*Original Research Article*

On The Spectrum Of Some Infinite Matrix As An Operator On the Sequence Space

.

ABSTRACT

*In various papers some authors have previously investigated [2], [3], [4], [5], [6] and determined the spectrum of weighted mean matrices considered as bounded operators on various sequence spaces. In this study, we determine the spectrum of a Norlund matrix as a bounded operator over the sequence space . This will be achieved by applying spectral theory, Banach space theorems of functional analysis as well as summability methods of summability theory. In which case it is shown that the spectrum of , that is . Also it is shown that .*

*Keywords: [ Spectrum, Norlund means, Sequence spaces, Boundedness }*

1. INTRODUCTION

Summability theory has various applications in functional analysis. Summability is typically the rule of assigning limits, which is central to analysis. The findings of this study will help engineers make improvements in the engineering fields where spectral values are applied. Mathematicians will find it meaningful when addressing issues of the same kind.

Some authors have studied the spectrum of a Norlund infinite matrix over different spaces. We introduce knowledge in the existing literature concerning the spectrum of Norlund infinite matrix.

Dorff and Wilansky [1] presented that the spectrum of a certain mercerian Norlund matrix with . In 1965, Brown et al [2] discovered the spectrum and eigenvalues of the Cesaro operator of space of square summable sequences. Wenger [3] calculated the fine spectra of Holder summability operators on the space of convergent sequences . Deddens [4] discovered the spectrum of all Hausdorff operators on and Rhodes [5] extended Weger’s work by determining the fine spectra of weighted mean operators on . Reade [6] strongminded the spectrum of Cesaro operator on the space of null sequences . Gonzale[7] worked on the fine spectrum of . Mutekhele[8] in his PhD dissertation extended Okutoyi’s work by determining the spectrum of operator on c(c) - the space of double sequences which converge. He further determined the fine spectra of operator on c(c) - the space of double sequence which converge. Coskun [9] determined the set of eigenvalues of a special Norlund Matrix as a bounded operator over some sequence spaces. Akanga [10] evaluated the spectrum of a special Norlund matrix as a bounded operator on . Irene [11] determined the spectrum of a special Norlund means as an operator on .

A number of research has been done on the spectrum of weighted mean matrices such as Cesaro and Holder means based on the review of the literature. However, not much has been accomplished through Norlund means. The spectrum of a Norlund matrix acting as an operator on the sequence spaces is determined in this study.

2. preliminaries and methods

**2.1 Spectrum**

Let and be Banach spaces and be a bounded linear operator. By , we denote the range of , i. e.

By ,we denote the set of all bounded linear operators on into itself. If , then the adjoint is a bounded linear operator on the dual defined by

 for all . Let be a complex normed linear space, where is the zero element and be a linear operator with domain . With , we associate the operator

 ,

where is a complex number and is the identity operator on . If has an inverse which is linear, we denote it by , that is

 ,

and call it the resolvent operator of . A regular value is a complex number such that;

1. exists
2. is bounded
3. is defined on a set which is dense in i.e

The resolvent set of , denoted by , is the set of all regular values . Its complement in the complex plane is called the spectrum of .

**2.2 Classical Summability**

The central problem in summability is to find means of assigning a limit to a divergent sequence or sum to a divergent series. In such a way that the sequence or series can be manipulated as though it converges, (Ruckel, 1981), pp. 159-161. The most common means of summing divergent series or sequences, is that of using an infinite matrix of complex numbers or by a power series.

***2.2.1. Definition***: *Sequence to Sequence transformation*

Let be an infinite matrix of complex numbers. Given a sequence define . If the series, converges for all , then we call the sequence , the of the sequence . If further,

 , we say that is summable .

There are various sequence to sequence transformations, here we state Norlund means below which is the matrix of intrest in this paper.

***2.2.2 (Norlund means)***

*The transformation given by*

*where , is called a Norlund means and is denoted by ( N,p ).*

*Its matrix is given by*

*In the matrix above if . i.e*

**

*or*

****

***2.2.3 Adjoint of A***

*It is the transpose of the matrix A and we denote it here by* *.*

***2.2.4 Dual space of***

*It is denoted and it is the space ; the space of absolutely convergent series.*

**2.3 General Results in Classical Summability**

**Definition 2.3.1** *(regular method, conservative method)*

Let be an infinite matrix of complex numbers.

1. If the transform of any convergent sequence of complex numbers exists and converges then is called a conservative method. We then write
2. If the transform of any convergent sequence of complex numbers exists and converges, then is called regular.

**Theorem 2.3.1** if and only if

1. for each fixed
2.

Proof: (Hardy, 1948), pp. 42 - 60; (Maddox, 1970), pp. 165 - 167.

3. results and discussion

Here we first show that exists and , secondly the spectrum of Norlund matrix is determined in the space .

**3.1 Exists and**

**Proposition 3.1.1**

*Proof:* This is done by solving the system for in terms of so as to find . That is

****

This gives the system

Now solving this system by working out in terms of gives;

This yields the matrix of denoted by



It is observed that , that is

**Corollary 3.1.1**

*Proof.* From theorem 2.3.1 it is clear that ; From if then generally

Also if , then

Hence matrix

The columns converges to zero if by ratio test for each

From matrix if we have that

Similarly for columns , by ratio test we have

So that all columns converge to zero for all

For the second condition we have a remark;

***Remark* 3.1.1,** For any matrix If then,

 Maddox 1970 pg 164, Reade 1985 pg 266

Now summing the entries of along the we have,

By the remark, hence condition two is filled.

This implies that if such that

**3.2 The Spectrum of**

**Theorem 3.2.1** Let such that exists. Then is the set

*Proof*. First we find . Then the complement of this is our spectrum, that is;

 From corollary 3.1.1, we find out that such that

 We also note that when , then the first column is infinite. That is

 Therefore the inverse does not exist hence the spectrum of is the set

 A disc in the complex plane centered at of radius 2.

**Theorem 3.2.2.**The spectrum is the set

*Proof.* Goldberg (1996)

**3.3 Conclusion**

 Exists and , further the spectrum is the set

Also**,** the spectrum is the set

**Disclaimer** (Artificial intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT) has been used in the manuscript.

**References**

‘

[1] Dorff, E. K. & Wilansky, A. (1960). Remarks on Summability, J L M S 35, 234 – 236.

[2] Brown A., Halmos, P & Shields, A. (1965). Cesaro operators, Acta Scientiarum Mathematicarum 26, 125 – 137.

[3] Wenger, R. B. (1975). The fine Spectra of Holder Summability operators, Indian Journal of Pure and Applied Math. 6, 695 - 710.

[4] Deddens, J. A. (1978). On the Spectra of Hausdorff operators on l 2 +, Proceedings of the American Mathematical Society 72(1).

[5]Rhoads, B. E. (1983). The Fine Spectra of Weighted mean Operators, Pacific Journal of Maths 104, 219 - 230.

[6] Reade, J. (1985). On the Spectrum of Cesaro Operator, Bull. Lond. Math. Soc 17, 263 – 276.

[7] Gonzalez, M. (1985). The Fine Spectrum of the Cesaro operator in (1 < p < ∞), Arch. Math 44, 355 - 358.

[8] Mutekhele, J. S. K. (1999). On the Spectrum of the Cesaro Operator of Order 2 (C11), Phd thesis, Department of Mathematics, Egerton University; Kenya.

[9] Coskun, C. (2003). A Special Norlund means and its Eigenvalues, Communication Faculty University Ankara, Ser. Al, Math. Stat 52(1), 27 – 31.

[10] Akanga, J. R. (2014). On the Spectrum of a special Norlund Means, jour. of Mathematical sciences. 25(2) 59-67.

[11] I. A. Okello et al (2017) A special Norlund Means and its Spectrum, International Journal of Mathematical Analysis Vol. 11, 2017, no. 13, 647 - 656

[12] Vermes, P. (1949). Series to series transformations and analytic continuation by matrix methods, Amer,. Journal. Math. 71, 541 – 562.

[13] Hardy, G. (1948). Divergent Series, Oxford University Press.

Electronic