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Density Analysis of Basic Graphs and Their Corona Products

Original research papers

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Abstract

This research paper explores the concept of **density of graph**, with a main focus on different graphs and their corona products. We have started with defining a few graphs like; path, cycle, ladder, kite, triangular snake, H, and circular ladder graph along with terms like order, size, and corona product of graph. Then we have investigated the density property of some basic graphs, the corona product of some basic graph with K_1, and corona product of two or more identical basic graphs. This newly derived findings can contribute to understanding some basic properties of graphs. These results can help in future research on graph theory related to density.

Keywords: Density, Corona product, Graph theory, Properties of graph.

1 Introduction

Graph theory is a fundamental branch of discrete mathematics that provides a powerful framework for modeling relationships and interactions in complex systems. Graph theory has evolved into a study with profound significance from Euler's work on the Königsberg bridge problem in the 18th century. All graphs considered in this research are assumed to be undirected, simple, and finite. **Density** is one of the essential structural properties of a graph. In simple language, the density of a graph quantifies the ratio of the number of edges available in the graph to the maximum number of edges. The value of density ranges between 0 and 1. Density of a graph serves as an indicator of the structural complexity and connectivity within a network. This makes it highly valuable in various real world applications. Applications includes social networks, computer networks, biological networks, and transportation systems.

In social networks, a higher density of graphs often reveals tightly connected communities or groups with frequent interactions. In computer networks, analyzing the density helps assess the robustness and fault tolerance of communication infrastructures. In biological networks, dense regions can signify critical functional modules such as protein-protein interaction clusters. In transportation systems, understanding the density of the network aids in evaluating route availability and optimizing infrastructure design. In general, the density of a graph provides meaningful insights into the resilience, efficiency, and patterns of complex systems across multiple disciplines.

One of the core concepts in graph theory is graph density. This measurement offers valuable insight into the structural tightness and connectedness of networks. Among the many ways to generate new graphs, the corona product is noteworthy. While introducing new characteristics, this intricate structures that maintain certain properties of the original graph. By studying the density of corona products of fundamental graphs — like paths, cycles, and complete graphs —researchers can determine key parameters, which are vital for creating realistic models of real-world networks.

These findings examines the density of both basic graphs and their corona products. The goal is to deepen the theoretical understanding of these graph families and to open up new avenues for applications in computer science, network research, and mathematical modeling.

2 Basic Terminology

2.1 Order, Size: (Sango and Jr., 2025)

The **size** of a graph G is the cardinality of its edge set and is denoted by $|E(G)|$.
The **order** of a graph G is the cardinality of its vertex set and is denoted by $|V(G)|$.

2.2 Path Graph (P_n): (Griffin, 2011-2021; West, 2002)

A **path** graph, denoted by P_n, is a graph consisting of a sequence of n distinct vertices in which each vertex(except the first and last) is connected to exactly two others, forming a single, straight-line structure, where $n \geq 2$.

2.3 Cycle Graph (C_n): (Griffin, 2011-2021; West, 2002)

A **cycle** graph, denoted by C_n, is a graph that consists of a single cycle with n vertices. In which each vertex is connected to exactly two other vertices, forming a closed loop, where $n \geq 3$.

2.4 Ladder Graph $(L_{n,1})$: (Gallian, 2000; Moussa and Badr, 2016)

A **ladder** graph, denoted by $L_{n,1}$, is a graph that resembles the structure of a ladder. It is constructed by taking two parallel path graphs P_n and connecting corresponding pairs of vertices with edges, where $n \geq 2$.

2.5 Kite Graph $(K_{n,m})$: (Joshi and Parmar, 2021)

A **kite** graph, denoted by $K_{n,m}$, is a graph formed by joining a complete graph K_n to a path graph P_m by identifying one vertex of K_n with one endpoint of P_m, where $n \geq 3$ and $m \geq 2$.

2.6 Triangular Snake Graph (T_n): (Parmar and Joshi, 2019)

A **triangular snake** graph, denoted by T_n, is a graph constructed by replacing each edge of a path graph P_n with a triangle, where $n \geq 2$.

2.7 H-Graph (H_n): (Joshi and Parmar, 2021)

A **H**-graph, denoted by H_n, is a graph constructed by taking two copies of the path graph P_n and connecting corresponding pairs of middle vertices with an edge, forming a structure resembling the letter "H", where $n \geq 3$.

2.8 Circular Ladder Graph (CL_n): (M. et al., 2014; Chen et al., 2013)

A **circular ladder** graph, denoted by CL_n, is constructed by connecting corresponding vertices of two n-vertex cycle graphs C_n with edges, effectively forming two concentric cycles with additional edges joining corresponding pairs of vertices, where $n \geq 3$.

2.9 Corona Product: (Gallian, 2000; Sango and Jr., 2025)

A **corona product** of two graphs G_1 and G_2, denoted by $G_1 \circ G_2$, is constructed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2, for all $i = 1, 2, \ldots, |V(G_1)|$.

2.10 Density of Graph: (Sango and Jr., 2025; Singh et al., 2020; Frucht and Harary, 1970)

For a graph $G(V, E)$ with n vertices, the **density** D is mathematically expressed as below, where m is the total number of edges in the graph:

$$D = \frac{2m}{n(n-1)}$$

3 Results on Density of Some Graph

Theorem 3.1. *(Triangular Snake Graph T_n) (Parmar and Joshi, 2019; Sango and Jr., 2025)*
For every positive integer $n \geq 2$, the density of the triangular snake graph T_n, denoted by $D(T_n)$, is precisely characterized by the below mathematical expression:

$$D(T_n) = \frac{3}{2n-1}$$

Proof. From the construction and schematic layout of the T_n as depicted, it can be deduced that the size of the graph is $3(n-1)$, while the order of the graph is $2n-1$.

$$\therefore D(T_n) = \frac{2m}{n(n-1)} = \frac{2 \cdot 3(n-1)}{(2n-1)(2n-1-1)} = \frac{3}{2n-1}.$$

□

Theorem 3.2. *(H-Graph H_n) (Joshi and Parmar, 2021; Sango and Jr., 2025)*
For every positive integer $n \geq 3$, the density of the H-graph H_n, denoted by $D(H_n)$, is precisely characterized by the below mathematical expression:

$$D(H_n) = \frac{1}{n}$$

Proof. From the construction and schematic layout of the H_n as depicted, it can be deduced that the size of the graph is $2n-1$, while the order of the graph is $2n$.

$$\therefore D(H_n) = \frac{2m}{n(n-1)} = \frac{2 \cdot (2n-1)}{2n(2n-1)} = \frac{1}{n}.$$

□

Theorem 3.3. *(Kite Graph $K_{3,n}$) (Joshi and Parmar, 2021; Sango and Jr., 2025)*
For every positive integer $n \geq 2$, the density of the kite graph $K_{3,n}$, denoted by $D(K_{3,n})$, is precisely characterized by the below mathematical expression:

$$D(K_{3,n}) = \frac{2}{n+2}$$

Proof. From the construction and schematic layout of the $K_{3,n}$ as depicted, it can be deduced that the size of the graph is $n+3$, while the order of the graph is $n+3$.

$$\therefore D(K_{3,n}) = \frac{2m}{n(n-1)} = \frac{2 \cdot (n+3)}{(n+3)(n+3-1)} = \frac{2}{n+2}.$$

□

Theorem 3.4. *(Circular Ladder Graph CL_n) (M. et al., 2014; Chen et al., 2013; Sango and Jr., 2025)*
For every positive integer $n \geq 3$, the density of the circular ladder graph CL_n, denoted by $D(CL_n)$, is precisely characterized by the below mathematical expression:

$$D(CL_n) = \frac{3}{2n-1}$$

Proof. From the construction and schematic layout of the CL_n as depicted, it can be deduced that the size of the graph is $3n$, while the order of the graph is $2n$.

$$\therefore D(CL_n) = \frac{2m}{n(n-1)} = \frac{2 \cdot 3n}{2n(2n-1)} = \frac{3}{2n-1}.$$

□

Corollary 3.1. *(T_n and CL_n)*
Density of the triangular snake graph T_n and the circular ladder graph CL_n are the same; $n \geq 3$.

$$D(T_n) = D(CL_n) = \frac{3}{2n-1}$$

4 Results on Density of Corona Product of Some Graph with K_1

Theorem 4.1. *($P_n \circ K_1$) (Gallian, 2000; Sango and Jr., 2025; West, 2002)*
For every positive integer $n \geq 2$, the density of the corona product of P_n with K_1, denoted by $D(P_n \circ K_1)$, is precisely characterized by the below mathematical expression:

$$D(P_n \circ K_1) = \frac{1}{n}$$

Proof. From the construction and schematic layout of the $P_n \circ K_1$ as depicted, it can be deduced that the size of the graph is $2n-1$, while the order of the graph is $2n$.

$$\therefore D(P_n \circ K_1) = \frac{2m}{n(n-1)} = \frac{2 \cdot (2n-1)}{2n(2n-1)} = \frac{1}{n}.$$

□

Corollary 4.1. *(H_n and $P_n \circ K_1$)*
Density of the H-graph H_n and the corona product of P_n with K_1 are the same; $n \geq 3$.

$$D(H_n) = D(P_n \circ K_1) = \frac{1}{n}$$

Theorem 4.2. *($C_n \circ K_1$) (Gallian, 2000; Sango and Jr., 2025; West, 2002)*
For every positive integer $n \geq 3$, the density of the corona product of C_n with K_1, denoted by $D(C_n \circ K_1)$, is precisely characterized by the below mathematical expression:

$$D(C_n \circ K_1) = \frac{2}{2n-1}$$

Proof. From the construction and schematic layout of the $C_n \circ K_1$ as depicted, it can be deduced that the size of the graph is $2n$, while the order of the graph is $2n$.

$$\therefore D(C_n \circ K_1) = \frac{2m}{n(n-1)} = \frac{2 \cdot 2n}{2n(2n-1)} = \frac{2}{2n-1}.$$

□

Theorem 4.3. *($L_{n,1} \circ K_1$) (Gallian, 2000; Moussa and Badr, 2016; Sango and Jr., 2025)*
For every positive integer $n \geq 2$, the density of the corona product of $L_{n,1}$ with K_1, denoted by $D(L_{n,1} \circ K_1)$, is precisely characterized by the below mathematical expression:

$$D(L_{n,1} \circ K_1) = \frac{5n-2}{2n(4n-1)}$$

Proof. From the construction and schematic layout of the $L_{n,1} \circ K_1$ as depicted, it can be deduced that the size of the graph is $5n-2$, while the order of the graph is $4n$.

$$\therefore D(L_{n,1} \circ K_1) = \frac{2m}{n(n-1)} = \frac{2 \cdot (5n-2)}{4n(4n-1)} = \frac{5n-2}{2n(4n-1)}.$$

□

Theorem 4.4. *($K_{3,n} \circ K_1$) (Gallian, 2000; Joshi and Parmar, 2021; Sango and Jr., 2025)*
For every positive integer $n \geq 2$, the density of the corona product of $K_{3,n}$ with K_1, denoted by $D(K_{3,n} \circ K_1)$, is precisely characterized by the below mathematical expression:

$$D(K_{3,n} \circ K_1) = \frac{2}{2n+5}$$

Proof. From the construction and schematic layout of the $K_{3,n} \circ K_1$ as depicted, it can be deduced that the size of the graph is $2(n+3)$, while the order of the graph is $2(n+3)$.

$$\therefore D(K_{3,n} \circ K_1) = \frac{2m}{n(n-1)} = \frac{2 \cdot 2(n+3)}{2(n+3)(2(n+3)-1)} = \frac{2}{2n+5}.$$

□

Theorem 4.5. *($T_n \circ K_1$) (Gallian, 2000; Parmar and Joshi, 2019; Sango and Jr., 2025)*
For every positive integer $n \geq 2$, the density of the corona product of T_n with K_1, denoted by $D(T_n \circ K_1)$, is precisely characterized by the below mathematical expression:

$$D(T_n \circ K_1) = \frac{5n-4}{(2n-1)(4n-3)}$$

Proof. From the construction and schematic layout of the $T_n \circ K_1$ as depicted, it can be deduced that the size of the graph is $5n-4$, while the order of the graph is $2(2n-1)$.

$$\therefore D(T_n \circ K_1) = \frac{2m}{n(n-1)} = \frac{2 \cdot (5n-4)}{2(2n-1)(2(2n-1)-1)} = \frac{5n-4}{(2n-1)(4n-3)}.$$

□

Theorem 4.6. *($H_n \circ K_1$) (Gallian, 2000; Joshi and Parmar, 2021; Sango and Jr., 2025)*
For every positive integer $n \geq 3$, the density of the corona product of H_n with K_1, denoted by $D(H_n \circ K_1)$, is precisely characterized by the below mathematical expression:

$$D(H_n \circ K_1) = \frac{1}{2n}$$

Proof. From the construction and schematic layout of the $H_n \circ K_1$ as depicted, it can be deduced that the size of the graph is $4n-1$, while the order of the graph is $4n$.

$$\therefore D(H_n \circ K_1) = \frac{2m}{n(n-1)} = \frac{2 \cdot (4n-1)}{4n(4n-1)} = \frac{1}{2n}.$$

□

5 Results on Density of Corona Product of two same Graph

Theorem 5.1. *($T_n \circ T_n$) (Gallian, 2000; Parmar and Joshi, 2019; Sango and Jr., 2025)*
For every positive integer $n \geq 2$, the density of the corona product of two T_n graphs, denoted by $D(T_n \circ T_n)$, is precisely characterized by the below mathematical expression:

$$D(T_n \circ T_n) = \frac{10n(n-1)+1}{n(2n-1)(4n^2-2n-1)}$$

Proof. From the construction and schematic layout of the $T_n \circ T_n$ as depicted, it can be deduced that the size of the graph is $10n^2-10n+1$, while the order of the graph is $2n(2n-1)$.

$$\therefore D(T_n \circ T_n) = \frac{2m}{n(n-1)} = \frac{2 \cdot (10n^2-10n+1)}{2n(2n-1)(2n(2n-1)-1)} = \frac{10n(n-1)+1}{n(2n-1)(4n^2-2n-1)}.$$

□

Example 5.1. *For the graph $T_3 \circ T_3$*

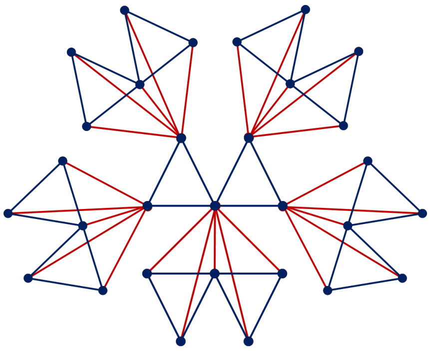

Figure 1: Graph of $T_3 \circ T_3$

Using definition,

$$D(T_3 \circ T_3) = \frac{2m}{n(n-1)} = \frac{2 \cdot 61}{30 \cdot 29} = \frac{61}{435}$$

Using theorem,

$$D(T_3 \circ T_3) = \frac{10n(n-1)+1}{n(2n-1)(4n^2-2n-1)} = \frac{10 \cdot 3(3-1)+1}{3(2 \cdot 3-1)(4 \cdot 3^2 - 2 \cdot 3 - 1)} = \frac{61}{435}$$

Theorem 5.2. *($H_n \circ H_n$) (Gallian, 2000; Joshi and Parmar, 2021; Sango and Jr., 2025)*
For every positive integer $n \geq 3$, the density of the corona product of two H_n graphs, denoted by $D(H_n \circ H_n)$, is precisely characterized by the below mathematical expression:

$$D(H_n \circ H_n) = \frac{8n^2-1}{n(2n+1)(4n^2+2n-1)}$$

Proof. From the construction and schematic layout of the $H_n \circ H_n$ as depicted, it can be deduced that the size of the graph is $8n^2 - 1$, while the order of the graph is $2n(2n+1)$.

$$\therefore D(H_n \circ H_n) = \frac{2m}{n(n-1)} = \frac{2 \cdot (8n^2-1)}{2n(2n+1)(2n(2n+1)-1)} = \frac{8n^2-1}{n(2n+1)(4n^2+2n-1)}.$$

□

Example 5.2. *For the graph $H_3 \circ H_3$*

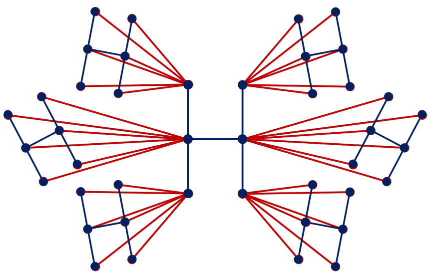

Figure 2: Graph of $H_3 \circ H_3$

Using definition,

$$D(H_3 \circ H_3) = \frac{2m}{n(n-1)} = \frac{2 \cdot 71}{42 \cdot 41} = \frac{71}{861}$$

Using theorem,

$$D(H_3 \circ H_3) = \frac{8n^2 - 1}{n(2n+1)(4n^2 + 2n - 1)} = \frac{8 \cdot 3^2 - 1}{3(2 \cdot 3 + 1)(4 \cdot 3^2 + 2 \cdot 3 - 1)} = \frac{71}{861}$$

Theorem 5.3. *($K_{3,n} \circ K_{3,n}$) (Gallian, 2000; Joshi and Parmar, 2021; Sango and Jr., 2025)*
For every positive integer $n \geq 2$, the density of the corona product of two $K_{3,n}$ graphs, denoted by $D(K_{3,n} \circ K_{3,n})$, is precisely characterized by the below mathematical expression:

$$D((K_{3,n} \circ K_{3,n}) = \frac{2(2n+7)}{(n+4)(n^2+7n+11)}$$

Proof. From the construction and schematic layout of the $K_{3,n} \circ K_{3,n}$ as depicted, it can be deduced that the size of the graph is $(n+3)(2n+7)$, while the order of the graph is $(n+3)(n+4)$.

$$\therefore D(K_{3,n} \circ K_{3,n}) = \frac{2m}{n(n-1)} = \frac{2(n+3)(2n+7)}{(n+3)(n+4)((n+3)(n+4)-1)} = \frac{2(2n+7)}{(n+4)(n^2+7n+11)}.$$

□

Example 5.3. *For the graph $K_{3,2} \circ K_{3,2}$*

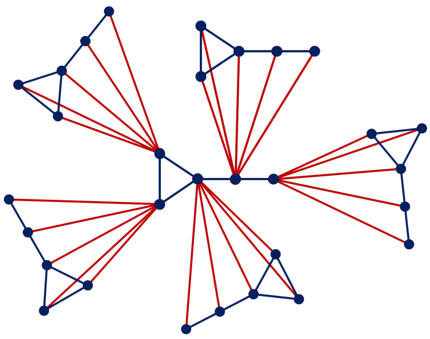

Figure 3: Graph of $K_{3,2} \circ K_{3,2}$

Using definition,

$$D(K_{3,2} \circ K_{3,2}) = \frac{2m}{n(n-1)} = \frac{2 \cdot 55}{30 \cdot 29} = \frac{11}{87}$$

Using theorem,

$$D(K_{3,2} \circ K_{3,2}) = \frac{2(2n+7)}{(n+4)(n^2+7n+11)} = \frac{2(2 \cdot 2+7)}{(2+4)(2^2+7 \cdot 2+11)} = \frac{11}{87}$$

Theorem 5.4. *($(P_n \circ K_1) \circ (P_n \circ K_1)$) (Gallian, 2000; Sango and Jr., 2025; West, 2002)*
For every positive integer $n \geq 2$, the density of the corona product of two $P_n \circ K_1$ graphs, denoted by $D((P_n \circ K_1) \circ (P_n \circ K_1))$, is precisely characterized by the below mathematical expression:

$$D((P_n \circ K_1) \circ (P_n \circ K_1)) = \frac{8n^2 - 1}{n(2n+1)(4n^2+2n-1)}$$

Proof. From the construction and schematic layout of the $(P_n \circ K_1) \circ (P_n \circ K_1)$ as depicted, it can be deduced that the size of the graph is $8n^2 - 1$, while the order of the graph is $2n(2n + 1)$.

$$\therefore D((P_n \circ K_1) \circ (P_n \circ K_1)) = \frac{2m}{n(n-1)} = \frac{2 \cdot (8n^2 - 1)}{2n(2n+1)(2n(2n+1)-1)} = \frac{8n^2 - 1}{n(2n+1)(4n^2+2n-1)}.$$

□

Example 5.4. *For the graph* $(P_3 \circ K_1) \circ (P_3 \circ K_1)$

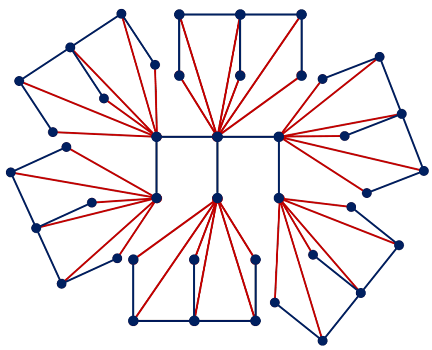

Figure 4: Graph of $(P_3 \circ K_1) \circ (P_3 \circ K_1)$

Using definition,

$$D((P_3 \circ K_1) \circ (P_3 \circ K_1)) = \frac{2m}{n(n-1)} = \frac{2 \cdot 71}{42 \cdot 41} = \frac{71}{861}$$

Using theorem,

$$D((P_3 \circ K_1) \circ (P_3 \circ K_1)) = \frac{8n^2 - 1}{n(2n+1)(4n^2+2n-1)} = \frac{8 \cdot 3^2 - 1}{3(2 \cdot 3 + 1)(4 \cdot 3^2 + 2 \cdot 3 - 1)} = \frac{71}{861}$$

Corollary 5.1. *(* $H_n \circ H_n$ *and* $(P_n \circ K_1) \circ (P_n \circ K_1)$ *)*
Density of the two corona product of H_n *and the two corona product of* $P_n \circ K_1$ *are the same;* $n \geq 3$.

$$D(H_n \circ H_n) = D((P_n \circ K_1) \circ (P_n \circ K_1)) = \frac{8n^2 - 1}{n(2n+1)(4n^2+2n-1)}$$

6 CONCLUSION

This study presents the density analysis of the corona product of various standard graphs with the complete graph K_1 as well as the corona product of two identical graphs. Furthermore, the investigation of the corona product of two similar graphs highlights the significant structural changes that occur in terms of connectivity and compactness. These findings may also be helpful for researchers working in areas where graph density plays a key role, such as network analysis, computer science, and structural modeling. Future work may involve extending this approach to other graph classes or analyzing the practical relevance of these density changes in real-world networks.

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