**IMPACT OF INCLINED MAGNETIC FIELD ON THERMO-CONCENTRATION BOUNDARY LAYERS IN UNSTEADY MICROPOLAR MAGNETOHYDRODYNAMIC FLOW OVER A NONLINEAR STRETCHING SHEET**

# ABSTRACT

This work investigates the influence of an inclined magnetic field on thermo-concentration boundary layers in a micropolar magnetohydrodynamic (MHD) fluid flow over a nonlinear stretching surface. The magnetic field is applied at an inclination angle , altering the Lorentz force and hence affecting flow, heat, and mass transfer characteristics. Similarity transformations reduce the governing partial differential equations to ordinary differential form, which are further converted to a set of first-order ordinary differential equations and solved numerically using a collocation method in MATLAB. The results, which describe the impact of varying the angle of inclination of the magnetic field on the fluid velocity, angular velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number of the micropolar fluid, are presented in graphical and tabular form. They show that increasing ,increases the fluid velocity and temperature profiles while the particle’s angular velocity and concentration decrease. Further, the skin friction coefficient and Nusselt number increase with an increase in the Sherwood number. All this is attributed to the weakening of the Lorentz force. Since the applied magnetic field aligns with the direction of fluid flow. The findings have significant implications for the design and control of MHD-based thermal systems where the orientation of the magnetic field plays a crucial control role. The results are consistent with existing literature on magnetic field orientation in fluid flow systems.

*Keywords:* Inclined magnetic field, Micropolar fluid, Thermo-concentration boundary layer, Nusselt number, Sherwood number, Stretching sheet

1. **INTRODUCTION**

Inclined magnetic fields are increasingly used in industrial MHD applications such as cooling systems, plasma technology, and electromagnetic separation. In micropolar fluids, the orientation of the magnetic field alters the interaction between microstructure rotation and fluid convection. This study focuses on how varying the inclination angle $α$influences heat and mass transfer via the Nusselt and Sherwood numbers. A numerical study of natural convection of a nanofluid in an inclined L-shaped cavity in the presence of an inclined magnetic field was done by [1], where the emphasize was to identify the symmetrical behavior in fluid motion and temperature distribution at specific magnetic field angles. [2] conducted a study on the flow characteristics of blood via an inclined tapered porous artery with minor stenosis under an inclined magnetic field. [3] examined how radiation, an angled magnetic field, and cross-diffusion affect flow over a stretching surface. The study found that aligning the angle strengthens the magnetic field, reducing fluid flow, friction factor, and mass transfer rate while boosting heat transmission inside the fluid. Heat transfer and inclined magnetic field analysis for peristaltically driven motion of tiny particles was investigated by [4], while [5] investigated how an angled magnetic field affects magnetohydrodynamic (MHD) boundary layer flow on a porous exponentially stretched sheet exposed to thermal radiation.[6] investigated how an angled magnetic field affects flow, heat, and mass transfer of Williamson nanofluids across a stretching sheet.[7] analyzed heat transfer on micropolar alumina-silica-water nanofluid flow in an inclined square cavity with inclined magnetic field and radiation efficiency. Irreversibility of micropolar nanofluid flow in a vertical channel under the influence of an angled magnetic field and a heat source or sink was investigated by [8] while [9] studied the effects of a ternary nanofluid on a micropolar fluid with angled MHD, slip flow, and heat transfer. The main focus here was to establish how the orientation of the magnetic field affects fluid flow and thermal characteristics. [10] studied the effect of an inclined magnetic field and heat transfer on peristaltic flow of the Rabinowitsch fluid model in an inclined channel whereby the observation was that the magnetic field inclination act as a controlling mechanism for peristaltic flow. [11] investigated the impact of an inclined magnetic field, heat generation/absorption, and radiation on the peristaltic flow of a micropolar fluid through a porous non-uniform channel with slip velocity, while [12] studied the peristaltic motion of micropolar fluid with slip velocity in a tapered asymmetric channel in the presence of an inclined magnetic field and thermal radiation. Heat and mass transfer on free convective flow of a micropolar fluid through a porous surface with an inclined magnetic field and Hall effects was investigated by [13], where the main focus was to examine how an inclined magnetic field together with the Hall current affects the heat and mass transfer of the micropolar fluid. [14] analyzed the impact of an inclined magnetic field on non-isothermal vertical surface flow of micropolar fluid embedded in porous stratum. The inclined magnetic field and thermal radiation effect on electroosmotic flow of a micropolar fluid through a porous microchannel was investigated by [15], where the main focus of the study was on electro**osmotic propulsion** in microfluidic channels. Based on the literature above, the angle of inclination of the magnetic field has a considerable influence on the flow, temperature, and concentration properties of micropolar MHD flows across nonlinear stretching sheets. It impacts velocity profiles, heat transmission, and boundary layer thicknesses, all of which are important in a variety of industrial applications. Therefore, this area has not been fully explored; hence, understanding it well enables better control and optimization of micropolar fluid-based operations.

# MATHEMATICAL FORMULATION

* 1. **Description of the Problem**

In this study, unsteady and, incompressible micropolar fluid flow over a nonlinearly stretching sheet subjected to an inclined magnetic field is considered. The flow analysis is performed in a two-dimensional Cartesian coordinate system, with the x-axis aligned with the stretching surface at y=0 and the y-axis perpendicular to it, as shown in **Figure 1**. The fluid moves with the velocity , where n > 1 represents the nonlinearity parameter and $γ$ signifies unsteadiness. The transverse magnetic field  is applied at an angle α with the y axis. The magnetic component introduces a modified Lorentz force proportional to .



**Figure 1: Sketch of the physical Problem**

The stretched sheet's temperature , is controlled by a heated fluid behind the wall, with a convective temperature and a constant wall concentration . The free stream conditions are represented by  and, which reflect the ambient temperature and concentration respectively.

## **GOVERNING EQUATIONS**

The flow is governed by continuity, momentum, Angular momentum, Energy and Concentration equations as shown:

Continuity equation;

 . (1)

Momentum equation;


 (2)

Angular momentum equation;

. (3)

Energy equation;



 (4)

And Concentration equation.

. (5)

The boundary equations are:



 (6)

And

 (7)

**2.3 SIMILARITY TRANSFORMATION**

Using similarity variables,

 (8)

The governing PDEs are transformed as:

Momentum;



 (9)

Angular momentum;

 (10)

Energy equation;



 (11)

And concentration equation.

 (12)

Key dimensionless parameters in the resulting equations above are; the unsteadiness parameter,

 , the permeability parameter , the micropolar parameter  , the magnetic parameter  , the Grashof number based on temperature differences  , the Reynolds number ,the temperature mixed convection parameter , the Grashof number for mass transfer  , Concentration mixed convection parameter  , the Prandtl number , the Eckert number  , Thermal radiation  , Schmidt number , chemical reaction parameter  ,Brownian parameter,  and Thermophoresis parameter 

The non-dimensionalised boundary conditions are given as below;

 as  (13)

 as  (14)

# NUMERICAL METHOD

**3.1 Conversion of A Higher Order Odes to a System of First Order Odes**

To solve the resulting ODEs numerically, they are first converted into a system of first order odes as below:

Let  and 

So that;



,

 ,

,

,

, (15)



 



 .

The boundary conditions are;

 as  (16)

 as  (17)

The physical quantities crucial to this study are the skin friction, Nusselt number and Sherwood number as described by the equations below.

skin friction,

 (18)

Nusselt number,

 (19)

Sherwood number

 (20)

**3.2 Numerical Solution**

To solve the system of first-order equations obtained above, rewrite it in vector form as

 ,  . (21)

where  is a vector of unknown parameters, and  and .
To simplify the solution, the boundary conditions are stated as  after suppressing . The system is numerically solved using a collocation approach based on piecewise cubic polynomials over a discretized mesh . At the endpoints and midpoints of each subinterval, the approximate solution  meets the system requirements. The results obtained is a nonlinear algebraic system that is solved repeatedly through linearization. To ensure accuracy, the residual is minimized using . This method generates precise and computationally efficient approximations for velocity, temperature, angular velocity, and concentration profiles under a variety of parameters and their conditions. The angle is varied from 0◦ to 90◦ to assess the impact on transport rates.

# RESULTS AND DISCUSSION

**4.1 Effects of Angle of inclination of the Magnetic Feld on Fluid Velocity, Angular Velocity, Temperature and Concentration Profiles.**

This section looks at how the magnetic field's angle of inclination affects the fluid flow's velocity, temperature, angular velocity, and concentration profiles. The findings are presented graphically, followed by a thorough discussion. Furthermore, the impact of the angle of inclination of the magnetic field on the skin friction, Nusselt number, and Sherwood number of the micropolar fluid is assessed.

**Figure 2** shows that the velocity profile of the micropolar fluid increases with the magnetic field's inclination. As the angle α increases, the Lorentz force decreases as the magnetic field aligns with the fluid flow. This reduces resistance and enhances fluid velocity thereby improving the velocity profile. **Figure 3** shows that the angular velocity of micropolar fluid particles decreases with an increasing angle of inclination of the magnetic field. This is because increasing α minimizes the magnetic field's effect on fluid flow, as it aligns with the flow direction. This lessens the Lorentz force and increases fluid velocity. Fluid particles become more resistive to rotation as their velocity increases, leading to a drop in the angular velocity profile.

**Figure 4** depicts how the overall temperature profile of the micropolar fluid increases with increasing magnetic field inclination. This is because, as the angle of inclination increases, the applied magnetic field aligns with the direction of fluid flow. This alignment reduces Lorentz force, allowing fluid velocity to increase. Higher velocity improves convective heat transfer between the surface and fluid particles, raising the overall fluid temperature. However, with a minor angle of inclination of the magnetic field, the temperature near the surface drops. This is due to a larger Lorentz force, which slows fluid motion and so reduces convective heat transmission. As a result, temperatures decrease more quickly near the surface.

In **Figure 5**, increasing the inclination of the magnetic field reduces the concentration profile of the micropolar fluid away from the surface. However, no alteration can be seen near the surface. The reason behind this is that, increasing the angle α aligns the magnetic field with the fluid flow direction, minimizing the Lorentz force. Reducing the fluid's velocity away from the surface improves convective transport and lowers its concentration profile.



Figure 2: Effect of angle of inclination of magnetic field on velocity profile



Figure 3: Effect of angle of inclination of magnetic field on Angular velocity profile



Figure 4: Effect of angle of inclination of magnetic field on Temperature profile



Figure 5: Effect of angle of inclination of magnetic field on Concentration profile

**4.2 Effects of Angle of inclination of the magnetic field on skin friction, Nusselt number and Sherwood number.**

**Table 1** shows how changes in the angle of inclination affect the skin friction, Nusselt number, and Sherwood number of the micropolar fluid. The results show that increasing the angle of inclination of the applied magnetic field causes an increase in skin friction. This is due to a decrease in Lorentz force as the magnetic field aligns with the flow direction. The reduced opposing force allows fluid to flow more easily, increasing its velocity near the wall. The velocity gradient near the wall increases, causing shear stress and increased skin friction. The Nusselt number increases as the applied magnetic field's angle of inclination increases. This is because the magnetic field aligns with the flow direction, making the Lorentz force decrease. This lowers the resistance to fluid flow, improving the convective heat transmission, which results in a thinner thermal boundary layer and stronger temperature gradients, increasing the Nusselt number. The Sherwood number decreases with increase in the angle of inclination of the applied magnetic field. This is due to the weakening of the Lorentz force, which reduces magnetic resistance to flow. This leads to increased fluid velocity and thicker velocity boundary layers at the surface which enhance hydrodynamic resistance near the surface, slowing convective mass transfer and resulting in a lower Sherwood number.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Parameter** | **value** |  |  Nu | Sh |
|  |  | -1.8272 | 0.4501 | 1.2589 |
|  | -1.5920 | 0.4620 | 1.2425 |
|  | -1.3921 | 0.4717 | 1.2407 |

Table 1: Numerical values of  ,Nu and Sh for Angle of inclination of the Magnetic Field

# CONCLUSION

The Impact of Inclined Magnetic Field on Thermo-Concentration Boundary Layers in Unsteady Micropolar MHD Flow over a nonlinear stretching sheet was explored. The governing equations for continuity, momentum, angular momentum, energy, and concentration were developed and converted into higher-order ordinary differential equations via similarity transformations. The nonlinear equations were then transformed into a set of first-order ordinary differential equations. The resulting system was numerically solved using MATLAB's collocation method, and the results were presented graphically and in tabular form. The effects of the magnetic field's angle of inclination on the fluid velocity, angular velocity, temperature, and concentration profiles were investigated. Furthermore, numerical values of the skin friction coefficient, Nusselt number and Sherwood number were determined. The main conclusions are:

* The fluid velocity and Temperature increases with increase in the angle of inclination of the magnetic field.
* The particle’s angular velocity and concentration decreases with increase in the angle of inclination of the magnetic field.
* The skin friction coefficient and Nusselt number increases with increase in the angle of inclination of the magnetic field.
* The Sherwood number decreases with increase in the angle of inclination of the magnetic field.

Further research may be carried out to expand the model and include a three-dimensional fluid flow for a more realistic representation of practical engineering systems.

**Disclaimer (Artificial Intelligence)**

Author(s) hereby declares that generative AI technologies such as Large Language Models, etc. have not been used during the writing or editing of manuscripts

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