Modelling of a Steady Micropolar Nanofluid flow along a wedge

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ABSTRACT

Micropolar fluids are polarized fluids hence have better thermal conductivity properties. The incorporation of nanoparticles into micropolar fluids further enhances their thermal conductivity performance. The gyration characteristic of these fluids is significant in fields such as astrophysics, stellar dynamics, and dynamic theory. Flow along wedge structures has important application in aerodynamics, hydrodynamics heat transfer and industrial processes. Advancements in technology have heightened the development of predictive models for advanced machine tools, including synthetic lubricants and power bearings. This study investigates the steady flow of a micropolar nanofluid over a wedge with a perpendicularly applied magnetic field. By incorporating gyration and inertial effects into the Navier-Stokes equations, the flow is modeled and converted to ordinary differential equations through similarity transformation. Then solved numerically by Fourth-Order Runge-Kutta method, in combination with the shooting technique and the bvp5c solver in MATLAB.

Results reveal that increase in magnetic and micropolar parameters reduces the fluid velocity due to higher rotational viscosity, however, micropolar effects increases the temperature, solute concentration, energy and mass transfer. Additionally, skin friction is highest at the least wedge parameter and magnetic parameter, fluid velocity and enhances mass heat and mass transfer regardless of the magnetic field strength.

The findings of this study will help in thin film lubrication, which is crucial in the designing of chemical processing equipment, coolants, and heat exchangers in engineering applications.

Key word: Steady flow, Magnetohydrodynamics, micropolar nanofluid, Runge-kutta fouth order method

INTRODUCTION

Heat and mass transfer in fluids have wide applications in industrial and chemical processes. Due to this, several researchers have developed an interest in understanding and improving the industrial processes. This has helped in designing chemical processing equipment such as the cooling process towers, refrigeration, air conditioning and heat exchangers. Moreover, to improve heat transfer, micropolar nanofluids research has been carried out by many researchers because of their real-life applications.

A flow in which velocity remains constant over time is termed steady. In steady flow, all fluid properties remain unchanged with time. Thermodynamics uses this principle to analyze energy changes in mechanical processes. Laminar flow in pipes is a classic example. Steady flow is applied in economics, electronics, mechanical systems, fiber optics, and pharmaceuticals. In engineering, steady flows are vital in designing turbines, compressors, nozzles, and pumps used in power generation, automotive systems, and HVACs. Steady flow is guided by the steady flow equation, which can be used to analyze the fluid flow of a system, calculate the efficiency of a device, evaluate the performance of a machine or device, and conduct analysis of a power plant.

Research on micropolar fluids was first introduced by Erigen (1965), who defined them as non-symmetric stress tensors with microscopic characteristics during their motion. He stated that their particles exhibit microscopic velocity and atomic gyration within the fluid. Lukasiewicz (1999) extended this work and demonstrated that micropolar fluids have five viscosity coefficients, in which the angular momentum effects play a vital role. This breakthrough has led to more advanced research in the area.

Falkner and Skan (1931) approximated the solution of the boundary equations. In their work they investigated the steady laminar flow past a wedge and demonstrated the importance of Prandtl boundary layer theory. They derived the differentiated Falker-Skan equations by reducing the boundary layer equation. They noted that these equations are composed of non-uniform flows that could be approximated at the wall and take the form Cx^m.

Rajagopal et al. (1983) studied the Falkner-Skan flows of non-Newtonian fluids past a uniformly heated wedge through a forced convection. They concluded that forced convection flows in a uniformly and isothermally heated flux for different numbers.

Buongiorno (1983) , through his model, showed that thermophoresis intensity enhances the temperature of micropolar nanofluid. In separate studies, Wanateba (1991) and Ishak et al. (2007) analyzed the flow over a moving wedge with injection and suction using the Keller box technique and came up with the solutions for large values of wedge parameters. They further stated that, despite the results and solution, there is need to consider the heat produced by the working bodies in the micropolar fluid. This gave an insight in the aerodynamics engineering and the hydrodynamics field. Yir (1999), investigated the effect of an induced magnetic current under thermophoresis effect by a non-heated wedge. He concluded that the magnetic field is intense on shear stress but not in a heat transfer rate.

Agarwal et al. (1990) determined the conduction of heat on a micropolar fluid over a porous stationary wall. Kim (1999) analyzed the boundary layer along a wedge with constant surface heat flux. Talukdar (2012) studied the perturbation techniques for unsteady MHD mixed convection periodic heat flow and mass transfer in micropolar fluids with chemical reaction in the presence of thermal radiation. In these studies, the researchers were interested in the heat transfer through a boundary layer defined by amount of heat injected. However, the conditions failed to work, and hence they incorporated the Newtonian heating conditions. From the studies chemical reactions reduce the concentration, velocity and viscous drag of a fluid.

Ishak and Yao (2011) studied the heat conduction as a result of surface convection under different geometrics due to its application in transpiration, cooling and material drying processes. They concluded that the surface convection parameter is proportional to the surface temperature of the body.

Kuznetsova et al. (2011) showed that micropolar rotation contributes to the development of bio-micro system and show a significant role in mixing and increasing mass movement. Rahman Mim (2012) studied the hydromagnetic movement of unsteady bio magnetic fluid along a wedge under convection. He discovered that the magnetic field affects the temperature and gyration of the blood capsule. According to the study, heat transfer is directly proportional to the wedge angle but indirectly proportional to the unsteadiness parameter. It was noted that the strong unsteadiness of the fluid usually triggers sanction on the wedge surface.

Scatter (2011) studied a 2D boundary layer flow over a wedge. In the study, he established new sets of transformations in finding the local similarity situations. He concluded that separation of the boundary layer may be enhanced by enough unsteadiness in an accelerated flow. These results were used by Rahman et al. (2012) and Hassan (2013) to analyze heat transfer and their characteristics. In these studies, they concluded that fluid velocity increases with increase in unsteadiness parameter.

Khat et al. (2014) presented the flow of fluids and heat conduction on carbon nanotubes with momentum boundary conditions. They established that the heat conduction in oiled engine carbo nanotubes is faster compared to kerosene-based carbon nanotubes. Kumar (2017) established the effect of thermophoresis in a conducting micropolar fluid over a wedge. He concluded that the rise of chemical parameters and Schmidt number increases the rate of mass transfer.

Moh'd Rijalet et al. (2018) investigated the unsteady linear MHD boundary layer flow over a wedge and concluded that enhancement of the magnetic parameter of a wedge angle and thermal buoyance enhances the fluid flow, while nanoparticles volume fraction decreases the fluid velocity. Zaid et al. (2019) determined the effect of a $\mathrm{Ti}O_2$ on a mixed convection flow of micropolar fluid along a wedge. They established that fluid flow is enhanced by microrotation profiles for the first and second solution, but this decreased the nanofluid velocity in the first solution and increased it for the second solution. Zaid et al. (2019) investigated the effect of viscosity, thermal conductivity and Prandtl number in a mixed convection of a micropolar fluid They established that multiple solutions can only be obtained for opposable boundary layer flow.

Zulkifl et al. (2020) studied the MHD micropolar nanofluid flow along a wedge and concluded that wedge angle m and magnetic parameter M are proportional to the fluid velocity.

Chandra and Sudarsana (2020) comparatively analyzed steady and unsteady flow of a Buongiorno's Williamson nanofluid with slip effect and established that increase in wedge angle parameter intensifies the temperature in both steady and unsteady flows.

Zeshan Zulifqar et al(2023) studied the approximate closed form solution for mass transfer flow due to a rotating rough and porous disk. In this work he compared an analytical method and numerical method for a non-Newtonian fluid. This work was complemented by his work in (2025) where he studied an approximate closed-form solution of the flow and heat transfer over a static wedge.

R. Suhasini et al (2025). conducted the numerical study of a magneto-micropolar nanofluid flow across a stretching shrinking surface on impact of heat source, viscous dissipation and chemical reaction using the Runge-Kutta-Fehlberg (RKF-45) method. He concluded that heat reduction depends on the Prandtl number. However, Lewis and brownian motion function factors increase.

Although a lot of work has been done on steady flow, none of the researchers numerically analyzed the micropolar nanofluid flow along a wedge using the Runge-kutta method with the BVP5C solver. In this work therefore we shall study the steady flow of a micropolar nanofluid along a wedge using the fourth order Runge kutta method coupled with the shooting technique and the bvp5c solver of the MATLAB.

Mathematical Formulation

In this study, a two-dimensional steady flow of a micropolar nanofluid along the surface of a wedge of an angle $\Omega = \pi\beta$ with a uniform surface temperature T_w and a uniform upstream velocity, pressure and temperature are considered. The pressure velocity outside the viscous boundary layer varies with distance x along the wedge such that $u_\infty = Cx^m$ as shown in the diagram. The magnetic field B = $B_0 x^{\left(\frac{m-1}{2}\right)}$ is applied normal to the x-axis as illustrated in the diagram.

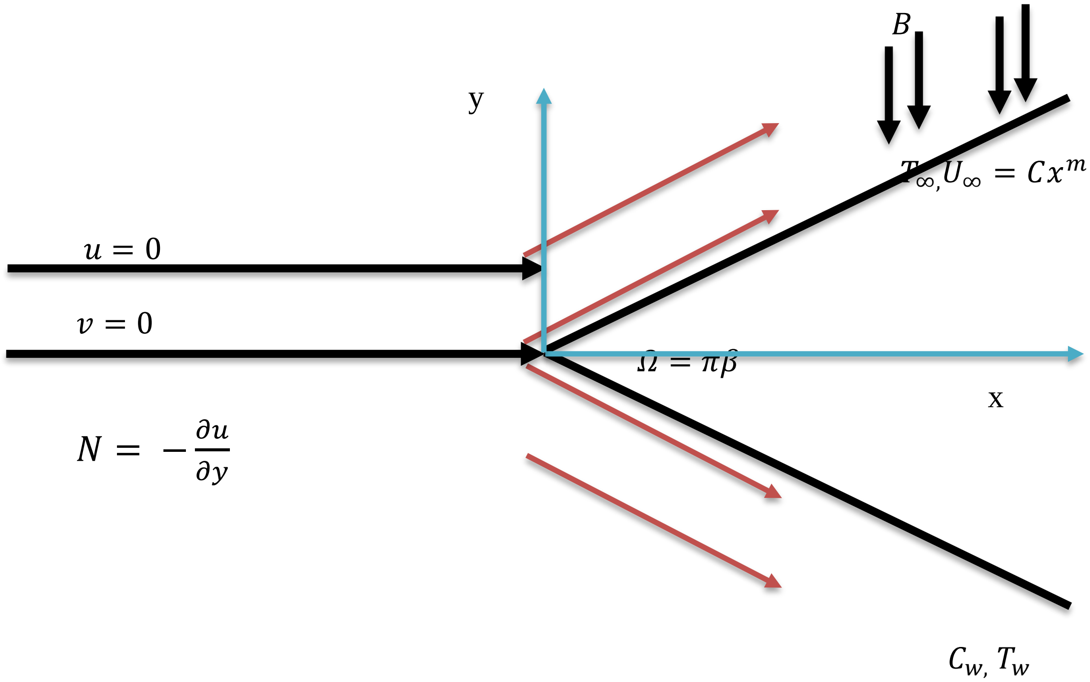

Figure 1: Schematic diagram of the flow

Assumption of the study

i. The magnetic Reynolds number is small enough to neglect the induced magnet field.
ii. The magnetic Reynolds number is small enough to neglect the induced magnet field.
iii. The flow is symmetrical over the wedge of constant angle $\pi\beta$.
iv. The surface temperature T_W is constant.
v. Both pressure and velocity outside the viscosity boundary layer vary with distance x along the wedge.
vi. There is uniform upstream velocity, pressure and temperature

Considering the assumption and using the Buongiorno's model (2006) and Eringen (1965), the equations governing the flow are;

Continuity equation
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_{\infty}\frac{\partial u_{\infty}}{x} + \left(\frac{\mu + s}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{s}{\rho}\frac{\partial N}{\partial y} + \frac{\sigma B^2}{\rho}(u_{\infty} - u) \tag{2}$$

Microrotation equation
$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \left(\frac{\gamma}{j\rho}\right)\frac{\partial^2 N}{\partial y^2} - \left(\frac{s}{j\rho}\right)\left(2N + \frac{\partial u}{\partial y}\right) \tag{3}$$

The energy equation $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left(\frac{D_B}{\Delta C}\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)$ (4)

Concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T\Delta T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-kx^{m-1}(C-C_\infty) \quad (5)$$

From the Bongiorno's model (2006) for a spin proportional to shear then the boundary conditions at the wall and at the stream are given by;

$$u=0,\ \ v=0,\ \ N=-\frac{1}{2}\frac{\partial u}{\partial y},\ \ T=T_w=T_0,\ \ \frac{D_B}{\Delta C}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0;\quad \text{for } y=0$$
$$u\to u_\infty=cx^m, N\to 0, T\to T_\infty, C\to C_\infty;\qquad \text{for}\ \ y\to\infty. \quad (6)$$

Where u and v are the velocities in x and y direction respectively, B is the magnetic current, σ is the Stefan- Boltzman constant and ρ is the density of the micropolar nanofluid, $u_\infty=Cx^m$ is the stream velocity, N is the microrotation vector normal to $x-y$ plane , T is the temperature of the fluid, T_w is the temperature of the wall, j is microrotation (inertia) density, γ is microrotation constant, D_B is Brownian diffusion coefficient, s is the vortex viscosity, μ is the dynamic viscosity, C is the ambient concentration at any reference point, C_W is variable concentration, k is the thermal Conductivity of the fluid and τ is the ratio of the effective heat capacity of the base fluid to the effective heat capacity of the micropolar fluid.

Non-dimesionalization of governing equations

Defining the velocity component in terms of the stream function $\psi=\psi\ (x, y)$ and Letting $u=\frac{\partial\varphi}{\partial y}$ and $v=-\frac{\partial\varphi}{\partial x}$ with the similarity variable,

$$\eta=\left(\frac{c}{\vartheta}\right)^{\frac{1}{2}}x^{(m-1)/2}y,\qquad \psi=(c\vartheta)^{\frac{1}{2}}x^{(m+1)/2}f(\eta),\qquad N=\left(\frac{c^3}{\vartheta}\right)^{\frac{1}{2}}x^{\frac{3m-1}{2}}g(\eta),$$
$$T=T_\infty+\Delta T\cdot\Theta,\qquad C=C_\infty+\Delta C\cdot\Phi \quad (7)$$

where ψ is the stream function, θ is the dimensionless temperature, N is the microrotation velocity and ϕ is the dimensionless nanofluid concentration.
The steady equations are $(1-5)$, subject to the boundary conditions (6), with the similarity variable (7) are non-dimensionalized into the equations;

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=mcx^{m-1}\frac{df}{d\eta}+\frac{m-1}{2}c\left(\frac{c}{\vartheta}\right)^{\frac{1}{2}}x^{\frac{3m-3}{2}}y\frac{d^2f}{d\eta^2}-mcx^{m-1}\frac{df}{d\eta}$$
$$-\frac{m-1}{2}cy\left(\frac{c}{\vartheta}\right)^{\frac{1}{2}}x^{\frac{3m-3}{2}}\frac{d^2f}{d\eta^2}=0 \quad (8)$$

Momentum equation

$$(1+\ R)\frac{d^3f}{d\eta^3}+R\frac{dg}{dy}+M\left(1-\frac{df}{d\eta}\right)-m\left(\frac{df}{d\eta}\right)^2+\frac{m+1}{2}f\frac{d^2f}{d\eta^2}+m=0 \quad (9)$$

Microrotation equation

$$\left(1+\frac{1}{2}R\right)\frac{d^2g}{d\eta^2}-R\left(2g+\frac{d^2f}{d\eta^2}\right)-\frac{3m-1}{2}\frac{df}{d\eta}g+\frac{m+1}{2}\frac{dg}{d\eta}f=0 \quad (10)$$

Energy equation

$$\frac{d^2\Theta}{d\eta^2}+\Pr\left(N_b\frac{d\Theta}{d\eta}\frac{d\Phi}{d\eta}+N_t\left(\frac{d\Theta}{d\eta}\right)^2+\frac{m+1}{2}\frac{d\Theta}{d\eta}f\right)=0 \quad (11)$$

Concentration equation

$$\frac{d^2\Phi}{d\eta^2}+\frac{N_t}{N_b}\frac{d^2\Theta}{d\eta^2}-k^*Sc\Phi+\frac{m+1}{2}Sc\,f\frac{d\Phi}{d\eta}=0 \quad (12)$$

The boundary conditions are non-dimensionalized into:

$$\eta=0:\ \ f=0,\quad \frac{df}{d\eta}=0,\ \ g=-\frac{1}{2}\frac{d^2f}{d\eta^2},\ \ \Theta=1,\ \ \frac{d\Phi}{d\eta}+\frac{N_t}{N_b}\frac{d\Theta}{d\eta}=0 \quad (13)$$
$$\eta=\infty:\ \ \frac{df}{d\eta}=1,\ \ g\to 0,\ \ \Theta\to 0,\ \ \Phi\to 0 \quad (14)$$

and the dimensionless parameters are defined as

$$R=\frac{s}{\rho\vartheta},\ \ M=\frac{\sigma B^2}{\rho c x^{m-1}},\ \ \frac{1}{Pr}=\frac{\alpha}{\vartheta},\quad N_b=\frac{\tau D_B}{\vartheta},\quad N_t=\tau\frac{D_T}{T_\infty}\frac{\Delta T}{\vartheta},$$
$$Sc=\frac{\vartheta}{D_B},\ \ k^*=\frac{k}{c}. \quad (15)$$

Where R is Micropolar parameter, M is Magnetic parameter, P_r Prandtl number, N_b is Brownian motion number, N_t is thermophoresis number and Sc is Schmidt number.

Numerical Solution

The obtained ordinary differential equations (8),(9),(10),(11), and (12)will be solved using the Runge Kutta Forth Order method by taking;

$$\frac{dy}{dx} = f(x, y), y(x_o) = y_o \tag{16}$$

From Taylor series then

$$K_1 = hf(x_{i,}y_i)K_2 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1)$$
$$K_3 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_2)$$
$$K_4 = hf(x_i + h, y_i + K_3)$$

Then $y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$ where $i = 0,1 \dots \dots N - 1$ (17)

This trial and error method converts the boundary value problem to initial value problem. The available boundary conditions determine the initial value problem formed. The resultant boundary value problem is compared with the actual boundary value problem using the trial and error. As the iterations increase the results get closer to the boundary value.

Considering;

$y'' = f(x, y, y'), y(a) = \alpha, y(b) = \beta$ with a unique solution $y(x)$ (18)

Using the shooting technique assuming (17) as the initial value problem, x being the time variable and a and b being are the initial and final values respectively then (18) reduces to

$$y'' = f(x, y, y'), y(a) = \alpha, y'(a) = t \tag{19}$$

Where it is chosen so that the solution satisfies the remaining boundary condition $y(b) = \beta$

The resulting non-dimensionalized equations are solved using the Fourth Order Runge-Kutta technique coupled with the shooting technique which converts the boundary value problems to initial value problem then subjected to the bvp5c solver in MATLAB

Letting

$$x_1 = f, x_2 = f', x_3 = f'', x_4 = g, x_5 = g', x_6 = \Theta, x_7 = \Theta', x_8 = \Phi, x_9 = \Phi' \tag{20}$$

So, we have

$$x_1' = x_2$$
$$x_2' = x_3$$
$$x_3' = -\frac{1}{(1+R)}\left(Rx_5 + M(1 - x_2) - mx_2^2 + \frac{m+1}{2}x_1x_3 + m\right)$$
$$x_4' = x_5$$
$$x_5' = -\frac{1}{\left(1 + \frac{1}{2}R\right)}\left(-R(2x_4 + x_3) - \frac{3m-1}{2}x_2x_4 + \frac{m+1}{2}x_1x_5\right)$$
$$x_6' = x_7$$
$$x_7' = -\Pr\left(N_b x_7 x_9 + N_t x_7^2 + \frac{m+1}{2}x_7x_1\right)$$
$$x_8' = x_9$$

$$x_9' = -\left(\frac{N_t}{N_b}x_7' - k^*Scx_8 + \frac{m+1}{2}Sc\,x_1x_9\right) \tag{21}$$

with the initial and boundary conditions

$$x_1(0) = 0,\ \ x_2(0) = 0,\ \ x_4(0) = -\frac{1}{2}x_3(0),\ \ x_6(0) = 1,$$

$$x_9(0) + \frac{N_t}{N_b}x_7 = 0.$$

$$x_2(\infty) \to 1,\ \ x_4(\infty) \to 0,\ \ x_6(\infty) \to 0,\ \ x_8(\infty) \to 0. \tag{22}$$

Results and Discussion

The interactions between various physical parameters and the flow velocity, microrotation, temperature and nanoparticle concentration profiles are analyzed by solving the dimensionless equations (8) to (12) with the associated condition (13) and (14). The results are illustrated as graphs and the discussion are as follows.

Effect of magnetic parameter on velocity profile, microrotation and temperature

Figures 2 – 3 show that an increase in the magnetic parameter reduces the flow velocity, microrotation, and temperature while the concentration produces dual response. The reduction in velocity due to the increased magnetic field strength shown (Figure 2) can be attributed to the influence of the Lorenz force. The imposition of a transverse magnetic field induces a Lorentz force that ulters the velocity distribution within the boundary layer. The graph indicates that the magnetic parameter M provides practical control over fluid dynamics by reducing both primary and secondary velocities. This reduction occurs due because the Lorentz force generated by the interaction between the magnetic field and the electrically conductive fluid, act as a resistive force that slows down the fluid's motion. As M increases, this resistive force becomes stronger, countering fluid movement, lowering kinetic energy, and thereby reducing flow speeds. These results concur with the findings of Ishak et al (2008) and that of Falkner and Skan (1931) as described in the literature.

The reduced momentum also results in a thinner thermal boundary layer, causing a drop in temperature (Figure 3) and suppressing the microrotation component as shown in Figure 4. A stronger magnetic field results in a decrease in flow temperature. This reduction in temperature can be attributed to the Lorentz force, that acts as a resistive influence on the fluid flow, reducing kinetic energy and thereby diminishing the fluid's ability to transport thermal energy efficiently. Consequently, the fluid experiences significant decrease in temperature. These results are consistent with the findings Ishak (2008) and Falkner and Skan (1931).

Figure 5 demonstrates that an increase in magnetic field strength reduces the concentration of reactive species within the flow. This effect arises because the slower flow, induced by the magnetic resistance, limits the dispersion and mixing of solutes. As a result, the concentration distribution is suppressed, leading to a lower overall concentration gradient within the fluid results that marry with the work of Zulkifli *et al* (2020) who studied of magnetohydrodynamics micropolar nanofluid flow over a wedge with chemical reaction. In their study the concentration increases near the boundary layer but reduces in the free stream concurring with the results in Figure 5.

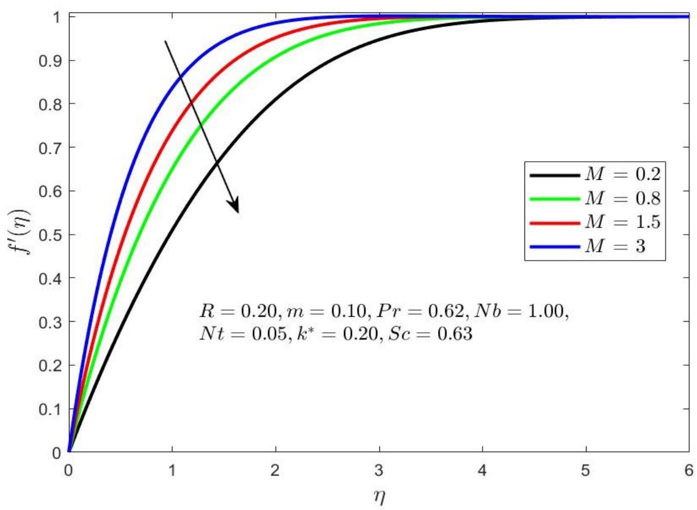

Figure 2: Effect of magnetic field strength on velocity profile

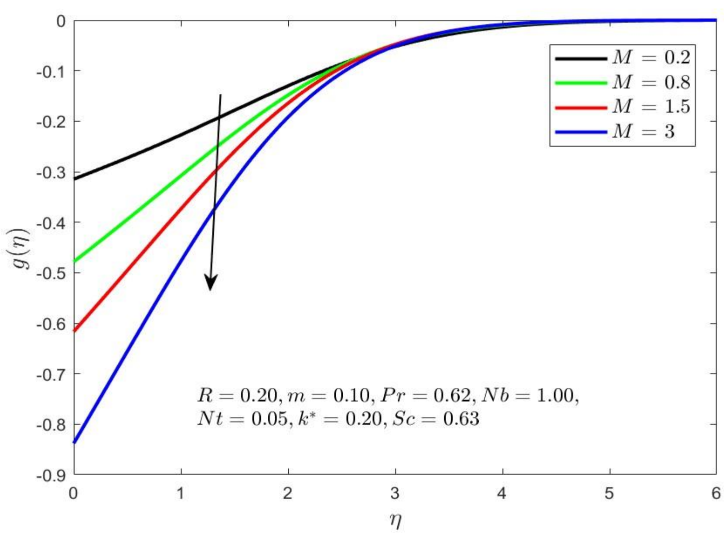

Figure 3: Effect of magnetic field on microrotation profile

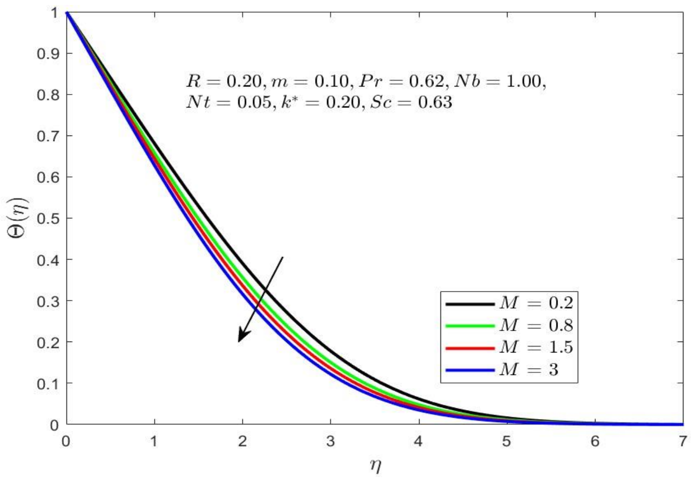

Figure 4: Effect of magnetic field on temperature profile

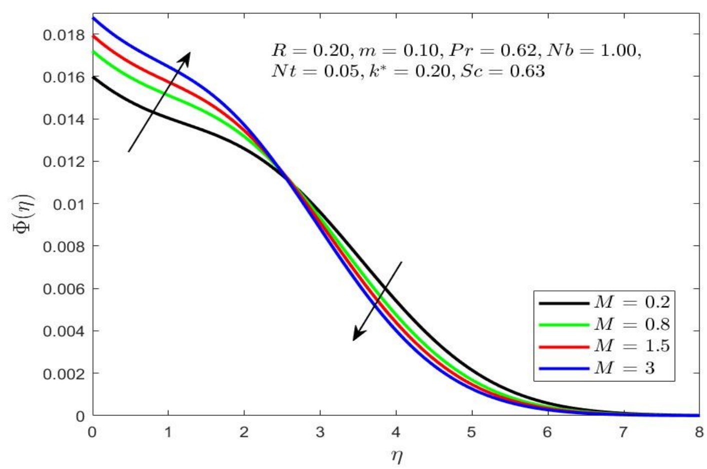

Figure 5: Effect of magnetic field on nanoparticle concentration profile

Effect of wedge parameter on the fluid flow

Figures 6–9 illustrate the responses of the flow to changes in the wedge shape parameter m. With increasing values of the m which characterizes the nature of the pressure gradient along the wedge, the velocity and microrotation profiles increase significantly, as shown in Figures 6 and 7. This is due to the favorable pressure gradient associated with wedge angles, which accelerates the boundary layer flow and intensifies microstructural activity. Correspondingly, the temperature profile decreases as shown in Figure (8), which is expected due to the reduced thermal boundary layer thickness associated with accelerated flow. The concentration profile Φ exhibited non-monotonic behaviour; it initially increases with m but eventually begins to decline. This suggests a competition between enhanced convective transport at lower values of m and reduced nanoparticle diffusion at higher values, possibly due to boundary layer thinning and thermal effects.

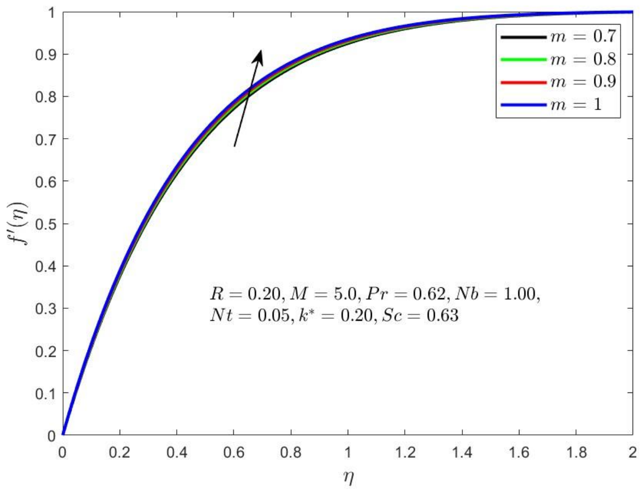

Figure 6: Effect of wedge shape parameter on velocity profile

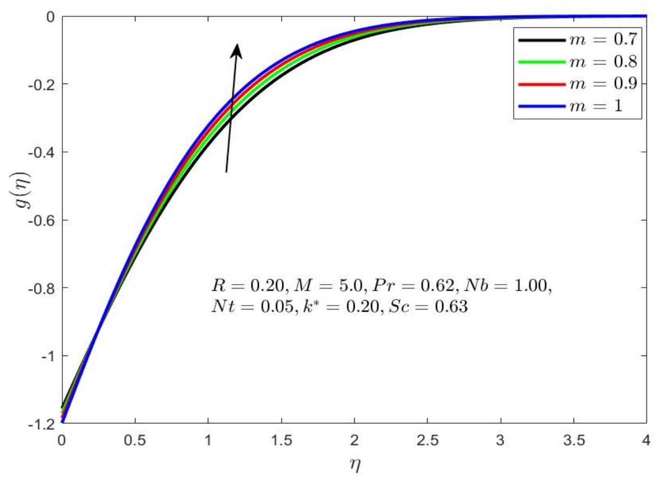

Figure 7: Effect of wedge shape parameter on microrotation profile

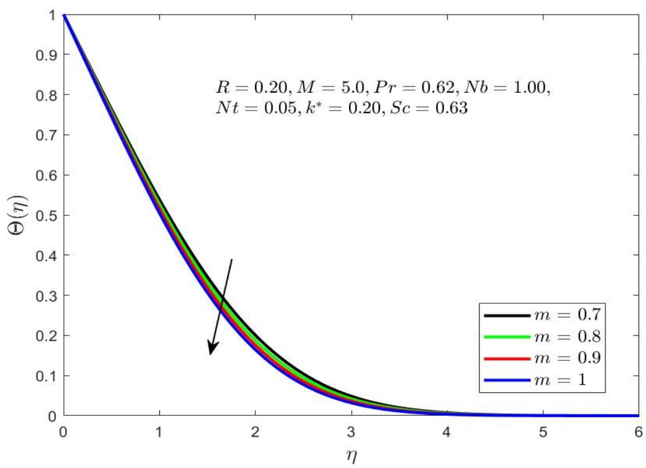

Figure 8: Effect of wedge shape parameter on temperature profile

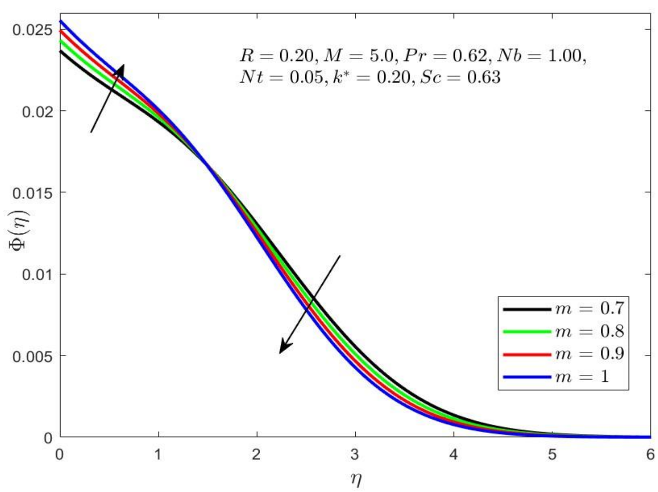

Figure 9: Effect of wedge shape parameter on nanoparticle concentration profile

Effect of microparameter and inertia on velocity profile

The micropolar parameter R reflects the significance of microstructure and spin inertia effects in the fluid, and its impacts on the flow are illustrated in Figures 10 – 13. As R increases, both the velocity and concentration profiles decrease (Figures 10 and 13). The suppression of velocity is a direct consequence of the internal resistance introduced by microelements in the fluid, which tends to dampen the overall flow. The microrotation h, on the other hand, increased due to the stronger influence of microstructural dynamics. The thermal profile θ also rose with increasing R, indicating that the reduction in convective heat transfer caused by lower velocity allowed for greater heat accumulation within the fluid domain. This reduction can be attributed to the increased rotational viscosity within the micropolar fluid, which imposes additional resistance against the fluid's translational motion. These results concur with the findings of Alkavan *et al.* (2012) in their experimental investigation on the convective heat transfer of nanofluid flow inside vertical helically coiled tube under uniform wall temperature. As a result, the fluid particles experience a dampening effect, slowing down the flow in both primary and secondary directions.

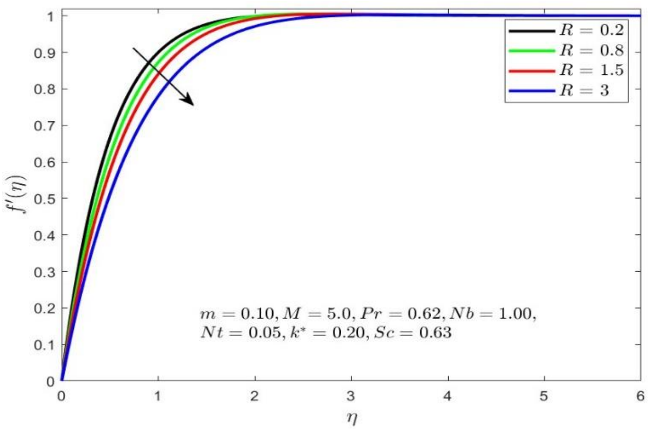

Figure 10: Effect of micropolar parameter on velocity profile

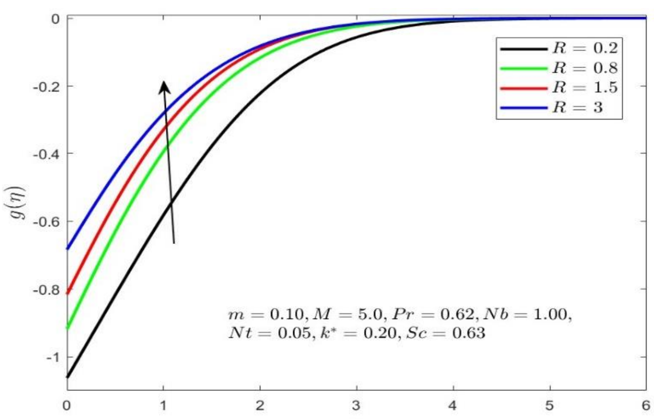

Figure 11: Effect of micropolar parameter on microrotation profile

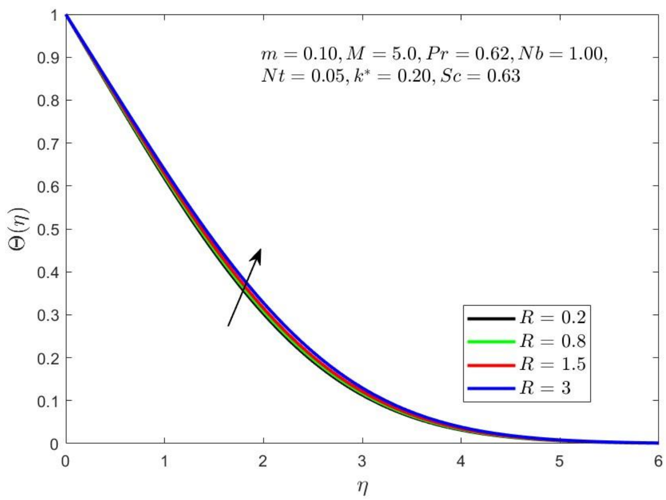

Figure 12: Effect of micropolar parameter on temperature profile

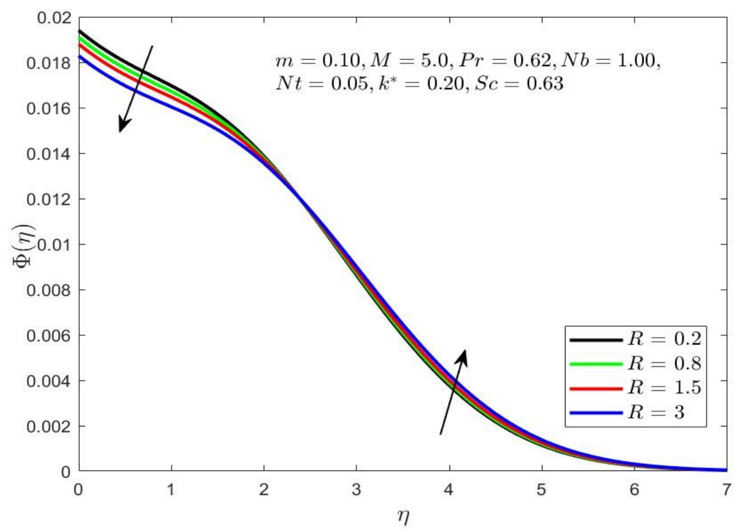

Figure 13: Effect of micropolar parameter on nanoparticle concentration profile

Effect of the skin friction, heat transfer rate and Sherwood number to the wedge parameter and magnetic field parameter

Figures 14 – 16 show the response of the skin friction, heat transfer rate and Sherwood number to the wedge parameter and magnetic field parameter. Figure 14 illustrates the skin friction coefficient as it responds to variations in the magnetic field M and wedge parameter m. The skin friction attains the highest value ($Re^{\frac{1}{2}}C_f = -4.61571$) when m and M are the least (in this case, $m = M = 0.1$). Furthermore, as observed in Figure 14, the wedge parameter increases the skin friction while the magnetic field counteracts this effect by reducing it. The heat transfer and Sherwood numbers are maximum at the highest wedge parameter irrespective of the value of the magnetic field parameter.

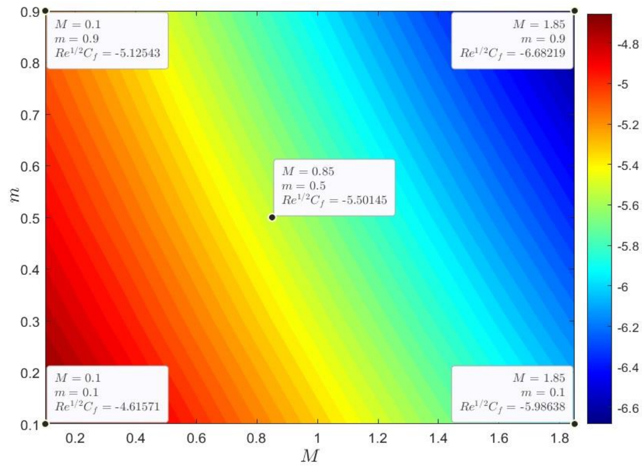

Figure 14: Response of skin friction to both m and M

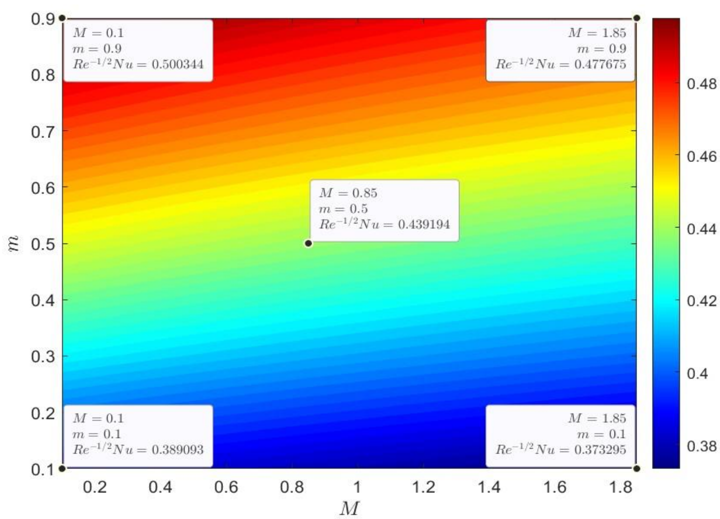

Figure 15: Response of heat transfer to both m and M

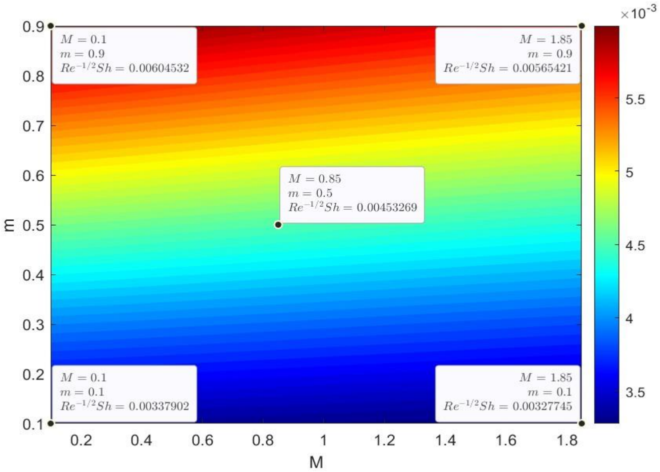

Figure 16: Response of mass transfer to both m and M

Conclusion

This study analyzed the steady flow of a micropolar fluids under the influence of a magnetic field, emphasizing velocity, temperature, and concentration distributions. The governing equations were formulated to account for the effects of magnetic fields, micropolar fluid properties, reaction rates, and the Schmidt number. These equations were nondimensionalized for simplification and solved numerically using the bvp5c solver in MATLAB.

Key findings from the study are as follows:

1. Magnetic Field Influence: An increase in the magnetic parameter reduced both primary and secondary velocities due to the Lorentz force. This decrease in velocity lowered convective heat and mass transfer efficiency, resulting in reduced temperature and solute dispersion. This indicates that the magnetic parameter can be employed as a control variable in regulating fluid flow and transport properties.

2. Micropolar Fluid Properties: Higher values of the micropolar parameter led to reduced velocities in both primary and secondary directions, attributed to enhanced rotational viscosity. Additionally, increased micropolar effects raised the temperature and solute concentration, highlighting the role of micro-rotational effects in modulating energy and mass transfer.

3. Micropolar nanofluid concentration: An increase in the reaction rate parameter reduced temperature and solute concentration due to endothermic reactions and accelerated consumption of reactants. These results underscore the importance of reaction rate control in processes requiring thermal regulation and efficient reactant utilization.

4. Wedge angle parameter boosts the skin friction, fluid velocity and enhances heat and mass transfer regardless of the magnetic field strength.

The numerical approach demonstrated the applicability of MATLAB's bvp5c solver for solving boundary value problems associated with micropolar fluid dynamics. The findings provide a theoretical basis for optimizing systems involving reactive micropolar fluids, such as chemical reactors, cooling systems, and other industrial processes.

Recommendation

Future studies could extend this analysis to incorporate three-dimensional flows, time-dependent boundary conditions, or turbulent flow regimes to enhance the understanding of reactive micropolar fluids under varied conditions.
Further research can be done to include the effects of the nanoparticle properties on the flow of the fluid.

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