**DEVELOPMENT OF EMPIRICAL CORRELATIONS FOR PORE PRESSURE PREDICTION FROM WELL LOGS USING MULTIPLE LINEAR REGRESSION**

**Abstract**

Accurate pore pressure prediction is essential for safe drilling operations and effective reservoir modeling, especially in regions where overpressure from disequilibrium compaction poses significant challenges. These challenges can lead to issues such as fluid loss, kicks, differential pipe sticking, heaving shale, and blowouts. Traditional methods often fail to capture the complex relationships between formation parameters and pore pressure. This study utilizes a machine learning (ML) approach to capture these intricate relationships, developing two empirical correlations for pore pressure prediction. The first correlation includes lithological information (sand and shale), and the second does not. Both correlations are derived from a linear regression model fitted to the well-log datasets. The data used in this research was obtained from three wells in the Northern Carnarvon Basin of Australia. It includes parameters such as sonic interval transit time, density, gamma-ray, depth, and well diameter. Wells 1 and 2 contributed approximately 22,038 data points, which were divided into 85% for model training and 15% for validation. The 8,860 data points from the Well 3 were used to test the model's accuracy.

The results, as evaluated by statistical metrics like mean absolute relative error (MARE), mean relative error (MRE), and root mean square error (RMSE), show that the newly developed correlations perform better than existing ones. The first model, which includes lithological data, demonstrated promising accuracy with an RMSE of 352.208. The second model, which does not include lithological data, surpassed the first with an RMSE of 342.105. These developed correlations offer enhanced predictive capabilities and effectiveness, making them suitable for estimating pore pressure in real-world drilling scenarios.

**Keywords**: Formation pore pressure, Abnormal pressure, Multiple linear regression, Reservoir modeling, Machine learning

**1. INTRODUCTION**

Pore pressure, also referred to as formation pressure, is the force that fluids exert within the pore spaces of rock formations. Pore pressure can be classified as normal or abnormal based on its magnitude. “With normal pore pressure ranging from 0.433 psi/ft in freshwater to 0.465 psi/ft in salt water, while abnormal pore pressure, due to geological and hydrodynamic factors, can be overpressured (higher) or under pressured (lower) than normal pressure” (Swarbrick et al., 1998). Overpressure in oil and gas drilling can be generated by different types of mechanisms, including compaction disequilibrium, chemical diagenesis, differential density effect, and fluid expansion which poses significant risks, including kicks, blowouts, lost circulation, and mud loss. These events can lead to increased non-productive time, hazardous accidents, and increased project costs. Therefore, Accurate pore pressure estimation is essential for ensuring the safety and cost-effectiveness of drilling operations. It plays a vital role in minimizing risks and enhancing overall project efficiency.

Abnormal formation pressure can be detected and estimated using predictive (pre-drill) methods like correlations from nearby wells and seismic data analysis. During drilling, monitoring drilling parameters and using measurement tools can provide real-time indications. Post-drill, wireline logging, and formation testing can verify the accuracy of pore pressure estimates.

Many researchers have conducted studies on detecting abnormal pore pressures. The significant focus on this issue highlights both the critical nature of this information and the challenges encountered in developing a reliable method to deliver it when urgently required. "Hottmann and Johnson (1965) first predicted pore pressure from shale properties using well log data. This technique identifies deviations from normal compaction trends as abnormal pressure." Afterward, other researchers presented empirical equations using sonic transit time, resistivity, porosity, and other well-logs data for pore pressure prediction. "Ham's (1966) equivalent depth method identifies abnormal pressure zones by comparing it to normal pressure in a nearby formation." "Eaton (1975) proposed two empirical equations to quantify the pore pressure using well log data from sonic log and resistivity log presented in equations 1.0 and 2.0."

…………………………………………….1.0

………………………………………………2.0

“where Is the pore pressure gradient (psi/ft); OBG is the overburden gradient (psi/ft); is the hydrostatic gradient (psi/ft); Is the normal compaction trend line (s/ft); Is the observed sonic log value (s/ft); X is the exponent value which is dependent on formation properties).” is the resistivity value obtained from logs; Is resistivity of the normal compaction trend line. "Bowers (1995) introduced a power relationship between sonic velocity and effective stress in petroleum basins, using well logging data to calculate vertical effective stress and pore pressure." "Eaton's and Bower’s methods are the most commonly used traditional approaches for pore pressure prediction, which attribute compaction disequilibrium as the primary mechanism for generating overpressure."

In recent decades, various data-driven methods have been developed to estimate subsurface pressure using well-log data, seismic data, and drilling parameters. The implementation of diverse machine learning techniques for predicting pore pressure often falls short of providing an empirical correlation for real-time practical application, unless integrated into software, highlighting the drawbacks of black-box ML models. "Hao Yu et al. (2020) developed a machine learning method for pore-pressure prediction from well logs, using a nonparametric multivariate model of petrophysical properties." A year later, "Abdelaal et al. (2021) developed three predictive models for real-time pore pressure gradient prediction from mechanical and hydraulic drilling parameters, using support vector machines (SVM), functional networks, and random forest (RF)." "AE Radwan et al. (2022) employed machine learning techniques to predict pore pressure from geophysical logs in the Mangahewa gas field, New Zealand, using a decision tree, Adaboost (ADA), random forest (RF), and transparent open box (TOB)." "Huayang Li et al. (2023) also employed machine learning to predict pore pressure in high-pressure reservoir zones, achieving 95% accuracy using KNN, Extra Trees, Random Forest, and LightGBM algorithms." "Deng S. et al. (2024) improved pore pressure prediction accuracy using machine learning and optimization algorithms, integrating the IGWO-MLP model for superior performance and high R-squared values."

**2.0 METHODOLOGY**

**2.1 DATA DESCRIPTION AND ANALYSIS**

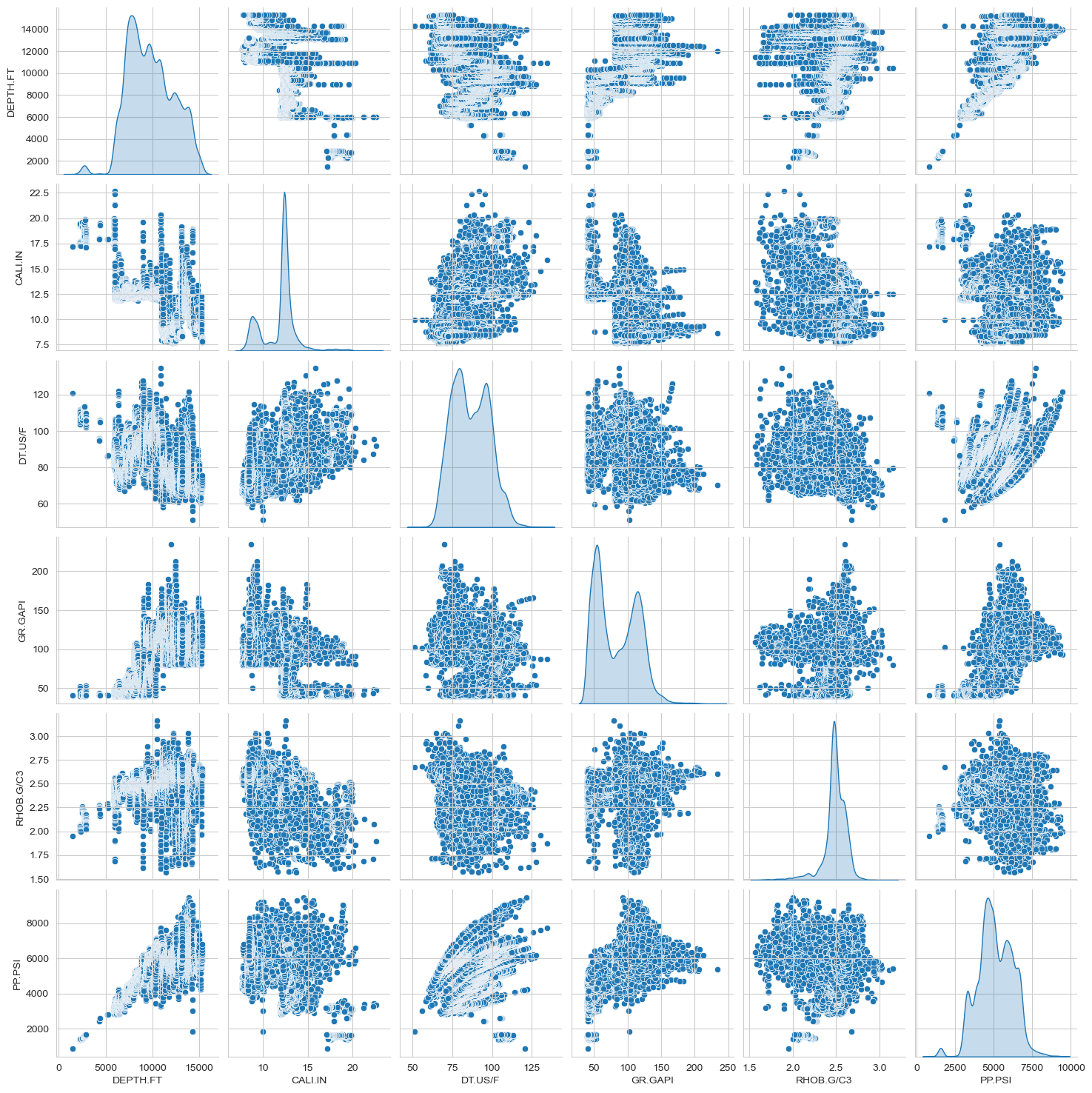
The datasets utilized in this study comprise well-logging data from three wells located in the Northern Carnarvon Basin, Australia. A total of 30,898 data points from these wells were analyzed and employed to develop the workflow outlined in this paper. With well 1 contributing 11,876 data points with a depth range of 457.6572 – 4,016.1972m, well 2 contributed 10,162 data points with a depth range of 690.8292 – 4,657.9536m, and well 3 contributed 8,860 data points with a depth range of 805.434 – 4,104.894m. The dataset includes features such as sonic time difference, density, gamma-ray, depth, and well diameter, which are intrinsically linked to the physical properties of the rock layer. The dataset from wells 1 and 2, comprising 22,038 data points, is split into an 85% training set and a 15% validation set, with well 3 serving as the blind testing dataset for independent evaluation of the model's performance. Tables 1.0 and 2.0 show a statistical summary of the training and testing dataset. Fig 1. shows a scatter pair plot which helps to visualize the distribution of variables and identify potential trends.

**Table 1.** A Statistical Summary of The Training Dataset’s Features

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | DEPTH.FT | CALI.IN | DT.US/F | GR. GAPI | RHOB.G/C3 | PP.PSI |
| Count | 22038.0000 | 22038.0000 | 22038.0000 | 22038.0000 | 22038.0000 | 22038.0000 |
| Mean | 9806.456534 | 11.877427 | 86.504113 | 83.767319 | 2.486612 | 5101.192241 |
| Std | 2406.636361 | 1.883954 | 11.414946 | 31.265961 | 0.137513 | 1121.135852 |
| Min | 1501.500048 | 7.663616 | 51.351720 | 40.000000 | 1.568559 | 838.865505 |
| Max | 15282.0008 | 22.640000 | 134.619900 | 234.330000 | 3.160000 | 9472.135973 |

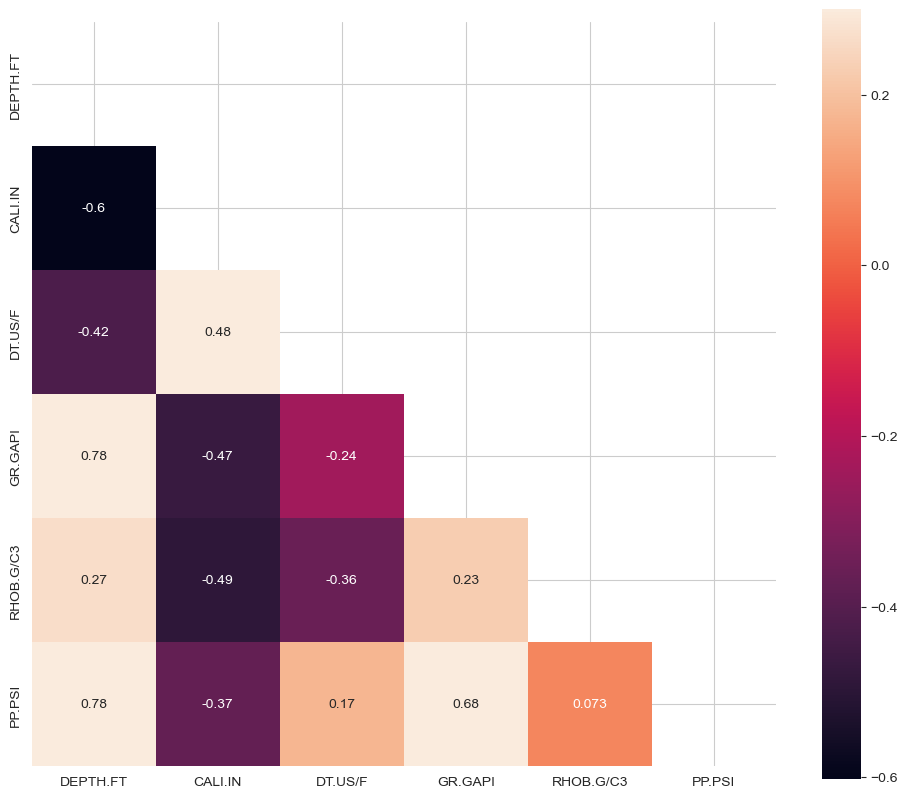
**Table 2.** A Statistical Summary of The Testing Dataset’s Features

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | DEPTH.FT | CALI.IN | DT.US/F | GR. GAPI | RHOB.G/C3 | PP.PSI |
| Count | 8860.0 | 8860.0 | 8860.0 | 8860.0 | 8860.0 | 8860.0 |
| Mean | 10569.1603 | 13.899537 | 89.653105 | 8860.0 | 2.486048 | 5933.361139 |
| Std | 1639.4371 | 2.216614 | 12.189492 | 34.602248 | 0.129586 | 843.429689 |
| Min | 2642.5000 | 8.735000 | 46.180000 | 40.000000 | 1.320000 | 1308.379001 |
| Max | 13467.5004 | 24.780000 | 133.350000 | 208.380000 | 3.336000 | 8981.648788 |

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**Fig 1** A Scatter pair-plot of the datasets utilized in this study

This study uses univariate and model-based feature selection to predict pore pressure. Pearson correlations in Fig. 2 show depth decreases pore pressure, larger caliper readings, and gamma-ray values indicate lower and higher pressures, and higher sonic travel times slightly increase pressure. Two predictive models were developed, one including all relevant features and the other excluding gamma-ray.



**Fig. 2.** Pearson correlation of the pore pressure features

**2.2 MATHEMATICAL CORRELATION MODELING**

Two new regression-based correlation models will be developed to compute pore pressure. The first model uses depth, well diameter, Interval Transit Time, gamma-ray, and bulk density as predictors, as shown in equation 3.0. The second model excludes gamma-ray, using depth, well diameter, Interval Transit Time, and bulk density, as described in equation 6.0. Both models aim to accurately predict pore pressure through these variables.

**2.2.1 First Correlation**

​……………………….3.0

Where , = Formation pore pressure, psi; = Depth, ft; = Well diameter, Inches; = Sonic interval transit time, ; = Gamma-ray value, GAPI; = Bulk density, ; = unknown correlation constants

The nonlinear function was linearized by introducing the logarithm function in equation 3.0:

+ + + …………………………………4.0

The above equation 4.0 corresponds to the multiple regression equation 5.0

y = + x1 + x2 + x3 + ……………………………………5.0

where; y = ; x1 = ; x2 = ; x3 = ; = ; = ; = residual error term

**2.2.2 Second Correlation**

………………………6.0

Where, = Formation pore pressure, psi; = Depth, ft; = Well diameter, Inches; = Sonic interval transit time, ; = Bulk density, ; = unknown correlation constants

The nonlinear function was linearized by introducing the logarithm function in equation 6.0:

+ + + …………………………………7.0

The above equation 7.0 corresponds to the multiple regression equation 8.0

y = + x1 + x2 + x3 + ……………………………………8.0

where; y = ; x1 = ; x2 = ; x3 = ; = ; = residual error term

It is therefore objective to determine the optimal values of constants to that render the residual term e negligible in the developed regression model. To achieve this, the study harnesses the power of Multiple linear regression Machine Learning algorithms to identify the best constants (global minimum) that minimize the residual error term to zero

**2.3 MACHINE LEARNING MODELS UTILIZED**

This study aims to develop a reliable and easy-to-use correlation for predicting pore pressure using

linear machine learning models. To achieve this, three linear machine learning models will be explored. This section provides a basic understanding of the mechanisms these models employed,

which will influence the results analyzed later in the study.

**2.3.1 Multiple Linear Regression**

Multiple linear regression is a statistical method used to model the relationship between a dependent variable and several independent variables. The goal is to derive the most accurate linear equation for prediction and analysis by estimating coefficients that reduce the sum of squared differences between the observed and predicted values. This process minimizes the L2 norm of the residuals, helping to determine the best-fitting coefficients.

**2.3.2 Regularized Lasso Regression**

Lasso Regression, an abbreviation for Least Absolute Shrinkage and Selection Operator, is a linear regression method that improves model performance and facilitates feature selection through regularization. It is especially effective for datasets with many predictors, as it identifies significant features while minimizing overfitting. The method introduces an L1 regularization penalty to the standard least squares objective function, limiting coefficient magnitude and encouraging sparse solutions.

**2.3.3 Regularized Ridge Regression**

Ridge Regression is a method in linear regression designed to enhance the stability of the model and manage multicollinearity by incorporating an L2 regularization term and a penalty to the standard least squares objective function. This penalty constrains the sum of coefficient squares, promoting

smaller values and reducing multi-collinearity impact. The objective function is to find values of beta that minimizes this function, balancing data fit and coefficient magnitude, while considering the L2 norm and regularization parameter.

**2.4 MODEL PERFORMANCE METRICS**

In this study, three statistical measures were employed to validate and evaluate the accuracy of the new correlations with the already existing ones.

**2.4.1 Root Mean Square Error**

Root Mean Square Error (RMSE) was calculated mathematically using Equation 9.0,

…………………………………………9.0

where 𝑛 denotes the number of observations, ​ represents the observed values, and ​ represents the predicted values.

**2.4.2 Mean Absolute Relative Error**

The Mean Absolute Relative Error (MARE), which measures the average percentage difference between predicted and actual values, was calculated using Equation 10.

……………………………………10

where 𝑛 denotes the number of observations, ​ represents the observed values, and ​ represents the predicted values.

**2.4.3 Mean Relative Error**

The Mean Relative Error (MRE), which quantifies the average deviation between predicted and actual values and provides insights into the predictive model's performance by normalizing the differences based on the magnitude of actual values, was mathematically calculated using Equation 11.

………………………………………………11

where 𝑛 denotes the number of observations, ​ represents the observed values, and ​ represents the predicted values.

**3.0 Result and Discussion**

**Result**

One important criterion for choosing a regression model was the need for an interpretable model with accessible coefficients and intercepts for empirical relationships. Three linear machine learning models were trained and validated on 85% and 15% of the training data set. The Simple Multiple linear regression model outperformed the Ridge and Lasso models, with an RMSE of 0.019514 and 0.019375 on the trained and validated set respectively as shown in Table 3.0. The unknown regression constants to were therefore extracted from the Multiple Linear Regression model which yielded the following correlations given below in equation 12 and 14.

**3.1 First Correlation:**

**\* ​………………….…….12**

Where; = 0.007557138824091286; = 0.99647091; = -0.0768239

= 1.03034302; = -0.03406814; = 0.02280915

**………….13**

**3.2 Second Correlation**

**…………………………………………14**

Where; = 0.009850685759312815; = 0.95617249; = -0.07235208;

= 1.01772954; = 0.02359403

**……15**

**Table 3.0** Models Performance on Pore pressure Training datasets

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Models | RMSE-Train | RMSE-Val | MAE-Train | MAE-Val |
| Linear Regression | 0.019514 | 0.019375 | 0.407732 | 0.405713 |
| Ridge | 0.022385 | 0.022266 | 0.444818 | 0.450455 |
| Lasso | 0.105746 | 0.099973 | 2.210725 | 2.128521 |

**3.4 MODEL COMPARISON WITH EXISTING CORRELATION**

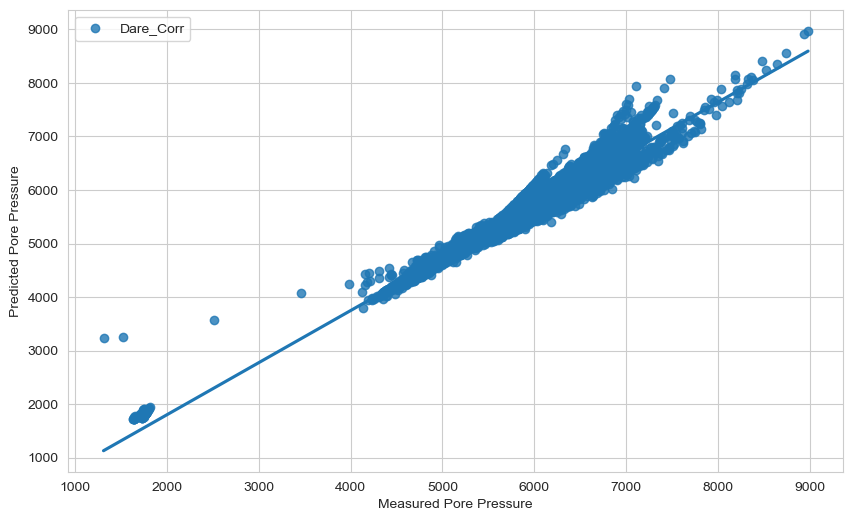
The performance of the newly developed empirical correlations is further compared with the widely used Eaton's correlations carried out on the test datasets, which is crucial to ascertain the extent of improvement achieved by the new correlations in predicting pore pressure using performance metrics.

**3.5 Comparison of the First Correlation with Eaton’s Correlation on the Test Datasets**

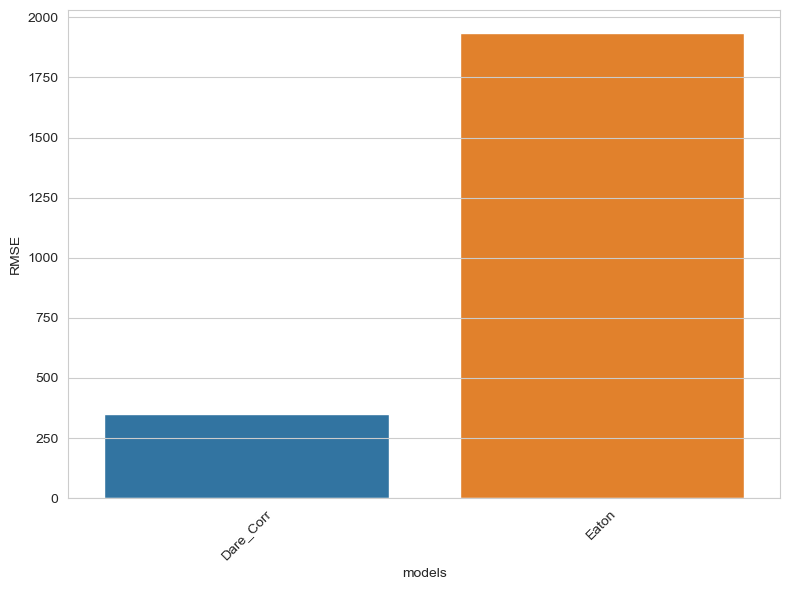
The results in Table 4.0 indicate that the new (first) Correlation model outperforms the Eaton correlation in predicting pore pressure, evidenced by its lower RMSE and MARE values. This new model provides a better fit and more accurate predictions, with a positive MRE indicating overestimation, contrasting with the negative MRE of the Eaton correlation. Figs. 3 and 4. visually support these findings, showing the new model's superior performance in a scatter plot of measured versus predicted pore pressure and a bar plot comparing RMSE scores, respectively.

**Table 4** a comparison of the First correlation developed with Eaton’s correlation on the Test datasets

|  |  |  |  |
| --- | --- | --- | --- |
|  | RMSE | MARE | MRE |
| New-Correlation | 352.207768 | 5.398975 | 5.120795 |
| Eaton | 1933.965552 | 29.506270 | -28.907812 |

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**Fig 3** a cross-plot between measured and predicted pore pressure on the entire test dataset using the first correlation extracted from the Multiple linear regression model.

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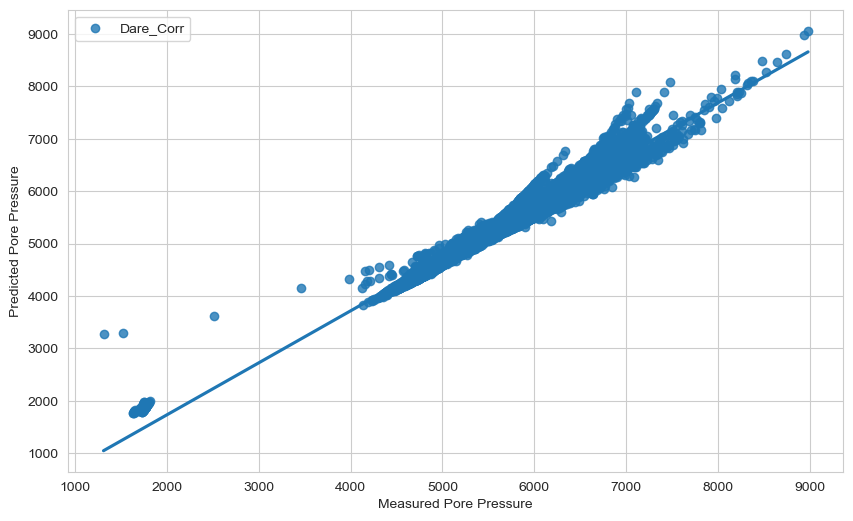
**Fig 4.** A bar plot showing the first correlation and Eaton ranking on the testing dataset based on RMSE score

**3.6 Comparison of the Second Correlation with Eaton’s Correlation on the Test Datasets**

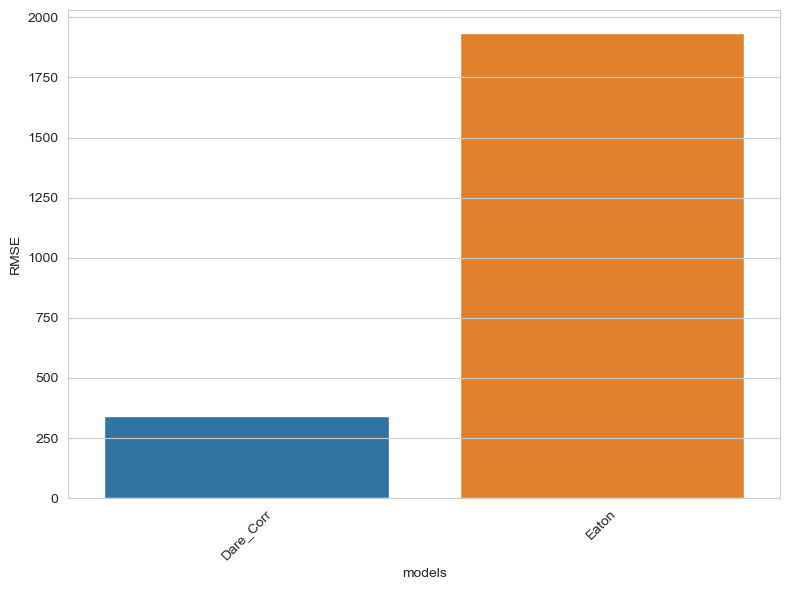
The result obtained from Table 5.0, shows that the New Correlation model outperforms the Eaton correlation in predicting pore pressure, indicated by lower RMSE and MARE values. The New Correlation model offers a better fit and more accurate predictions, with a positive MRE reflecting overestimated values, in contrast to the Eaton correlation's negative MRE, which suggests underestimated pore pressure levels. Figures 5 and 6 support these findings, depicting the relationships between measured and predicted pore pressures and comparing RMSE scores of the second correlation and Eaton correlation, respectively.

**Table 5.** a comparison of the Second correlation developed with Eaton’s correlation on the Test datasets

|  |  |  |  |
| --- | --- | --- | --- |
|  | RMSE | MARE | MRE |
| New-Correlation | 342.104553 | 5.385154 | 5.043188 |
| Eaton | 1933.965552 | 29.506270 | -28.907812 |

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**Fig 5** Cross plot between measured and predicted pore pressure on the entire testing dataset using the second correlation extracted from the Multiple linear regression model.

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**Fig 6** A bar plot showing the Second correlation and Eaton ranking on the testing dataset based on RMSE score

**3.6 Discussion**

In this research study, Eaton's method was applied to estimate pore pressure using acoustic log data from the existing test data, based on the assumption of a normal compaction trend line derived from the exponential relationship between interval transit time and depth. The overburden pressure was determined using density log measurements, and a mud weight of 9.7 ppg was employed to calculate the hydrostatic gradient. Pore pressure values were computed using Eaton's equation (1) for comparison purposes with an ‘x’ exponent of 3. The results obtained from both correlations show that the new empirical correlations predict pore pressure more accurately than Eaton's correlation. Their lower RMSE, MARE, and MRE values indicate greater accuracy and reliability, making them better suited for practical applications in regions of overpressure due to disequilibrium compaction in the Northern Carnarvon Basin.

**4.0 Conclusion**

This study employed machine learning models to establish two empirical correlations for predicting pore pressure using well-logging data from three wells in the Northern Carnarvon Basin, Australia. The two correlations were developed from multiple linear regression models by linearizing a nonlinear pore pressure function, and the best coefficient constants were extracted from the linear regression model fitted to the datasets. Both correlations were compared to an existing correlation, and the results showed that the new empirical correlations predict pore pressure more accurately than Eaton's correlation. The comparative analysis of the two developed correlations revealed that the second correlation was superior, with an RMSE of 342.104553 compared to the first correlation's RMSE of 352.207768, making it the optimal choice. Despite both correlations performing significantly well, the first correlation, which utilized lithology values from gamma-ray logs, is proposed for predicting pore pressure for new wells using existing data. Conversely, the second correlation, effective even without specific lithology information, is proposed as a robust solution for predicting pore pressure in new wells for future drilling operations. This versatility enables decision-makers to tailor their pore pressure prediction methods based on data availability, thereby supporting sound decision-making in oil and gas exploration.

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