Reviewer Report (2025-ARJOM-146955)

1. In Eq. (2.6), the left-hand side should be $\|f_1((a+1)x_1,(a+1)x_2) - f_1((a-1)x_1,(a-1)x_2) - af_1(2x_1,2x_2)\|$, but the norm symbol $\|\cdot\|$ is missing.

2. In Eq. (2.9), the expression $\|f_1(2ax_1,2ax_2) - a^2 f_1(2x_1,2x_2), z\|$ is not standard; 2-norms take two vector arguments, not a vector and a scalar z.

3. In Eq. (2.10), the denominator $\|a\|^2$ is missing in the second term: it should be $\frac{\varepsilon 2^{-2p}(\|a\|^{2p}+1)}{\|a\|^2}\|x_1,x_2\|^p$.

4. In Eq. (2.12), the summation index j starts at 0, but the term $\|a\|^{2(p-1)j}$ should be $\|a\|^{2pj-2j}$ for clarity.

5. In Eq. (2.13), the geometric series formula is misapplied; the denominator should be $1-\|a\|^{2(p-1)}$, but this is only valid if $\|a\|^{2(p-1)} < 1$, which requires justification since $0 < p < 1$.

6. In the derivation after Eq. (2.13), the factor 2^{-2p} is sometimes written as 2^{-p}; this is inconsistent.

7. In Theorem 2.1, the bound in (2.3) has denominator $2^{2p}(\|a\|^2 - \|a\|^{2p})$, but earlier steps suggest the denominator should be $\|a\|^2 - \|a\|^{2p}$ without the 2^{2p} factor.

8. In Eq. (2.16), the right-hand side should be $(2\|a\| + \|a\|^{2p} + 1)\varepsilon\|x_1,x_2\|^p$, but the grouping of terms is ambiguous.

9. In Eq. (2.17), the term $\|a\|^{-2p}$ appears, but since $a \in \mathbb{Z}\setminus\{-1,0,1\}$, $\|a\|$ is just $|a|$, so writing $|a|^{-2p}$ would be clearer.

10. In Eq. (2.19), the fraction $\frac{\|a\|^{-2p}(1-\|a\|^{2(-p+1)n})}{1-\|a\|^{2(-p+1)}}$ is undefined when $p=1$, but the theorem assumes $p>1$, so the exponent sign should be checked.

11. In Theorem 2.2, inequality (2.15) has denominator $2^{2p}(\|a\|^{2p} - \|a\|^2)$, but for $p>1$, $\|a\|^{2p} > \|a\|^2$, so the sign is correct, yet the factor 2^{2p} lacks justification from prior steps.

12. In Eq. (2.23), the factor of 2 on the left comes from $f_2(ay,ay) + f_2(ay,ay)$, but Df_2 evaluated at $(0,0)$ should be checked for symmetry.

13. In Eq. (2.25), division by a assumes $a \neq 0$, which is given, but the norm should be $|a|$, not $\|a\|$, since $a \in \mathbb{Z}$.

14. In Eq. (2.27), the exponent $(2p-1)j$ is negative since $0 < p < \frac{1}{2}$, so the geometric series converges, but the denominator $1-\|a\|^{2p-1}$ should be written as $1-|a|^{2p-1}$.

15. In Theorem 2.3, inequality (2.21) has denominator $2(|a| - |a|^{2p})$, but from Eq. (2.27), the constant appears to be $\frac{1}{2|a|(1-|a|^{2p-1})}$, which is not obviously equal.

16. In Eq. (2.30)–(2.32), the scaling $x_i \mapsto x_i/a$ is used, but a is an integer, not necessarily invertible in the space; this is acceptable only if scalar division is defined over $\mathbb{R}$, which should be clarified.

17. In Eq. (2.33), the sum $\sum_{j=0}^{n-1}|a|^{(-2p+1)j-2p}$ simplifies to $|a|^{-2p}\frac{1-|a|^{(-2p+1)n}}{1-|a|^{-2p+1}}$, but the manuscript writes the denominator as $1-|a|^{(-2p+1)}$, which is correct only if $|a|^{-2p+1} \neq 1$.

18. In Theorem 2.4, the bound (2.29) has denominator $2(|a|^{2p} - |a|)$, but the derivation yields $\frac{1}{2|a|(|a|^{2p-1}-1)}$, which is equivalent only after algebraic manipulation that is not shown.

19. In Theorem 2.5, the combined error bound (2.35) adds the quadratic and additive errors, but the cross-term from $f(0,0)$ is handled inconsistently between (2.36) and (2.37).

20. In the definition of Df above Eq. (2.1), the arguments are written as $((x_1,x_2),(y_1,y_2))$, but the functional equation (1.4) is for single-variable f, so the extension to $f: X\times X \to Y$ needs clearer motivation.