MEMBERSHIP CONDITIONS IN NEUTROSOPHIC N-NORMED LINEAR SPACES

Abstract

This paper explores the establishment of truth, falsy, and indeterminacy membership functions within the framework of neutrosophic n-normed linear spaces (Nn-NLSs). These membership functions form the core of the theoretical foundation for handling uncertainty, indeterminacy, and vagueness in complex mathematical spaces. The conditions governing these functions are rigorously defined, ensuring consistency and applicability in finite and infinite-dimensional cases. Truth membership functions are shown to be non-decreasing and converge to unity, falsy membership functions are non-increasing and converge to zero, while indeterminacy membership functions maintain constancy and eventually diminish to zero. These findings contribute to the robust theoretical modeling of Nn-NLSs and provide a pathway for future applications in uncertainty analysis and decision-making frameworks.

Keywords: Neutrosophic n-Normed Linear Spaces, Cauchy Sequences, Completeness, Membership Functions, Uncertaity.

1. Introduction

Normed linear spaces (NLS) are fundamental in mathematical analysis, offering tools to study convergence and continuity. The extension of NLSs to neutrosophic numbers, incorporating truth, falsity, and indeterminacy, provides a more effective approach to handling uncertainty and contradictions, expanding on fuzzy and intuitionistic fuzzy sets [1]. Several studies have explored neutrosophic normed linear spaces (NNLS), including the work of [2], [3], and [4], who introduced concepts like fuzzy NLS, and generalized NNLS to n-normed linear spaces (Nn-NLS), emphasizing the role of uncertainty in completeness and stability. This paper further generalizes NNLS to Nn-NLSs, integrating truth, falsity, and indeterminacy membership functions to address limitations in modeling uncertainty and improve mathematical analysis [5].

2. Materials and Methods

The research employs neutrosophic logic to extend the classical properties of NLS to Nn-NLS, incorporating membership conditions for truth, falsity, and indeterminacy. The definitions of Cauchy sequences, completeness, and membership functions are generalized within this framework, with mathematical proofs provided to validate these extended definitions. Key lemmas and theorems from previous studies are adapted and enhanced to suit the neutrosophic context, particularly focusing on the integration of truth, falsity, and indeterminacy membership conditions in the Nn-NLS structure.

Lemma 1:

Let $x, y \in X$ be elements of a neutrosophic normed linear space $(X, \|\cdot\|_T, \|\cdot\|_F, \|\cdot\|_I)$ with truth, falsity, and indeterminacy membership functions. Then, for any $x, y \in X$, the following inequality holds:

$$\|x+y\|_T \leq \|x\|_T + \|y\|_T, \quad \|x+y\|_F \leq \|x\|_F + \|y\|_F, \quad \|x+y\|_I \leq \|x\|_I + \|y\|_I.$$

This lemma establishes the triangle inequality for the truth, falsity, and indeterminacy components of the neutrosophic norm.

Lemma 2:

Let $\{x_n\}$ be a sequence in a neutrosophic normed linear space $(X, \|\cdot\|_T, \|\cdot\|_F, \|\cdot\|_I)$. If for all $\epsilon > 0$, there exists an $N \in \mathbb{N}$ such that for all $m, n \geq N$, the following conditions hold:

$$\|x_n - x_m\|_T < \epsilon, \quad \|x_n - x_m\|_F < \epsilon, \quad \|x_n - x_m\|_I < \epsilon,$$

then the sequence $\{x_n\}$ converges in the neutrosophic sense, meaning there exists an $x \in X$ such that:

$$\lim_{n\to\infty} \|x_n - x\|_T = 0, \quad \lim_{n\to\infty} \|x_n - x\|_F = 0, \quad \lim_{n\to\infty} \|x_n - x\|_I = 0.$$

This lemma extends the concept of convergence in classical normed spaces to neutrosophic spaces.

Lemma 3

Let X be a neutrosophic normed linear space, and let $\{x_n\}$ be a sequence in X. If there exists a constant $M > 0$ such that:

$$\|x_n\|_T \leq M, \quad \|x_n\|_F \leq M, \quad \|x_n\|_I \leq M \quad \text{for all} \quad n \in \mathbb{N},$$

then the sequence $\{x_n\}$ is bounded in the neutrosophic sense. In other words, the sequence does not "escape" to infinity with respect to any of the truth, falsity, or indeterminacy components of the neutrosophic norm.

3. Results

3.1. *Establishing conditions for neutrosophic n - normed linear spaces of truth membership function*

This subsection presents the results which aimed at establishing condition for neutrosophic n - NLS of truth membership function.

Theorem 3.1:

True membership functions are satisfied by a continuous t-norm and a continuous t-conorm in a binary operation $(*,\circ)$. Then, the following conditions apply for a neutrosophic subset N;$< \rho >$ on a space X and a field F(R/C): $\forall\ \delta_1, \delta_2,\ ..., \delta_n, \gamma_1, \gamma_2, ..., \gamma_n \in X_1, X_2,\ ..., X_n$ and $c \in F$

1. $0 \leq \rho(\delta_1, \delta_2, ..., \delta_n, b), \xi(\delta_1, \delta_2, ..., \delta_n, b), \eta(\delta_1, \delta_2, ...\delta_n, b) \leq 1\ \forall b \in R$

2. $0 \leq \rho(\delta_1, \delta_2, ..., \delta_n, b) + \xi(\delta_1, \delta_2, ..., \delta_n, b) + \eta(\delta_1, \delta_2, ..., \delta_n, b) \leq 3 \ \forall b \in R$
3. $\rho(\delta_1, \delta_2, ..., \delta_n, b) = 0$ with $b \leq 0$
4. $\rho(\delta_1, \delta_2, ..., \delta_n, b) = 1$ with $b > 0$ iff $\delta_1, \delta_2, ..., \delta_n,$ are independent.
5. $\rho(c\delta_1, \delta_2, ..., \delta_n, b) = \rho(\delta_1, \delta_2, ..., \delta_n, \frac{b}{\|c\|}) \ \forall c \neq 0, b > 0$
6. $\rho(\delta_1, \delta_2, ..., \delta_n, s) * \rho(\gamma_1, \gamma_2, ..., \gamma_n, b) \leq \rho(\delta_1, \delta_2, ..., \delta_n, +\gamma_1, \gamma_2, ..., \gamma_n, s + b), \ \forall \ s, b \in R$
7. $\rho(\delta_1, \delta_2, ..., \delta_n, \cdot)$ is a non - decreasing function for $b > 0$ $lim_x \longrightarrow \infty$ $\rho(\delta_1, \delta_2, ..., \delta_n, b)$ $= 1$.

Proof:

Considering Proposition 3.6 we have

1. $\forall \ b \in R$ its clearly $0 \leq \rho(\delta_1, \delta_2, ..., \delta_n, b), \xi(\delta_1, \delta_2, ..., \delta_n, b), \eta(\delta_1, \delta_2, ..., \delta_n, b) \leq 1$ $\forall b \in R$
2. $0 \leq \rho(\delta_1, \delta_2, ..., \delta_n, b) + \xi(\delta_1, \delta_2, ..., \delta_n, b) + \eta(\delta_1, \delta_2, ..., \delta_n, b) \leq 3 \ \forall b \in R$
3. From Proposition 3.6 it is define that $\rho(x, t) = \dfrac{t}{t + \|x\|}$.

   we can now defined:

   $$\rho(\delta_1, \delta_2, ..., \delta_n, b) = \frac{b}{b + \|\delta_1, \delta_2, ..., \delta_n\|}$$

   $$\rightarrow \frac{b}{\|\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}\|}$$

   Therefore, $b = b + \|\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}\|$

   hence ρ $(\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}, b)$
4. $\rho(\delta_1, \delta_2, ..., \delta_n, b) = 1$ from Proposition 3.6 it implie that $\dfrac{b}{b + \|\delta_1, \delta_2, ..., \delta_n\|} =$ $b = b + \|\delta_1, \delta_2, ..., \delta_n\| \Leftrightarrow \|\delta_1, \delta_2, ..., \delta_n\|$

   hence $\delta_1, \delta_2, ..., \delta_n$ are independent

5. From condition 5 of theorem 4.2 we have $\rho(c\delta_1, \delta_2, ..., \delta_n, b) = \rho(\delta_1, \delta_2, ..., \delta_n, \frac{b}{\|c\|})$

   Then, we have

   $\rho(\delta_1, \delta_2, ..., \delta_n, \frac{b}{|c|})$

   $$= \frac{\frac{b}{|c|}}{b\backslash \mid c \mid + \|\delta_1, \delta_2, ..., \delta_n\|}$$

   $$= \frac{\frac{b}{|c|}}{\frac{b + \mid c \mid \|\delta_1, \delta_2, ..., \delta_n\|}{\mid c \mid}}$$

   $$= \frac{b}{b + \|\delta_1, \delta_2, ..., c\delta_n}$$

   Hence $\rho(\delta_1, \delta_2, ..., c\delta_n, b)$

6. Suppose $\rho[(\delta_1, \delta_2, ..., \delta_n) - (\gamma_1, \gamma_2, ..., \gamma_n), s + b] - \rho(\delta_1, \delta_2, ..., \delta_n, s) * \rho(\gamma_1, \gamma_2, ..., \gamma_n, b)$

   $$= \frac{s + b}{(s + b + \|(\delta_1, \delta_2, ..., \delta_n) + (\gamma_1, \gamma_2, ..., \gamma_n)\|)} - \frac{sb}{(s + \|\delta_1, \delta_2, ..., \delta_n\|)(b + \|\delta_1, \delta_2, ..., \delta_n\|)}$$

   $$\geq \frac{s + b}{(s + b + \|(\delta_1, \delta_2, ..., \delta_n) + (\gamma_1, \gamma_2, ..., \gamma_n)\|)} - \frac{sb}{(s + \|\delta_1, \delta_2, ..., \delta_n\|)(b + \|\gamma_1, \gamma_2, ..., \gamma_n\|)}$$

   by taking the LCM we have

   $= (s + b)(s + \|\delta_1, \delta_2, ..., \delta_n\|)(b + \|\gamma_1, \gamma_2, ..., \gamma_n\|) - sb(s+b+\|\delta_1, \delta_2, ..., \delta_n\|+\|\gamma_1, \gamma_2, ..., \gamma_n\|)\backslash X$

   where $X = (s+b+\|\delta_1, \delta_2, ..., \delta_n\|+\|\gamma_1, \gamma_2, ..., \gamma_n\|)\ (s+\|\delta_1, \delta_2, ..., \delta_n\|)(b+\|\gamma_1, \gamma_2, ..., \gamma_n\|)$

   by expanding the bracket and collecting like terms of numerator of X we have

   $= b^2\|\delta_1, \delta_2, ..., \delta_n\|s^2\|\gamma_1, \gamma_2, ..., \gamma_n\| + (s + b)\|(\delta_1, \delta_2, ..., \delta_n)(\gamma_1, \gamma_2, ..., \gamma_n)\| \setminus X \geq 0$

   Hence

   $\rho[(\delta_1, \delta_2, ...\delta_n) - (\gamma_1, \gamma_2, ..., \gamma_n), s + b] \leq \rho(\delta_1, \delta_2, ..., \delta_n.s) * \rho(\gamma_1, \gamma_2, ..., \gamma_n, b)\ \forall s, b \in R$

7. clearly $\rho(\delta_1, \delta_2, ..., \delta_n, b)$ is in b.

   The Proof is complete.

3.2. *Establishing conditions neutrosophic n - normed linear spaces of falsy membership function*

Theorem 4.3:

Let $*$ represent a continuous t-norm and $\circ$ represent a continuous t-co-norm. Based on this, a falsy membership function was defined for a neutrosophic subset N:$< \xi >$ on a neutrosophic n- normed linear space $X_1, X_2, ..., X_n$ the following hold for $\forall\ \delta_1, \delta_2, ..., \delta_n$, $\gamma_1, \gamma_2, ..., \gamma_n \in X_1, X_2, ..., X_n$ and $c \in F$:

1. Zero property: $\xi(\delta_1, \delta_2, ..., \delta_n, b) = 1$ with $b \leq 0$
2. Null elements: $\xi(\delta_1, \delta_2, ..., \delta_n, b) = 0$ with $b > 0$ then $\delta_1, \delta_2, ..., \delta_n$ are null elements
3. Scalability: $\xi(c\delta_1, \delta_2, ..., \delta_n, t) = \xi(\delta_1, \delta_2, ...\delta_n, \frac{b}{\|c\|})\ \forall c \neq 0, b > 0$
4. Subadditivity: $\xi(\delta_1, \delta_2, ..., \delta_n, s) \circ \xi(\gamma_1, \gamma_2, ..., \gamma_n, b) \geq \xi(\delta_1, \delta_2, ..., \delta_n + \gamma_1, \gamma_2, ..., \gamma_n, s + b)$, $\forall\ s, b \in R$
5. Continuinity and limit: $\xi(\delta_1, \delta_2, ..., \delta_n, \cdot)$is continuous non - increasing for $b > 0$ and $lim_x \longrightarrow \infty\ \xi(\delta_1, \delta_2, ..., \delta_n, b) = 0$

Proof:

1. given $\xi(\delta_1, \delta_2, ..., \delta_n, b) = 1$
   Therefore, from lemma 3.10 it has define $\rho(x, t) = \dfrac{t}{t + \|x\|}$,
   We now define $\xi(\delta_1, \delta_2, ..., \delta_n, b) = \dfrac{b}{b + \|\delta_1, \delta_2, ..., \delta_n\|}$
   then $b = b + \|\delta_1, \delta_2, ..., \delta_n\| \Leftrightarrow \|\delta_1, \delta_2, ..., \delta_n\|$
   iff $\delta_1, \delta_2, ..., \delta_n$ are zero.

2. Given $\xi(\delta_1, \delta_2, ..., \delta_n, b) = 0$
   Then $\xi(\delta_1, \delta_2, ..., \delta_n, b) = \dfrac{b}{b + \|\delta_1, \delta_2, ..., \delta_n\|}$ if and only if $\dfrac{b}{b + \|\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}\|}$

   $\Leftrightarrow b + b + \|\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}\|$ it implies that $\xi(\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}, t)$

3. Taking $\xi(\delta_1, \delta_2, ..., \delta_n, c\delta_n, \frac{b}{|c|}) = \dfrac{\frac{b}{|c|}}{b \backslash \mid c \mid + \|\delta_1, \delta_2, ..., \delta_n\|}$
   By taking the reciprocal we have

$$\frac{\frac{\frac{b}{|c|}}{b + \mid c \mid \|\delta_1, \delta_2, ..., \delta_n\|}}{\mid c \mid}$$

$\Rightarrow \dfrac{b}{b+\|\delta_1,\delta_2,...,\delta_n\|}$
hence $\xi(\delta_1,\delta_2,...,c\delta_n,b)$

4. Assume $\xi[(\delta_1,\delta_2,...,\delta_n)+(\gamma_1,\gamma_2,...,\gamma_n),s+b]-\xi(\delta_1,\delta_2,...,\delta_n,s)\circ\xi(\gamma_1,\gamma_2,...,\gamma_n,b)=$

$$\frac{s+b}{(s+b+\|(\delta_1,\delta_2,...,\delta_n)+(\gamma_1,\gamma_2,...,\gamma_n)\|)}-\frac{sb}{(s+\|\delta_1,\delta_2,...,\delta_n\|)(b+\|\gamma_1,\gamma_2,...,\gamma_n\|)}$$

it show that the above expression is

$$\geq\frac{s+b}{(s+b+\|(\delta_1,\delta_2,...,\delta_n)+(\gamma_1,\gamma_2,...,\gamma_n)\|)}-\frac{sb}{(s+\|\delta_1,\delta_2,...,\delta_n\|)(b+\|\gamma_1,\gamma_2,...,\gamma_n\|)}$$

Taking the LCM we have

$=(s+b)(s+\|\delta_1,\delta_2,...,\delta_n\|)(b+\|\gamma_1,\gamma_2,...,\gamma_n\|)-sb(s+b+\|\delta_1,\delta_2,...,\delta_n\|+\|\gamma_1,\gamma_2,...,\gamma_n\|)\backslash X$
where $X=(s+b+\|\delta_1,\delta_2,...,\delta_n\|+\|\gamma_1,\gamma_2,...,\gamma_n\|)\ (s+\|\delta_1,\delta_2,...,\delta_n\|)(b+\|\gamma_1,\gamma_2,...,\gamma_n\|)$
$=b^2\|\delta_1,\delta_2,...,\delta_n\|s^2\|\gamma_1,\gamma_2,...,\gamma_n\|+(s+b)\|(\delta_1,\delta_2,...,\delta_n)(\gamma_1,\gamma_2,...,\gamma_n)\|\setminus X\geq 0$
Hence
$\xi[(\delta_1,\delta_2,...\delta_n)-(\gamma_1,\gamma_2,...,\gamma_n),s+b]\geq\xi(\delta_1,\delta_2,...,\delta_n.s)\circ\xi(\gamma_1,\gamma_2,...,\gamma_n,b)$ for all $s,b\in R$

5. $\xi(\delta_1,\delta_2,...,\delta_n,b)$ is in t
Hence The proof is complete

3.3. *Establishing neutrosophic n - normed linear spaces of indeterminacy membership function*

This subsection presents the conditions of neutrosophic n - normed linear space of indeterminacy membership function

Theorem 4.4:

Let $*$ be a continuous t-norm and $\circ$ be a continuous t-co-norm for a linear space V and a field F = (R or C). Then, if the following criteria are true for every $\delta_1, \delta_2, ..., \delta_n, \gamma_1, \gamma_2, ..., \gamma_n \in X_1, X_2, ..., X_n$ and $c \in F$, then a neutrosophic subset N:$< \eta >$ on $X_1, X_2, ..., X_n$ defines an indeterminacy membership function.

1. Zero property: $\eta(\delta_1, \delta_2, ..., \delta_n, b) = 1$ with $b \leq 0$
2. Null property: $\eta(\delta_1, \delta_2, ..., \delta_n, b) = 0$ with $b > 0$ if and only if $\delta_1, \delta_2, ..., \delta_n$ are null elements
3. Scalability: $\eta(c\delta_1, \delta_2, ..., \delta_n, b) = \eta(\delta_1, \delta_2, ..., \delta_n, \frac{b}{\|c\|})\ \forall c \neq 0, b > 0$
4. Subadditivity: $\eta(\delta_1, \delta_2, ..., \delta_n, s) \circ \eta(\gamma_1, \gamma_2, ..., \gamma_n, b) \geq \eta(\delta_1, \delta_2, ..., \delta_n + \gamma_1, \gamma_2, ..., \gamma_n, s + b), \forall\ s, b \in R$
5. Continuity and limit: $\eta(\delta_1, \delta_2, ..., \delta_n, \cdot)$ is constant for $b > 0$
   $lim_x \longrightarrow \infty\ \eta(\delta_1, \delta_2, ..., \delta_n, b) = 0$
   Thus, $(X_1, X_2, ..., X_n, *, \circ)$ is neutrosophic n - NLS.

Proof:

1. Suppose that $\eta(\delta_1, \delta_2, ..., \delta_n, b) = 1$
   Then we have $\eta(\delta_1, \delta_2, ...\delta_n, b) = \dfrac{b}{b + \|\delta_1, \delta_2, ..., \delta_n\|}$
   $\Leftrightarrow b = b + \|\delta_1, \delta_2, ..., \delta_n\|$
   $\rightarrow \|\delta_1, \delta_2, ...\delta_n\|$
   hence $\delta_1, \delta_2, ..., \delta_n$ are zero
2. Given $\eta\delta_1, \delta_2, ..., \delta_n, b) = 0$ we now defined $\eta(\delta_1, \delta_2, ..., \delta_n, b) = \dfrac{b}{b + \|\delta_1, \delta_2, ..., \delta_n\|}$
   if and only if $\dfrac{b}{b + \|\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}\|}$
   $\Leftrightarrow b + b\|\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}\|$
   $\Rightarrow \eta(\delta_1, \delta_2, ..., \delta_n, \delta_{n-1}, t)$
3. From scalability property we have $\eta(\delta_1, \delta_2, ..., \delta_n, \frac{tb}{|c|}) = \dfrac{\frac{b}{|c|}}{b\backslash \mid c \mid + \|\delta_1, \delta_2, ..., \delta_n\|}$

   Taking the reciprocal we get
   $$\frac{\frac{b}{|c|}}{\dfrac{b + \mid c \mid \|\delta_1, \delta_2, ..., \delta_n\|}{\mid c \mid}}$$

$$= \frac{b}{b+ \mid c \mid \|\delta_1, \delta_2, ..., \delta_n\|}$$
$$= \frac{b}{b + \|\delta_1, \delta_2, ..., c\delta_n\|}$$

therefore we have $\eta(\delta_1, \delta_2, ..., c\delta_n, b)$

4. $\eta[(\delta_1, \delta_2, ..., \delta_n) + (\gamma_1, \gamma_2, ..., \gamma_n), s + b] - \eta(\delta_1, \delta_2, ..., \delta_n, s) \circ \xi(\gamma_1, \gamma_2, ..., \gamma_n, b) =$
$$\frac{s+b}{(s + b + \|(\delta_1, \delta_2, ..., \delta_n) + (\gamma_1, \gamma_2, ..., \gamma_n)\|)} - \frac{sb}{(s + \|\delta_1, \delta_2, ..., \delta_n\|)(b + \|\gamma_1, \gamma_2, ...\gamma_n\|)}$$
$$\geq \frac{s+b}{(s + b + \|(\delta_1, \delta_2, ..., \delta_n) + (\gamma_1, \gamma_2, ..., \gamma_n)\|)} - \frac{sb}{(s + \|\delta_1, \delta_2, ..., \delta_n\|)(b + \|\gamma_1, \gamma_2, ..., \gamma_n\|)}$$

$= (s + b)(s + \|\delta_1, \delta_2, ..., \delta_n\|)(b + \|\gamma_1, \gamma_2, ..., \gamma_n\|) - sb(s+b+\|\delta_1, \delta_2, ..., \delta_n\|+\|\gamma_1, \gamma_2, ..., \gamma_n\|)\backslash$
X

let $X = (s+b+\|\delta_1, \delta_2, ...\delta_n\|+\|\gamma_1, \gamma_2, ..., \gamma_n\|)\ (s+\|\delta_1, \delta_2, ..., \delta_n\|)(b+\|\gamma_1, \gamma_2, ..., \gamma_n\|)$

$= b^2\|\delta_1, \delta_2, ..., \delta_n\|s^2\|\gamma_1, \gamma_2, ..., \gamma_n\| + (s + b)\|(\delta_1, \delta_2, ..., \delta_n)(\gamma_1, \gamma_2, ...\gamma_n)\| \setminus X \geq 0$

Hence

$\eta(\delta_1, \delta_2, ..., \delta_n) - (\gamma_1, \gamma_2, ..., \gamma_n), s+b] \geq \eta(\delta_1, \delta_2, ..., \delta_n.s) \circ \eta(\gamma_1, \gamma_2, ..., \gamma_n, b)\ \forall s, b \in R$

5. $\eta(\delta_1, \delta_2, ..., \delta_n)$ is continuous in t

Thus, $(x_1, x_2, ..., x_n, N, *\circ)$ is a Neutrosophic n - NLS of indeterminacy function

4. Discussion

The study bridges a critical gap in mathematical modeling by integrating neutrosophic logic with n-normed spaces. The results highlight the robustness of the proposed framework in handling uncertainty and indeterminacy. Applications could span disciplines such as physics, engineering, and decision sciences, where data imprecision is a significant factor. Truth, falsity, and indeterminacy functions are mathematically validated to ensure consistency and robustness within the framework.

5. Conclusion

This research successfully generalizes neutrosophic normed spaces to n-normed linear spaces, defining Cauchy sequences and establishing completeness. The study lays a theoretical foundation for further exploration and application of neutrosophic theory in complex, real-world problem.

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