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Detecting Mean Shifts in Financial Time Series with Application to Financial Data

Abstract

In this paper, we propose a fully automated method for detecting changes in the mean of piecewise CHARN models. The approach combines an adaptive model selection algorithm with a robust changepoint detection procedure based on local power estimation. Applied to financial datasets, the method identifies both known and previously undetected structural breaks. These changes often correspond to real-world events, confirming the sensitivity and reliability of the approach.

Keywords: CHARN model; changepoint; Algorithm; S&P 500; FSTE 100 2020 Mathematics Subject Classification: 62M10; 62G10; 62F05; 62P20; 91B84.

1 Introduction

Detecting changepoints in time series is a crucial problem in many disciplines, particularly in economics and finance, where sudden shifts in market behavior, volatility, or underlying structural regimes can have significant impacts. A changepoint refers to a moment when the statistical properties of a series such as its mean, variance, or dependence structure experience a fundamental alteration. Understanding these shifts is essential not only for accurate modeling and forecasting but also for identifying the underlying causes driving such changes, whether they stem from policy interventions, macroeconomic shocks, or structural market transformations.

The study of structural breaks was initiated by Page (1954) in the context of quality control and has since evolved into a major field of research spanning economics Perron et al. (2006), finance Andreou and Ghysels (2009), climatology Beaulieu et al. (2012); Reeves et al. (2007), and engineering Stoumbos et al. (2000). Techniques such as the CUSUM test Page (1954), and its least-squares adaptation $\text{CUSUM}^{\text{ols}}$ by Brown et al. (1975), have laid the foundation for change detection in various time series settings. Subsequent advancements, including p-value corrections Zeileis (2001, 2004) and robust methods for dependent data Aue and Horváth (2013), have further enhanced detection capabilities.

In economic and financial data, detecting changepoints is particularly challenging due to features such as serial dependence, heteroskedasticity, and short-lived or weak changes. Offline methods optimizing information criteria Horváth (1993); Yao (1987), efficient algorithms like PELT Killick et al. (2012), and segmentation approaches Fryzlewicz (2014); Vostrikova (1981) have been proposed to address these issues. Moreover, Bayesian methods Hadj-Amar et al. (2020) and likelihood ratio scanning Yau and Zhao (2016) provide flexible frameworks for identifying multiple changes in complex financial environments.

More recent contributions have focused on the detection of subtle or localized changes in financial time series, addressing the significant challenges posed by nonstationarity and weak signals Diop and Kengne (2023); Hallgren et al. (2022). In particular, detecting weak structural changes, such as small shifts in the mean under conditional heteroskedasticity, has attracted growing attention Ltaifa (2021); Ngatchou-Wandji and Ltaifa (2023); Salman et al. (2024b). In financial contexts, it is crucial not only to determine *when* a change occurs, but also to understand *why*, whether it results from monetary policy interventions, geopolitical events, or endogenous market dynamics. Such understanding is essential for effective risk management and informed decision-making.

Despite these advances, the problem of detecting weak changes where shifts are of small magnitude and potentially transient remains relatively under-explored. Ltaifa (2021) and Ngatchou-Wandji and Ltaifa (2023) specifically addressed this issue within the framework of Conditional Heteroskedastic Autoregressive Nonlinear (CHARN) models, with further generalizations proposed by Salman et al. (2024b). When changes are brief and atypical, distinguishing between false alarms and genuine structural breaks becomes particularly challenging. Developing robust methodologies capable of reliably detecting weak and transient changes is therefore crucial for improving the analysis of piecewise stationary financial data.
To address this issue, Salman et al. (2024a) proposed a new automatic algorithm for the detection of weak changes in the mean, incorporating operations designed to distinguish between *true changepoints* and false alarms. A *true changepoint* is defined as a structural shift that results in a piecewise stationary process, whereas a *false alarm* refers to a transient change that affects only a small number of observations without altering the overall stationarity of the series.

In this paper, we first revisit the method and theoretical results developed in Salman et al. (2024b), along with the algorithm proposed in Salman et al. (2024a). Building on this foundation, we propose a complementary algorithm designed to automatically select the appropriate time series model for the real data under investigation. The combination of both algorithms forms a unified framework capable of handling a wide variety of data across different fields. Furthermore, we demonstrate the enhanced performance of the complete method through applications to real-world financial data, with comparisons drawn against results from other studies in the literature.
The structure of the paper is organized as follows. In Section 2, we recall the essential theoretical results from Salman et al. (2024b) and review the algorithm introduced in Salman et al. (2024a). In Section 3, we present our complementary algorithm and highlight its significance. Section **??** illustrates the application of the full method to a real financial dataset. Finally, Section 5 concludes the paper.

2 Approach and Algorithm

In this section, we provide a concise overview of the method developed in Salman et al. (2024b), which generalizes the approaches introduced in Ngatchou-Wandji and Ltaifa (2023). These methods aim to detect weak changes in the mean by leveraging the theoretical power properties of a likelihood ratio test. In addition, we review the algorithm proposed in Salman et al. (2024a) and detail its underlying mechanisms.
The statistical model used in Salman et al. (2024b) belongs to the class of Conditional Heteroscedastic Autoregressive Nonlinear (CHARN) models (see, e.g., Härdle et al. (1998)). More specifically, let $d, p, k, n \in \mathbb{N}$, with $k \ll n$. Assume that the observations $X_1, \ldots, X_n$ are generated from a piecewise stationary CHARN model.

$$X_t = T(\rho_{\mathbf{0}} + \gamma \odot \omega(t); \mathbf{X}_{t-1}) + V(\mathbf{X}_{t-1})\varepsilon_t, t \in \mathbb{Z}, \tag{2.1}$$

with

$$X_t = Y_{t,j} = T(\rho_{\mathbf{0}} + \boldsymbol{\gamma}_j \omega_j(t); \mathbf{X}_{t-1,j}) + V(\mathbf{X}_{t-1,j})\varepsilon_t, \quad \tau_{j-1} \le t < \tau_j, \quad j = 1, \dots, k+1, \tag{2.2}$$

where for $j = 1, \dots, k$, $(Y_{t,j})_{t\in\mathbb{Z}}$ is a stationary and ergodic process; $\boldsymbol{\rho}_0 \in \mathbb{R}^p$, $T(\boldsymbol{\rho}_0, .)$ and $V(.)$ are real-valued functions with $\inf_{x\in\mathbb{R}^d} V(x) > 0$; the τ_j, $j = 0, \dots, k+1$, are potential instants of changes with $\tau_0 = 1$ and $\tau_{k+1} = n+1$; for $j = 1, \dots, k$, $\mathbf{X}_{t,j} = (Y_{t,j}, \dots, Y_{t-d+1,j})^\top$, $\mathbf{X}_{\tau_{j-1}+\ell} = \mathbf{X}_{\tau_{j-1}+\ell,j}$, $\ell = 0, \dots, d-1$ and for $t \in [\tau_{j-1} + d - 1, \tau_j)$, $\mathbf{X}_t = (X_t, \dots, X_{t-d+1})^\top$; for $j, \ell = 1, \dots, k$, $j \neq \ell$, the process $(Y_{t,j})_{t\in\mathbb{Z}}$ and $(Y_{t,\ell})_{t\in\mathbb{Z}}$ are mutually independent (Yau and Zhao (2016) noted that this assumption can be extended to some weak dependence assumption); $(\varepsilon_t)_{t\in\mathbb{Z}}$ is a standard white noise with density f. $\boldsymbol{\gamma} = \left(\boldsymbol{\gamma}_1^\top, \dots, \boldsymbol{\gamma}_{k+1}^\top\right)^\top$, $\boldsymbol{\gamma}_j \in \mathbb{R}^p$, $j = 1, \dots, k+1$; $\omega(t) = (\mathbb{1}_{[\tau_0,\tau_1)}(t), \mathbb{1}_{[\tau_1,\tau_2)}(t), ..., \mathbb{1}_{[\tau_{k-1},\tau_k)}(t), \mathbb{1}_{[\tau_k,\tau_{k+1})}(t))^\top = (\omega_1(t), \dots, \omega_{k+1}(t)) \in \{0,1\}^{k+1}$; for $\boldsymbol{\gamma} = (\boldsymbol{\gamma}_1^\top, \dots, \boldsymbol{\gamma}_{k+1}^\top)^\top$ and $\omega(t) = (\omega_1(t), \dots, \omega_{k+1}(t))^\top$, $\boldsymbol{\gamma} \odot \omega(t)$ stands for $\boldsymbol{\gamma} \odot \omega(t) = \boldsymbol{\gamma}_1\omega_1(t) + \cdots + \boldsymbol{\gamma}_{k+1}\omega_{k+1}(t) \in \mathbb{R}^p$, and $\boldsymbol{\gamma}_i\omega_i = (\gamma_{i,1}\omega_i, \dots, \gamma_{i,p}\omega_i) \in \mathbb{R}^p$.
This class of models is broad and includes various structures such as AR(p), ARCH(p), EXPAR(p), and GEXPAR(p) models. Their statistical and probabilistic properties have been extensively studied in the literature (see, for example, Chen et al. (2018) for an analysis of the ergodicity of GEXPAR models).

For $\boldsymbol{\gamma}_0 \in \mathbb{R}^{p(k+1)}$ and $\boldsymbol{\beta} \in \mathbb{R}^{p(k+1)}$ depending on the τ_j's, Salman et al. (2024b) construct a likelihood ratio test for testing

$$H_0 : \boldsymbol{\gamma} = \boldsymbol{\gamma}_0 \quad \text{against} \quad H_{\boldsymbol{\beta}}^{(n)} : \boldsymbol{\gamma} = \boldsymbol{\gamma}_n = \boldsymbol{\gamma}_0 + \frac{\boldsymbol{\beta}}{\sqrt{n}}. \tag{2.3}$$

Note that the norm of $\boldsymbol{\beta}$ is small relative to n, and thus the two hypotheses under consideration become increasingly close as the sample size n grows.
First, the authors establish that the constructed test satisfies the locally asymptotically normal (LAN) property, and that the hypotheses are contiguous in the sense of Le Cam (see Le Cam (1986) and Droesbeke and Fine (1996)). These properties enable the study of the theoretical power of the proposed test and lead to an explicit expression for it. Specifically, under certain technical assumptions, they show that the constructed likelihood ratio test is asymptotically optimal and that its asymptotic power is given by

$$\mathcal{P}_{k,\tau^k} = 1 - \Phi(z_\alpha - \vartheta(\boldsymbol{\rho}_0, \boldsymbol{\gamma}_0, \boldsymbol{\beta})) \tag{2.4}$$

where $\boldsymbol{\rho}_0$ represents the true nuisance parameter, $\alpha \in (0,1)$ denotes the level of significance, z_α is the $(1-\alpha)$-quantile of the standard Gaussian distribution with cumulative distribution function ϕ, ϑ is a real-valued function defined on $\mathbb{R}^{p(k+1)\times p(k+1)}$, whose explicit form is provided in Salman et al. (2024b).

In practice, the model parameters are unknown and must be estimated. Salman et al. (2024a) summarized and clearly explained the methodology for estimating the power of the test under unknown parameters. It is worth noting that parameter estimation has been extensively studied in the literature; for instance, Chen et al. (2018) discusses the estimation of both linear and nonlinear components in GExpAR models, a particular case of the CHARN model considered in Salman et al. (2024b), while Brockwell et al. (1990) addresses parameter estimation in linear models such as ARMA. In Salman et al. (2024b), the decision procedure for the testing problem is based on the estimated power $\widehat{\mathcal{P}}_{k,\tau^k}$, obtained by substituting the true parameters with their estimators in the expression of $\mathcal{P}_{k,\tau^k}$.

To describe the estimation procedure, let $1 \leq j \leq k+1$ and $1 \leq h \leq p$. Denote by $\widehat{\rho}_{j,h}$ a consistent estimator (e.g., the maximum likelihood estimator) of $\rho_{0,h} + \beta_{j,h}/\sqrt{n}$, based on the observations within the interval $[\tau_{j-1}, \tau_j)$. The estimator of $\beta_{j,h}$ is then defined as $\widehat{\beta}_{j,h} = \sqrt{n}(\widehat{\rho}_{j,h} - \widehat{\rho}_{0,h})$, where $\widehat{\rho}_{0,h}$ denotes the estimator of the stationary parameter $\rho_{0,h}$ based on the first segment of observations $[1, \tau_1)$. By replacing the true parameters with their estimates, it is shown that the constructed test retains its asymptotic optimality, and the corresponding estimated power is explicitly expressed as $\widehat{\mathcal{P}}_{k,\tau^k}$.

To determine whether a given observation X_t corresponds to a changepoint, we rely on the estimated local power of the test, denoted by $\mathcal{P}_{k,t}$, as previously described. Estimating this quantity requires computing all its components, which motivates the need for a dedicated algorithm. One of the most challenging components to estimate is the vector $\boldsymbol{\beta}$, which, by construction, reflects the contrast between two sets of estimated parameters: one based on a segment of stationary observations, and the other on a segment that includes the candidate changepoint. This comparison enables the local power $\mathcal{P}_{k,t}$ to serve as a statistical decision criterion for identifying potential structural changes at time t.

To implement this idea, Salman et al. (2024a) assume that the first m observations, $X_1, \ldots, X_m$, are stationary. Based on this initial segment, the underlying statistical model is estimated/assumed to be a particular CHARN model providing a reference for comparison.

To detect the first potential changepoint at a time $t > m$, the algorithm defines two intervals: $I_1 = \{X_1, \ldots, X_{t-1}\}$, and $I_2 = \{X_1, \ldots, X_t\}$, where X_t is the observation under investigation. Parameter estimation is carried out for both intervals, and the contrast between them yields an estimate of $\boldsymbol{\beta}$, from which the local power $\widehat{\mathcal{P}}_{k,t}$ is computed.

If this estimated power exceeds a predefined threshold, the observation X_t is flagged as a critical point. To distinguish between a true changepoint and a false alarm, the algorithm removes X_t from I_2 and sequentially replaces it with future observations $X_{t+\iota}$, for $\iota = 1, \ldots, \ell$, where $1 \leq \ell < m$, repeating the same local power calculation. If the power remains above the threshold across replacements, a changepoint is confirmed. The algorithm then assumes that the following m observations are stationary and restarts the process for subsequent detection.

Conversely, if the local power does not exceed the threshold, X_t is not considered a changepoint. In that case, the algorithm updates both intervals I_1 and I_2 by appending the next observation, and continues automatically.

3 Proposed complementary algorithm

The approach proposed by Salman et al. (2024b) assumes that the data are piecewise stationary. The algorithm proposed in Salman et al. (2024a) first adjusts a CHARN model to the initial m stationary observations, then selects a model intended for application across the entire dataset. Nevertheless, once changepoints are detected, the initially chosen model may no longer remain appropriate for the newly stationary segments. However, this assumption is relatively weak, as the selected model may only be well-suited for this initial limited segment of the data. An alternative idea would be to fit a model using all available observations; however, this approach is also problematic, as it fails to account for potential nonstationarities that the detection procedure precisely aims to uncover. Therefore, it is essential to develop an adaptive algorithm capable of updating the model selection dynamically as new stationary regimes are identified.

3.1 Algorithm

Consider a set of n observations, denoted as $X_1, X_2, \ldots, X_n$. We use m as the minimum number of observations assumed to be stationary and maintain us estimating the parameters of the statistical model. Our additional complementary algorithm will be use before applying that of Salman et al. (2024a), and it can be explained by the following steps:

1. Consider h independent Uniform random variables $u \in U[1, n-m]$, $1 \leq h < n-m$, where m denotes the minimum number of observations assumed to be stationary as per the algorithm mentioned above:
   (a) Select a subset S_u, which contains the observations $X_{u+1}, X_{u+2}, \ldots, X_{u+m}$.
   (b) Fit the CHARN model to the time series subset S_u.
   (c) Based on different selection criteria (such as AIC, BIC, etc.), extract the best-fitting time series model that explains the behavior of the observations in S_u, denoted as M_u.
2. The optimal model to be use for applying the algorithm described in Salman et al. (2024a) is:

$$M = \text{ Most frequent model } M_u \text{ among } h \text{ models}$$

4 Application to real world financial data

In this section, we apply the approach proposed by Salman et al. (2024b) to detect structural changes in real-world financial data, specifically focusing on the Standard & Poor's 500 and the Financial Times Stock Exchange 100 indices. To achieve this, we employ the technique presented in Subsection 3.1, combined with the algorithm proposed by Salman et al. (2024a) and summarized in Section 2.

4.1 Standard & Poor's 500 Index (S&P 500)

Here, we apply our newly approach to financial data, specifically the S&P 500 index's daily stock prices. We utilize daily data from January 1992 to December 2000, the same dataset used in Salman et al. (2024b), to determine whether the new algorithm detects subtle changes that may have gone undetected by the previously proposed algorithm.

Since the data exhibits a trend indicating that the S&P 500 index is non-stationary, we work with the transformed series X_t, defined as:

$$X_t = \log\left(\frac{P_t}{P_{t-1}}\right),$$

where P_t represents the S&P 500 price index at time t. This transformation ensures that any potential breaks in the series P_t are preserved in X_t, due to the continuity property of the logarithmic function.

We begin by assuming stationarity over m observations, where we set $m = 25$ based on the approximate number of trading days in a month. Additionally, we consider a vector of 200 different random variables u ($h = 200$), $u \in U[1, n - m = 2020 - 25]$, representing randomly selected sub-samples from the dataset.

Next, to identify the most suitable particular CHARN model, we implement our algorithm presented in Section 3.1. The dominant repetitive model is summarized in Table 1.

Model	Frequency
ARIMA(0,0,0)	166
ARIMA(1,0,0)	5
ARIMA(2,0,0)	2

Table 1: The most frequently selected particular CHARN models for S&P 500 Index.

Finally, using our methodology, we adopt the same model proposed in Salman et al. (2024b), which is defined as follows:

$$X_t = \frac{\beta_j}{\sqrt{n}} + \theta_j \varepsilon_t, \quad t \in [\tau_j, \tau_{j+1}[, \quad \varepsilon_t \sim N(0, 1),$$

where β_j and θ_j represent the model parameters, and ε_t is a standard normally distributed error term. Then, applying the algorithm for changes detection using this model, we detects the changes presented in this series and explained with Figure 1.

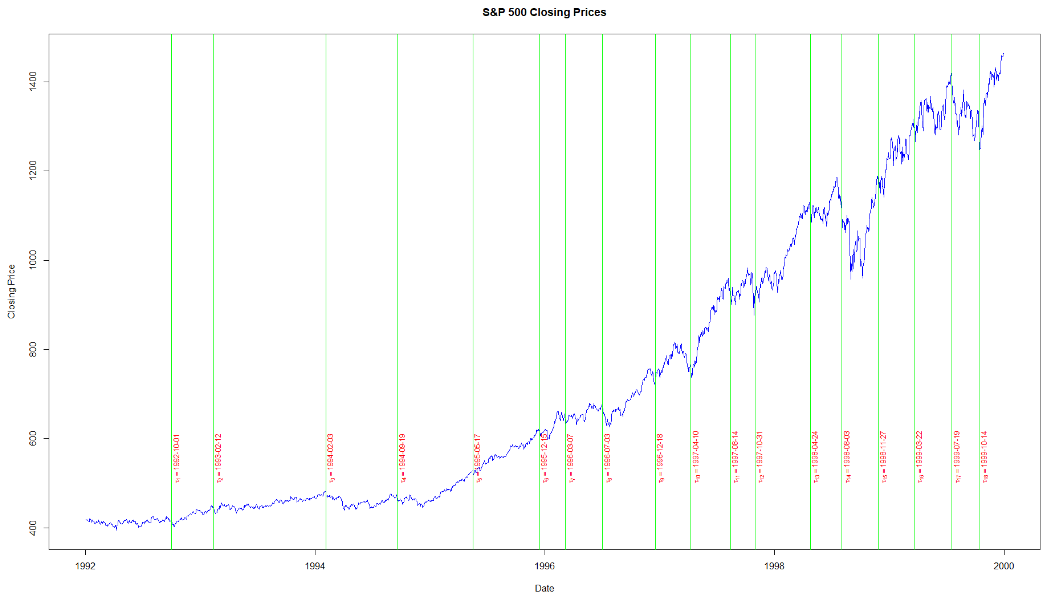

Figure 1: Estimated breaks detected in S&P 500 indices.

While the algorithm of Salman et al. (2024b) successfully identified several significant changes in the S&P 500 index, the new algorithm detects additional important breaks that correspond to events not captured by the previous technique.

For instance, the new algorithm identified **1993-02-12**, a changepoint which could be linked to the inauguration of President Bill Clinton and the anticipation surrounding his economic policies aimed at reducing the federal deficit. This period marked the beginning of shifts in fiscal policy, which could explain the market's response. However, this subtle change was not captured by the old algorithm.

Similarly, the date **1994-09-19**, identified by the new algorithm, corresponds to the bond market crisis, also known as the "Bond Market Massacre" of 1994. The old algorithm identified a related point in **1994-03-03**, which we linked to the U.S. lifting of the trade embargo on Vietnam, but failed to capture the subsequent market turbulence later in the year.

Moreover, the detected changepoint at **1997-04-10**, which likely reflects the market's reaction to early concerns about inflation and potential interest rate hikes. This date is not detected by the previous algorithm, which only identified **1997-07-14**, likely driven by the deepening Asian financial crisis.

Another example is the detection of **1996-03-07**, a subtle change linked to the Federal Reserve's decision to hold interest rates steady after prior hikes. This change was missed by the old algorithm, which only captured a later date in July 1996 (**1996-07-11**), possibly linked to strong corporate earnings at the time.

Additionally, **1998-08-03**, coinciding with the start of the Russian financial crisis, whereas the algorithm of Salman et al. (2024b) only captured **1998-06-22** and **1998-11-02**, which were connected to market reactions to the Long-Term Capital Management (LTCM) crisis and the Federal Reserve's rescue operation.

An important remark must be made regarding the dates 1997-04-10 and 1997-07-19, when the estimated local power crossed the threshold, signaling the presence of a critical point. It subsequently fell below the threshold after 4 and 5 observations, respectively. We can conclude that, in real-world data, the events occurring at these times had a distinct, short-term impact, unlike those that caused more persistent changes at other dates.

In conclusion, the new combined algorithm not only confirms several changepoints identified by the old one but also detects additional, earlier, or subtler changes linked to significant market events. This enhancement demonstrates the improved sensitivity of the new algorithm in capturing weak changes in financial markets that the previous algorithm overlooked. The improved sensitivity is primarily attributed to the selection of an appropriate model, identified from a large number of subsets, and its integration with the automatic detection algorithm proposed by Salman et al. (2024a).

4.1.1 Financial Times Stock Exchange 100 Index (FTSE 100)

In this part, we apply our approach to detects the change in the FTSE 100 index. We use daily data from July 27th, 2005, to July 13th, 2009. Instead of using the original data where the trend appears clearly, we use the logarithm return as defined in the previous section. As we explain in the previous section, we begin by assuming the stationarity of the first m observations, where we set again $m = 25$ based on approximate number of trading days in a month. Using the same values and following the same procedure as in the previous section, the dominant repetitive model among 300 subset considered is summarized in the Table 2.

Model	Frequency
ARIMA(0,0,0)	226
ARIMA(0,1,0)	11
ARIMA(1,0,0)	16
ARIMA(3,0,0)	13

Table 2: The most frequently selected particular CHARN models for a subset of FTSE 100 Index.

Out of 300 subsets derived from the original dataset of 1004 observations, 16 subsets suggest an Autoregressive AR(1) model, indicating that the daily price depends on the previous day's price value. Additionally, 13 subsets suggest that the price depends on the price values from the past three days. However, the dominant model identified is the shifted model, which consists of a mean plus an error term. This shifted model will be the one utilized for further analysis. Using these results, we adjust the following particular CHARN model

$$X_t = \frac{\beta_j}{\sqrt{n}} + \theta_j \varepsilon_t, \quad t \in [\tau_j, \tau_{j+1}[, \quad \varepsilon_t \sim N(0,1),$$

where β_j and θ_j represent the model parameters, and ε_t is a standard normally distributed error term. By applying our change detection algorithm using this model, we successfully identified the structural changes present in the series, as illustrated in Figure 2. Fryzlewicz and Subba Rao (2014)

Figure 2: Estimated breaks detected in FTSE 100 Index.

proposed a technique for detecting changes, referred to as the BASTA-res technique, which the authors applied to the FTSE 100 index over the same period analyzed here. The corresponding dates detected by their method are notably June 5, 2007, August 18, 2008, and December 4, 2008. These dates align with significant financial events, such as the onset of the subprime mortgage crisis and the collapse of Lehman Brothers. While their method successfully identifies these key turning points, our approach provides additional insights by detecting earlier and intermediate structural changes, such as on February 26, 2007, and July 25, 2007, indicating shifts in market dynamics preceding the financial crisis.

For instance, the changepoint on May 11, 2006, detected by our algorithm, likely reflects global market concerns about inflation and interest rates, which triggered a broader sell-off. Similarly, the changepoint on July 25, 2007, reflects early signs of market volatility linked to the subprime crisis, a significant moment that went undetected by Fryzlewicz and Subba Rao (2014). Furthermore, our algorithm detected important changes on January 18, 2008, March 20, 2008, and September 18, 2008, providing a more detailed representation of market turbulence during the 2008 financial crisis.

These additional changepoints underscore the increased sensitivity and granularity of our approach. By identifying earlier and more frequent structural changes, it offers a deeper understanding of the market's evolving behavior, which could be crucial for more proactive risk management. The results demonstrate that our approach not only captures major financial disruptions, as found by Fryzlewicz and Subba Rao (2014), but also identifies critical early warning signals and minor shifts, providing a more comprehensive view of the market's volatility."

5 Conclusion

In this study, we combined our proposed algorithmic component with that introduced by Salman et al. (2024a) to apply, in a fully automated manner, the approach developed by Salman et al. (2024b) for selecting and detecting breaks in the mean of piecewise CHARN models. Applied to financial datasets, this integrated framework effectively identifies structural changes driven by major market events. Our method not only reproduces known breakpoints found in previous studies but also uncovers additional, previously undetected changes. These newly identified shifts can often be linked to specific historical events, underscoring the robustness and enhanced sensitivity of our approach in detecting nuanced transitions in time series data.

A key contribution of this work is the development of an automated procedure for selecting the optimal threshold tailored to the domain under investigation. This innovation addresses a long-standing challenge in changepoint detection and will be a central focus in our future research efforts.

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