Nonparametric Estimation of Laplace Transform of the Ruin Probability from Empirical Data Using symmetric Kernel

Abstract

This paper addresses the estimation of ruin probability, a central issue in actuarial science and financial risk management. A major difficulty lies in the limited knowledge of the claim size distribution, which often restricts the applicability of classical parametric methods. To overcome this, we introduce a nonparametric kernel estimator for the Laplace transform of the new finite-time ruin probability in the classical Cramér–Lundberg model. The asymptotic properties of the estimator are investigated through the computation of the Mean Integrated Squared Error (MISE), highlighting improved accuracy and stability compared to traditional approaches. Simulation studies confirm the method's reliability and robustness across different scenarios. Beyond its theoretical contribution, the results provide insurers with more flexible and accurate tools for risk evaluation and financial stability assessment.

keywords: Nonparametric estimation, Laplace transform, symmetric kernel, unknown density, ruin probability .

2020 AMS Subject Classification: 60E10; 62G05; 62G07.

*

1 Introduction

The assessment of ruin probability is a central problem in insurance mathematics, where accurate modeling of the claim size distribution plays a crucial role. Nonparametric density estimation has therefore attracted considerable attention. Kernel methods, in particular, have been applied in various contexts: Touazi et al. (2016) studied strong stability of ruin probabilities under the Sparre Andersen model, while Touazi et al. (2014) considered the compound Poisson risk model with an unknown claim size distribution, estimated using modified gamma kernels. Extensions with inverse gamma kernels (Mousa, Hassan, and Fathi 2016) and semi-parametric approaches for large claims (Harfouche and Bareche 2021) further enriched this line of research.

More recently, advances include distributed nonparametric estimation achieving minimax optimal rates (Yuan, Guo, and Huang 2025), and the *Score-Debiased Kernel Density Estimation (SD-KDE)* method (Epstein et al. 2025), which reduces bias in kernel density estimation and improves performance, particularly in terms of the Mean Integrated Squared Error (MISE).

In the context of ruin probability, the latter is most often expressed via its Laplace transform. Recently, Sun and Zhang (2024) investigated the Laplace transform of ruin time in models with Parisian delays, while Antipov (2025) applied Laplace transform techniques to stochastic claims and dynamic reserves. These studies underscore the versatility of Laplace-based approaches for modeling complex risk processes. The main challenge thus remains the estimation of the Laplace transform of the claim size distribution, a crucial step before approximating the Laplace transform of the ruin probability. Mnatsakanov, Ruymgaart, and Ruymgaart (2008) developed an inversion method for the Laplace transform that enables the estimation of a probability density function from its Laplace transform. Building on this idea, Shimizu (2012) investigated a nonparametric estimation of the Gerber–Shiu discounted penalty function in the framework of Lévy processes. Along similar lines, Mnatsakanov and Elmagbri (2018) proposed a novel nonparametric procedure for density estimation of positive random variables based on Laplace transform inversion and moment recovery techniques.

More recently, Kabore et al. (2025) introduced a nonparametric estimator of the Laplace transform of an unknown density via symmetric kernels (e.g., Gaussian). Their work provides an explicit expression of the AMISE and a data-driven bandwidth selection rule, validated through simulations and practical applications.

In this article, we build on the results of Kabore et al. (2025) to estimate the Laplace transform of the claim size distribution, and then extend this approach to the estimation of the Laplace transform of the ruin probability. We consider the surplus process of an insurance company, defined under the classical Cramér–Lundberg model in continuous time t:

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i, \quad t \geq 0 \tag{1}$$

where : u and c are positive real numbers , representing respectively the initial reserves of the company and the premium rate;
$\{N(t)\}_{t\geq 0}$ is a homogeneous Poisson process with intensity λ, modeling the number of claims occurring up to time t;
$\{X_i\}_{i\geq 1}$ is a sequence of positive, independent and identically distributed (i.i.d.) random variables, with distribution function F_X, density f_X, and mean m, representing the claim amounts.
The finite-time ruin probability at time T is defined as:

$$R(u,T) = \mathbb{P}(\exists t \in [0,T], \quad U(t) < 0). \tag{2}$$

This quantity satisfies the following integro-differential equation:

$$\begin{aligned} R(u,T) = &\int_0^T \lambda e^{-\lambda t} (1 - F_X(u+ct))\, dt \\ &+ \int_0^T \lambda e^{-\lambda t} \int_0^{u+ct} R(u+ct-x,T)\, dF_X(x)\, dt. \end{aligned} \tag{3}$$

We focus on the Laplace transforms of the ruin probability and of the claim size density:

$$\begin{cases} \rho(s,T) = \int_0^\infty R(u,T) e^{-us}\, du \\ \varphi(s) = \int_0^\infty f_X(u) e^{-us}\, du \end{cases}, \quad s > 0. \tag{4}$$

These quantities are estimated respectively by $\hat{\rho}_n(s,T)$ and $\hat{\varphi}_n(s)$ using the continuous symmetric kernel method. In Kabore et al. (2025), the AMISE and the convergence rate of the estimator $\hat{\varphi}_n(s)$ are expressed in terms of the Laplace transform:

$$\begin{aligned} \text{AMISE}(h) = &\frac{\theta_K}{nh} \int_{\mathbb{T}_{\mathbb{C}_+}} \left(\int_{\mathbb{T}} e^{-xs} \varphi(x,s)\, dx \right) ds \\ &+ \frac{\sigma_K^4 h^4}{4} \int_{\mathbb{T}_{\mathbb{C}_+}} \left(\int_{\mathbb{T}} \varphi''(x,s)\, dx \right)^2 ds, \end{aligned} \tag{5}$$

where :K is the continuous symmetric kernel, $\sigma_K^2 = \int_{\mathbb{T}} w^2 K(w)\, dw < \infty$, $\theta_K = \int_{\mathbb{T}} K^2(w)\, dw < \infty$, $\varphi(x,s) = e^{-sx} f_X(x)$, $\varphi''(x,s) = \frac{\partial^2}{\partial x^2} \varphi(x,s)$. The optimal bandwidth is given by:

$$h_{\text{opt}} = n^{-1/5} \left(\frac{\theta_K \int_{\mathbb{T}_{\mathbb{C}_+}} \left(\int_{\mathbb{T}} e^{-xs} \varphi(x,s)\, dx \right) ds}{\sigma_K^4 \int_{\mathbb{T}_{\mathbb{C}_+}} \left(\int_{\mathbb{T}} \varphi''(x,s)\, dx \right)^2 ds} \right)^{1/5}. \tag{6}$$

The main objective of this article is to estimate the Laplace transform of the ruin probability $\rho(s,T)$ using the continuous symmetric kernel method, and then to apply the inversion procedure of Abate and Whitt (1995) to obtain an estimate of the finite-time ruin probability $R(u,T)$.

Recent developments in numerical Laplace transform inversion have introduced efficient and stable approaches. Maréchal, Triki, and Lee (2023) proposed a regularization method via mollification to improve convergence in unstable numerical environments. Henríquez and Hesthaven (2024) combined Laplace transforms with reduced-order modeling for rapid and accurate approximation of linear hyperbolic problems, with potential applications to solvency modeling. Alexopoulos (2025) investigated uniform convergence in Fourier inversion formulas applied to Laplace transforms, extending results to Banach-space valued functions and semigroup frameworks. These studies collectively enhance both the theoretical understanding and practical computation of Laplace transform inversions in risk and actuarial contexts.

The originality of our contribution lies in the direct estimation of $\rho(s,T)$ using a Gaussian kernel while taking into account the complex variable s. Our results are of practical importance for insurance companies: when the claim size distribution is unknown, our approach provides a method to estimate this distribution, thereby allowing the anticipation and evaluation of the ruin probability.

The article is organized as follows. Section 2 presents an explicit formulation of the Laplace transform of the ruin probability, derived here for the first time. Section 3 is devoted to the convergence analysis and the study of the Integrated Squared Error (ISE) of the estimators $\hat{\rho}_n(s,T)$ and $\hat{R}_n(u,T)$. Section 4 provides numerical simulation results. Finally, Section 5 concludes the paper.

We work within the framework of the Lebesgue measure, under the following notations and assumptions.

Notations

$K(\cdot)$ denotes a continuous symmetric kernel with support $\mathbb{S}$, bounded on at least one side. $\mathbb{R}^+$ represents the set of positive real numbers. $\mathbb{E}$ and $\mathbb{V}\mathrm{ar}$ denote the expectation and variance operators, respectively. $\mathbb{C}$ is the set of complex numbers, and $\mathbb{T}_{\mathbb{C}_+} = \{s \in \mathbb{C} \mid \Re(s) > 0\}$. L is the space of integrable functions, while L^2 is the space of square-integrable functions.

Assumptions

(A_1): f_X admits a Laplace transform. (A_2): $\varphi(s) \in L(\mathbb{T}_{\mathbb{C}_+})$. (A_3): $\varphi(s) \in L^2(\mathbb{T}_{\mathbb{C}_+})$. (A_4): $\int_{\mathbb{T}_{\mathbb{C}_+}} \varphi(s)\,ds \leq 1$. (A_5): the estimators $\hat{\varphi}_1(s),\ldots,\hat{\varphi}_n(s)$ are i.i.d.

2 Laplace transform of the finite-time ruin probability

In this section, we seek an expression for the Laplace transform of the ruin probability that does not depend on the premium rate c, on the ruin probability at $u=0$, $R(0,T)$, and in which λ and ξ are treated as constants. Following the approach of

García (2005), we then consider equality (3) to establish the Laplace transform of the finite-time ruin probability T through the following lemma:

Lemma 1. *(Finite-Time Ruin Probability)*
Let $R(u,T)$ denote the finite-time ruin probability for T, with $u \geq 0$ representing the initial surplus. Its Laplace transform is given by:

$$\rho(s,T) = \frac{\left(1-e^{-\lambda T}\right)\left[1-ms-\varphi(s)\right]}{s\left[1+\left(e^{-\lambda T}-1\right)\varphi(s)-(1+\xi)\,ms\right]}, \quad s>0, \tag{7}$$

where:
$\lambda > 0$ *is the parameter of the homogeneous Poisson process,*
$\xi > 0$ *is the safety loading,*
m is the mean of the claim amounts,
$\varphi(s)$ *is the Laplace transform of the claim size density f_X.*

Proof
The finite-time ruin probability, denoted by $R(u,T)$, is defined as the probability that the surplus of an insurance company reaches zero or falls below zero within a given time interval $t \in [0,T]$, with u representing the initial surplus.
By conditioning on the time and on the occurrence of the first two claims, we have:
The probability of ruin after the occurrence of the first claim, without ruin occurring beforehand, is:

$$\begin{aligned}
R_1(u,T) &= P\{u+ct-X_1<0; t\in[0;T]\} \\
&= P\{X_1>u+ct; t\in[0;T]\} \\
&= P\{(X_1>u+ct)\cap(t\in[0;T])\} \\
&= \int_0^T \lambda e^{-\lambda t}\int_{u+ct}^{\infty} f_X(x)\,dx\,dt \\
&= \int_0^T \lambda e^{-\lambda t}\left[\int_{u+ct}^{0} f_X(x)\,dx+\int_0^{\infty} f_X(x)\,dx\right]dt \\
&= \int_0^T \lambda e^{-\lambda t}\left[\int_{u+ct}^{0} f_X(x)\,dx+1\right]dt \\
&= \int_0^T \lambda e^{-\lambda t}\left[1-\int_0^{u+ct} f_X(x)\,dx\right]dt \\
&= \int_0^T \lambda e^{-\lambda t}\left[1-F_X(u+ct)\right]dt. \qquad (8)
\end{aligned}$$

Since ruin can occur depending on the size of the first claim, and given that the process restarts at zero after the occurrence of a claim, the probability of ruin after the occurrence of the first claim and after the occurrence of the second claim is:

$$
\begin{aligned}
R(u,T) &= R_1(u,T)+P\{u+ct-X_1-X_2<0; t\in[0;T]\}\\
&= R_1(u,T)+P\{X_2>u+ct-X_1; t\in[0;T]\}\\
&= R_1(u,T)+P\{(X_2>u+ct-X_1)\cap(t\in[0;T])\}\\
&= R_1(u,T)+\int_0^T \lambda e^{-\lambda t}\int_0^{u+ct} R(u+ct-x,T)\,dF_X(x)dt\\
&= \int_0^T \lambda e^{-\lambda t}[1-F_X(u+ct)]\,dt+\int_0^T \lambda e^{-\lambda t}\int_0^{u+ct} R(u+ct-x,T)\,dF_X(x)dt.
\end{aligned} \tag{9}
$$

By setting $y=u+ct$, equality (9) becomes:

$$
\begin{aligned}
R(u,T) &= \frac{\lambda}{c}\int_u^{u+cT} e^{-\frac{\lambda}{c}(y-u)}[1-F_X(y)]\,dy\\
&\quad+\frac{\lambda}{c}\int_u^{u+cT} e^{-\frac{\lambda}{c}(y-u)}\int_0^y R(y-x,T)dF_X(x)dy\\
&= \frac{\lambda}{c}e^{\frac{\lambda}{c}u}\left(\int_u^{u+cT} e^{-\frac{\lambda}{c}y}[1-F_X(y)]\,dy+\int_u^{u+cT} e^{-\frac{\lambda}{c}y}\int_0^y R(y-x,T)dF_X(x)dy\right).
\end{aligned} \tag{10}
$$

By differentiating both sides of equality (10) with respect to u, we obtain:

$$
\begin{aligned}
\frac{d}{du}R(u,T) &= \left(\frac{\lambda}{c}\right)^2 e^{\frac{\lambda}{c}u}\left(\int_u^{u+cT} e^{-\frac{\lambda}{c}y}[1-F_X(y)]\,dy+\int_u^{u+cT} e^{-\frac{\lambda}{c}y}\int_0^y R(y-x,T)dF_X(x)dy\right)\\
&\quad+\frac{\lambda}{c}e^{\frac{\lambda}{c}u}\left(\frac{d}{du}\int_u^{u+cT} e^{-\frac{\lambda}{c}y}[1-F_X(y)]\,dy+\frac{d}{du}\int_u^{u+cT} e^{-\frac{\lambda}{c}y}\int_0^y R(y-x,T)dF_X(x)dy\right),
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
\frac{d}{du}\int_u^{u+cT} e^{-\frac{\lambda}{c}y}[1-F_X(y)]\,dy &= e^{-\frac{\lambda}{c}(u+cT)}[1-F_X(u+cT)]\left[\frac{d}{du}(u+cT)\right]\\
&\quad-e^{-\frac{\lambda}{c}u}[1-F_X(u)]\left(\frac{d}{du}u\right)\\
&\quad+\int_u^{u+cT}\frac{d}{du}\left([1-F_X(y)]e^{-\frac{\lambda}{c}y}\right)dy\\
&= \left([1-F_X(u+cT)]e^{-\lambda T}-1+F_X(u)\right)e^{-\frac{\lambda}{c}u},
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
\frac{d}{du}\int_u^{u+cT} e^{-\frac{\lambda}{c}y}\int_0^y R(y-x,T)dF_X(x)dy &= e^{-\frac{\lambda}{c}(u+cT)}\int_0^{u+cT} R(u+cT-x,T)dF_X(x)\left[\frac{d}{du}(u+cT)\right] \\
&\quad -e^{-\frac{\lambda}{c}u}\int_0^u R(u-x,T)dF_X(x)\left(\frac{d}{du}u\right) \\
&\quad +\int_u^{u+cT}\frac{d}{du}\left(e^{-\frac{\lambda}{c}y}\int_0^y R(y-x,T)dF_X(x)\right)dy \\
&= e^{-\lambda T}e^{-\frac{\lambda}{c}u}\int_0^{u+cT} R(u+cT-x,T)dF_X(x) \\
&\quad -e^{-\frac{\lambda}{c}u}\int_0^u R(u-x,T)dF_X(x). \qquad (13)
\end{aligned}
$$

From equalities (10), (11), (12), and (13), we obtain:

$$
\begin{aligned}
\frac{d}{du}R(u,T) &= \frac{\lambda}{c}R(u,T) \\
&\quad +\frac{\lambda}{c}e^{\frac{\lambda}{c}u}\left([1-F_X(u+cT)]e^{-\lambda T}-1+F_X(u)\right)e^{-\frac{\lambda}{c}u} \\
&\quad +\frac{\lambda}{c}e^{\frac{\lambda}{c}u}\left(e^{-\lambda T}e^{-\frac{\lambda}{c}u}\int_0^{u+cT} R(u+cT-x,T)dF_X(x)\right) \\
&\quad -\frac{\lambda}{c}e^{\frac{\lambda}{c}u}\left(e^{-\frac{\lambda}{c}u}\int_0^u R(u-x,T)dF_X(x)\right) \\
&= \frac{\lambda}{c}R(u,T) \\
&\quad +\frac{\lambda}{c}\left([1-F_X(u+cT)]e^{-\lambda T}-1+F_X(u)\right) \\
&\quad +\frac{\lambda}{c}e^{-\lambda T}\int_0^{u+cT} R(u+cT-x,T)dF_X(x) \\
&\quad -\frac{\lambda}{c}\int_0^u R(u-x,T)dF_X(x).
\end{aligned}
$$

(14)

By applying the Laplace transform to equality (14) and using the properties of the Laplace transform of a derivative, let us set:

$$
\begin{cases}
\rho(s,T) = \int_0^\infty R(u,T)e^{-us}du \\
\varphi(s) = \int_0^\infty f_X(u)e^{-us}du,
\end{cases} \qquad (15)
$$

we have:

$$
\int_0^\infty \left\{\frac{d}{du}R(u,T)\right\}e^{-us}du = -R(0,T)+s\rho(s,T), \qquad (16)
$$

$$\begin{aligned}
\int_0^\infty \left\{[1-F_X(u+cT)]\,e^{-\lambda T}-[1-F_X(u)]\right\}e^{-us}du &= \\
e^{-\lambda T}\int_0^\infty [1-F_X(u+cT)]\,e^{-us}du \\
-\int_0^\infty [1-F_X(u)]\,e^{-us}du &= \\
\frac{1}{s}[1-\varphi(s)]\,e^{-\lambda T}-\frac{1}{s}[1-\varphi(s)] &= \\
\frac{1}{s}\left(e^{-\lambda T}-1\right)[1-\varphi(s)], \qquad (17)
\end{aligned}$$

$$\begin{aligned}
\int_0^\infty \left\{e^{-\lambda T}\int_0^{u+cT} R(u+cT-x,T)dF_X(x)\right\}e^{-us}du &= \\
e^{-\lambda T}\int_0^\infty \left\{\int_x^\infty e^{-us}R(u+cT-x,T)dudF_X(x)\right\} &= \\
\left(e^{-xs}e^{xs}\right)e^{-\lambda T}\int_0^\infty \left\{\int_0^{u+cT} R(u+cT-x,T)dF_X(x)\right\}e^{-us}du &= \\
e^{-\lambda T}\int_0^\infty e^{-xs}dF_X(x)\left\{\int_x^\infty R(u+cT-x,T)\right\}e^{-us}e^{xs}du &= \\
e^{-\lambda T}\varphi(s)\int_x^\infty R(u-x+cT,T)e^{-(u-x)s}du &= \\
e^{-\lambda T}\varphi(s)\int_0^\infty R(w+cT,T)e^{-ws}dw &= \\
e^{-\lambda T}\varphi(s)\int_0^\infty R(w+cT,T)e^{-ws}dw &= \\
e^{-\lambda T}\varphi(s)\rho\,(s,T), \qquad (18)
\end{aligned}$$

$$\begin{aligned}
\int_0^\infty e^{-us}\int_0^u R(u-x,T)dF_X(x)du &= \int_0^\infty e^{-us}\int_x^\infty R(u-x,T)dudF_X(x) \\
&= \int_0^\infty dF_X(x)\int_x^\infty R(u-x,T)e^{-us}du \\
&= e^{-xs}e^{xs}\int_0^\infty dF_X(x)\int_x^\infty R(u-x,T)e^{-us}du \\
&= \int_0^\infty e^{-xs}dF_X(x)\int_x^\infty R(u-x,T)e^{-(u-x)s}du \\
&= \varphi(s)\int_0^\infty R(w,T)e^{-ws}dw \\
&= \varphi(s)\rho\,(s,T). \qquad (19)
\end{aligned}$$

From equalities (15), (16), (17), (18), and (19), the Laplace transform of equality

(14) is:

$$-R(0,T)+s\rho(s,T) = \frac{\lambda}{c}\rho(s,T)+\frac{\lambda}{cs}\left(e^{-\lambda T}-1\right)[1-\varphi(s)]$$

$$+\frac{\lambda}{c}e^{-\lambda T}\varphi(s)\rho(s,T)-\frac{\lambda}{c}\varphi(s)\rho(s,T), \quad (20)$$

Finally, we obtain the Laplace transform of the finite-time ruin probability:

$$\rho(s,T)=\frac{R(0,T)+\frac{\lambda}{cs}\left(e^{-\lambda T}-1\right)[1-\varphi(s)]}{s-\frac{\lambda}{c}[1+q(s,T)-\varphi(s)]}. \quad (21)$$

where $q(s,T)=e^{-\lambda T}\varphi(s)$.

The denominator of (21) is similar to the difference between the two sides of Lundberg's equation $\lambda\left(\mathbb{E}\left[e^{VX}\right]-1\right)=cV$, where $V>0$ is the Lundberg adjustment root in the favorable case $c>\lambda\mathbb{E}[X]$. In our case, Lundberg's equation takes the form $\lambda\varphi(s)=\lambda[1+q(s,T)]-cs$.

According to Gerber and Shiu (1998), for $s=0$ and $\lim_{T\to\infty}q(s,T)$, $s=0$ is an obvious solution for the denominator. Therefore, $\rho(0,T)<\infty$ implies that the numerator of $\rho(0,T)$ is zero. However, taking $T\to\infty$ corresponds to calculating $R(0,\infty)=R(0)$. To be able to compute $R(0,T)$, we impose the condition $\rho(0,T)<\infty$ if $q(0,T)\neq 0$ and $s=0$ is a zero of the numerator. For the Laplace transform of the claim size distribution, $\varphi(s)=\mathbb{E}[e^{-sX}]$, we have the following series expansion:

$$\varphi(s)=1-sm+o(s), \qquad m:=\mathbb{E}[X].$$

Thus,

$$\frac{1-\varphi(s)}{s} \longrightarrow m \quad \text{as } s\to 0.$$

Taking the limit as $s\to 0$, the numerator of (21) tends to:

$$R(0,T)+\frac{\lambda}{c}\left(e^{-\lambda T}-1\right)m,$$

and the denominator tends to:

$$-\frac{\lambda}{c}q(0,T).$$

Since $q(0,T)=e^{-\lambda T}\neq 0$, we have:

$$\rho(0,T)=\frac{R(0,T)+\frac{\lambda}{c}\left(e^{-\lambda T}-1\right)m}{-\frac{\lambda}{c}e^{-\lambda T}}.$$

Since $s=0$ is a zero of the numerator, we have:

$$R(0,T)+\frac{\lambda}{c}\left(e^{-\lambda T}-1\right)m=0.$$

Hence,

$$R(0,T) = \frac{\lambda m}{c}\left(1-e^{-\lambda T}\right)$$

$$(22) \qquad = \frac{1-e^{-\lambda T}}{1+\xi}.$$

From equalities (21) and (22), we obtain:

$$(23) \qquad \rho\,(s,T) = \frac{\left(1-e^{-\lambda T}\right)\left[1-ms-\varphi(s)\right]}{s\left[1+\left(e^{-\lambda T}-1\right)\varphi(s)-(1+\xi)\,ms\right]}.$$

□

We remark that

$$\rho\,(s) = \lim_{T\to\infty}\rho\,(s,T)$$

and (23) derived here for the first time.

3 Estimation of the Laplace Transform of the Finite-Time Ruin Probability

We consider (23) and define

$$(24) \qquad \hat{\rho}_n(s,T) = \frac{(1-e^{-\lambda T})(1-\hat{m}s-\hat{\varphi}(s))}{\hat{\Theta}(s,T)}, \quad s>0,$$

as the estimator of the Laplace transform of the finite-time ruin probability given in (23). where

$$(25) \qquad \hat{\Theta}(s,T) = s\left[1+(1+e^{-\lambda T})\hat{\varphi}(s)-(1+\xi)\hat{m}s\right]$$

such that $\hat{m}\to m$ as $n\to\infty$. In this section, we study the behavior of Θ given by (25) near 0 by adopting the same strategy as Mnatsakanov, Ruymgaart, and Ruymgaart (2008).
By applying a second-order Taylor expansion of the function Θ around 0, we obtain:

$$(26) \qquad \Theta(s,T) = \Theta(0,T)+s\frac{d}{ds}\Theta(0,T)+\frac{1}{2}s^2\frac{d^2}{ds^2}\Theta(\mathring{s},T), \quad 0\leqslant\mathring{s}<s$$

where we calculate its different terms as follows:

$$(27) \qquad \Theta(0,T) = 0$$

$$\Theta(\mathring{s},T) = s - s\left(e^{-\lambda T}-1\right)\varphi(\mathring{s}) - (1+\xi)ms^2 \tag{28}$$

$$\frac{d}{ds}\Theta(\mathring{s},T) = 1+\left(e^{-\lambda T}-1\right)\left[\varphi(\mathring{s})+s\frac{d}{ds}\varphi(\mathring{s})\right]-2s(1+\xi)m \tag{29}$$

$$\begin{aligned}
\frac{d}{ds}\Theta(0,T) &= 1+\left(e^{-\lambda T}-1\right)\left[\varphi(0)+s\frac{d}{ds}\varphi(0)\right]-2s(1+\xi)m \\
&= 1+\left(e^{-\lambda T}-1\right)(1-ms)-2s(1+\xi)m \\
&= e^{-\lambda T}-\left(1+2\xi+e^{-\lambda T}\right)ms \qquad (30)
\end{aligned}$$

since $\frac{d}{ds}\varphi(0) = -m$.
From (30) , we have

$$s\frac{d}{ds}\Theta(0,T) = se^{-\lambda T}-\left(1+2\xi+e^{-\lambda T}\right)ms^2. \tag{31}$$

We calculate the second-order derivative of $\Theta(\mathring{s},T)$ starting from relation (29):

$$\frac{d^2}{ds^2}\Theta(\mathring{s},T) = s\left(e^{-\lambda T}-1\right)\frac{d^2}{ds^2}\varphi(\mathring{s})-2m\left(\xi+e^{-\lambda T}\right), \tag{32}$$

from equalities (26), (28), (31) and (32) we obtain

$$\begin{aligned}
\Theta(s,T) &= \Theta(0,T)+s\frac{d}{ds}\Theta(0,T)+\frac{1}{2}s^2\frac{d}{ds^2}\Theta(\mathring{s},T); 0 \leqslant \mathring{s} \leqslant s \\
&= se^{-\lambda T}-\left(1+3\xi+2e^{-\lambda T}\right)ms^2+\frac{1}{2}s^3\left(e^{-\lambda T}-1\right)\frac{d^2}{ds^2}\varphi(\mathring{s}) \\
&\leqslant -\left(1+3\xi+2e^{-\lambda T}\right)ms^2 \\
&\leqslant -2e^{-\lambda T}ms^2. \qquad (33)
\end{aligned}$$

Starting from the inequality $0 \le \varphi(s) \le 1$ and considering (24), by successive bounding we have: From inequality (33), we observe that when s is close to 0:

$$\Theta(s,T) \approx se^{-\lambda T} \quad \text{et} \quad \hat{\Theta}_n(s,T) \approx se^{-\lambda T},$$

since the quadratic terms s^2 become negligible compared to the linear term s as $s \to 0$.
The inverse of the Laplace transform $F(s)$ of a function $f(x)$ is well-defined (in the L^2 sense) only if $F(s)$ is square-integrable along a vertical line in the complex plane where it is analytic. For $\rho(s,T)$ and $\hat{\rho}_n(s,T)$, the behavior of $\Theta(s,T)$ and $\hat{\Theta}_n(s,T)$ as $s \to 0$ suggests that:

$$\Theta(s,T) \sim s \quad \text{and} \quad \hat{\Theta}_n(s,T) \sim s \quad \text{as} \quad s \to 0.$$

If $R(u,T)$ and $\hat{R}_n(u,T)$ are obtained via the inverse Laplace transform of $\rho(s,T)$ and $\hat{\rho}_n(s,T)$, their behavior at infinity is determined by that of their Laplace transforms at $s=0$.
The functions $\rho(s,T)$ and $\hat{\rho}_n(s,T)$ are not square-integrable, since their $\Theta(s,T)$ and $\hat{\Theta}_n(s,T)$ tend to s as $s \to 0$, making their inverses too singular to belong to L^2. Therefore, recovering a function from a noisy image is ill-posed, which motivates the need for a regularized inverse. This consists of slightly modifying the functions $\rho(s,T)$ and $\hat{\rho}_n(s,T)$ to ensure that they belong to $L^2[0,\infty)$, then $R(u,T)$ can be recovered by first applying a regularized inverse to the modified $\rho_n(s,T)$. See Mnatsakanov, Ruymgaart, and Ruymgaart (2008), Chauveau, Rooij, and Ruymgaart (1994), Shimizu (2012) for more details.
We then formulate the following theorem:

Theorem 1. *Let $\rho(s,T)$ be the Laplace transform of the finite-time ruin probability. Under the assumptions:*

1. *$\hat{m}$ is an estimator of m with $\mathbb{E}[(\hat{m}-m)^2] = \mathscr{O}(n^{-1})$,*

2. *$\hat{\varphi}(s)$ is an estimator of $\varphi(s)$ with $\sup_s \mathbb{E}[(\hat{\varphi}_n(s)-\varphi(s))^2] = \mathscr{O}(n^{-\alpha})$, $0 < \alpha \le 1$,*

3. *$\hat{\Theta}_n(s,T)$ is an estimator of $\Theta(s,T)$ such that:*

   - *$|\hat{\Theta}_n(s,T)| \ge C_\Theta |s|^2$ almost surely,*
   - *$\mathbb{E}\left[\sup_s |\hat{\Theta}_n(s,T)^{-1} - \Theta(s,T)^{-1}|^2\right] = \mathscr{O}(n^{-\beta})$ with $\beta > 0$.*

Then the integrated squared error satisfies:

$$ISE\{\hat{\rho}_n(s,T)\} = \mathscr{O}\left(n^{-\min(1,\alpha,\beta)}\right)$$

Proof.

Let us decompose the error:

$$\begin{aligned}
\hat{\rho}_n(s,T)-\rho(s,T) &= (1-e^{-\lambda T})\left(\frac{1-\hat{m}s-\hat{\varphi}_n(s)}{\hat{\Theta}(s,T)}-\frac{1-ms-\varphi(s)}{\Theta(s,T)}\right)\\
&= (1-e^{-\lambda T})\Bigg\{(1-\hat{m}s-\hat{\varphi}_n(s))\Big(\frac{1}{\hat{\Theta}(s,T)}-\frac{1}{\Theta(s,T)}\Big)\\
&\qquad + \frac{(1-\hat{m}s-\hat{\varphi}_n(s))-(1-ms-\varphi(s))}{\Theta(s,T)}\Bigg\}\\
&= (1-e^{-\lambda T})\Bigg\{(1-\hat{m}s-\hat{\varphi}_n(s))\Big(\frac{1}{\hat{\Theta}(s,T)}-\frac{1}{\Theta(s,T)}\Big)\\
&\qquad + \frac{-\hat{m}s-\hat{\varphi}_n(s)+ms+\varphi(s)}{\Theta(s,T)}\Bigg\}\\
&= (1-e^{-\lambda T})\Bigg\{(1-\hat{m}s-\hat{\varphi}_n(s))\Big(\frac{1}{\hat{\Theta}(s,T)}-\frac{1}{\Theta(s,T)}\Big)\\
&\qquad + \frac{(\hat{m}-m)s+(\hat{\varphi}_n(s)-\varphi(s))}{\Theta(s,T)}\Bigg\}.
\end{aligned}$$

$$\begin{aligned}
\hat{\rho}_n(s,T)-\rho(s,T) &= \frac{(1-e^{-\lambda T})}{\Theta(s,T)}[(\hat{m}-m)s+(\hat{\varphi}_n(s)-\varphi(s))]\\
&\quad + (1-e^{-\lambda T})(1-\hat{m}s-\hat{\varphi}_n(s))\left(\frac{1}{\hat{\Theta}(s,T)}-\frac{1}{\Theta(s,T)}\right).
\end{aligned}$$

By applying the Cauchy-Schwarz inequality for integrals:

$$\begin{aligned}
\mathrm{ISE}\{\hat{\rho}_n(s,T)\} &\le 2\int_{\mathbb{T}_{\mathbb{C}_+}}\left|\frac{(1-e^{-\lambda T})}{\Theta(s,T)}[(\hat{m}-m)s+(\hat{\varphi}_n(s)-\varphi(s))]\right|^2 ds\\
&\quad + 2\int_{\mathbb{T}_{\mathbb{C}_+}}\left|(1-e^{-\lambda T})(1-\hat{m}s-\hat{\varphi}_n(s))\left(\frac{1}{\hat{\Theta}_n(s,T)}-\frac{1}{\Theta(s,T)}\right)\right|^2 ds\\
&\le C\left[(\hat{m}-m)^2\underbrace{\int_{\mathbb{T}_{\mathbb{C}_+}}\frac{s^2}{|\Theta(s,T)|^2}ds}_{\text{convergent}}+\underbrace{\int_{\mathbb{T}_{\mathbb{C}_+}}\frac{|\hat{\varphi}_n(s)-\varphi(s)|^2}{|\Theta(s,T)|^2}ds}_{\text{to bound}}\right]\\
&\quad + C'\underbrace{\int_{\mathbb{T}_{\mathbb{C}_+}}\left|\frac{1}{\hat{\Theta}_n(s,T)}-\frac{1}{\Theta(s,T)}\right|^2 ds}_{\text{to bound}}.
\end{aligned}$$

For the term in $\hat{\varphi}_n(s) - \varphi(s)$, by Cauchy-Schwarz:

$$\int_{\mathbb{T}_{\mathbb{C}_+}} \frac{|\hat{\varphi}_n(s) - \varphi(s)|^2}{|\Theta(s,T)|^2} ds \leq \left(\int_{\mathbb{T}_{\mathbb{C}_+}} |\hat{\varphi}_n(s) - \varphi(s)|^4 ds \right)^{1/2} \left(\int_{\mathbb{T}_{\mathbb{C}_+}} |\Theta(s,T)|^{-4} ds \right)^{1/2} = \mathscr{O}(n^{-\alpha})$$

The last term is bounded directly by assumption 3. Combining these results:

$$\begin{aligned}\mathbb{E}[\mathrm{ISE}\{\hat{\rho}_n(s,T)\}] &\leq C_1\,\mathbb{E}\left[(\hat{m} - m)^2\right] + C_2 \sup_s \mathbb{E}\left[|\hat{\varphi}_n(s) - \varphi(s)|^2\right] \\ &\quad + C_3\,\mathbb{E}\left[\int_{\mathbb{T}_{\mathbb{C}_+}} \left|\frac{1}{\hat{\Theta}_n(s,T)} - \frac{1}{\Theta(s,T)}\right|^2 ds\right] \\ &= \mathscr{O}\left(n^{-\min(1,\alpha,\beta)}\right).\end{aligned}$$

□

The regularized inverse of the Laplace transform $\mathscr{L}_\ell^{-1}$, $\ell = \ell(n) > 0$, was introduced by Chauveau, Rooij, and Ruymgaart (1994). It was later reformulated by Mnatsakanov, Ruymgaart, and Ruymgaart (2008) and Shimizu (2012). It is applied to allow convergence of the inverse Laplace transform in the context of empirical data estimation.

In general, $\| \mathscr{L}^{-1} \|_{L^2}$ is not bounded, but for each $\ell = \ell(n) > 0$, $\| \mathscr{L}_\ell^{-1} \|_{L^2}$ is bounded such that

(34) $$\| \mathscr{L}_\ell^{-1} \|_{L^2} \leqslant \frac{1}{\ell} \| \mathscr{L}^{-1} \|_{L^2} = \frac{1}{\ell}\sqrt{\pi},$$

$$\ell = \ell(n) \to 0 \quad \text{si} \quad n \to \infty.$$

Mnatsakanov, Ruymgaart, and Ruymgaart (2008) uses the regularization function $\Phi(s) = e^{-\theta u}$ to modify the Laplace transform of the ruin probability. Proceeding in the same way, from equality (7) we obtain:

(35) $$\rho_\theta(s,T) = e^{-\theta u}\rho(s,T), \quad \theta > 0$$

$$\theta = \theta(n) \to 0 \quad \text{if} \quad n \to \infty.$$

whose estimator is given by

(36) $$\widehat{\rho}_{\theta(n)}(s,T) = e^{-\theta u}\widehat{\rho}_n(s,T).$$

By applying the regularized inversion, we formulate the following proposition which gives the equality satisfied by the ISE of the ruin probability estimator.

Proposition 1. *Let $\ell, \theta \in \mathbb{R}_+^*$, such that $\ell \to 0$ and $\theta \to 0$ as $n \to \infty$. The integrated squared error (ISE) of the ruin probability estimator satisfies the following equality:*

$$ISE\left\{\hat{R}_n(u,T)\right\} = \mathscr{O}\left(\frac{1}{\ell^2} n^{-\min(1,\alpha,\beta)}\right), \quad 0 < \alpha \leq 1, \quad \beta > 0;$$

where $\hat{R}_n(u,T) := \mathscr{L}_\ell^{-1}\left\{\hat{\rho}_{\theta(n)}(s,T)\right\}(u,T)$.

Proof.
By using Theorem 1, inequality (34), and equality (36), we have:

$$
\begin{aligned}
\mathrm{ISE}\left\{\widehat{R}_n(u,T)\right\} &= \int_0^\infty \left(\widehat{R}_n(u,T) - R(u,T)\right)^2 du \\
&= \int_0^\infty \left(e^{\theta u}\widehat{R}_{\theta(n)}(u,T) - R(u,T)\right)^2 du \quad (\text{since } \widehat{R}_n(u,T) = e^{\theta u}\widehat{R}_{\theta(n)}(u,T)) \\
&= \int_0^\infty \left(e^{\theta u}\widehat{R}_{\theta(n)}(u,T) - e^{\theta u}R_\theta(u,T)\right)^2 du \quad (\text{since } R(u,T) = e^{\theta u}R_\theta(u,T)) \\
&= \int_0^\infty e^{2\theta u}\left(\widehat{R}_{\theta(n)}(u,T) - R_\theta(u,T)\right)^2 du \\
&\leq \sup_{u\geq 0} e^{2\theta u} \int_0^\infty \left(\widehat{R}_{\theta(n)}(u,T) - R_\theta(u,T)\right)^2 du \\
&= \sup_{u\geq 0} e^{2\theta u} \cdot \mathrm{ISE}\left\{\widehat{R}_{\theta(n)}(u,T)\right\} \\
&= \sup_{u\geq 0} e^{2\theta u} \cdot \left\|\mathscr{L}_\ell^{-1}\left\{\widehat{\rho}_{\theta(n)}(s,T)\right\} - \mathscr{L}_\ell^{-1}\left\{\rho_\theta(s,T)\right\}\right\|_{L^2}^2 \\
&\leq \sup_{u\geq 0} e^{2\theta u} \cdot \left\|\mathscr{L}_\ell^{-1}\right\|_{L^2}^2 \left\|\widehat{\rho}_{\theta(n)}(s,T) - \rho_\theta(s,T)\right\|_{L^2}^2 \\
&\leq \sup_{u\geq 0} e^{2\theta u} \cdot \frac{\pi}{\ell^2} \cdot \mathrm{ISE}\left\{\widehat{\rho}_{\theta(n)}(s,T)\right\} \\
&= \sup_{u\geq 0} e^{2\theta u} \cdot \frac{\pi}{\ell^2} \cdot \int_0^\infty \left(e^{-\theta u}\widehat{\rho}_n(s,T) - e^{-\theta u}\rho(s,T)\right)^2 ds \\
&= \sup_{u\geq 0} e^{2\theta u} \cdot \frac{\pi}{\ell^2} \cdot \int_0^\infty e^{-2\theta u}\left(\widehat{\rho}_n(s,T) - \rho(s,T)\right)^2 ds \\
&\leq \frac{\pi}{\ell^2} \cdot \sup_{u\geq 0} e^{2\theta u}e^{-2\theta u} \cdot \mathrm{ISE}\left\{\widehat{\rho}_n(s,T)\right\} \\
&= \frac{\pi}{\ell^2} \cdot \mathscr{O}\left(n^{-\min(1,\alpha,\beta)}\right) \\
&= \mathscr{O}\left(\frac{1}{\ell^2} n^{-\min(1,\alpha,\beta)}\right)
\end{aligned}
$$

□

The parameters ℓ (regularized inversion) and h (kernels) play a similar role in controlling the bias and variance of the estimator. In contrast, θ is introduced to ensure the square-integrability of the transform, without a direct effect on smoothing. In our simulations, we set $\theta = 0$ and choose $\ell(n) = h(n)$, using the method of Dubner and Abate (1968) for the inversion.

4 Numerical Simulation Results

To facilitate numerical simulations and the interpretation of results, the complex variable s of the Laplace transform is restricted to positive real values, as has also been adopted in several recent works such as Todorov and Tauchen (2012), Curato, Mancini, and Recchioni (2018), Mnatsakanov and Elmagbri (2018), Gadallah et al. (2022), Sebastian and Joseph (2024), and Sheng and Henao (2025). We consider a known theoretical Weibull(1;3) density, compute the optimal smoothing parameter h_{opt}, which will be used to evaluate the MISE (Mean Integrated Squared Error) and the variance of $\hat{\rho}_n(s,T)$ and $\widehat{R}_n(u,T)$ after N replications. The Laplace variable s is discretized over a given interval to allow numerical evaluation of the estimated Laplace transform. All integrals over u, x, and s are approximated using Euler's method (constant step discretization) implemented in MATLAB R2022b.

Table 1: MISE and Variance of $\hat{\rho}_n(s,T)$ for a Weibull density with $N = 20$ replications and $T = \infty$.
The table shows that as the sample size n increases, the optimal bandwidth h_{opt} decreases, improving estimation accuracy (lower MISE). The variance remains very low, indicating good stability of the estimator $\hat{\rho}_n(s,T)$.

n	h_{opt}	**MISE**	$\mathbb{V}$ar
50	0.184392	8.173779e-02	3.764790e-03
500	0.116343	9.660399e-05	2.330473e-09
1000	0.101283	6.905566e-05	1.622024e-09
2000	0.088172	1.225762e-02	1.408268e-03
3000	0.081304	1.860030e-02	2.044473e-03
5000	0.073408	5.608379e-02	4.065526e-03

Table 2: MISE and Variance of $\hat{R}_n(u,T)$ for Weibull density with $N = 20$ replications and $T = \infty$.
The MISE remains stable at $\approx 5.09 \times 10^{-4}$ regardless of sample size, while variance decreases exponentially with n, confirming the estimator's asymptotic consistency.

n	h_{opt}	**MISE**	$\mathbb{V}$ar
50	0.184392	5.092804e-04	1.147693e-13
500	0.116343	5.092332e-04	1.079562e-14
1000	0.101283	5.092234e-04	3.948377e-15
2000	0.088172	5.092160e-04	3.823890e-15
3000	0.081304	5.092419e-04	1.564314e-15
5000	0.073408	5.092126e-04	1.573224e-15

We consider $n = 6773$ claim values X_i from an auto insurance company data set from the CASdatasets (Casualty Actuarial Society) extracted from R. The optimal choice of the smoothing parameter h is made using cross-validation on the sample

of size n. We used the Laplace transform of the Gaussian kernel for selecting h via cross-validation.
In our simulations with the real data, the mean m of the claims was estimated from the claim amounts, whereas we fixed the value of the claim arrival intensity λ as well as the safety loading parameter ξ. We obtain the following curves after simulation:

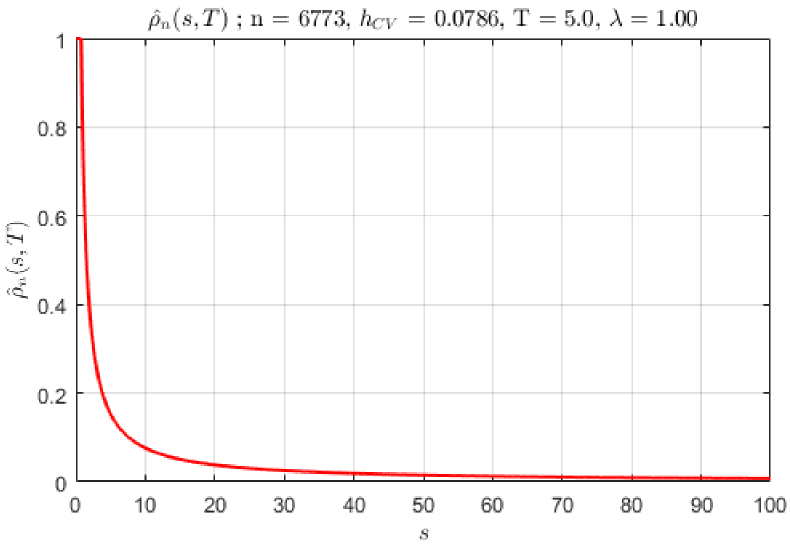

Figure 1: Estimation of the Laplace transform of the ruin probability using the Gaussian kernel.

Figure 1 shows the evolution of the Gaussian kernel estimation of the Laplace transform of the probability of ruin $\hat{\rho}_n(s,T)$ as a function of s for $n = 6773$ and the optimal smoothing value $h_{CV} = 0.0786$. The MISE cannot be computed in this case, as the theoretical density is random (unknown).

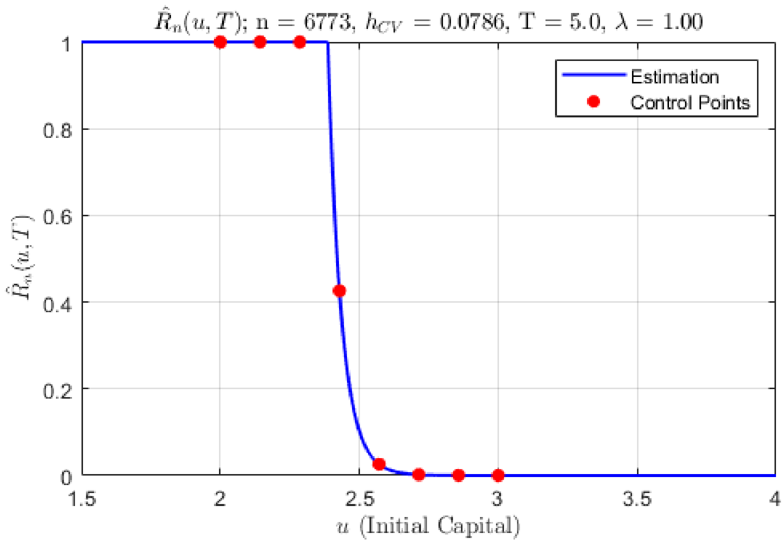

Figure 2: Estimation of the ruin probability using the Gaussian kernel.

Figure 2 shows the evolution of the Gaussian kernel estimation of the ruin probability $\hat{R}_n(u,T)$ as a function of u for $n = 6773$ and the optimal smoothing value $h_{CV} = 0.0786$. The MISE cannot be computed in this case, as the theoretical density is random (unknown).

Table 3: Gaussian kernel: Estimated ruin probability $\hat{R}_n(u,T)$ for different values of T and $T = \infty$.
The estimated ruin probabilities show rapid convergence as T increases, stabilizing at $T = 10$ for all u values. The results demonstrate that infinite-horizon ruin probabilities ($T = \infty$) are effectively approximated by finite $T = 20$ estimates. The Gaussian kernel provides stable estimates across all time horizons, with ruin probabilities decreasing exponentially as the initial capital u increases.

u	2.00	2.14	2.29	2.43	2.57	2.71	2.86	3.00
$\hat{R}_n(u,1)$	1	1	1	0.271 113	0.016 475	0.001 165	0.000 094	0.000 009
$\hat{R}_n(u,3)$	1	1	1	0.407 541	0.024 765	0.001 751	0.000 142	0.000 013
$\hat{R}_n(u,5)$	1	1	1	0.426 004	0.025 887	0.001 830	0.000 148	0.000 014
$\hat{R}_n(u,10)$	1	1	1	0.428 875	0.026 061	0.001 843	0.000 149	0.000 014
$\hat{R}_n(u,20)$	1	1	1	0.428 894	0.026 062	0.001 843	0.000 149	0.000 014
$\hat{R}_n(u,\infty)$	1	1	1	0.428 894	0.026 062	0.001 843	0.000 149	0.000 014

5 Conclusion

We have proposed a robust nonparametric method for estimating the ruin probability via its Laplace transform, offering promising prospects for risk assessment in insurance. Theoretical and numerical results demonstrate the effectiveness of the approach, paving the way for extensions to more complex models.As perspectives, the nonparametric estimation of the Laplace transform could be extended to multiline models to better capture interactions between different claims, and integrated into dynamic reserve processes with random payments in order to provide robust tools for risk management and the optimization of financial strategies.

References

Abate, J. and W. Whitt (1995). “Numerical inversion of Laplace transforms of probability distributions”. In: *ORSA Journal on Computing* 7.1, pp. 36–43.

Alexopoulos, Joannis (2025). “Uniformity in the Fourier inversion formula with applications to Laplace transforms”. In: *Archiv der Mathematik* 65.2. Accès libre, pp. 123–135. DOI: 10.1007/s00013-025-02153-7. URL: https://link.springer.com/article/10.1007/s00013-025-02153-7.

Antipov, V. (2025). *On the Application of Laplace Transform to the Ruin Problem with Random Insurance Payments and Investments in a Risky Asset*. arXiv preprint arXiv:2508.07235. URL: https://arxiv.org/abs/2508.07235.

Chauveau, D. E., A. C. M. van Rooij, and F. H. Ruymgaart (1994). “Regularized Inversion of Noisy Laplace Transforms”. In: *Advances in Applied Mathematics* 15, pp. 186–201. DOI: 10.1006/aama.1994.1007.

Curato, Imma V., Maria E. Mancini, and Maria C. Recchioni (2018). “Spot volatility estimation using the Laplace transform”. In: *Econometrics and Statistics* 6, pp. 22–43. ISSN: 2452-3062. DOI: 10.1016/j.ecosta.2016.07.002.

Dubner, Harvey and Joseph Abate (1968). “Numerical Inversion of Laplace Transforms by Relating Them to the Finite Fourier Cosine Transform”. In: *Journal of the ACM* 15, pp. 115–123. DOI: 10.1145/321439.321446.

Epstein, E. L. et al. (2025). “Score-Debiased Kernel Density Estimation”. In: *arXiv preprint*. arXiv:2504.19084.

Gadallah, A. M. et al. (2022). “Modeling various survival distributions using a nonparametric hypothesis testing based on Laplace transform approach with some real applications”. In: *Computational and Mathematical Methods in Medicine* 2022.1, p. 5075716.

García, Jesús L. (2005). “Explicit Solutions for Survival Probabilities in the Classical Risk Model”. In: *ASTIN Bulletin* 35.1, pp. 19–44. DOI: 10.2143/AST.35.1.583162.

Gerber, Hans U. and Elias S. W. Shiu (1998). “On the Time Value of Ruin”. In: *North American Actuarial Journal* 2.1, pp. 48–72. DOI: 10.1080/10920277.1998.10595671.

Harfouche, Zineb and A. Bareche (2021). “Semi-parametric approach for approximating the ruin probability of classical risk models with large claims”. In: *Communications in Statistics - Simulation and Computation*. DOI: 10.1080/03610918.2021.1992636.

Henríquez, Fernando and Jan S. Hesthaven (2024). “Fast Numerical Approximation of Linear, Second-Order Hyperbolic Problems Using Model Order Reduc-

tion and the Laplace Transform". In: *arXiv preprint arXiv:2405.19896*. Accès libre. URL: https://arxiv.org/abs/2405.19896.
Kabore, R. F. E. J. et al. (2025). "Nonparametric Estimation of the Laplace Transform from Empirical Data Using Symmetric Kernels Method". In: *Journal of Advances in Mathematics and Computer Science* 40, pp. 23–32.
Maréchal, Pierre, Faouzi Triki, and Walter C. Simo Tao Lee (2023). "Regularization of the inverse Laplace transform by Mollification". In: *arXiv preprint arXiv:2304.08057*. Accès libre. URL: https://arxiv.org/abs/2304.08057.
Mnatsakanov, R., L. Ruymgaart, and F. Ruymgaart (2008). "Nonparametric estimation of ruin probabilities given a random sample of claims". In: *Mathematical Methods of Statistics* 17, pp. 35–43. DOI: 10.3103/S1066530708010031.
Mnatsakanov, R. M. and F. Elmagbri (2018). "Nonparametric density estimation based on the scaled Laplace transform inversion". In: *Travel Medicine and Infectious Disease*. DOI: 10.1016/j.trmi.2018.09.003.
Mousa, A. M., M. Kh. Hassan, and A. Fathi (2016). "A new non parametric estimator for Pdf based on inverse gamma distribution". In: *Communications in Statistics - Theory and Methods* 45.23, pp. 7002–7010. DOI: 10.1080/03610926.2014.972575.
Sebastian, S. and B. Joseph (2024). "Generalized Discrete Laplace Distribution: Estimation and Applications". In: *Advances and Applications in Statistics* 91.6, pp. 699–717.
Sheng, D. and R. Henao (2025). "Learning Survival Distributions with the Asymmetric Laplace Distribution". In: *arXiv preprint*. arXiv:2505.03712.
Shimizu, Yasutaka (2012). "Non-parametric estimation of the Gerber–Shiu function for the Wiener–Poisson risk model". In: *Scandinavian Actuarial Journal* 1, pp. 56–69.
Sun, T. and X. Zhang (2024). "Laplace Transformation of the Ruin Time for a Risk Model with a Parisian Implementation Delay". In: *Mathematics* 12.4, p. 551. DOI: 10.3390/math12040551.
Todorov, V. and G. Tauchen (2012). "Realized Laplace transforms for pure-jump semimartingales". In: *The Annals of Statistics* 40.2, pp. 1233–1262.
Touazi, A. et al. (2014). "Méthode du noyau dans l'analyse de stabilité forte d'un modèle de risque". In: *Colloque International, USTHB*. URL: https://hal.science/hal-01086342.
Touazi, A. et al. (2016). "Méthodes du noyau et stabilité forte dans l'approximation de la probabilité de ruine". In: *MOSIM (Modélisation, Optimisation and Simulation)*. École Supérieure de Technologie, pp. 5–14.
Yuan, D., T. Guo, and Z. Huang (2025). "Distributed Nonparametric Estimation: from Sparse to Dense Samples per Terminal". In: *arXiv preprint*. arXiv:2501.07879.