**Modelling Fluid Flow in an Open Circular Channel with Three Lateral Inflows**

**Abstract**

Kenya's recent heavy rainfall events underscore the urgent need for effective flood management infrastructure. While flooding—whether gradual or sudden—causes significant damage to infrastructure and agriculture, current research on circular channels with three lateral inflows remains limited. This study numerically investigates incompressible Newtonian fluid flow in circular channels with lateral inflow angles ranging from 0° to 90° and varying entry lengths. Using similarity transformations and finite difference methods, the continuity and momentum equations were solved and analyzed using Python. Results indicate that increasing the number of lateral inflows leads to a measurable decrease in main channel velocity. Additionally, as lateral inflow angles increase, the main channel velocity declines further. These findings inform the design of efficient drainage and irrigation systems critical for flood mitigation, climate resilience, and sustainable agriculture. Advancements in such water management systems enhance food security, create employment, and drive socioeconomic development by optimizing resource utilization.

**Keywords**: Lateral input, Open Channel, Circular Channel, Cross-section area, Fluid, Laminar Flow.

**1.0. Introduction**

**1.1. Background Information**

Recently, Kenya has faced significant heavy rainfall, resulting in the destruction of bridges, buildings, livestock, and farms due to flooding rivers and overflowing lakes. In February 2024, a major environmental disaster struck when Lake Victoria in Kisumu breached its banks, leading to the death of at least thirty individuals (Attfield, 2024). This catastrophe swept away essential resources, including schools, homes, trees, vehicles, and agricultural produce in the affected village. Furthermore, in 2023, the Suswa-Naivasha landslide blocked roads and damaged several bridges. The latest incident occurred on April 29, 2024, in Mai Mahiu, Nakuru County, claiming 51 lives and causing significant damage to infrastructure (Githeko *et al* ., 2024).

Flooding continues to be a major issue in Kenya, highlighting the need to design channels that effectively manage this environmental challenge. Furthermore, it’s important to use the same water resources for both irrigation and hydroelectric power generation. The flow in open channels is driven by differences in potential energy. The persistent flooding problems, along with the demand for water transportation for irrigation and hydroelectric generation, emphasize the necessity for an efficient channel model featuring three lateral inflows to maximize water conveyance.

In Kenya, many rural towns and roads lack engineered drainage systems, leading to inefficient surface runoff management during rainfall events. From a fluid mechanics perspective, the absence of properly designed open-channel or sub-surface drainage leads to excessive overland flow, increased shear stress on road surfaces, and localized ponding. This unregulated runoff results in hydraulic overloading of existing natural channels and culverts, often exceeding their flow capacity and causing backflow or flooding. During peak rainfall, high Reynolds number flows and rapid accumulation of surface water lead to erosive forces that degrade unpaved roads and agricultural land. The resulting impassability of transport routes disrupts the movement of goods and raw materials, while inundation of farmland causes sediment transport, nutrient leaching, and root-zone saturation—ultimately damaging crops and reducing agricultural productivity. These hydrodynamic failures underscore the need for efficient fluid flow modeling and optimized drainage system design to mitigate flow resistance, enhance conveyance, and ensure hydraulic stability under variable precipitation intensities.

This research is grounded in Newton’s second law of motion and the Navier - Stokes equation, which have been simplified under the open channels of arbitrary shaped assumption where the characteristic length scale is much smaller than the flow depth. The analysis has focused on determining the appropriate cross-sectional area, lateral inflow length, and angle to effectively manage three inflow channels, thereby helping to prevent blockages in the drainage channel.

**1.2. Specific objective**.

(i) To examine how changing the three lateral inflow channels' lengths affects the main open channel's velocity.

(ii)To examine how changing the lateral inflow channels' angle affects the main open channel's velocity

(iii) To find out how changing the cross-sectional size of three lateral inflow channels affects the main open channel's velocity

**1.3. Literature review**

Tuitoek and Hicks (2001) investigated flood management by simulating compound channels with erratic flow in order to better manage floods. By developing a model based on the Saint Venant equation of flow, they added some terminology like flow depth (h), flow rate (Q), friction slope (*sf*), and hydraulic radius (R) in order to account for the momentum phenomenon of move to integrate an inconsistency in the flow in the circular channel and for open channel flows with uniform and localized lateral inflow Chirchir (2021) investigated fluid flow in an open channel with a horseshoe cross-section.

Ojiambo *et al*., (2018) and Kinyanjui *et al*., (2011) did an investigation that focused on unsteady non-uniform flow on open channels with circular cross-section. The continuity equation embodies the concept that a conserved quantity can shift in location but cannot be generated or destroyed, combining the transport theorem and the law of mass conservation (Chow, 1959). Saint- Venants equation of continuity, which was developed by two mathematicians, De Saint Venant and Bousinnesque. In the 19th century from Navier equation for Shallow water and one dimension. Dynamic routing is a solution to the St Venant equation; and it is often used to measure or compare other techniques (Singh, 2017).

Guo *et al*., (2022) examined the flow of fluid in an open channel with an elliptical cross-section. The findings showed a greater hydraulic depth due to an increased hydraulic radius; because of the buildup of eroded particles, the depth of fluid flow decreases throughout the channel, decreasing the fluid velocity in the process. Velocity of flow is also impacted by variations in friction slope. The flow velocity reduces as the friction increases. Shear pressure on the channel bed and walls produces friction, which prevents the water from flowing smoothly. The impact of lateral inflow on the channel’s velocity was not examined in this study. This study will model a circular channel to address this gap.

Simegnaw *et al.,* (2021) researched on open channel flows with parabolic cross-section. The results showed that higher channel slopes and energy coefficients result in higher flow velocities. Conversely, a reduction in top breadth results in a rise in velocity. The effects of lateral inflow on the channel’s velocity were not examined in this study. This study will help in modeling of open circular channel that will address this gap.

Rotich *et al* ., (2021) studied fluid flow in an open rectangular and triangular channel. The findings showed that hydraulically, open channels with rectangular cross-sections are more efficient than those with triangular cross-sections. Also, the research showed that an increase in the channel’s energy coefficient, top width, and slope causes a rise in flow velocity for both rectangular and triangle channels. Moreso, the flow velocity increases with depth and reaches its maximum just below the free surface. The rectangular channel moves more water faster than an open triangular channel at the same depth and width, according to the velocity profile for both types of channels. The impact of lateral intake on the primary flow velocity was not examined in this study.

Thiong’o *et al* ., (2013) focused on open rectangular and triangular channel flows. The goal was to ascertain the hydraulic efficiency of the open rectangular and triangular channels. There are non-linear partial differential equations as a result of the conservation of mass and momentum rules. The finite difference approach was adopted since such equations cannot be solved analytically. The depth and velocity of the flow are crucial variables in figuring out discharge. Research has been done on how altering different parameters affects velocity. It has also been studied how fluid velocity changes with depth. The finite element approach can yield findings that are more accurate than those obtained by the finite difference method utilized in this study to solve its equations.

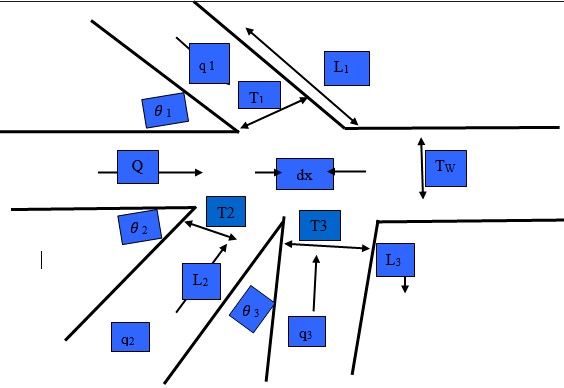
Yadav et al., (2021) studied State of art of different kinds of fluid flow interactions with piezo for energy harvesting considering experimental, simulations and mathematical modelling”.The study studied the different kinds of fluid flow interactions with piezo smart materials have been discussed for energy harvesting. The present work has been classified into the following categories:(i) experimental investigations (ii) simulation and (iii) mathematical modelling. In section (i) different experimental set-ups such as harvesting of energy with the help of vortex flow, turbulent flow, cross flow, flow in an open channel and closed channel and flow through nozzles have been examined. In section (ii) simulations studies performed with different tools/software like ANSYS/fluent, COSMOL etc. have been detailed. Lastly, in section (iii) different mathematical equations such as Navier-Stokes equation of motion, Continuity equation, finite element method, numerical methods, transport equations, Bernoulli equation, equation of linear elasticity, fluid structural equations, piezoelectric equations and coupled-wave equations are described for generation of energy with fluid’s interaction.

Many studies on open channel flow have focused on rectangular, parabolic, trapezoidal, and horseshoe-shaped channels, while circular-shaped channels with three lateral inflows have received minimal attention, indicating the need for further study to fill the gap. This research aims to investigate the modeling of a circular channel that can move the maximum amount of water from the most flooded area into irrigation land.

The discussed research studies above show that the effects of three lateral inflows in the velocity inside the main channel have not been adequately addressed. Therefore, the existing reservoir of literature is inadequate for addressing the complex dynamics pertain to the interplay between circular channels and three lateral inflows, highlighting the importance of more thorough investigations in this particular field

**2.0. Mathematical Model Development**

Let Q, be the discharges into the main circular open channel as well as: *q*1, *q*2, and *q*3 be the discharges onto first, second and third lateral inflow respectively. Let *L*1, *L*2, and *L*3 and *θ*1, *θ*2, and *θ*3 denote the length of the first, second and third lateral inflow channels and inclination angles of the first, second and third lateral inflows, respectively. The top diameter of the first, second and third lateral inflow channel is *T*1, *T*2, and *T*3 respectively (McGuirk and Rodi, 1978). At a time interval dt, the total amount of fluid that reaches the cell dx is taken into account as shown in the figure below



**Picture 1 : Development of a Mathematical Model**

**2.1. Model Assumption**

(i) Between the main channel and the three lateral inflow channels, there is not much solid particle buildup.

(ii)The three lateral inflows have a direct proportionality in length, angles, depth, and lateral inflow velocities. Q1 = q2 = q3, L1 = L2 = L3, and θ1 = θ2 = θ3,

(iii) The main canal and the three lateral inflows have circular cross sections.

**3.0. Governing Equations**

**3.1. Equation of Continuity**

A differential equation that characterizes the movement of a conserved variable inside a specified mass is called an equation of continuity. The basic notion behind all instances of continuity equations is the same: the amount that enters or exits an area through its border is the sole factor that may alter the overall amount of the conserved quantity inside that region (Serrin, 1959). A conserved amount can only travel from one place to another; it cannot grow or shrink. Unsteady flow in open channels of any shape is governed by the Saint-Venant equation of continuity, which is:

(1) Net volume of the fluid is , lateral inflow is and discharge for 3 lateral inflows is . Increment of the fluid is . Considering that our fluid's density is constant and consistent with the fluid's conservation law then,

. (2)

Dividing equation (2) through out by equation (3) is obtained

(3)

Discharge is given by the product of cross-sectional area and velocity

(4)

Substituting equation (4) above into equation (3) and differentiating partially with respect to *e*quation (5) is obtained

(5)

The flow area can be assumed to be a known function of the depth and therefore the derivatives of A can be expressed in terms of y.

(6)

Where *T* is thechannel top width and Franz (1982) assumed that T is determined by

(7)

Substituting equations (6) into (5), equation (8) is obtained,

(8)

Dividing equation (8) throughout by T and rearranging, equation (9) is obtained,

(9)

Equation (9) is the general equation of continuity for open channel flow with 3 lateral inflows S channel at an angle.

**3.2 Momentum equation**

Fluid motion is described by momentum equations. These formulas are based on Newton's second rule of motion and the idea that fluid stress is made up of a pressure term and a diffusing viscous term that is proportional to the gradient of velocities. They establish a connection between a fluid element's acceleration or rate of change of momentum and the total force exerted on it. Convective acceleration, which is linked to variations in velocity over location, is the cause of the non-linearity in these non-linear partial differential equations (White and Majdalani, 2006).

According to the conservation law in the momentum equation;

(10)

Dividing equation (10) through out by to obtain equation (11)

(11)

Substituting equation (4) into equation (11) above and differentiating partially with respect to x considering the area A is a constant to obtain equation (12).

(12)

Rearranging equation (12) to obtain equation (13)

(13)

Substituting Equation (3) into equation (13) to obtain equation (14)

(14)

Dividing equation (14) throughout by  to obtain equation (15)

(15)

Equation (15) is rearranged to obtain equation (16) we get,

(16)

Equation (16) is the general momentum equation of an open channel with 3 lateral inflow channels at varying angles.

**4.0 Equations Governing the Fluid Fow in Finite Difference Form**

Velocity profiles for three Lateral Inflows

The Continuity and Momentum equations is given by equations (17) and (18) and solved numerically by the Finite method

(17)

(18)

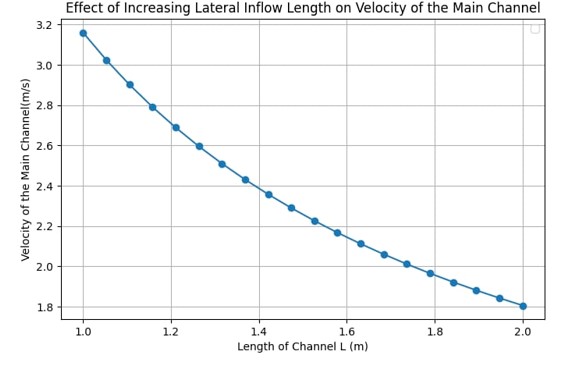
Subject to the condition

0.002;

**5.0 Results and Discussion**

**Effects of Variation of Length of Lateral Inflow on velocity of the Main Channel**

The effects of increasing Lateral Inflow Length on Velocity of the main channel is illustrated



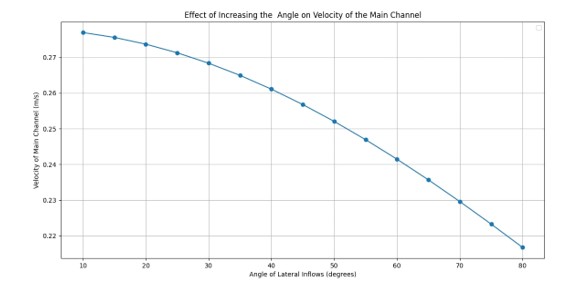
**Figure 1: Effect of variation of lateral inflow length on velocity in the main open channel**

Figure 1 demonstrates how the velocity of the main channel decreases when lateral inflows are introduced. This is because the water flows over a larger total cross-sectional area. In laminar flow, velocity and cross-sectional area are inversely correlated at a constant flow rate, according to the principle of continuity. Consequently, a drop in velocity is required for a bigger cross-sectional area. The average velocity is lowered because the additional water from the lateral inputs efficiently disperses the flow across a larger region. Laminar flow, in which water flows in smooth, stratified streams, is characterized by this redistribution of flow in addition to a decrease in velocity. Because turbulence is more likely to arise at higher speeds, lower velocity also aids in maintaining the laminar flow regime. By adding water along their path, lateral flows essentially redistribute the flow of the main channel. As a result, the concentrated flow from the channel's centre is dispersed, resulting in a more even distribution of water across its breadth. More water enters the system as the inflow length grows, which may increase the main channel's overall discharge (flow rate). The velocity may initially increase as a result of the increased discharge if the amount of water entering the main channel increases noticeably.

Depending on the terrain's characteristics (such as vegetation, roughness, or channel shape), a longer inflow length may result in more friction for the water as it passes through the channel. Once the water enters the main channel, this increased resistance may cause its velocity to decrease. If the channel is small and rough, the flow will encounter more drag and turbulence, which lowers the velocity over a longer distance. Wider and smoother channels, on the other hand, can offer less resistance, enabling the water to sustain a greater pace.

**Effects of Varying Angles**

The effect of increasing the angle on velocity of the main channel is illustrated in figure2.



**Figure 2: Effects of varying angle on velocity at**

Figure 2 demonstrates how flow dynamics, including the main channel's velocity, may be greatly impacted by increasing the angle of lateral inflow channels into the main channel. The water's behavior while interacting with the major flow is significantly influenced by the angle at which it enters the main channel from a lateral inflow channel. The possible impacts of raising the angle of lateral input on the main channel's velocity are broken down as follows:

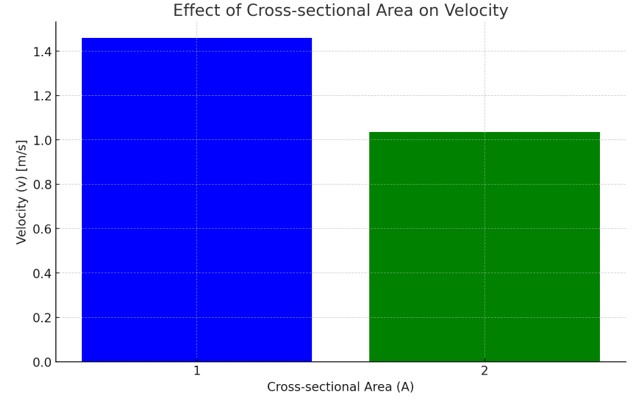
Turbulence is increased by sharp angles: The flow from the tributary enters the main channel flow more abruptly when a lateral inflow channel meets the main channel at a sharper angle (near perpendicular). As a result of the two flows interacting from opposite directions, there is more turbulence. Localized speed variations can result from turbulence-induced changes in the velocity of the main channel.

The amount of water entering the main channel gets more direct as the angle of lateral inflow rises, especially as the angle approaches perpendicular. The water's velocity downstream will rise if the lateral inflow is substantial since it will increase the discharge in the main channel, provided the channel can handle the extra flow. Stream power, or the energy available to move sediment and form the channel, is closely correlated with the main channel velocity. Higher lateral input angles and the corresponding rise in discharge cause stream power to rise, which can raise the velocity, especially downstream from the confluence.

The flow may split if the lateral inflow meets the main channel at a steeper angle, particularly if the flow direction changes abruptly. Dead zones or recirculating regions areas of low velocity in the main channel where the water does not move quickly can result from this separation. Furthermore, the main channel may develop spinning eddies or vortices as a result of abrupt lateral inflows.   
In general, raising the angle of lateral inflows at the top of a circular channel results in a more efficient and successful merging of flows. By lowering turbulence and energy losses, this maintains laminar properties and could even raise the overall flow velocity.

**Effects of Cross-Sectional Variation on Velocity in the Main Channel**

The effects of increasing the Cross- Sectional Area of Lateral Inflows on Velocity is illustrated in figure 3.



**Figure 3: Effect of cross – section area on velocity in the main open channel**

Figure 3 demonstrates how basic concepts like the continuity equation, flow distribution, and momentum conservation may have a substantial impact on the velocity of the main flow in an open channel system by varying the cross-sectional area of the lateral inflow channels. Any change in a channel's cross-sectional area (A) will result in a proportional change in the flow's velocity (V), according to the continuity equation, which stipulates that the flow rate (Q) in an incompressible fluid must remain constant. In particular, the formula is Q = AV, which states that, under the assumption that the discharge rate remains constant, a decrease in the cross-sectional area of lateral inflows will result in an increase in the main flow velocity.

When the lateral inflow channels have a smaller cross-sectional area, the volume of water entering the main channel is reduced. This leads to a decreased lateral inflow, and consequently, the velocity of the main flow generally increases to maintain the same discharge rate. On the other hand, if the lateral channel has a larger cross-sectional area, it will contribute more water to the main flow, causing an increase in the flow volume that needs to be accommodated by the main channel. In this case, the main flow velocity will decrease as the water is distributed across a larger cross-sectional area.

In addition to the changes in cross-sectional area, the rate of lateral inflow also plays a crucial role in determining the main flow velocity. A large, sudden lateral inflow whether from increased discharge or larger channel areas can lead to a temporary decrease in the velocity of the main flow, as the system adjusts to accommodate the additional water. Conversely, if the lateral inflow diminishes, the velocity of the main flow will increase, as less water enters the system. The interaction between the main flow and the lateral inflows also introduces turbulence and mixing, which can cause local fluctuations in the velocity profile of the main channel, especially near the junction.

**6.0 Conclusion**

An open Circular cross- section channel with three lateral inflows has been developed with the resulting partial differential equations solved using python software to obtain velocity profiles. The research found out that the velocity reduces in the main channel as the lateral inflows and angles increases. It is recommended that future research should be carried on;

(i) Effects of lateral outflow on discharge.

(ii) The flow in trapezoidal, rectangular and triangular with three lateral inflow channels in different point of the main flow.

(iii) Fluid flow through elliptic channels.

**Nomenclature:**

**Q** Discharge

**H** Radius

**A** Area

**V**  Velocity of the Channel

**S**  Slope of the Channel bottom

**Y** Depth of Flow

**Disclaimer (Artificial intelligence**)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

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