau')}_{LJ'}F^{5(\tau)}_{EJ})\}$$

$$Y_8^L = -2\sqrt{2}\sum_{J'J} A^L_{1,0}\{P^+_{J'+J}(\beta_V^{(\tau')}\beta_V^{(\tau)}F^{(\tau')}_{CJ'}F^{(\tau)}_{EJ}+\beta_A^{(\tau')}\beta_A^{(\tau)}F^{5(\tau')}_{CJ'}F^{5(\tau)}_{EJ})+P^-_{J'+J}(\beta_A^{(\tau')}\beta_A^{(\tau)}F^{5(\tau')}_{CJ'}F^{5(\tau)}_{MJ}- \beta_V^{(\tau')}\beta_V^{(\tau)}F^{(\tau')}_{CJ'}F^{(\tau)}_{MJ})\}$$

$$Y_9^L = -2\sqrt{2}\sum_{J'J} A^L_{1,0}\{P^+_{J'+J}(\beta_V^{(\tau')}\beta_A^{(\tau)}F^{(\tau')}_{CJ'}F^{5(\tau)}_{MJ}+\beta_A^{(\tau')}\beta_V^{(\tau)}F^{5(\tau')}_{CJ'}F^{(\tau)}_{MJ})+P^-_{J'+J}(\beta_A^{(\tau')}\beta_V^{(\tau)}F^{5(\tau')}_{CJ'}F^{(\tau)}_{EJ}- \beta_V^{(\tau')}\beta_A^{(\tau)}F^{(\tau')}_{CJ'}F^{5(\tau)}_{EJ})\}$$

$$Y_{10}^L = -\sum_{J'J} A^L_{1,1}\{P^+_{J'+J}\{\beta_V^{(\tau')}\beta_V^{(\tau)}(F^{(\tau')}_{EJ'}F^{(\tau)}_{EJ}-F^{(\tau')}_{MJ'}F^{(\tau)}_{MJ})+\beta_A^{(\tau')}\beta_A^{(\tau)}(F^{5(\tau')}_{EJ'}F^{5(\tau)}_{EJ}-F^{5(\tau')}_{MJ'}F^{5(\tau)}_{MJ})\}+ P^-_{J'+J}\{\beta_A^{(\tau')}\beta_A^{(\tau)}(F^{5(\tau')}_{EJ'}F^{5(\tau)}_{MJ}+F^{5(\tau')}_{MJ'}F^{5(\tau)}_{EJ})-\beta_V^{(\tau')}\beta_V^{(\tau)}(F^{(\tau')}_{EJ'}F^{(\tau)}_{MJ}+F^{(\tau')}_{MJ'}F^{(\tau)}_{EJ})\}\}$$

$$\bar{Y}^L_{10} = -\sum_{J'J} A^L_{1,1}\{P^+_{J'+J}\{(\beta^{(\tau')}_V \beta^{(\tau)}_A (F^{(\tau')}_{EJ'} F^{5(\tau)}_{MJ} - F^{(\tau')}_{MJ'} F^{5(\tau)}_{EJ}) + \beta^{(\tau')}_A \beta^{(\tau)}_V (F^{5(\tau')}_{EJ'} F^{(\tau)}_{MJ} - F^{5(\tau')}_{MJ'} F^{(\tau)}_{EJ})\} +$$
$$P^-_{J'+J}\{\beta^{(\tau')}_A \beta^{(\tau)}_V (F^{5(\tau')}_{MJ'} F^{(\tau)}_{MJ} + F^{5(\tau')}_{EJ'} F^{(\tau)}_{EJ}) - \beta^{(\tau')}_V \beta^{(\tau)}_A (F^{(\tau')}_{MJ'} F^{5(\tau)}_{MJ} + F^{(\tau')}_{EJ'} F^{5(\tau)}_{EJ})\}\}$$

$$\bar{Y}^L_1 = Y^L_2, \quad \bar{Y}^L_2 = Y^L_1, \quad \bar{Y}^L_6 = -Y^L_7, \quad \bar{Y}^L_7 = -Y^L_6\ , \quad \bar{Y}^L_8 = -Y^L_9, \quad \bar{Y}^L_9 = -Y^L_8.$$

F_{MJ},F_{EJ},F_{CJ} and F_{LJ}(F^5_{MJ},F^5_{EJ},F^5_{CJ} and F^5_{LJ}) are matrix elements of the magnetic, electrical, coulombic and longitudinal multipole vector (axial-vector) operators calculated in the core layer model.

$$F^5_C = \frac{1}{3\sqrt{2\pi}} \cdot \frac{q}{M}\Bigg[-\frac{3}{2}F_A(\Psi_{1,1}(\frac{3}{2},\frac{1}{2}) + \Psi_{1,1}(\frac{1}{2},\frac{3}{2}))$$
$$+ \frac{1}{2}(q_0 F_P - 2\eta M F_T)\Bigg(\sqrt{5}\left(1-\frac{2}{5}y\right)\Psi_{1,1}(\frac{3}{2},\frac{3}{2}) - 2(1-y)(\Psi_{1,1}(\frac{3}{2},\frac{1}{2}) + \Psi_{1,1}(\frac{1}{2},\frac{3}{2}))$$
$$-\frac{1}{\sqrt{2}}(1+2y)\Psi_{1,1}(\frac{1}{2},\frac{1}{2})\Bigg)\Bigg]e^{-y}$$

$$F_M = -\frac{1}{6\sqrt{2\pi}} \cdot \frac{q}{M}\Bigg[F_1\big(\sqrt{10}\Psi_{1,1}(\frac{3}{2},\frac{3}{2}) + \sqrt{2}(\Psi_{1,1}(\frac{3}{2},\frac{1}{2}) + \Psi_{1,1}(\frac{1}{2},\frac{3}{2})) + 2\Psi_{1,1}(\frac{1}{2},\frac{1}{2})\big)$$
$$+\mu\Bigg(\sqrt{10}\left(1-\frac{4}{5}y\right)\Psi_{1,1}(\frac{3}{2},\frac{3}{2}) - 2\sqrt{2}\left(1-\frac{1}{2}y\right)(\Psi_{1,1}(\frac{3}{2},\frac{1}{2}) + \Psi_{1,1}(\frac{1}{2},\frac{3}{2}))$$
$$-(1-2y)\Psi_{1,1}(\frac{1}{2},\frac{1}{2})\Bigg)\Bigg]e^{-y}$$

$$F^5_L = -\frac{1}{3\sqrt{2\pi}}\left(F_A - \frac{q^2}{2M}F_P\right)\Bigg[\sqrt{5}\left(1-\frac{2}{5}y\right)\Psi_{1,1}(\frac{3}{2},\frac{3}{2}) - 2(1-y)(\Psi_{1,1}(\frac{3}{2},\frac{1}{2}) + \Psi_{1,1}(\frac{1}{2},\frac{3}{2}))$$
$$-\frac{1}{\sqrt{2}}(1+2y)\Psi_{1,1}(\frac{1}{2},\frac{1}{2})\Bigg]e^{-y}$$

$$F^5_E = -\frac{1}{3\sqrt{2\pi}}F_A\Bigg[\sqrt{10}\left(1-\frac{4}{5}y\right)\Psi_{1,1}(\frac{3}{2},\frac{3}{2}) - 2\sqrt{2}\left(1-\frac{1}{2}y\right)(\Psi_{1,1}(\frac{3}{2},\frac{1}{2}) + \Psi_{1,1}(\frac{1}{2},\frac{3}{2}))$$
$$-(1-2y)\Psi_{1,1}(\frac{1}{2},\frac{1}{2})\Bigg]e^{-y}$$

where ψ is a nuclear parameter ; $\mu = F_1 + 2MF_2$; q_0 is the transition energy ; $y = (bq/2)^2$ where b is the harmonic oscillator parameter.

The coefficient $A^L_{m'm}$ is given by :

$$A^L_{m'm} = (-)^{J_f+J_n}[J'][J][L]\left(\frac{(L-M)!}{(L+M)!}\right)^{1/2}\begin{pmatrix} J' & J & L \\ m' & m & M \end{pmatrix}\begin{Bmatrix} J' & J & L \\ J_n & J_n & J_i \end{Bmatrix} \qquad (25)$$