

MODELLING AND FORECASTING OF CLIMATE DATA IN NIGERIA USING SEASONAL AUTOREGRESSIVE INTEGRATED MOVING AVERAGE VECTOR METHOD: A CASE OF AKWA IBOM STATE

Abstract: *Accurate and reliable information on weather and climate is crucial to support the smooth operation of various sectors, including agriculture, aviation, transportation, water resources management, and disaster risk reduction. These sectors are highly vulnerable to climate variability and change, where extreme weather events such as storms, floods, and droughts have devastating impacts. The purpose of this study is to model climatic data, such as rainfall, humidity, wind speed, temperature, using a Seasonal Autoregressive Integrated Moving average Vector (SARIMAV) model. Seasonal monthly Rainfall, humidity, wind speed and temperature data in Uyo Akwa Ibom State were collated from 2004-2024 for the research. The autocorrelation and partial autocorrelation functions of the differenced series suggested different orders of SARIMAV models for the four Climate data. Ordinary least squares was adopted to estimate parameters of the models. The analysis produced two sets of models; the SARIMAV models suggested from the correlogram and its reduced form to ensure model invertibility, From the MSE, AIC and BIC, the reduced parameter models outperformed the earlier SARIMAV models. The study's findings provide valuable insights for climate forecasting and decision-making in the region.*

Keywords: Multivariate; Seasonal; Autoregressive; Moving Average; Rainfall; Humidity; Wind Speed and Temperature

1. INTRODUCTION

Climate is one of the challenging tasks in weather forecasting. Weather data consists of various atmospheric features such as wind, precipitation, humidity, pressure, and temperature among others. Accurate and timely climate prediction can be very helpful for effective security measures for planning water resources management, issuance of early flood warning, construction activities, transportation activities, agricultural tasks, managing the flight operations and flood situation. The variability of weather is a crucial phenomenon in today's world. It is ever challenging and a topic of interest because prediction is not always accurate. It is a continuous, high dimensional, dynamic and complicated process because it involves many factors of the atmosphere. The parameters required to predict the weather are enormously complex such that there is uncertainty in prediction even for a short period (Geetha & Nasira, 2014). In Nigeria and on the worldwide scale, large numbers of attempts have been made by different researchers to predict climate accurately using various techniques, but due to the nonlinear nature of some climate variables like rainfall, prediction accuracy obtained by these techniques is still below the satisfactory level. The climate has a critical slice in the long-term viability of life on Earth. Due to the influence of varying climatic conditions, the pattern of climate is always changing. Flooding, landslides, and drought are all significant penalties of this change (Shivhare et al (2017)) As climate patterns are unexpected and significant, precise forecasting of them is necessary for efficient planning and decision-making (Ambildhuke & Gupta, 2022).

Owing to global warming, droughts are becoming more frequent, while rising temperatures and limited precipitation are posing major concerns for food security (Lobell and Hammer et al (2013)). Climate change is jeopardizing humanity and biodiversity by distorting agricultural productivity (Ahmad & Alam, 2009). Extreme temperatures, for instance, are anticipated to become more frequent and intense, posing a threat to food production systems (Deryng & Conway, 2019). Agro-physical modelling entails the prediction of meteorological parameters and future trends based on previous time series (Kryzszczak & Baranowski, 2017). The sensitivity of crop production models to climatic and environmental variations has been underscored in various studies (Fronzek. 2018; Pirttioja et al., 2018)).

The work is developing and applying improved weather prediction models, The Seasonal Autoregressive Integrated Moving Average Vector (SARIMAV) model capable of accurately forecasting climate events in Akwa Ibom State.

Multivariate time series analysis is an analysis of multi-variables, with a response vector as a dependent and predictor vectors as independent variables Wei (1990). In multivariate time series analysis, both the response and predictor vectors are modeled with corresponding lagged parameters of the response and predictor vectors. What distinguish multivariate time series from multivariate regression is that, only the response variable is modeled in regression, while all the vectors are modeled in multivariate time series. Dufour (2006) defined an m -dimensional vector process $(X_t : t \in \mathbb{Z})$ as an Autoregressive (p) model or Vector Autoregressive (VAR) of order ' p ' if it satisfies an equation of the form $X_t = \mu + \sum \phi_p X_{t-k} + a_t$ for very t , where $1 \leq k \leq p$, $\phi_1, \phi_2, \dots, \phi_p$, are $m \times m$ fixed matrices and $(a_t : t \in \mathbb{Z})$ is a white noise process.

SARIMA Model is an acronym for Seasonal Autoregressive Integrated Moving Average. The benefit of the SARIMA model over ARIMA is that it accounts for the seasonal component of the data. (Wang & Guo, 2009). This seasonal component ("S" in SARIMA) of the data, increases the model overall predictive ability, widely serves as a tool for time series modeling and prediction. The following seven parameters are the core of SARIMA model: $SARIMA(p, d, q)(P,)$. In modeling climatic data, Seasonal Autoregressive Integrated Moving Average (SARIMA) series models have been found suitable for analysis and forecasts. Within the SARIMA model, there are multiple models which are characterized by different orders of autoregressive and/or moving average components in the data set. Sequel to this development, this paper is motivated by the need to fit classical models to the rainfall, Humidity, Wind Speed and Temperature data in Uyo of the Akwa Ibom State..

2. LITERATURE REVIEW

Many contributions have been offered in the analysis of climatic data, on rainfall, temperature, humidity, drought, with case studies within and outside Nigeria. These investigations have employed various methodologies, including SARIMA, ARIMA, and Fourier series models, to understand and forecast climatic trends.

In Nigeria, researchers have explored rainfall patterns, applying SARIMA models to data from different regions.

Usoro (2014) fitted SARIMA models to quarterly rainfall data. In this study, n-dimensional Seasonal autoregressive integrated moving average vector (SARIMAV) models are compared with univariate models, ordinary least squares method was adopted to estimate the parameters, it was established the SARIMA model is not only applicable to univariate case, hence SARIMAV models were established, verified valid and useful in modelling multiple season series.

Usoro (2016) compared SARIMA model to Fourier series model, using rainfall data in Akwa Ibom State. The finding was that the two models were good and compared favourably. Fourier model was found to give smoother estimates because of the sinusoidal behaviour characterised by the model.

Aghelpour et al. (2019) conducted a comparison between the SARIMA, SVR, and SVR-FA models. The study used monthly temperature data of five stations from various climates in Iran from 1951- 2011, with a proportion of 75% training data and 25% testing data. Despite using the SVR model and its meta-innovative type, SVR-FA, the results showed that the SARIMA model still produced better performance in forecasting long-term temperatures.

Chen et.al (2021) investigates on Time Series Forecasting of Temperatures using SARIMA: An Example from Time series modelling and forecasting, which predicts future values by analysing previous values, is useful in many practical applications. In the research, the average monthly temperature in Nanjing, China, from 1951 to 2017 was evaluated using SARIMA (Seasonal Autoregressive Integrated Moving Average) techniques. The training set consists of data from 1951 to 2014, whereas the testing set consists of data from 2015 to 2017. The choice of a model and the precision of prediction are thoroughly discussed. The findings suggest that proposed research method achieves excellent predicting accuracy.

Aliyu S. et al (2021) Applied SARIMA model in modelling and forecasting monthly rainfall in Nigeria was considered in this study. The study utilizes the Nigerian monthly rainfall data between 1980-2015 obtained from World Bank Climate Portal. The Box-Jenkin's methodology was adopted. SARIMA (2, 0, 1) (2, 1, 1)₁₂ was the best model among others that fit the Nigerian rainfall data (1980-2015) with maximum p-value from BoxPierce Residuals Test. The study forecasts Nigeria's monthly rainfall from 2018 through 2042. It was discovered that the month of April is the period of onset of rainfall in Nigeria and November is the period of retreat. Based on the findings, Nigeria will experience approximately equal amount of rainfall between 2018 to 2021 and will experience a slight increase in rainfall amount in 2022 to about 1137.078 (mm). There will be a decline of rainfall at 2023 to about 1061 (mm). Rainfall values will raise again to about 1142.756 (mm) in 2024 and continue to fluctuate with decrease in variation between 2024 to 2042, then remain steady to 2046 at approximately 1110.0 (mm). Nigerian Government should provide a more mechanized and drier season farming methods to ease the outage of rainfall in future that may be caused due to natural (or unpredictable) variation.

Dongyao (2023) explored the utilization of the Seasonal Autoregressive Integrated Moving Average (SARIMA) model for forecasting global climate change. The SARIMA model is a machine learning algorithm that can effectively capture seasonal patterns and non-linear characteristics of climate data. The study initiates by performing data preprocessing tasks, which encompass data cleaning, managing missing values, and converting the data into a suitable format for analysis. The SARIMA model is then constructed, considering the seasonality and autocorrelation of the climate data. Historical climate data is used to train the SARIMA models, which are then utilized to forecast future global climate changes.

Dahiya et al (2024) inspects the time series analysis of the monthly precipitation and mean temperatures for the Punjab and Haryana states of India. For Punjab state, data was taken for the years 1951–2020 and for Haryana state, data was collected for the years 1957–2020. In order to anticipate the subsequent 15 years (2016–2030), using the seasonal ARIMA (SARIMA) model and fitted to the data up to the year 2015. For precipitation, ARIMA (1,1,0) (0,1,2)₁₂ and ARIMA (1,1,2) (0,1,1)₁₂ were the most suitable model for precipitation data analysis of Haryana and Punjab states, respectively. For temperature, best selected model was ARIMA (2,1,2) (0,1,1)₁₂, and ARIMA (2,1,2) (2,1,2)₁₂ for Haryana and Punjab states, respectively. The predicted outcomes establish that the predicted data closely matches the data's pattern.

In this work, the intention is to fit a Multivariate time series (SARIMAV) models to the climate data in the four climatic variables of Uyo-Akwa Ibom State.

This paper is building on Usoro et al. (2014) and extending the findings who fitted to rainfall data in two extreme states (north and south) with the assumption that the models can be adequate for the rainfall data. What justifies the use of SARIMA models is that rainfall, Humidity, Wind and Temperature data do not exhibit linearity characteristics to warrant the use of the popular linear ARIMA models. The plot of the original series and autocorrelation functions gives better explanation to non-linearity characteristics of the climatic variables data. In addition to the fact that SARIMA models are multiplicative models, findings from many contributors have given SARIMA models more advantage than ARIMA models when modelling rainfall and other seasonal series. The assumption in this paper is that a particular SARIMA

model cannot be adequate to fit the climatic data (rainfall, humidity, wind speed, and temperature) in all geographical zones in Nigeria. The justification behind this assumption is that a SARIMA process, expressed as orders of each autoregressive and moving average for the seasonal and non-seasonal parts, may vary significantly across different zones due to unique spatial patterns of each climatic variable. Consequently, each location is expected to have a distinct SARIMA model that accurately captures the local patterns of rainfall, humidity, wind speed, and temperature. This study aims to contribute to the existing body of research by applying Multivariate time series (SARIMAV) models to climate data in Uyo, Akwa Ibom State, Nigeria, focusing on four climatic variables: rainfall, humidity, wind speed, and temperature.

3. METHODOLOGY:

3.1 Source of Data for the Research

Average Temperature, Wind Speed, Relative Humidity, Rainfall were obtained from the Nigerian Meteorological Agency (NIMET) Akwa Ibom State, Nigeria. The data for Uyo were collected on a monthly basis from January 2004 to December 2024 a period of 20 years.

3.2 Model Identification

The autocorrelations function (ACF) and the partial autocorrelation functions (PACF) are the two most useful tools in any attempt at time series model identification (Granger and Newbold, 1986).

TABLE 1: SARIMA MODELS (ACF and PACF based) of the mixed ARIMA

| S/N | CLIMATE VARIABLE | MODEL |
|-----|---------------------|-----------------------------|
| 1. | Rainfall | $SARIMA(p, d, q)(P, D, Q)s$ |
| 2. | Humidity | $SARIMA(p, d, q)(P, D, Q)s$ |
| 3. | Wind Speed | $SARIMA(p, d, q)(P, D, Q)s$ |
| 4. | Average Temperature | $SARIMA(p, d, q)(P, D, Q)s$ |

3.3 SEASONAL AUTOREGRESSIVE MOVING AVERAGE VECTOR (SARIMAV)

Matrix Presentation of Vector Models

The general non-multiplicative SARIMAV models are presented as:

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ \vdots \\ X_{nt} \end{bmatrix} = \begin{bmatrix} \phi_{1.11} & \phi_{1.12} & \dots & \phi_{1.1n} \\ \phi_{1.21} & \phi_{1.22} & \dots & \phi_{1.2n} \\ \vdots & \vdots & \dots & \vdots \\ \phi_{1.m1} & \phi_{1.m2} & \dots & \phi_{1.mn} \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ \vdots \\ X_{nt-1} \end{bmatrix} + \begin{bmatrix} \phi_{2.11} & \phi_{2.12} & \dots & \phi_{2.1n} \\ \phi_{2.21} & \phi_{2.22} & \dots & \phi_{2.2n} \\ \vdots & \vdots & \dots & \vdots \\ \phi_{2.m1} & \phi_{2.m2} & \dots & \phi_{2.mn} \end{bmatrix} \begin{bmatrix} X_{1t-2} \\ X_{2t-2} \\ \vdots \\ X_{nt-2} \end{bmatrix} + \\
 + \begin{bmatrix} \phi_{p.11} & \phi_{p.12} & \dots & \phi_{p.1n} \\ \phi_{p.21} & \phi_{p.22} & \dots & \phi_{p.2n} \\ \vdots & \vdots & \dots & \vdots \\ \phi_{p.m1} & \phi_{p.m2} & \dots & \phi_{p.mn} \end{bmatrix} \begin{bmatrix} X_{1t-p} \\ X_{2t-p} \\ \vdots \\ X_{nt-p} \end{bmatrix} + \begin{bmatrix} \theta_{1.11} & \theta_{1.12} & \dots & \theta_{1.1n} \\ \theta_{1.21} & \theta_{1.22} & \dots & \theta_{1.2n} \\ \vdots & \vdots & \dots & \vdots \\ \theta_{1.m1} & \theta_{1.m2} & \dots & \theta_{1.mn} \end{bmatrix} \begin{bmatrix} e_{1t-1} \\ e_{2t-1} \\ \vdots \\ e_{nt-1} \end{bmatrix}$$

$$\begin{aligned}
 & + \begin{bmatrix} \theta_{2.11} & \theta_{2.12} & \dots & \theta_{2.1n} \\ \theta_{2.21} & \theta_{2.22} & \dots & \theta_{2.2n} \\ \vdots & \vdots & \dots & \vdots \\ \theta_{2.m1} & \theta_{2.m2} & \dots & \theta_{2.mn} \end{bmatrix} \begin{bmatrix} e_{1t-2} \\ e_{2t-2} \\ \vdots \\ e_{nt-2} \end{bmatrix} + \dots + \begin{bmatrix} \theta q_{.11} & \theta q_{.12} & \dots & \theta q_{.1n} \\ \theta q_{.21} & \theta q_{.22} & \dots & \theta q_{.2n} \\ \vdots & \vdots & \dots & \vdots \\ \theta q_{.m1} & \theta q_{.m2} & \dots & \theta q_{.mn} \end{bmatrix} \begin{bmatrix} e_{1t-q} \\ e_{2t-q} \\ \vdots \\ e_{nt-q} \end{bmatrix} + \\
 & \begin{bmatrix} \gamma_{12.11} & \gamma_{12.12} & \dots & \gamma_{12.1n} \\ \gamma_{12.21} & \gamma_{12.22} & \dots & \gamma_{12.2n} \\ \vdots & \vdots & \dots & \vdots \\ \gamma_{12.m1} & \gamma_{12.m2} & \dots & \gamma_{12.mn} \end{bmatrix} \begin{bmatrix} X_{1t-12} \\ X_{2t-12} \\ \vdots \\ X_{nt-12} \end{bmatrix} + \begin{bmatrix} \gamma_{24.11} & \gamma_{24.12} & \dots & \gamma_{24.1n} \\ \gamma_{24.21} & \gamma_{24.22} & \dots & \gamma_{24.2n} \\ \vdots & \vdots & \dots & \vdots \\ \gamma_{24.m1} & \gamma_{24.m2} & \dots & \gamma_{24.mn} \end{bmatrix} \begin{bmatrix} X_{1t-24} \\ X_{2t-24} \\ \vdots \\ X_{nt-24} \end{bmatrix} + \\
 & \dots + \begin{bmatrix} \gamma_{PS.11} & \gamma_{PS.12} & \dots & \gamma_{PS.1n} \\ \gamma_{PS.21} & \gamma_{PS.22} & \dots & \gamma_{PS.2n} \\ \vdots & \vdots & \dots & \vdots \\ \gamma_{PS.m1} & \gamma_{PS.m2} & \dots & \gamma_{PS.mn} \end{bmatrix} \begin{bmatrix} X_{1t-PS} \\ X_{2t-PS} \\ \vdots \\ X_{nt-PS} \end{bmatrix} + \begin{bmatrix} \beta_{12.11} & \beta_{12.12} & \dots & \beta_{12.1n} \\ \beta_{12.21} & \beta_{12.22} & \dots & \beta_{12.2n} \\ \vdots & \vdots & \dots & \vdots \\ \beta_{12.m1} & \beta_{12.m2} & \dots & \beta_{12.mn} \end{bmatrix} \begin{bmatrix} e_{1t-12} \\ e_{2t-12} \\ \vdots \\ e_{nt-12} \end{bmatrix} \\
 & + \begin{bmatrix} \beta_{24.11} & \beta_{24.12} & \dots & \beta_{24.1n} \\ \beta_{24.21} & \beta_{24.22} & \dots & \beta_{24.2n} \\ \vdots & \vdots & \dots & \vdots \\ \beta_{24.w1} & \beta_{24.w2} & \dots & \beta_{24.mn} \end{bmatrix} \begin{bmatrix} e_{1t-24} \\ e_{2t-24} \\ \vdots \\ e_{nt-24} \end{bmatrix} + \dots + \begin{bmatrix} \beta_{QS.11} & \beta_{QS.12} & \dots & \beta_{QS.1n} \\ \beta_{QS.21} & \beta_{QS.22} & \dots & \beta_{QS.2n} \\ \vdots & \vdots & \dots & \vdots \\ \beta_{QS.m1} & \beta_{QS.m2} & \dots & \beta_{QS.mn} \end{bmatrix} \begin{bmatrix} e_{1t-QS} \\ e_{2t-QS} \\ \vdots \\ e_{nt-QS} \end{bmatrix} + \begin{bmatrix} V_{1t} \\ V_{2t} \\ \vdots \\ V_{nt} \end{bmatrix} \quad (1)
 \end{aligned}$$

3.3.1 The above SARIMAV models are reduced to the form:

$$\begin{aligned}
 X_{it} = & \sum_{k=1}^p \sum_{i=1}^m \sum_{j=1}^n \phi_{k.ij} X_{jt-k} + \sum_{l=1}^q \sum_{u=1}^m \sum_{v=1}^n \theta_{l.uv} e_{jt-l} + \sum_{s=12}^{PS} \sum_{i=1}^m \sum_{j=1}^n \gamma_{S.ij} X_{jt-s} \\
 & + \sum_{s=12}^{QS} \sum_{u=1}^m \sum_{v=1}^n \beta_{S.uv} e_{jt-s} + V_{jt} \quad (2)
 \end{aligned}$$

Where $\phi_{k.ij}$ and $\theta_{l.uv}$ are matrices of coefficients of non-seasonal autoregressive and moving average models respectively, while $\gamma_{S.ij}$ and $\beta_{S.uv}$ are matrices of coefficients of seasonal autoregressive and moving average models respectively and V_{jt} is the error associated with the model.

3.4 Estimation of parameters of the model

In this section, estimates of the parameters of the multivariate SARIMA is considered. The Climate data used included monthly data of Rainfall, Humidity, Wind Speed and temperature for Akwa Ibom State; represented by X_{1t} , X_{2t} , X_{3t} , and X_{4t} respectively.

3.5. Probabilistic model selection criteria

3.5.1 Akaike's information criterion (AIC) AIC value is computed as given below:

$$AIC = \frac{2K}{n} + \ln \left(\frac{RSS}{n} \right) \quad (3)$$

Where k is the number of parameters in the model, RSS is the residual sum of squares, n is the number of observations in the model.

3.5.2 Bayesian Information Criterion (BIC)

$$(BIC) = \{(\sigma_e^2)\} + k \{\ln(n)\} \quad (4)$$

n is the sample size of sample residual and k is the total number of estimated parameters in the fitted model while σ_e^2 represents the error variance.

3.6 Forecast errors

Mean square error (MSE) and mean absolute error (MAE) is one scale-dependent forecast errors used in the study to evaluate the performance of the best-fit SARIMA models.

$$MSE = \frac{\sum_{t=1}^T (\hat{X}_{it} - X_{it})^2}{T_i} \quad (5)$$

Where \hat{X}_{it} is the predicted rainfall, Humidity, Wind Speed and Temperature values, X_{it} is the test rainfall, Humidity, Wind Speed and Temperature values and T is the number of rainfall, Humidity, Wind Speed and Temperature data points.

4.0 DATA ANALYSIS

4.1. ACF and PACF plot of climate data in Uyo.

Fig. 1 – Fig.8 shows the ACF and PACF of Seasonally differenced climate data series.

Figure 1: ACF of seasonally differenced rainfall series

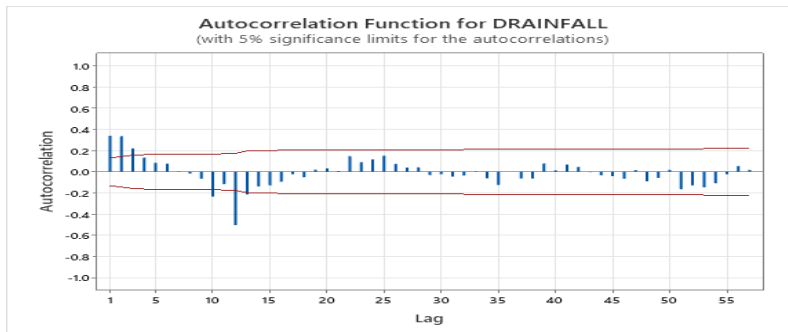


Figure 2: PACF of seasonally differenced rainfall series

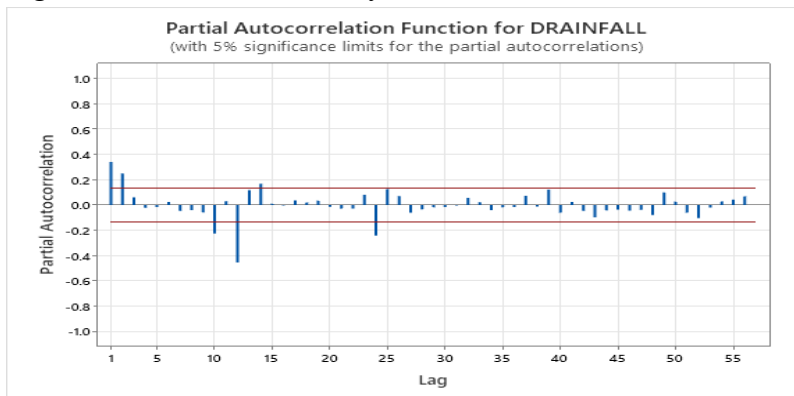


Figure 3: ACF of seasonally differenced Humidity series

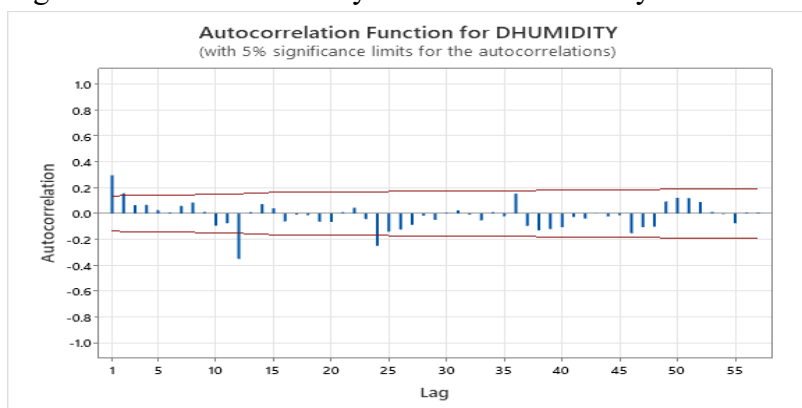


Figure 4: PACF of seasonally differenced Humidity series

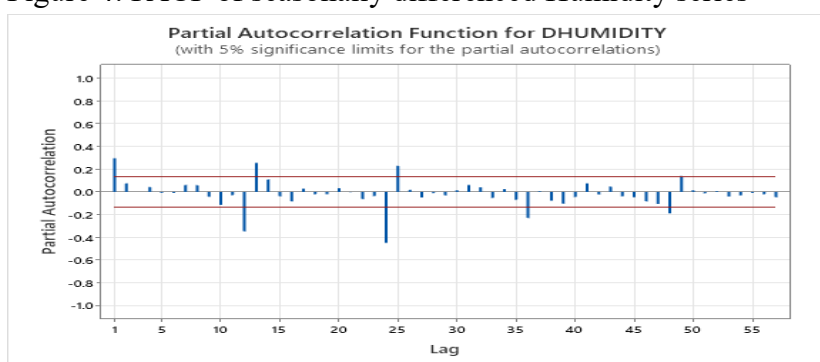


Figure 5: ACF of seasonally differenced Wind Speed series

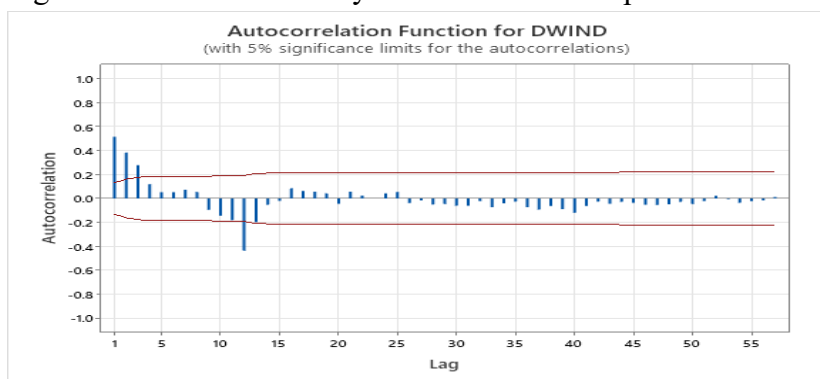


Figure 6: PACF of seasonally differenced Wind Speed series

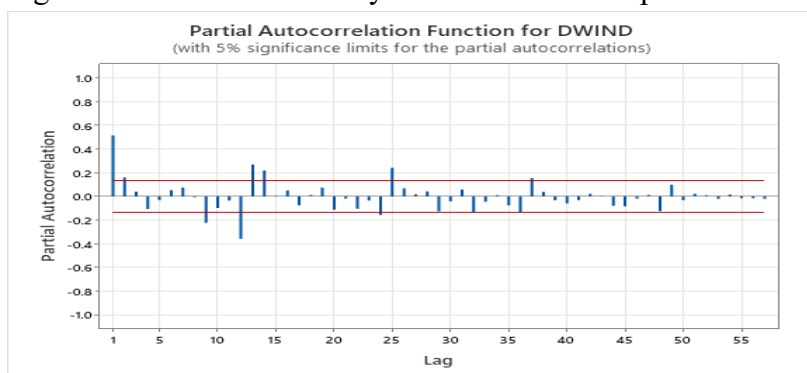


Figure 7: ACF of seasonally differenced Temperature series

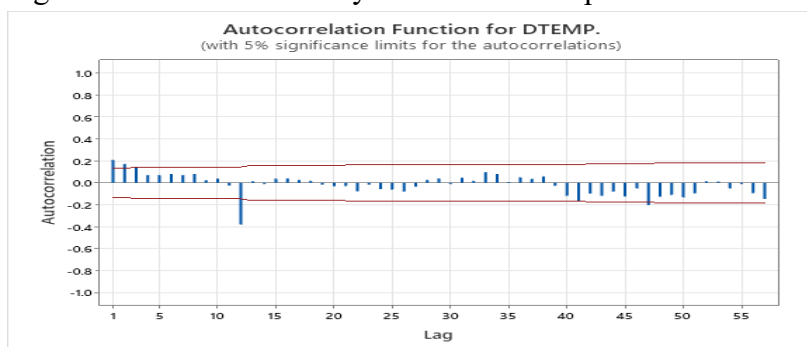


Figure 8: PACF of seasonally differenced Temperature series

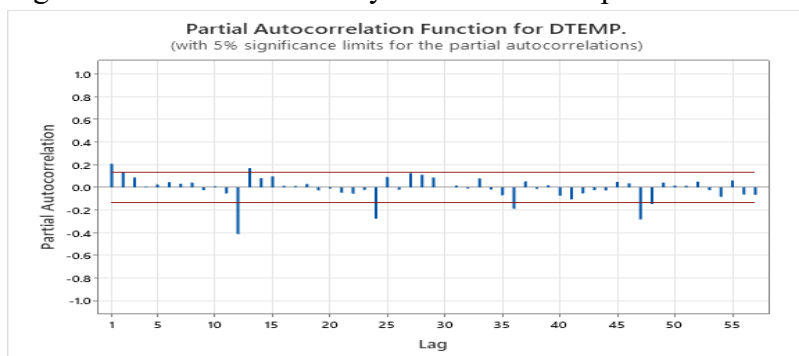


Figure 1-8 representing the . ACF and PACF of the seasonally differenced rainfall, Humidity, Wind Speed, and Temperature data in Uyo, Akwa Ibom state.

TABLE 2 : SARIMA MODELS

| S/ N | CLIMATE VARIABLE | MODEL |
|---------|---------------------------------|---|
| 1. | Rainfall (X_{1t}) | <i>SARIMA</i> (2, 0, 2) (2, 1, 1) ₁₂ . |
| 2. | Humidity (X_{2t}) | <i>SARIMA</i> (1, 0, 1)(2,1, 2) ₁₂ . |
| 3. | WindSpeed (X_{3t}) | <i>SARIMA</i> (1, 0, 2) (1, 1, 1) ₁₂ . |
| 4. | Ave.Temperature (X_{4t}) | <i>SARIMA</i> (1, 0, 2)(2, 1, 1) ₁₂ . |

4.2. Descriptive statistics

This statistics includes mean, Variance, Median, standard deviation, minimum and maximum and quartile values of the Climate data in each of the variables.

TABLE 3: Descriptive Statistics: Rainfall, Humidity, Wind, Temperature

| Climate variable | N | Mean | Variance | Median | Std. Deviation | Minimum | Maximum | Q ₁ | Q ₂ |
|--------------------------------|-----|-------|-----------|--------|----------------|---------|---------|----------------|----------------|
| Rainfall X_{1t} | 240 | 221.1 | 35323.658 | 208.56 | 187.94 | 0.0 | 929.0 | 54.00 | 312.31 |
| Humidity X_{2t} | 240 | 81.03 | 54.089 | 82.00 | 7.35 | 47.00 | 94.00 | 77.25 | 86.00 |
| Wind Speed X_{3t} | 240 | 8.58 | 3.824 | 8.00 | 1.95 | 2.0 | 20.0 | 8.00 | 9.00 |
| Temperature X_{4t} | 240 | 27.21 | 1.508 | 27.20 | 1.22 | 20.6 | 30.5 | 26.31 | 28.03 |

4.3. Parameters Estimation

Once the model is tentatively established, the parameters and the corresponding standard errors can be estimated using statistical techniques, least square estimation method.

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} \phi_{1.11} & \phi_{1.12} & \phi_{1.13} & \phi_{1.14} \\ \phi_{1.21} & \phi_{1.22} & \phi_{1.23} & \phi_{1.24} \\ \phi_{1.31} & \phi_{1.32} & \phi_{1.33} & \phi_{1.34} \\ \phi_{1.41} & \phi_{1.42} & \phi_{1.43} & \phi_{1.44} \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \end{bmatrix} + \begin{bmatrix} \phi_{2.11} & 0 & 0 & 0 \\ \phi_{2.21} & 0 & 0 & 0 \\ \phi_{2.31} & 0 & 0 & 0 \\ \phi_{2.41} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_{1t-2} \\ X_{2t-2} \\ X_{3t-2} \\ X_{4t-2} \end{bmatrix} + \\
 \begin{bmatrix} \theta_{1.11} & \theta_{1.12} & \theta_{1.13} & \theta_{1.14} \\ \theta_{1.21} & \theta_{1.22} & \theta_{1.23} & \theta_{1.24} \\ \theta_{1.31} & \theta_{1.32} & \theta_{1.33} & \theta_{1.34} \\ \theta_{1.41} & \theta_{1.42} & \theta_{1.43} & \theta_{1.44} \end{bmatrix} \begin{bmatrix} e_{1t-1} \\ e_{2t-1} \\ e_{3t-1} \\ e_{4t-1} \end{bmatrix} + \begin{bmatrix} \theta_{2.11} & 0 & \theta_{2.13} & \theta_{2.14} \\ \theta_{2.21} & 0 & \theta_{2.23} & \theta_{2.24} \\ \theta_{2.31} & 0 & \theta_{2.33} & \theta_{2.34} \\ \theta_{2.41} & 0 & \theta_{2.43} & \theta_{2.44} \end{bmatrix} \begin{bmatrix} e_{1t-2} \\ e_{2t-2} \\ e_{3t-2} \\ e_{4t-2} \end{bmatrix} +$$

$$\begin{bmatrix} \gamma_{12.11} & \gamma_{12.12} & \gamma_{12.13} & \gamma_{12.14} \\ \gamma_{12.21} & \gamma_{12.22} & \gamma_{12.23} & \gamma_{12.24} \\ \gamma_{12.31} & \gamma_{12.32} & \gamma_{12.33} & \gamma_{12.34} \\ \gamma_{12.41} & \gamma_{12.42} & \gamma_{12.43} & \gamma_{12.44} \end{bmatrix} \begin{bmatrix} X_{1t-12} \\ X_{2t-12} \\ X_{3t-12} \\ X_{4t-12} \end{bmatrix} + \begin{bmatrix} \gamma_{24.11} & \gamma_{24.12} & 0 & \gamma_{24.14} \\ \gamma_{24.21} & \gamma_{24.22} & 0 & \gamma_{24.24} \\ \gamma_{24.31} & \gamma_{24.32} & 0 & \gamma_{24.34} \\ \gamma_{24.41} & \gamma_{24.42} & 0 & \gamma_{24.44} \end{bmatrix} \begin{bmatrix} X_{1t-24} \\ X_{2t-24} \\ X_{3t-24} \\ X_{4t-24} \end{bmatrix} + \\
 \begin{bmatrix} \beta_{12.11} & \beta_{12.12} & \beta_{12.13} & \beta_{12.14} \\ \beta_{12.21} & \beta_{12.22} & \beta_{12.23} & \beta_{12.24} \\ \beta_{12.31} & \beta_{12.32} & \beta_{12.33} & \beta_{12.34} \\ \beta_{12.41} & \beta_{12.42} & \beta_{12.43} & \beta_{12.44} \end{bmatrix} \begin{bmatrix} e_{1t-12} \\ e_{2t-12} \\ e_{3t-12} \\ e_{4t-12} \end{bmatrix} + \begin{bmatrix} 0 & \beta_{24.12} & 0 & 0 \\ 0 & \beta_{24.22} & 0 & 0 \\ 0 & \beta_{24.32} & 0 & 0 \\ 0 & \beta_{24.42} & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1t-24} \\ e_{2t-24} \\ e_{3t-24} \\ e_{4t-24} \end{bmatrix} + \begin{bmatrix} V_{1t} \\ V_{2t} \\ V_{3t} \\ V_{4t} \end{bmatrix} \quad (6)$$

4.3.1 PRESENTATION OF THE PARAMETERS ESTIMATES OF MULTIVARIATE SEASONAL MODELS

The use of four vectors X_{1t} , X_{2t} , X_{3t} and X_{4t} introduced us to a multivariate. As a special case of the SARIMAV model, given four vectors X_{1t} , X_{2t} , X_{3t} and X_{4t} in multivariate time series,

$$\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} = \begin{bmatrix} 0.346 & 0.021 & -0.0386 & 2.8 \\ -0.0035 & 0.147 & 0.319 & -0.258 \\ -0.00141 & 0.0412 & 0.410 & -0.046 \\ -0.000066 & -0.0293 & -0.0138 & 0.056 \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \end{bmatrix} + \begin{bmatrix} 0.124 & 0 & 0 & 0 \\ 0.0028 & 0 & 0 & 0 \\ 0.00203 & 0 & 0 & 0 \\ -0.00151 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_{1t-2} \\ X_{2t-2} \\ X_{3t-2} \\ X_{4t-2} \end{bmatrix} + \\
 \begin{bmatrix} -0.148 & -1.47 & 0.0024 & -2.8 \\ 0.00421 & 0.1248 & -0.233 & 2.392 \\ 0.00138 & -0.0441 & -0.011 & -0.002 \\ -0.000178 & 0.0151 & -0.0106 & -0.001 \end{bmatrix} \begin{bmatrix} e_{1t-1} \\ e_{2t-1} \\ e_{3t-1} \\ e_{4t-1} \end{bmatrix} + \begin{bmatrix} -0.005 & 0 & 9.64 & -8.9 \\ 0.00142 & 0 & -0.073 & 0.751 \\ -0.00139 & 0 & 0.1012 & 0.096 \\ 0.001864 & 0 & -0.0297 & 0.1090 \end{bmatrix} \begin{bmatrix} e_{1t-2} \\ e_{2t-2} \\ e_{3t-2} \\ e_{4t-2} \end{bmatrix} + \\
 \begin{bmatrix} -0.529 & 0.98 & -5.76 & -15.1 \\ 0.00837 & -0.476 & 0.376 & -0.23 \\ -0.00082 & 0.0020 & -0.2509 & -0.461 \\ -0.00131 & -0.0029 & 0.0082 & -0.674 \end{bmatrix} \begin{bmatrix} X_{1t-12} \\ X_{2t-12} \\ X_{3t-12} \\ X_{4t-12} \end{bmatrix} + \begin{bmatrix} -0.1897 & -0.001 & 0 & -15.700 \\ 0.00219 & -0.322 & 0 & -0.029 \\ 0.00094 & -0.0213 & 0 & -0.240 \\ -0.000344 & 0.0235 & 0 & -0.2727 \end{bmatrix} \begin{bmatrix} X_{1t-24} \\ X_{2t-24} \\ X_{3t-24} \\ X_{4t-24} \end{bmatrix} + \\
 \begin{bmatrix} -0.062 & -1.47 & 2.72 & -1.2 \\ -0.0036 & 0.0030 & -0.287 & 0.054 \\ 0.00030 & 0.0055 & -0.0457 & 0.446 \\ 0.000644 & -0.0034 & -0.0475 & 0.139 \end{bmatrix} \begin{bmatrix} e_{1t-12} \\ e_{2t-12} \\ e_{3t-12} \\ e_{4t-12} \end{bmatrix} + \begin{bmatrix} 0 & 1.83 & 0 & 0 \\ 0 & -0.102 & 0 & 0 \\ 0 & 0.0145 & 0 & 0 \\ 0 & -0.0059 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1t-24} \\ e_{2t-24} \\ e_{3t-24} \\ e_{4t-24} \end{bmatrix} \quad (7)$$

Expansion of the above matrices provides multivariate seasonal autoregressive moving average models for the four vector series X_{1t} , X_{2t} , X_{3t} and X_{4t} .

The matrix equation above, using regression notation can also be denoted as follows:

$$\begin{aligned}
 X_{1t} = & 0.346X_{1t-1} + 0.021X_{2t-1} - 0.0386X_{3t-1} + 2.8X_{4t-1} + 0.124X_{1t-2} - 0.148e_{1t-1} - 1.47e_{2t-1} + \\
 & 0.0024e_{3t-1} - 2.8e_{4t-1} + 0.005e_{1t-2} + 9.64e_{3t-2} - 8.9e_{4t-2} - 0.529X_{1t-12} + 0.98X_{2t-12} - \\
 & 5.76X_{3t-12} - 15.1X_{4t-12} - 0.1897X_{1t-24} - 0.001X_{2t-24} - 15.7X_{4t-24} - 0.062e_{1t-12} - 1.47e_{2t-12} + \\
 & 2.72e_{3t-12} - 1.2e_{4t-12} + 1.83e_{2t-24} \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 X_{2t} = & -0.00354X_{1t-1} + 0.147X_{2t-1} + 0.139X_{3t-1} - 0.258X_{4t-1} + 0.0028X_{1t-2} + 0.00421e_{1t-1} + \\
 & 0.1248e_{2t-1} - 0.233e_{3t-1} + 2.392e_{4t-1} + 0.00142e_{1t-2} - 0.073e_{3t-2} + 0.751e_{4t-2} + \\
 & 0.00837X_{1t-12} - 0.476X_{2t-12} + 0.376X_{3t-12} - 0.23X_{4t-12} + 0.00219X_{1t-24} - 0.322X_{2t-24} -
 \end{aligned}$$

$$0.029X_{4t-24} - 0.0036e_{1t-12} - 0.30e_{2t-12} - 0.287e_{3t-12} - 0.054e_{4t-12} - 0.1026e_{2t-24} \tag{9}$$

$$X_{3t} = -0.00141X_{1t-1} + 0.0412X_{2t-1} + 0.410X_{3t-1} - 0.046X_{4t-1} + 0.00203X_{1t-2} + 0.00138e_{1t-1} - 0.0441e_{2t-1} - 0.011e_{3t-1} - 0.002e_{4t-1} - 0.00139e_{1t-2} + 0.1012e_{3t-2} + 0.096e_{4t-2} - 0.00082X_{1t-12} + 0.0020X_{2t-12} - 0.2509X_{3t-12} - 0.461X_{4t-12} - 0.00094X_{1t-24} - 0.0213X_{2t-24} - 0.240X_{4t-24} + 0.00034e_{1t-12} + 0.0055e_{2t-12} - 0.0457e_{3t-12} + 0.446e_{4t-12} + 0.0145e_{2t-24} \tag{10}$$

$$X_{4t} = -0.000066X_{1t-1} - 0.0293X_{2t-1} - 0.0138X_{3t-1} + 0.056X_{4t-1} - 0.00151X_{1t-2} - 0.0000178e_{1t-1} + 0.0151e_{2t-1} - 0.01066e_{3t-1} - 0.001e_{4t-1} + 0.00186e_{1t-2} - 0.0297e_{3t-2} + 0.1090e_{4t-2} - 0.00131X_{1t-12} - 0.0029X_{2t-12} + 0.0082X_{3t-12} - 0.674X_{4t-12} - 0.000344X_{1t-24} - 0.0235X_{2t-24} - 0.2727X_{4t-24} + 0.000644e_{1t-12} - 0.0034e_{2t-12} - 0.0475e_{3t-12} + 0.139e_{4t-12} + 0.0059e_{2t-24} \tag{11}$$

4.4 Estimation of Model Parameters (Full SARIMA model)

Table 4. present the estimate and test of significance of the parameters of each SARIMA model.

TABLE 4. Estimates of Parameters

| Response Variables | Predictor variables | Model | Coeff. | P-value |
|-----------------------------|---------------------|----------|----------|---------|
| Rainfall X_{1t} | X_{1t} | AR(1) | 0.346 | 0.003 |
| | | AR(2) | 0.124 | 0.288 |
| | | MA(1) | -0.148 | 0.197 |
| | | MA(2) | 0.005 | 0.965 |
| | | SAR(12) | -0.529 | 0.000 |
| | | SAR(24) | -0.1897 | 0.033 |
| | | SMA(12) | -0.062 | 0.637 |
| | | X_{2t} | AR(1) | 0.021 |
| | MA(1) | | -1.47 | 0.551 |
| | SAR(12) | | 0.98 | 0.751 |
| | SAR(24) | | -0.001 | 0.047 |
| | SMA(12) | | -1.47 | 0.591 |
| | SMA(24) | | 1.83 | 0.480 |
| | X_{3t} | AR(1) | -0.0386 | 0.034 |
| | | MA(1) | 0.0024 | 0.047 |
| | | MA(2) | 9.64 | 0.084 |
| | | SAR(12) | -5.76 | 0.464 |
| | | SMA(12) | 2.72 | 0.702 |
| | X_{4t} | AR(1) | 2.8 | 0.909 |
| | | MA(1) | -2.8 | 0.894 |
| MA(2) | | -8.9 | 0.425 | |
| SAR(12) | | -15.1 | 0.598 | |
| SAR(24) | | -15.7 | 0.237 | |
| SMA(12) | | 1.2 | 0.960 | |
| Humidity X_{2t} | X_{1t} | AR(1) | -0.00354 | 0.380 |
| | | AR(2) | 0.0028 | 0.498 |
| | | MA(1) | 0.00421 | 0.302 |

| | | MA(2) | 0.00142 | 0.723 |
|--------------------------------|--------------------|---------|-----------|---------|
| | | SAR(12) | 0.00837 | 0.019 |
| | | SAR(24) | 0.00219 | 0.486 |
| | | SMA(12) | -0.0036 | 0.443 |
| | X_{2t} | AR(1) | 0.147 | 0.144 |
| | | MA(1) | 0.1248 | 0.154 |
| | | SAR(12) | -0.476 | 0.000 |
| | | SAR(24) | -0.322 | 0.002 |
| | | SMA(12) | -0.003 | 0.975 |
| | | SMA(24) | -0.1026 | 0.266 |
| | X_{3t} | AR(1) | 0.319 | 0.352 |
| | | MA(1) | -0.233 | 0.419 |
| | | MA(2) | -0.073 | 0.711 |
| | | SAR(12) | 0.376 | 0.179 |
| | | SMA(12) | -0.287 | 0.256 |
| | X_{4t} | AR(1) | -0.258 | 0.003 |
| | | MA(1) | 2.392 | 0.002 |
| | | MA(2) | 0.751 | 0.019 |
| | | SAR(12) | -0.23 | 0.818 |
| | | SAR(24) | -0.029 | 0.951 |
| | | SMA(12) | 0.054 | 0.948 |
| Response Variable | Predictor variable | Model | Coeff. | P-value |
| Wind Speed X_{3t} | X_{1t} | AR(1) | -0.00141 | 0.020 |
| | | AR(2) | 0.00203 | 0.163 |
| | | MA(1) | 0.00138 | 0.335 |
| | | MA(2) | -0.00139 | 0.323 |
| | | SAR(12) | -0.00082 | 0.651 |
| | | SAR(24) | 0.00094 | 0.396 |
| | | SMA(12) | 0.0003 | 0.853 |
| | X_{2t} | AR(1) | 0.0412 | 0.244 |
| | | MA(1) | -0.0441 | 0.152 |
| | | SAR(12) | 0.002 | 0.959 |
| | | SAR(24) | -0.0213 | 0.562 |
| | | SMA(12) | 0.0055 | 0.871 |
| | X_{3t} | SMA(24) | 0.0145 | 0.655 |
| | | AR(1) | 0.410 | 0.001 |
| | | MA(1) | -0.011 | 0.911 |
| | | MA(2) | 0.1012 | 0.145 |
| | | SAR(12) | -0.2509 | 0.011 |
| | X_{4t} | SMA(12) | -0.0457 | 0.607 |
| | | AR(1) | -0.046 | 0.880 |
| | | MA(1) | -0.002 | 0.995 |
| MA(2) | | 0.096 | 0.491 | |
| SAR(12) | | -0.461 | 0.198 | |
| SAR(24) | | -0.240 | 0.147 | |
| Temperature X_{4t} | X_{1t} | SMA(12) | 0.446 | 0.029 |
| | | AR(1) | 0.000066 | 0.923 |
| | | AR(2) | -0.001506 | 0.033 |

| | | | | |
|--|----------|---------|-----------|-------|
| | | MA(1) | -0.000178 | 0.797 |
| | | MA(2) | 0.001864 | 0.007 |
| | | SAR(12) | -0.00131 | 0.139 |
| | | SAR(24) | -0.000344 | 0.520 |
| | | SMA(12) | 0.000644 | 0.420 |
| | X_{2t} | AR(1) | -0.0293 | 0.088 |
| | | MA(1) | 0.0151 | 0.309 |
| | | SAR(12) | -0.0029 | 0.875 |
| | | SAR(24) | 0.0235 | 0.188 |
| | | SMA(12) | -0.0034 | 0.838 |
| | | SMA(24) | -0.0059 | 0.707 |
| | X_{3t} | AR(1) | -0.0293 | 0.813 |
| | | MA(1) | -0.0106 | 0.829 |
| | | MA(2) | -0.0297 | 0.376 |
| | | SAR(12) | 0.0082 | 0.862 |
| | | SMA(12) | -0.0475 | 0.270 |
| | X_{4t} | AR(1) | 0.056 | 0.704 |
| | | MA(1) | 0.001 | 0.994 |
| | | MA(2) | 0.109 | 0.107 |
| | | SAR(12) | -0.674 | 0.000 |
| | | SAR(24) | -0.2727 | 0.001 |
| | | SMA(12) | 0.139 | 0.328 |

Source: Statistical computation from original data 2004-2024

4.5 Diagnostic Testing

Once the multivariate model 4.21 has been acquired, the next step is to verify the correctness of the model fit. The following diagnostic techniques are used to this end.

4.5.1 Residual Autocorrelation Function plot

Figure 9: ACF PLOT OF X_{1t} (Full SARIMA Model) Residual

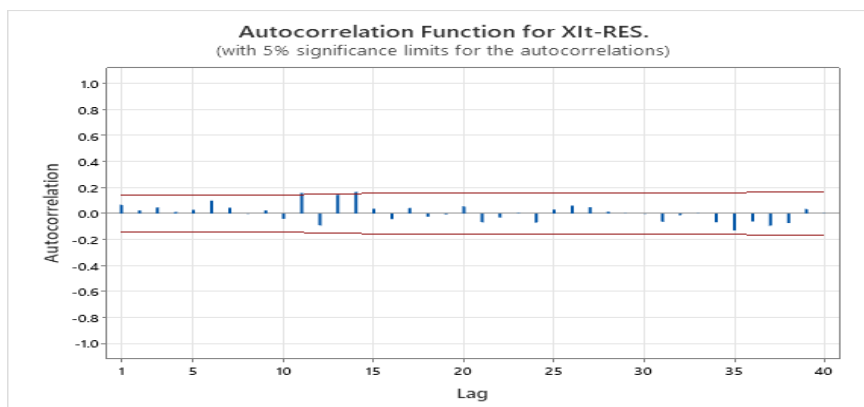


Figure 10: ACF PLOT OF X_{2t} (Full SARIMA Model) Residual

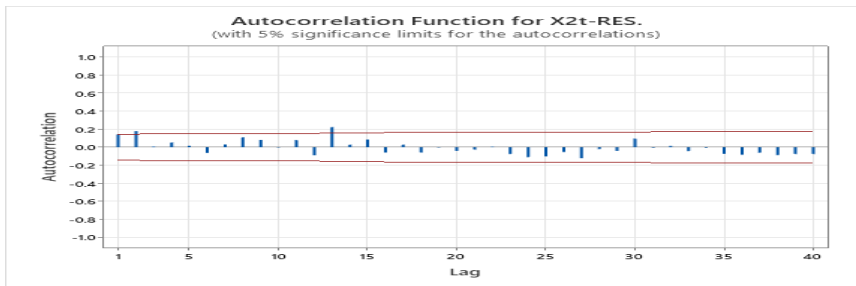


Figure 11: ACF PLOT OF X_{3t} (Full SARIMA Model) Residual

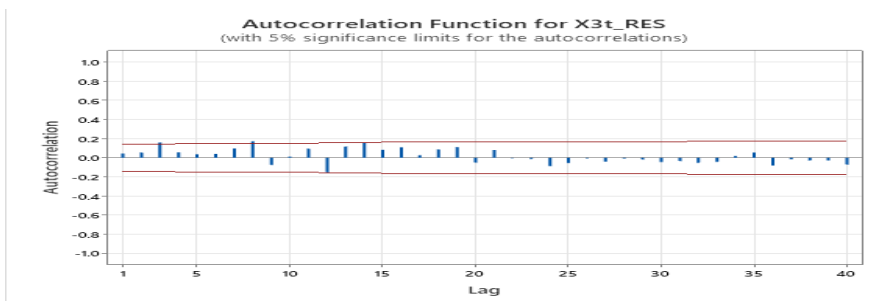


Figure 12: ACF PLOT OF X_{4t} (Full SARIMA Model) Residual

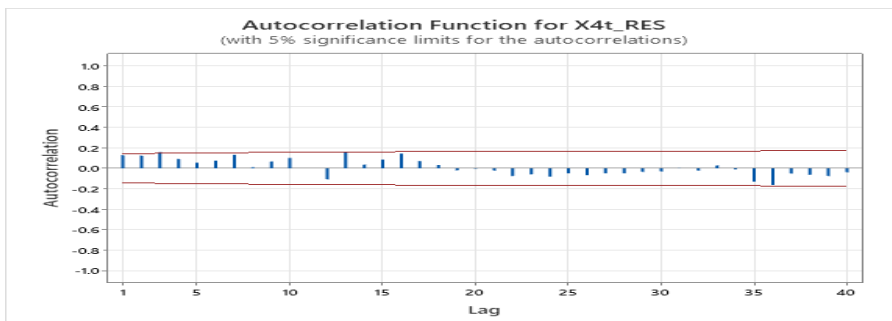


Figure 13: PACF PLOT OF X_{1t} (Full SARIMA Model) Residual

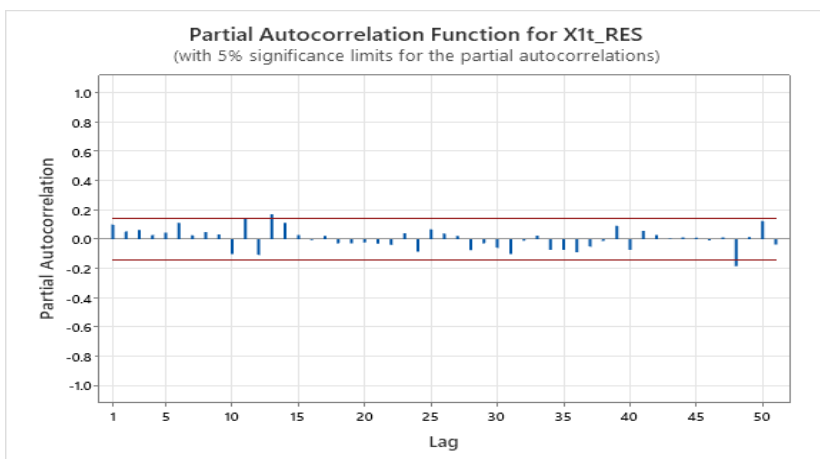


Figure 14: PACF PLOT OF X_{2t} (Full SARIMA Model) Residual

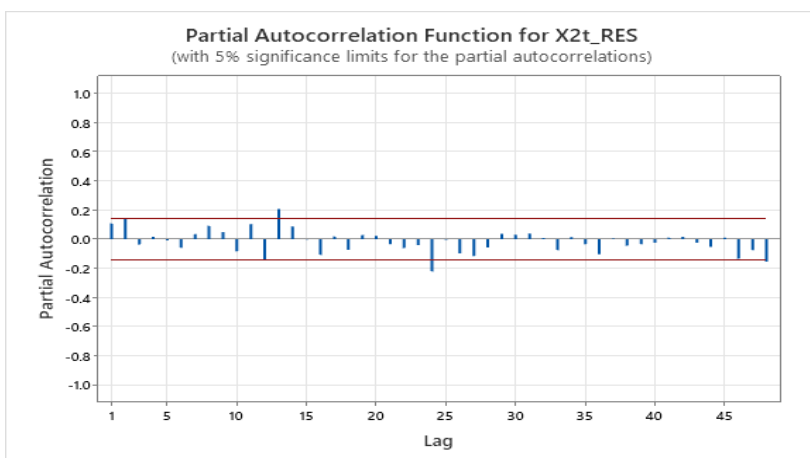
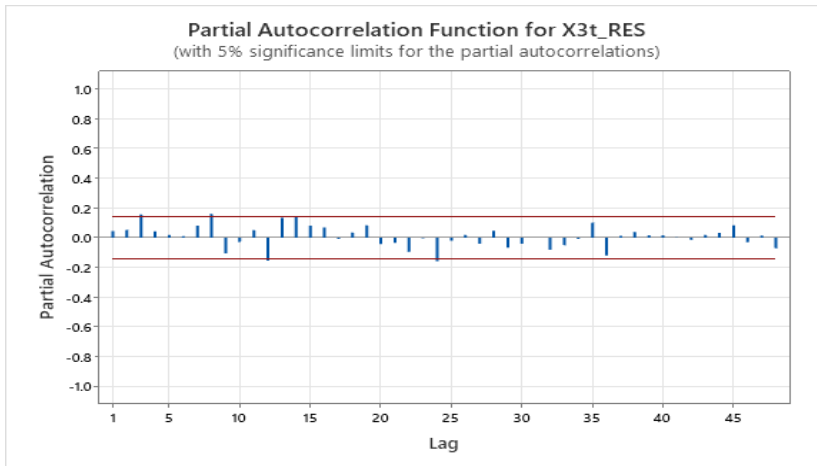
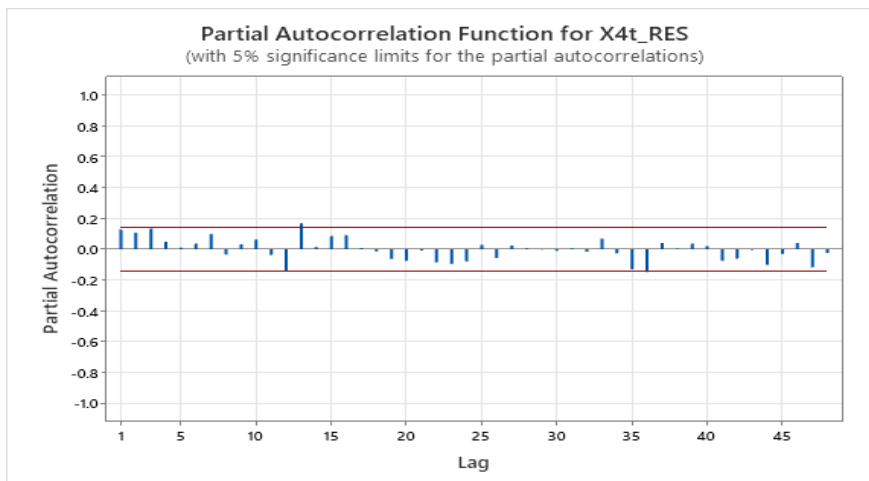


Figure 15: PACF PLOT OF X_{3t} (Full SARIMA Model) ResidualFigure 16: PACF PLOT OF X_{4t} (Full SARIMA Model) Residual

The residuals of the model are not totally identically and independently distributed, and they do not represent a pure white noise process.

4.6 REDUCED PARAMETER SARIMA MODELS

$$\begin{aligned}
 \begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \\ X_{4t} \end{bmatrix} &= \begin{bmatrix} 0.2687 & 0 & -0.88 & 0 \\ 0 & 0 & 0 & -0.826 \\ -0.4443 & 0 & 0 & 0 \\ 0 & -0.0223 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \\ X_{3t-1} \\ X_{4t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.00139 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_{1t-2} \\ X_{2t-2} \\ X_{3t-2} \\ X_{4t-2} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.79 & 0 \\ 0 & 0 & 0 & 0.615 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1t-1} \\ e_{2t-1} \\ e_{3t-1} \\ e_{4t-1} \end{bmatrix} + \\
 &\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.932 \\ 0 & 0 & 0 & 0 \\ 0.001686 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1t-2} \\ e_{2t-2} \\ e_{3t-2} \\ e_{4t-2} \end{bmatrix} + \begin{bmatrix} -0.6078 & 0 & 0 & 0 \\ 0.430 & -0.4758 & 0 & 0 \\ 0 & 0 & 0.3027 & 0 \\ 0 & 0 & 0 & -0.5065 \end{bmatrix} \begin{bmatrix} X_{1t-12} \\ X_{2t-12} \\ X_{3t-12} \\ X_{4t-12} \end{bmatrix} + \\
 &\begin{bmatrix} 0.1886 & -0.190 & 0 & 0 \\ 0 & -0.4263 & 0 & 0 \\ 0 & 0 & 0 & -0.190 \\ 0 & 0 & 0 & -0.2597 \end{bmatrix} \begin{bmatrix} X_{1t-24} \\ X_{2t-24} \\ X_{3t-24} \\ X_{4t-24} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.098 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{1t-12} \\ e_{2t-12} \\ e_{3t-12} \\ e_{4t-12} \end{bmatrix} \tag{12}
 \end{aligned}$$

Expansion of the above matrices provides multivariate seasonal autoregressive moving average models

for the four vector series X_{1t} , X_{2t} , X_{3t} and X_{4t} .

$$\begin{aligned}
 X_{1t} = & 0.2687X_{1t-1} - 0.88X_{3t-1} + 0.794e_{3t-1} - 0.6078X_{1t-12} + 0.1886X_{1t-24} - \\
 & 0.190X_{2t-24} \tag{13}
 \end{aligned}$$

$$\begin{aligned}
 X_{2t} = & -0.826X_{4t-1} + 0.615e_{4t-1} + 0.932e_{4t-2} - 0.430X_{1t-12} - 0.4758X_{2t-12} - \\
 & 0.4263X_{2t-24} \tag{14}
 \end{aligned}$$

$$\begin{aligned}
 X_{3t} = & 0.4443X_{1t-1} + 0.3027X_{3t-12} - 0.190X_{3t-24} + 0.098e_{4t-12} \tag{15}
 \end{aligned}$$

$$\begin{aligned}
 X_{4t} = & -0.0223X_{2t-1} - 0.00139X_{1t-2} + 0.001686e_{1t-2} - 0.5065X_{4t-12} - \\
 & 0.2597X_{4t-24} \tag{16}
 \end{aligned}$$

4.6.1 Estimation of Model Parameters

Table 5 present the estimate and test of significance of the parameters of each SARIMA model.

TABLE 5 Estimates of Parameters (Reduced SARIMAV)

| Response Variable | Predictor variable | Model | Coeff. | P-value |
|----------------------|--------------------|---------|---------|---------|
| Rainfall X_{1t} | X_{1t} | AR(1) | 0.2687 | 0.000 |
| | | SAR(12) | -0.6078 | 0.000 |
| | | SAR(24) | 0.1886 | 0.010 |
| | X_{2t} | SAR(24) | -0.190 | 0.004 |

| | | | | |
|--------------------------------|----------|-------------------------|--------------------------|-------------------------|
| | X_{3t} | AR(1) MA(1) | -0.880 0.790 | 0.002 0.004 |
| | X_{4t} | 0 | 0 | |
| Humidity X_{2t} | X_{1t} | SAR(12) | 0.430 | 0.019 |
| | X_{2t} | SAR(12) SAR(24) | -0.4758 -0.4263 | 0.000 0.000 |
| | X_{3t} | 0 | 0 | |
| | X_{4t} | AR(1) MA(1) MA(2) | -0.826 0.615 0.932 | 0.003 0.002 0.019 |
| Wind Speed X_{3t} | X_{1t} | AR(1) | 0.4443 | 0.000 |
| | X_{2t} | 0 | 0 | |
| | X_{3t} | SAR(12) | 0.3027 | 0.000 |
| | X_{4t} | SAR(24) SMA(12) | -0.190 0.098 | 0.000 0.014 |
| Temperature X_{4t} | X_{1t} | AR(2) MA(2) | -0.00139 0.001686 | 0.016 0.006 |
| | X_{2t} | AR(1) | -0.0223 | 0.035 |
| | X_{3t} | 0 | 0 | |
| | X_{4t} | SAR(12) SAR(24) | -0.5065 -0.2597 | 0.000 0.000 |

Source: Statistical computation from original data 2004-2024

4.7 Performance of the Estimated Models

The estimated MSE, AIC and BIC for Rainfall X_{1t} , Humidity X_{2t} , Wind X_{3t} and temperature X_{4t} were carried out with the help of the Minitab software. The results for are presented in Tables 6.

TABLE 6 : MSE, AIC and BIC comparison table between full ARIMA model and the reduced parameter model

| | Full SARIMA Model | | | Reduced SARIMA Model | | |
|----------|-------------------|---------|---------|----------------------|---------|---------|
| | MSE | AIC | BIC | MSE | AIC | BIC |
| X_{1t} | 389633 | 2319.70 | 2391.09 | 111481 | 2295.05 | 2316.75 |
| X_{2t} | 415.47 | 1118.0 | 1189.38 | 128.824 | 1100.59 | 1122.29 |

| | | | | | | |
|----------|--------|--------|--------|--------|--------|--------|
| X_{3t} | 73.328 | 741.13 | 812.51 | 14.390 | 712.03 | 727.65 |
| X_{4t} | 9.785 | 479.95 | 551.33 | 2.685 | 457.84 | 476.51 |

Source: Statistical computation from original data 2004-2024

4.8 Residual Autocorrelation and Partial Autocorrelation Function Plot (Reduced Parameter SARIMA Model)

Fig 17. ACF Residuals plot of Reduced SARIMA Model

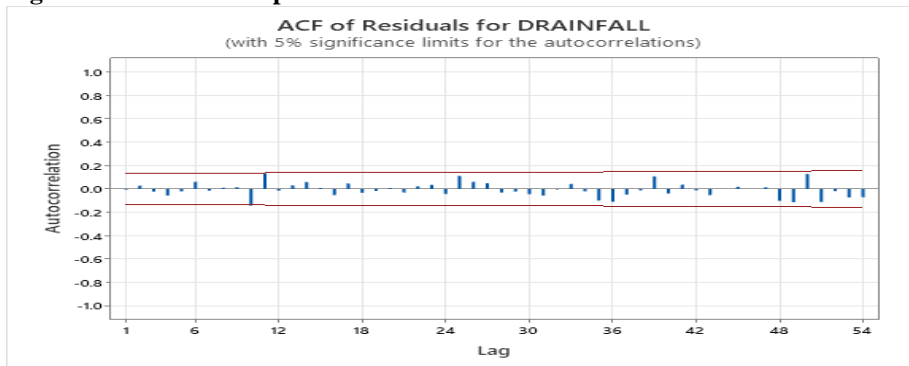


Fig 18. PACF Residuals plot of Reduced SARIMA Model

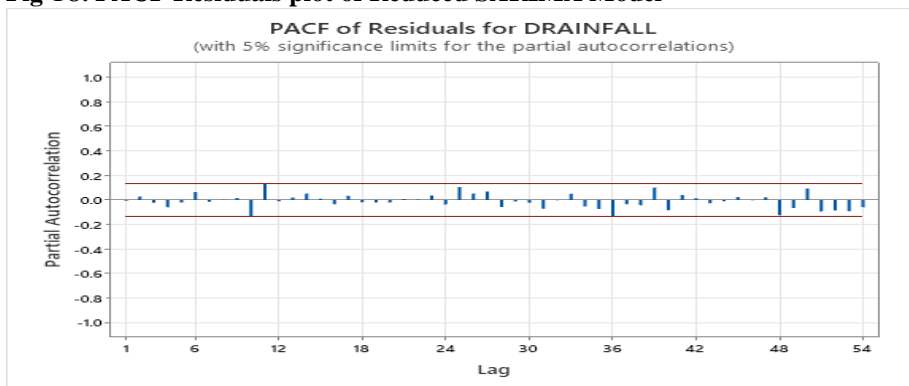


Fig 19. ACF Residuals plot of Reduced SARIMA Model

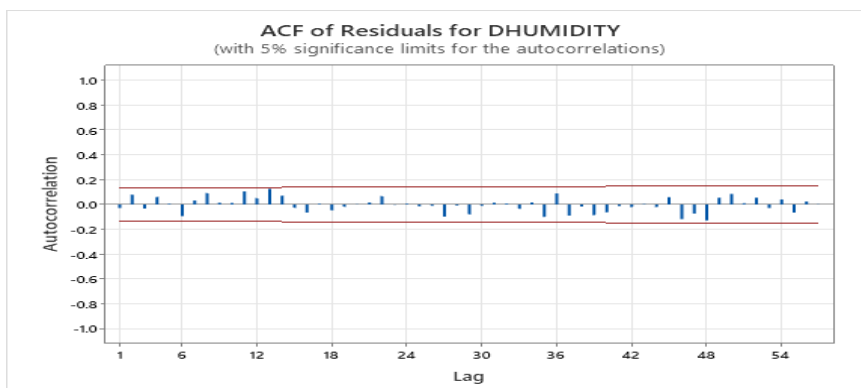


Fig 20. Residuals plot of Reduced SARIMA Model

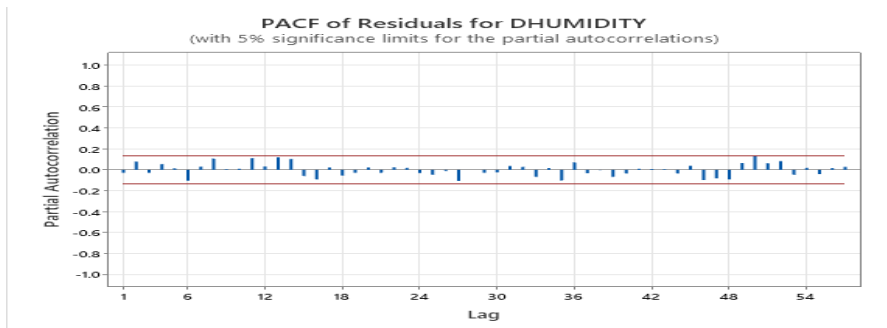


Fig .21. ACF Residuals plot of Reduced SARIMA Model

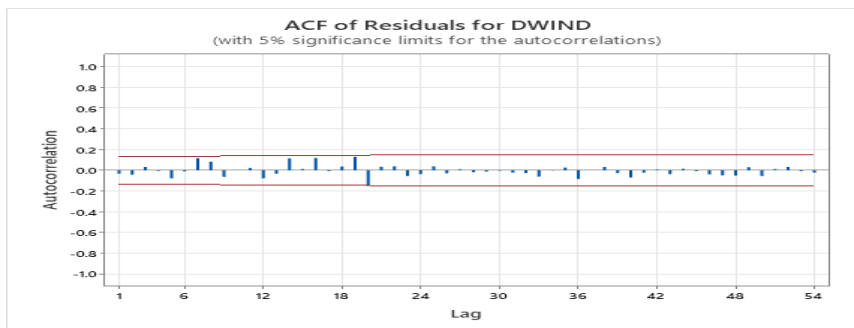


Fig 22. PACF Residuals plot of Reduced SARIMA Model

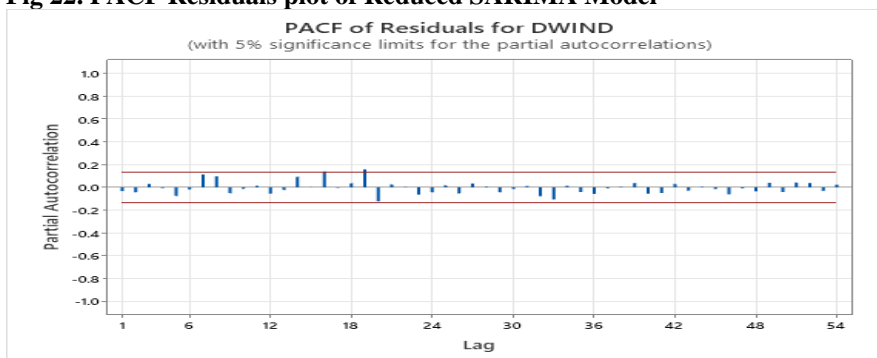
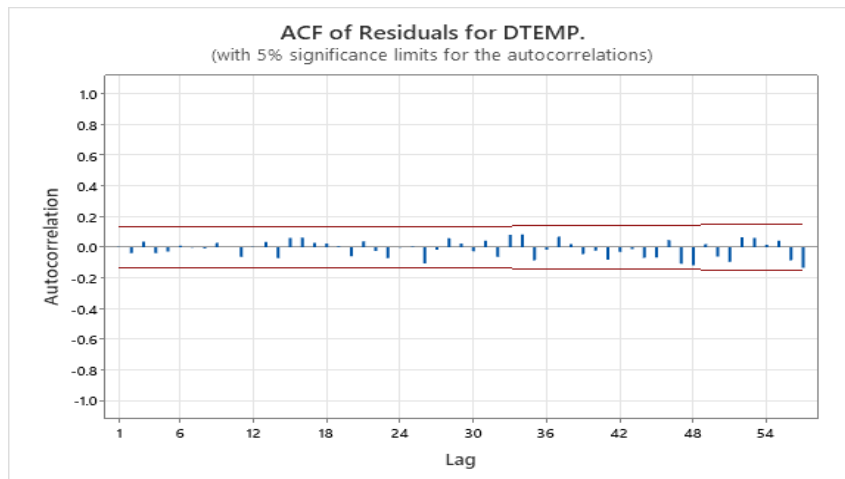
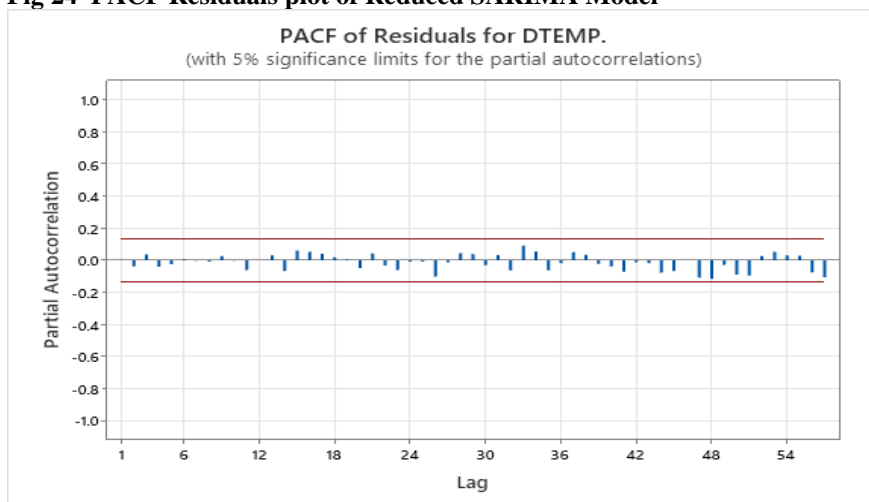


Fig 23. ACF Residuals plot of Reduced SARIMA Model**Fig 24 PACF Residuals plot of Reduced SARIMA Model**

The above autocorrelation and partial autocorrelation functions from the Reduced SARIMA model have shown that the residuals of the models are identically and independently distributed. These are pure white noise processes. Hence, each of the estimated models is adequate for the analysis and forecast of the respective set of data

5.0 CONCLUSION

This study investigated the application of Seasonal ARIMA Vector (SARIMAV) models in forecasting rainfall, humidity, wind speed, and temperature in Uyo, Akwa Ibom State, Nigeria. The results showed that the SARIMAV model, the generalized models presented in matrix form take care of the multivariate

seasonal series, irrespective of the number of vectors. Reliability of the SARIMAV models is established in time series analysis. Specifically the Reduced Parameter Model (RPM), outperformed the full SARIMAV model in all variables, demonstrating its superiority in capturing the underlying patterns in the data. The RPM's improved performance was attributed to its ability to eliminate insignificant parameters, resulting in a more parsimonious and stable model.

6.0 RECOMMENDATIONS

Based on the findings of this study, the following recommendations are made:

- (i) Adoption of SARIMAV models: SARIMAV models, particularly the RPM, should be considered for forecasting climate variables in Uyo, Akwa Ibom State, Nigeria.
- (ii) Model simplification: Model simplification techniques, such as reducing insignificant parameters, should be employed to improve model performance and stability.
- (iii) Future research: Future research should focus on comparing the performance of SARIMAV models with other multivariate forecasting models, such as Vector Autoregression (VAR) models.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

- Aghelpour, P., Mohammadi, B. & Biazar, S. M. (2019). Long-term Monthly Average Temperature Forecasting in some Climate types of Iran, using the models SARIMA, SVR, and SVR-FA. *Journal of Theoretical and Applied Climatology*, 138(3-4); 1471-1480. <https://doi.org/10.1007/s00704-019-02905-w>
- Ahmad, J; Alam, D; & Haseen M.S. (2011). Impact of Climate change on Agriculture and Food Security in India. *International Journal of Agriculture & Environment*, 4; 129–137.
- Aliyu, S; Abubakar, M. A. & Adenomon, M. O; (2021). Application of SARIMA Models in Modelling and Forecasti *Asian Journal of Probability and Statistics*, 13(3); 30-43.
- Ambildhuke, G.M; Gupta, B. B; (2022). Performance Analysis of Ensemble Techniques for Rainfall Prediction: a study based on the Current Atmospheric Parameters. *Journal of Climate Change*, 8 (3); 51–62. <https://doi:10.3233/JCC220021>
- Chen, P; Niu, A., Liu, D. & Jiang, W. (2021). Time series forecasting of temperatures using SARIMA: An example from Nanjing. *IOP Conference Series: Materials Science and Engineering*, volume 24 394(5); <https://doi.org/10.1088/1757-899X/394/5/052024>.

- Dahiya, P., Kumar, M. & Manhas, S. (2024). Time series study of climate variables utilising a seasonal ARIMA technique for the Indian states of Punjab and Haryana. *Journal of Discover Applied Sciences Research*, 6(650); <https://doi.org/10.1007/s42452-024-06380-5>
- Deryng, D; Conway, D; Ramankutty, N. & Price, J. (2014). Global Crop yield response to extreme heat stress under multiple climate change futures. *Environmental Research Letters*, 9(3); <https://doi.org/10.1088/1748-9326/9/3/034011>.
- Dongyao, L. (2023) The Prediction and Analysis of Global Climate change based on SARIMA Jinan Foreign Language School, Jinan. *China Proceedings of the International Conference on Machine Learning and Automation*, 40(1); 268-273. <https://doi.org/10.54254/2755-2721/40/20230665>
- Dufour, J.M, *Multivariate Time Series Modelling*, Cambridge University press, Cambridge, U.K, 2006. 2nd edition.
- Fronzek S; Pirttioja, N; Carter, T.R; Bindi, M; Hoffmann, H; & Palosuo, T. (2018). Classifying multi-model wheat yield impact response surfaces showing sensitivity to temperature and precipitation change. *Agricultural Systems*, 159; 209–224.
- Geetha. A. & Nasira, G. M. (2014). Artificial Neural Networks‘Application in Weather Forecasting. *International Journal of Computational Intelligence and Informatics*, Vol.4; No. 3. pp 177-182.
- Granger, W. J. & Newbold, P. (1986). *Forecasting Economic Time Series*. 2nd Edition. Academic Press.
- Krzyszczak, J., Baranowski, P., & Hoffmann, H. (2017). Temporal scale influence on multifractal properties of agro-meteorological time series. *Agricultural and Forest Meteorology*, 239; 223–235
- Lobell, D. B; Hammer, G. L; & Mclean, G. (2013). The critical role of extreme heat for maize production in the United States. *Journal of Nature Climate Change*, (3); 497–501. <https://doi.org/10.1038/nclimate1832>.
- Shivhare, N; Rahul, A. K; & Omar, P. J, Chauhan M.S. (2017) Identification of Critical Soil Erosion prone areas and Prioritization of micro watersheds using Geoinformatics techniques. *Journal of Ecological Engineering*, 121; 26–34. <https://doi.org/10.1016/j.ecoleng.2017.09.004>
- Usoro, A. E. & Awakessien, C. (2016) Time series modelling of Rainfall data in different locations in Nigeria *Journal of Mathematics and Statistics*, 6 (3); 372-380.
- Usoro, A. E; & John, E. E. (2019). Volatility of Internally Generated Revenue and Effects of its Major Components. A case of Akwa Ibom State, Nigeria. *American Journal of Theoretical and Applied Statistics*, Volume 8, Issue 6; pp. 276-286.
- Usoro, A. E. (2014), Seasonal Autoregressive Integrated Moving Average Vector Models and their Application to quarterly Rainfall series, *Journal of Statistical and Econometric Methods*, 3(4); 47-56.
- Wang, W., & Guo, Y. (2009). Air Pollution Data Analysis in Los Angeles long beach with seasonal ARIMA model. *International conference on energy and environment technology*, Vol. 3; pp. 7-10.
- Wei, W.S. (1990). *Time Series Analysis; Univariate and Multivariate Methods*, California, Addison-Wesley, *Journal of the American Statistical Association* Vol. 86, No. 413, pp. 245-246. <https://doi.org/10.2307/2289741>