DYNAMICS IN FLEET MANAGEMENT WITH COMPETITION FOR PASSENGERS AT OPTIMAL LEVELS.

Abstract

We present a dynamic model describing the time evolution of travelers number, fleet units effort and fleet fare price. The fare price is fixed by the gap between demand and supply and assume fast time scale. A system of two equations is derived from a system of three equations through aggregation of variable method. Long term behavior of the model is then studied by determination of equilibrium points, stability and bifurcation analysis. Three equilibrium points are obtained as; unstable origin, stable carrying capacity and a stable free equilibrium. In the later case, a stable equilibrium corresponds to high cost of running fleet units hence negative effort, whereas unstable equilibrium corresponds to high demand for public transport which allow positive effort hence a saddle.
Key words:Fleet units, time evolution, fare price.

1. Introduction

Product diffusion and market acceptance of new products, services and innovations are actually a few of the major concerns especially to the business people.This will not only determine their importance or weakness over other competitors but also provide plans that can be implemented when needed. Due to high prices of new goods in the market upon product diffusion, only those people who belong to better income group can afford to buy the products at higher prices and it is only by the time these goods depreciates that they can diffuse to all other low income groups. As soon as the product has flooded the market, the rate of product diffusion will decrease asymptotically for it is limited by market number. Thus the impression on the product diffusion can clearly show the characteristics of logistic equation. At first all the fleets at the respective time of diffusion used to give high returns as a

business due to little competition but soon when the fleets had saturated the market is when the income depreciated beyond zero returns to some extend, see [6].
A Resource can to be renewable if it poses self regenerate capacity which enhance commercial exploitation by humans. Economic aspects on management of any renewable resource was considered by clark [1] more specifically, most kind of single population exploiting strategies have been mimic within a simple model that is

$$\dot{X} = f(X) - H \tag{1}$$

with $X = X(t)$, amount of the resource at time t whereas $f(X)$ is a given function which represent the natural growth rate of the resource with time and $H = H(t)$ is the harvesting rate a function of time. Always whenever harvesting rate goes beyond the growth rate then the population declines and vise versa, clark et [2] studied the optimal exploitation of resources at about general logistic models with depensation and also critical depensation in a generalized logistic models. In order to achieve a biomathematical generality, a simple simple logistic model is assumed, that is

$$\dot{X} = rX(1 - \frac{X}{K}) - EX, \tag{2}$$

r and K are the intrinsic growth rate and environmental carrying capacity respectively, E being the resource harvesting effort. Whenever resource $X^* = \frac{K}{2}$ then we obtain the optimal effort E from the non-trivial fixed point of equation (2) as;

$$E = \frac{r}{2}$$

2. Public transport outlook

Public transport in Kenya begun during colonial period. In 1934, the overseas company of London brought the local 13 fleet buses on 12 routes. It was latter changed to Kenya bus service and enjoyed monopoly in the transport sector until 1970's when matatu owners association and the county bus owners association stepped in to bridge the gap and meet the demand steepened by rural urban migration, see [5].

Matatu transport industry started as an illegal venture due to stringiest government policies but was eventually legalized in 1973 by Kenya's first president Jomo kenyatta ,see [4]. Soon after legalizing matatu industry in Kenya, it became a blessing to Kenyan economy as they created jobs to many Kenyans. Since matatu operators were hardworking kenyans devoted to their work,this led to the growth of young republic in terms of economy.

Bodaboda business originated by early 1990's from Uganda through busia town in western part of the country Kenya and spread to the other towns in the country, see [3]. Until now the business has already rooted in all parts of the country. The transport services was a Ugandan initiative that grew from Busia on Kenya-Uganda border. The name bodaboda emerged from from the term 'boarder' otherwise Kenya-Uganda boarder. Businessmen used to transport passengers and goods across Kenya-Uganda roads. Bicycles were majorly used in a flat terrain roads and later the mode of transport shifted to motorcycles which was more convenient.

From the literature, much has been commented and written on the advantages of fleets to the transport sector but little attention has been paid to the profitability of the business and effects of over-investment in fleet units beyond an optimum number. Similarly, there isn't a dynamical model for fleet management as the number of fleet units increases.

3. Fleet management model and long term solutions

We consider a system of ordinary differential equations that describes the relationship between the dependent variables: Travelers number $T(t)$, Fleet units management effort $E(t)$, which is actually fleet units involve in transporting travelers and fleet fare $P(t)$ with time t. A general fleet model that can be used to study the relationship to these variables can take the form;

$$\begin{aligned} \dot{T} &= rT(1-\frac{T}{K}) - qTE \\ \dot{E} &= \beta E(qPT - C) \\ \dot{P} &= \alpha P(aP^{-b} - qTE). \end{aligned} \tag{3}$$

System (3) is then aggregated while considering the fact that the fleet fare evolves faster than the travelers population growth and the fleet gathering effort. From the third equation of system (3), we set the fleet fare adjustment dynamics b to take the value $b = 1$ then use nontrivial value $P^* = \frac{a}{qTE}$ in the effort equation to yield;

$$\begin{aligned} \dot{T} &= rT(1 - \frac{T}{K}) - qTE \\ \dot{E} &= E(\frac{a}{E} - C), \end{aligned} \tag{4}$$

which is a system of two dimensional differential equation.

The solutions of the system (4) which doesn't change with time are discovered, these are the points the system dynamics persist with time. The fixed points are basically the intersection of T and E nullclines, that is $E_0 = (T_0, E_0) = (0, 0)$,$E_1 = (T_1, E_1) = (K, 0)$ and $E_2 = (T^*, E^*) = (K(1 - \frac{aq}{rC}), \frac{a}{C})$.

We then analyze the stability of this fixed points, the system is first linearized to give;

$$J(T, E) = \begin{bmatrix} r - \frac{2Tr}{K} - qE & -qT \\ 0 & -C. \end{bmatrix} \tag{5}$$

At E_0,

$$J(0, 0) = \begin{bmatrix} r & 0 \\ 0 & -C. \end{bmatrix}$$

Since eigenvalues; r and $-C$, are of opposite sign, the fixed point E_0 is a saddle equilibrium point and is always unstable.
At E_1,

$$J(K, 0) = \begin{bmatrix} -r & -qK \\ 0 & -C \end{bmatrix}.$$

J(K,0) is a stable fixed point since both eigenvalues ;$-r$ and $-C$ are negative. Since Fleet gathering activities, reduce the number of travelers to be served at respective zones which in turns varies the carying capacity, then zero fleet gathering effort leads to huge traffic or the number of travelers being kept at a large carrying capacities hence stable equilibrium.
At E_2,

$$J(T^*, E^*) := \begin{bmatrix} -r + \frac{qa}{C} & -qK + \frac{q^2Ka}{rC} \\ 0 & -C \end{bmatrix}. \tag{6}$$

If $C > \frac{qa}{r}$, then $J(T^*, E^*)$ is a stable fixed point since both eigenvalues are negative whereas if $C < \frac{qa}{r}$ then $J(T^*, E^*)$ is a saddle equilibrium point since one of the eigenvalue is negative and the other one being positive. Increased cost of running fleet units as compared to the urgency of passengers to travel gives negative fleet unit investment, making the fleet business unfavorable hence E_2 is stable equilibrium. In the case when the willingness of passengers to travel is higher than the cost of running fleet units, allow positive effort, then there is increase in fleet units invesment which reduce the number of travellers at a given zone making the equilibrium point E_2 a saddle, which is unstable.

4.Bifurcation analysis of the aggregated model

Before we present bifurcation diagram for the aggregated model we make equation (4)dimensionless, we allow the following transformations on the variables:

$$\Upsilon = \sqrt{q}T, \xi = \sqrt{q}E, \tau = \sqrt{q}t,$$

we also introduce the following parameters;

$$\eta = \frac{r}{\sqrt{q}}, \mu = \frac{C}{\sqrt{q}}, \kappa = \sqrt{q}K.$$

Equation (4)become

$$\begin{aligned} \dot{\Upsilon} &= \Upsilon(\eta(1 - \frac{\Upsilon}{\kappa}) - \xi) \\ \dot{\xi} &= (a - \mu\xi). \end{aligned} \tag{7}$$

Upon setting $q = 1$ in system (4) we obtain system (7) with $T = \Upsilon$,$E = \xi$,$r = \eta$,$C = \mu$ and $K = \kappa$, thus we can use initial parameters to give the aggregated system;

$$\begin{aligned} \dot{T} &= rT(1 - \frac{T}{K}) - TE \\ \dot{E} &= (a - CE). \end{aligned} \tag{8}$$

Bifurcation analysis on the above aggregated model will show us long term dynamical behaviour of the original system.

Proposition 1 *There exist a value $C = C_0$ where the system (8) undergoes transcritical bifurcation while the travelers number vary with C (Fleet operating cost). Also, for $C < C_0$, the fixed point E_2 is unstable whereas when $C > C_0$, the fixed point is stable.*

Proof

Substituting non-trivial value $E^* = \frac{a}{C}$ of the second Equation of system (8) in the first Equation yields:

$$\dot{T} = T(r(1 - \frac{T}{K}) - \frac{a}{C}) =: \psi(T, C). \tag{9}$$

Clearly, $T = 0$ and the curve $T = K(1 - \frac{a}{Cr})$ are the two equilibrium points to eqution (9). First we differentiate equation (9) with respect to T at the equilibrium point (T^*, K^*) to obtain

$$\psi'(T, C) = -r + \frac{qa}{C}. \tag{10}$$

Equation (10) is stable when $\psi'(T, C) < 0$ and unstable when $\psi'(T, C) > 0$, and that we have change in stability at

$$C = C_0 := \frac{a}{r}, \tag{11}$$

which is a point where bifurcation occurs.

Figure 1 show a transcritical bifurcation diagram obtain from system (8).

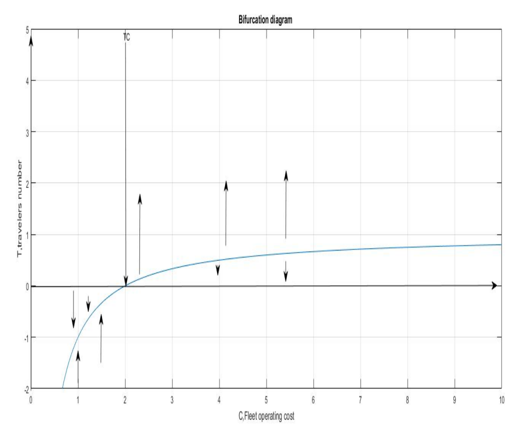

Figure 1: One parameter bifurcation diagram for r=0.2,a=0.4,k=1 and C as parameter

Bifurcation occur at the value

$$C = \frac{a}{r} = 2,$$

below this bifurcation value the solutions $T^* = 0$ is unstable while $T^* = K(1 - \frac{a}{Cr})$ is stable due to positive effort and beyond the bifurcation value, the solution $T^* = 0$ is stable while $T^* = K(1 - \frac{a}{Cr})$ is unstable due to negative effort. Increasing E(Fleet units) beyond certain threshold value, raise the cost of running fleet units as compared to willingness of the travelers to move from one zonal stage point to another, making the fleet business untenable hence stability. In the case when the cost of running fleet units is small, as a result of greater chance of travelers responding to movement between two zones, allows positive fleet gathering effort. The condition lead to increase in fleet units which reduce the number of travelers at a given zone hence instability.

5.Conclusion

From this study, a mathematical model was successfully formulated using a system of differential equations to describe the dynamic interactions between the travelers' number, fleet management effort, and fleet fare pricing. The models long-term behavior was investigated through local and global stability analyses, offering insights into the system's response to small disturbances and its overall stability under various conditions. Additionally, bifurcation analysis illustrate how variations in fleet number which in turn vary operating cost can lead to fundamentally different operational regimes. A key objective of determining the threshold value was successfully achieved. The analysis revealed that there exists an optimal threshold value E_{max} of fleet engagement beyond which additional fleet units do not significantly enhance service efficiency.

6.Recommendation

Based on the analysis, it is recommended that the government regulate the importation of fleet units in alignment with the dynamics of human population growth. This approach will help ensure an optimal number of fleet units are available to effectively serve the population. Additionally, the government should diversify economic opportunities by investing more significantly in sectors such as agriculture. This would not only stimulate economic growth but also attract and engage the youth in productive ventures
Further research should explore how pricing strategies interact with optimal fleet deployment under competitive pressure. Dynamic pricing, bundled services, or loyalty programs may help operators maximize revenue and re-

tain market share while operating within optimal resource limits, especially during high-demand periods.

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