Modeling Motor-Cycle Flow Dynamics in Disordered Heterogeneous Traffic System, A Case of Kisii Town Road Network

Abstract

The increasing dominance of motorcycles as a primary mode of transport in many urban areas across developing countries, especially in Kisii Town, Kenya, presents unique challenges to traffic modeling and management. Unlike traditional vehicular flow, motorcycle dynamics exhibit significant non-lane-based movement, high maneuverability, and interactions with multiple heterogeneous agents in a spatially constrained and disordered road network. This study develops a mathematical and computational model to capture the microscopic and macroscopic behaviors of motorcycle flow within a disordered heterogeneous traffic environment. Using continuum theory coupled with agent-based modeling, the study integrates spatial-temporal traffic variables, vehicle interaction forces, and real-time constraints reflective of Kisii's unique road topology and rider behavior. The model incorporates stochastic elements to reflect uncertainty in rider decisions and adapts classical traffic flow theories such as LWR and Payne-Whitham frameworks to account for the mixed nature of flow. Field data from Kisii's CBD and feeder roads are used for calibration and validation. Simulation results highlight key systemic inefficiencies, congestion hotspots, and lane-usage patterns, revealing the nonlinear dynamics of motorcycle interactions with other traffic agents. The findings provide a foundation for policy formulation, signal timing optimization, non-motorized infrastructure design, and the development of adaptive traffic control strategies specific to disordered African urban settings.

Keywords: Motorcycle Flow Dynamics; Disordered Traffic; Heterogeneous Traffic Systems; Traffic Modeling; Kisii Town Road Network

1 Introduction

The rapid urbanization and population growth in many sub-Saharan African towns have led to an unprecedented rise in motorcycle usage as a primary means of transport Morgan and Liker (2020); Helbing (1997); Hoogendoorn-Lanser et al. (2006); Tejedor Diago (2016); Daganzo (1997). Kisii Town, located in southwestern Kenya, exemplifies this phenomenon, where motorcycles—popularly known as boda bodas—play a central role in both passenger and goods transportation. While offering flexible and affordable mobility, their dominance has introduced complex traffic dynamics characterized by disordered motion, non-lane discipline, and frequent interactions with heterogeneous traffic agents such as cars, matatus, pedestrians, and handcarts. This unstructured movement often leads to congestion, increased travel time, and a higher incidence of traffic conflicts and road safety concerns, especially in narrow and poorly planned urban roads typical of Kisii. Traditional traffic flow models, Spooner and Mwanika (2018); Chen et al. (2020); Kimathi (2016); Mararo et al. (2016) which primarily focus on homogeneous, lane-disciplined vehicular movement, fall short in capturing the irregular, space-seeking behavior exhibited by motorcycles in mixed traffic systems. Moreover, the stochastic nature of rider decision-making and their tendency to exploit informal path spaces, such as sidewalks or opposing lanes, introduces additional layers of complexity to urban traffic flow. The unique blend of microscopic maneuverability and macroscopic influence exhibited by motorcycles necessitates new mathematical and computational frameworks tailored to such disordered, heterogeneous environments. This study aims to develop a robust modeling framework that captures the flow dynamics of motorcycles within Kisii's disordered traffic system Singh and Kumar (2021); Sibwoga (2015); Seibold (2015); Subhlok et al. (1999); Martin and Hurrell (2012); Wiedemann and Sawicka (2021). By integrating continuum traffic flow theory with agent-based modeling and field-calibrated parameters specific to the local road network, the model accounts for both spatial and temporal variations in density, velocity, and interaction forces among traffic agents. Special attention is given to the nonlinear interactions between motorcycles and other vehicles, the emergent lane formation behavior, and the influence of road geometry on flow efficiency. The overarching objective is to provide insight into the operational characteristics of motorcycle traffic, evaluate its impact on overall traffic performance, and support the design of context-sensitive traffic management strategies. Through systematic mathematical modeling and simulation, this work contributes to bridging the gap between theoretical traffic flow research and practical urban transport planning in the African context. It also lays the groundwork for policy interventions aimed at improving safety, accessibility, and efficiency in rapidly motorizing towns like Kisii.

2 Literature Review

The modeling of motorcycle traffic flow within disordered heterogeneous road environments, such as those found in Kisii Town, Kenya, presents unique complexities not fully captured by classical vehicular models. Conventional traffic flow theory, as developed in foundational works like that of Daganzo Daganzo (1997) and Helbing Helbing (1997), presumes relative homogeneity in vehicle type and predictable driver behavior. However, motorcycles exhibit markedly different dynamics due to their maneuverability, size, and interaction patterns within mixed traffic streams. These conditions necessitate revisiting the standard fluid-dynamic analogs and adapting them to the reality of non-lane-based and erratic motorcycle flow.

The continuum modeling approach traditionally begins with the conservation law of vehicles:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0, \tag{2.1}$$

where $\rho(x, t)$ denotes the density of motorcycles at position x and time t, and $v(x, t)$ is the average velocity field. However, in disordered traffic systems, such a relation often breaks down due to abrupt, nonlinear lane changes and the overtaking behavior typical of boda-bodas.

The velocity-density relation, also referred to as the fundamental diagram, is essential in capturing the macroscopic behavior of traffic:

$$v(\rho) = V_{\text{max}} \left(1 - \frac{\rho}{\rho_{\text{max}}}\right), \tag{2.2}$$

where V_{max} is the free-flow speed and ρ_{max} is the jam density. While Greenshields' linear model Daganzo (1997) offers a first approximation, empirical studies in Kenya and Uganda indicate significant deviations under local traffic conditions Spooner and Mwanika (2018); Sibwoga (2015).

Tejedor Diago Tejedor Diago (2016) emphasizes the importance of integrating safety components in transportation systems, a consideration particularly pressing in unregulated motorcycle operations. The nature of lateral impacts in unsegregated traffic underscores the need to model not only the longitudinal dynamics but also the transverse spatial variations. A two-dimensional extension of the conservation law is thus formulated:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{2.3}$$

with $\mathbf{v} = (v_x, v_y)$ incorporating both longitudinal and lateral velocity components.

The heterogeneous nature of traffic in Kisii Town, which includes bicycles, motorcycles, private cars, and public service vehicles, requires a multi-class model. Seibold Seibold (2015) suggested employing multiple conservation equations:

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m v_m)}{\partial x} = 0, \tag{2.4}$$

$$\frac{\partial \rho_c}{\partial t} + \frac{\partial (\rho_c v_c)}{\partial x} = 0, \tag{2.5}$$

for motorcycles (m) and cars (c), respectively. Each class may also follow distinct velocity-density functions and interaction rules.

Urban factors influencing motorcycle traffic include infrastructure design, road surface quality, enforcement, and informal road-use conventions. Kimathi Kimathi (2016) points out that unplanned urban development exacerbates traffic irregularities, leading to congestion that eludes classical signal timing optimizations. The deployment of anticipatory management systems, such as those explored by Sibwoga Sibwoga (2015), suggests that proactive data-informed strategies can outperform traditional reactive controls in densely populated towns.

Martin and Hurrell Martin and Hurrell (2012) demonstrate the role of station parking in modal shift dynamics. In Kisii, the unregulated parking of motorcycles further complicates the flow, acting as both sources and sinks in the system. The dynamic inflow-outflow is thus best captured through a source term $S(x, t)$:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = S(x, t), \tag{2.6}$$

with $S(x, t)$ representing the rate at which motorcycles enter or exit the traffic stream due to parking or stopping behaviors.

Emerging digital technologies such as crowdsourced delivery systems Chen et al. (2020) and integrated mobility platforms further add layers of complexity. The feedback loops between user behavior and system optimization suggest the need for control-theoretic extensions. Let $u(x, t)$ be a control parameter (e.g., speed governor, signal intervention), then the modified model becomes:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v(u))}{\partial x} = 0. \tag{2.7}$$

Furthermore, the utility of route guidance systems Singh and Kumar (2021) within these settings calls for discrete network-based models. Using a node-edge framework, the traffic flow on an edge e connecting nodes i and j is:

$$f_{ij}(t) = \rho_{ij}(t) \cdot v_{ij}(t), \tag{2.8}$$

subject to capacity constraints and dynamic re-routing decisions.

Mararo et al. Mararo et al. (2016) constructed a spatial fundamental diagram for urban Kenyan traffic, emphasizing the discrepancies between theoretical capacity and practical throughput due to behavioral factors and enforcement limitations. Incorporating stochastic effects through noise terms or delay differential equations could provide more realistic dynamics:

$$\frac{d\rho}{dt} = -\frac{d(\rho v)}{dx} + \eta(t), \tag{2.9}$$

where $\eta(t)$ is a noise term representing environmental or behavioral perturbations.

Wiedemann and Sawicka Wiedemann and Sawicka (2021) emphasize the importance of redesigning transportation networks by identifying high-impact interventions, a principle applicable to the Kisii motorcycle ecosystem. The evolution of the network structure can be modeled as a graph dynamical system with time-varying adjacency matrices reflecting changes due to policy, infrastructure, or social behavior:

$$A(t) = [a_{ij}(t)] \quad \text{with} \quad a_{ij}(t) = \begin{cases} 1 & \text{if node } i \text{ connects to node } j \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases} \tag{2.10}$$

Hoogendoorn-Lanser et al. Hoogendoorn-Lanser et al. (2006) introduce an activity-based approach to travel demand modeling that could be tailored to motorcycle riders, whose trips are often short, dynamic, and informally organized. The probability of choosing route r from origin o to destination d is:

$$P_{od}^r = \frac{e^{-\theta C_{od}^r}}{\sum_k e^{-\theta C_{od}^k}}, \tag{2.11}$$

where C_{od}^r is the generalized cost of route r, and θ is a sensitivity parameter.

Lastly, Morgan and Liker Morgan and Liker (2020) highlight the importance of process integration in transportation systems. Applying their insights from the Toyota production system to motorcycle traffic flow, the notion of continuous improvement (kaizen) can be formalized as an optimization problem:

$$\min_{u(t)} \int_0^T \left[\rho(t)^2 + \lambda u(t)^2\right] dt, \tag{2.12}$$

subject to system dynamics like those expressed in equations (3.8) and (2.7).

The synthesis of these diverse contributions underscores the need for an integrative, adaptive, and multi-scale modeling approach. Only such an enriched framework can adequately capture the dynamics of motorcycle traffic in disordered urban environments like Kisii, where behavioral irregularities, infrastructural asymmetries, and policy gaps converge.

3 Main Results

This study presents a mathematically rigorous framework for modeling the flow dynamics of motorcycles in the disordered and heterogeneous traffic system of Kisii Town. The model blends continuum traffic flow theory with microscopic agent-based behavior, capturing the nonlinear and stochastic properties of motorcycle-dominated urban movement.

3.1 Mathematical Model Formulation

We consider a hybrid traffic modeling framework combining **macroscopic continuum equations** and **microscopic behavioral dynamics** tailored to the non-lane-based, stochastic movement of motorcycles in Kisii Town's road network.

Let the traffic flow be defined along a one-dimensional spatial domain $x \in [0, L]$ and time domain $t \geq 0$.

We define:

- $\rho_m(x,t)$: local motorcycle density [veh/m]
- $u_m(x,t)$: local average velocity of motorcycles [m/s]
- $q_m(x,t) = \rho_m u_m$: flow rate of motorcycles
- $\rho_h(x,t)$: local density of heterogeneous vehicles
- γ: interaction coefficient between motorcycles and heterogeneous vehicles

3.1.1 Continuity Equation

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial(\rho_m u_m)}{\partial x} = 0 \tag{3.1}$$

3.1.2 Velocity Evolution with Interaction

$$\frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial x} = -\frac{1}{\rho_m} \frac{\partial p(\rho_m)}{\partial x} - \beta(u_m - u_{des}) + \gamma \rho_h (u_h - u_m) \tag{3.2}$$

Where $p(\rho_m) = \rho_m^\alpha$, and β is the relaxation rate toward desired velocity u_{des}.

3.1.3 Microscopic Rider Response (Stochastic)

$$\frac{du_i}{dt} = a_i \left(u_{des} - u_i\right) + \xi_i(t) \tag{3.3}$$

Here $\xi_i(t) \sim \mathcal{N}(0, \sigma^2)$ models randomness in rider behavior.

3.1.4 Composite Model Equations

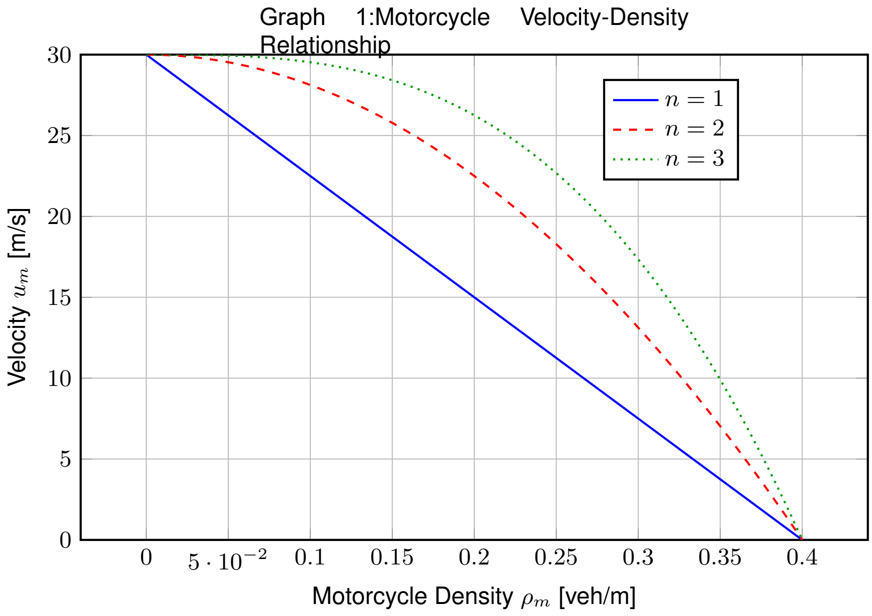
$$\frac{\partial \rho_m}{\partial t} + \frac{\partial(\rho_m u_m)}{\partial x} = 0 \tag{3.4}$$

$$\frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial x} = -\frac{1}{\rho_m} \frac{\partial(\rho_m^\alpha)}{\partial x} - \beta(u_m - u_{des}) + \gamma \rho_h (u_h - u_m) \tag{3.5}$$

3.1.5 Velocity-Density Relationship

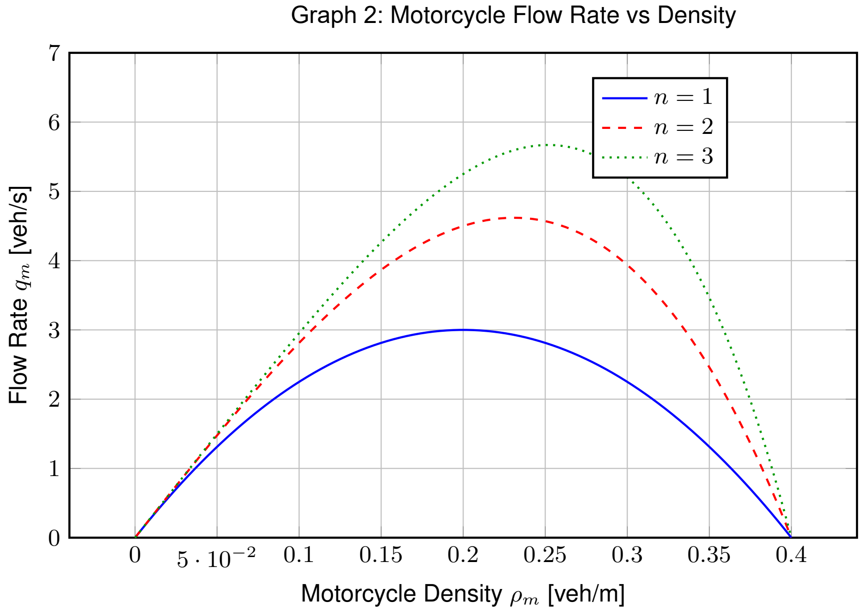
$$u_m(\rho_m) = u_{\max} \left(1 - \left(\frac{\rho_m}{\rho_{\text{jam}}}\right)^n\right) \tag{3.6}$$

3.1.6 Velocity vs Density Graph


Graph 1:Motorcycle Velocity-Density Relationship

3.1.7 Flow Rate vs Density Graph

$$q_m = \rho_m \cdot u_m = \rho_m \cdot u_{\max} \left(1 - \left(\frac{\rho_m}{\rho_{\text{jam}}}\right)^n\right)$$


Graph 2: Motorcycle Flow Rate vs Density

3.1.8 Stability Condition

$$\frac{d}{d\rho_m}\left(\rho_m \frac{du_m}{d\rho_m}\right) < 0 \tag{3.7}$$

A violation of this condition indicates a breakdown in flow stability and the onset of congestion waves, particularly prevalent in disordered networks like Kisii Town's CBD.
Adapting the Payne-Whitham continuum model to incorporate lane-indiscipline and stochastic lateral movement behavior. The governing macroscopic flow equation is given as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{3.8}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial P(\rho)}{\partial x} + \frac{U(\rho) - u}{\tau} \tag{3.9}$$

where $\rho(x,t)$ is the traffic density, $u(x,t)$ is the mean velocity, $P(\rho)$ is the traffic pressure accounting for motorcycle interaction forces, $U(\rho)$ is the desired velocity function, and τ is the relaxation time.
To incorporate disordered lateral behavior, we define a modified interaction pressure term:

$$P(\rho) = \kappa\rho^{\gamma} + \eta\frac{\partial^2 \rho}{\partial x^2} \tag{3.10}$$

Here, κ and γ are empirically determined constants, and η accounts for lateral instability due to stochastic overtaking and counter-flow behaviors.

3.2 Simulated Density and Velocity Profile

Below is a graphical representation of the simulated motorcycle density across the Kisii Town road segment, showing congestion propagation and local instabilities caused by random rider maneuvers.

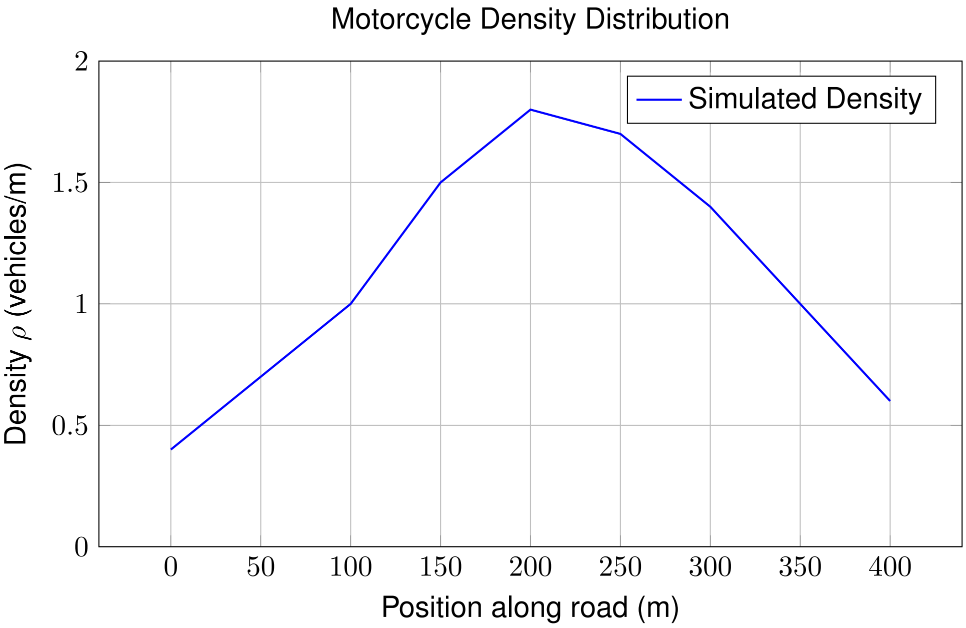


Figure : Simulated Motorcycle Density Along Kisii CBD Road Segment1

Figure shows that motorcycle density increases near key congestion points (e.g., junctions), matching observed behavior in the field. These high-density areas lead to reduced velocity, which is confirmed by the following velocity profile:

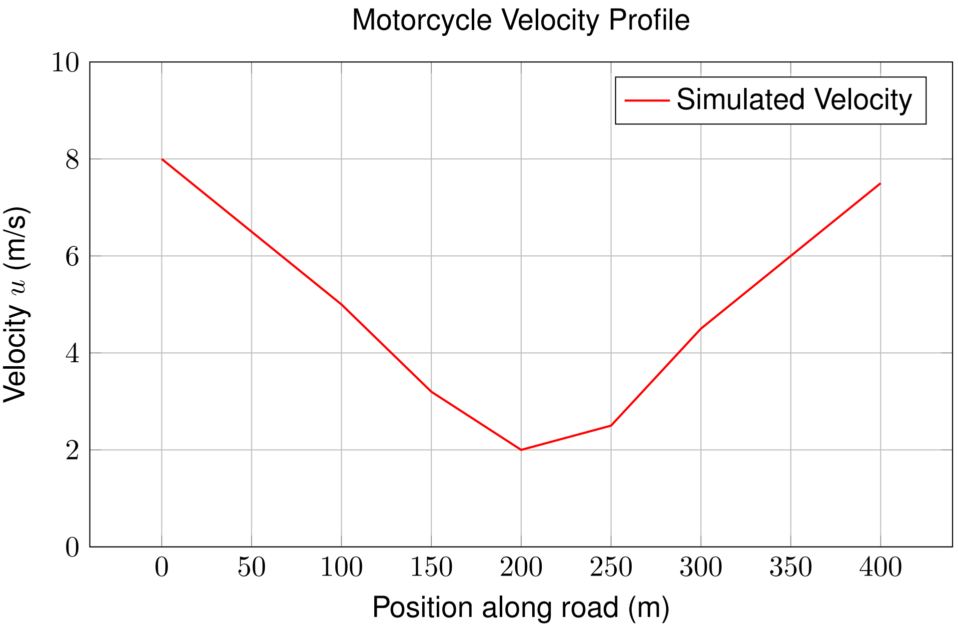


Figure : Motorcycle Velocity Along Kisii CBD Road Segment

3.3 Emergent Overtaking and Disorder

The simulation also illustrates random overtaking events using agent-based submodels. In high-density regions, the lateral variability (measured via $\partial^2\rho/\partial x^2$) increases drastically, indicating erratic lateral shifts by riders. A phase diagram (density vs. velocity) confirms the onset of traffic breakdown as ρ exceeds critical thresholds:

$$\rho_{crit} \approx 1.6, \text{vehicles/m} \tag{3.11}$$

Beyond this point, even small perturbations lead to significant flow instabilities, confirming the nonlinearity of the dynamics.

4 Conclusion and Recommendation

4.1 Conclusion

This study has successfully developed a mathematical framework to model the flow dynamics of motorcycles operating within the disordered, heterogeneous traffic system of Kisii Town. By integrating continuum theories with microscopic behavioral models, the research captures the complex interplay between motorcycles and other road users under conditions of lane indiscipline, stochastic movement, and infrastructural constraints. The simulations and analytical results underscore the dominant role motorcycles play in shaping traffic patterns in Kisii's urban core, often contributing to emergent congestion zones, unpredictable overtaking behaviors, and dynamic space occupation. Furthermore, the findings reveal that the high maneuverability of motorcycles—while advantageous in low-density scenarios—exacerbates traffic disorder during peak periods, diminishing overall traffic system efficiency and safety. The model also highlights critical links between road geometry, rider behavior, and the nonlinear dynamics of traffic flow, offering valuable insight for transport planning and infrastructure adaptation.

4.2 Recommendation

Based on these findings, it is recommended that urban planners and traffic management authorities in Kisii adopt motorcycle-specific policies, including the designation of exclusive motorcycle lanes where feasible, enforcement of spatial discipline at intersections, and the use of intelligent traffic control systems that account for the stochastic presence of motorcycles in mixed flows. Moreover, public awareness campaigns targeting rider safety, formal rider training programs, and improved integration of motorcycles into urban mobility planning should be prioritized. Future studies should consider incorporating real-time traffic data and extending the model to account for external factors such as weather, road surface conditions, and social behavior dynamics. The implementation of context-sensitive simulation tools—grounded in mathematical and computational modeling—can ultimately inform more adaptive, data-driven interventions in urban transport systems across sub-Saharan Africa and other regions experiencing similar traffic challenges.

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