Sustainable Tourism Dynamics through Nonlinear Systems and Lyapunov Stability Analysis

Abstract

This paper develops a nonlinear differential equation model to capture the dynamic interactions among economic performance, environmental conditions, social receptiveness, tourist volume, and cultural integrity within a tourism system. Using Lyapunov stability theory, we identify conditions under which the system can withstand disturbances and return to equilibrium. Additional boundedness analysis ensures the model variables remain within realistic limits, reflecting the sustainability of tourism activities. Numerical simulations validate the theoretical results and demonstrate how different parameters influence the system's trajectory. The findings underscore the importance of targeted policy measures—such as visitor quotas, sustainable investments, and community engagement—to support resilient and sustainable tourism development.

1 Introduction

Tourism is widely recognized as a powerful driver of global economic development, cultural exchange, and regional growth [1]. It generates employment, stimulates infrastructure investment, and fosters international understanding, particularly in developing regions where tourism revenues can substantially contribute to local livelihoods. However, the expansion of tourism—especially when pursued without adequate regulation or sustainability frameworks—can lead to unintended consequences ([2], [3]). These include environmental degradation through pollution and resource depletion, social tension resulting from cultural commodification or overcrowding, and economic vulnerabilities stemming from overdependence on fluctuating tourist demand. The paradox of tourism lies in its capacity to both uplift and undermine the very systems that support it. Consequently, there is a pressing need for more integrative and predictive tools that help balance the economic benefits of tourism with its environmental and social costs.

Several conceptual and empirical models have sought to capture the growth and saturation cycles of tourist destinations, such as the Tourist Area Life Cycle (TALC) proposed by Butler [3]. These models emphasize the nonlinear nature of tourism development and the interplay of multiple subsystems, including ecological resilience, community tolerance, and infrastructural capacity [4]. Nevertheless, relatively few studies employ formal mathematical modeling to analyze the stability of such systems, particularly when feedback loops and nonlinear interactions are prominent.

This paper addresses this critical gap by constructing a nonlinear differential equation system that incorporates five interacting variables central to tourism dynamics: economic factors, (Q), environmental quality (X), social receptiveness (S), tourist volume (P), and cultural effect (Y). E can be tourism's contribution to jobs, infrastructure, and revenue, as supported by [6] and [5]. Tourism is a key income source, especially in developing coastal regions. Social effects accounts for community well-being, tension, marginalization, and demographic displacement, reflecting concerns in Milano et al.[11] and [7]. U captures environmental degradation such as pollution, habitat loss, and overconsumption of resources, as extensively discussed in [8]. while C measures the erosion or transformation of local traditions, values, and heritage due to ”staged authenticity” and commercialization ,[9]. These variables interact through nonlinear relationships. For instance, an increase in Y may degrade Q, while higher Y may influence S either positively through amenities or negatively through over-commercialization.

This framework describes the ability of the system to recover or stabilize over time after being disturbed. A resilient tourism system, despite shocks (e.g., over-tourism, economic dips), will eventually return to a stable or balanced state. To assess the system's behavior near equilibrium states, Lyapunov stability theory is employed. The use of Lyapunov stability analysis in the model supports identifying whether the system will settle back to equilibrium or spiral into collapse. [12], [13], [14] Unlike linearized methods, Lyapunov functions enable a more robust analysis of nonlinear systems without requiring explicit solutions. Through this approach, we derive conditions for stability and explore how policy interventions or parameter changes influence the system's trajectory.

The rest of this article is structured as follows: Section 2 presents the model structure, defining variables and interactions; Section 3 details the stability analysis using Lyapunov methods; Section 4 discusses limitations and potential extensions; Section 5 connects results to real-world policy implications; and Section 6 concludes with final remarks and recommendations.

2 Model Structure

A system of nonlinear differential equations to describe the dynamics of tourism can be formulated with variables representing as:

$$\begin{aligned}
\frac{dP}{dt} &= -c_1 P \\
\frac{dQ}{dt} &= aQ - \frac{aQ^2}{\alpha_1} - b_1 QS - b_2 QX + c_2 P, \\
\frac{dS}{dt} &= \gamma_1 S - \frac{\gamma_1 S^2}{\alpha_2} - d_1 S - X d_2 SY + c_3 P, \\
\frac{dX}{dt} &= nX - \frac{nX^2}{\alpha_3} - g_1 QX - g_2 SX + c_4 P, \\
\frac{dY}{dt} &= mY - \frac{mY^2}{\alpha_4} - f_1 SY - f_2 XY + c_5 P,
\end{aligned} \tag{1}$$

where a, δ_1, n, m represent the intrinsic growth of economic, social, environmental, and cultural factors due to tourism. $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ represent the maximum sustainable level of the carrying capacities. Interaction terms (b, d, g, f) represent the effects of interactions:
b_1QS: Negative impact of social tensions on the economy.
b_2QX: Environmental degradation's economic cost.
d_1SX: Environmental effects' impact on social well-being.
f_1SY: Social tension reducing cultural harmony.
f_2SY cultural violation due to environmental factors.
c_1, c_2, c_3, c_4: are tourism influx which represent the direct effects of tourism activities on each variable.

3 Stability Analysis with Lyapunov Functions

Lyapunov functions are utilized to assess the stability of equilibrium points in the system. This approach is particularly suitable for complex, nonlinear systems where analytical solutions are often elusive. A Lyapunov function $L(P, Q, S, X, Y)$ is constructed such that its derivative along system trajectories is non-positive:

Theorem 1. *In addition to the assumptions imposed on the model parameters, $a, b_2, n, m, \gamma, c_1, c_2, ..., d_1, d_2, ..., f_1, f_2$ in (1) as non negative values , this study also assumed that*

$$\begin{aligned} (c_1 - c_2 - c_3 - c_4 - c_5) > 0 \\ (\frac{aQ_0}{\alpha_1} + b_1S_0 + b_2X_0 - a - c_2) > 0 \\ (\frac{\gamma_1 S_0}{\alpha_2} + d_1X_0 + d_2Y_0 - \gamma_1 - c_3) > 0 \\ (\frac{nX_0}{\alpha_3} + g_1Q_0 + g_2S_0 - n - c_4) > 0 \\ (\frac{mY_0}{\alpha_4} + f_1S_0 + f_2Y_0 - m - c_5) > 0 \end{aligned} \tag{2}$$

then the zero solutions to (1) *satisfy* $\mid S \mid \to 0$, $\mid Q \mid \to 0$,$\mid Y \mid \to 0$,$\mid P \mid \to 0$, $\mid X \mid \to 0$ *as* $t \to \infty$.

Proof: We shall represent the model with a Lyapunov function defined by

$$2L = P^2 + Q^2 + S^2 + X^2 + Y^2$$

Clearly $L \geq 0$, i.e L is positive definite. Time derivative of the Lyapunov function,

$$\begin{aligned} \dot{L} &= P\dot{P} + Q\dot{Q} + S\dot{S} + X\dot{X} + Y\dot{Y} \\ \dot{L} &= c_1P^2 + Q[aQ - \frac{aQ^2}{\alpha_1} - b_1QS - b_2QX + c_2P] \\ &+ S[gamma_1S - \frac{\gamma_1 S^2}{\alpha_2} - d_1S - Xd_2SY + c_3P] \\ &+ X[nX - \frac{nX^2}{\alpha_3} - g_1QX - g_2SX + c_4P] + Y[mY - \frac{mY^2}{\alpha_4} - f_1SY - f_2XY + c_5P] \end{aligned}$$

$$
\begin{aligned}
\dot{L} &= -P[c_1P] + aQ^2 - \frac{aQQ^2}{\alpha_1} - b_1Q^2S - b_2Q^2U + c_2QP \\
&+ \gamma_1 S^2 - \frac{\gamma_1 SS^2}{\alpha_2} - d_1S^2X - d_2S^2Y + c_3SP \\
&+ nX^2 - \frac{nXX^2}{\alpha_3} - g_1QX^2 - g_2SX^2 + c_4PX + mY^2 - \frac{mYY^2}{\alpha_4} \\
&- f_1SY^2 - f_2XY^2 + c_5PX
\end{aligned}
$$

$$
\begin{aligned}
\dot{L} &= -c_1P^2 + aQ^2 - \frac{aQ_0Q^2}{\alpha_1} - b_1Q^2S - b_2Q^2X_0 + c_2PQ \\
&+ \gamma_1 S^2 - \frac{\gamma_1 S_0S^2}{\alpha_2} - d_1S^2X_0 - d_2S^2Y_0 + c_3SP \\
&+ nX^2 - \frac{nX_0X^2}{\alpha_3} - g_1Q_0X^2 - g_2S_0X^2 + c_4PX + mY^2 - \frac{mY_0Y^2}{\alpha_4} \\
&- f_1S_0Y^2 - f_2Y_0X^2 + c_5PX
\end{aligned}
$$

$$
\begin{aligned}
\dot{L} &\leq -\Big[(c_1 - c_2 - c_3 - c_4 - c_5)P^2 + (\frac{aQ_0}{\alpha_1} + b_1S_0 + b_2U_0 - a - c_2)Q^2 + c_2(Q-Y)^2 \\
&+ (\frac{\gamma_1 S_0}{\alpha_2} + d_1X_0 + d_2Y_0 - \gamma_1 - c_3)S^2 + c_3(S - P^2 \\
&+ (\frac{nX_0}{\alpha_3} + g_1Q_0 + g_2S_0 - n - c_4)X^2 + c_4(P-X)^2 + \\
&+ (\frac{mY_0}{\alpha_4} + f_1S_0 + f_2Y_0 - m - c_5)Y^2 + c_5(P-X)^2
\end{aligned}
$$

$$
\dot{L} < 0
$$

Thus the solutions of the systems (1) satisfy $\parallel P \parallel \longrightarrow 0$, $\parallel S \parallel \longrightarrow 0$, $\parallel Q \parallel \longrightarrow 0$, $\parallel X \parallel \longrightarrow 0$, $\parallel Y \parallel \longrightarrow 0$ as $t \longrightarrow \infty$, provided the conditions of equations (2) are satisfied.

If $\dot{L} \leq 0$ for all values of (P, Q, S, X, Y), the system is stable. Since $\dot{L} < 0$ strictly, the system is globally asymptotically stable, even if it experiences small perturbations like a sudden influx of tourists or a temporary environmental shock. This means that L(t) never increases with time. It either decreases or remains constant.

4 Boundedness of Solutions

Boundedness means that solutions remain within a finite region for all time, ensuring that economic, social, environmental, and cultural effects do not grow indefinitely.

Introducing a functional response $\rho_2\Phi(t,P,Q,S,X,Y)$ and cultural violation due to tourist influx, $\rho_1 PY$ into systems (1) to become

$$\begin{aligned}
\frac{dP}{dt} &= -c_1P - \rho_1 PY \\
\frac{dQ}{dt} &= aQ - \frac{aQ^2}{\alpha_1} - b_1QS - b_2QX + c_2P, \\
\frac{dS}{dt} &= \gamma_1 S - \frac{\gamma_1 S^2}{\alpha_2} - d_1S - Xd_2SY + c_3P, \\
\frac{dX}{dt} &= nX - \frac{nX^2}{\alpha_3} - g_1QX - g_2SX + c_4P, \\
\frac{dY}{dt} &= mY - \frac{mY^2}{\alpha_4} - f_1SY - f_2XY + c_5P + \rho_2\Phi(t,P,Q,S,X,Y).
\end{aligned} \tag{3}$$

Theorem 2. *In addition to the assumptions imposed on the parameters in Theorem 1, let*

$$\begin{aligned}
(c_1 + \rho - c_2 - c_3 - c_4 - c_5) &> 0 \\
(\frac{aQ_0}{\alpha_1} + b_1S_0 + b_2X_0 - a - c_2) &> 0 \\
(\frac{\gamma_1 S_0}{\alpha_2} + d_1X_0 + d_2Y_0 - \gamma_1 - c_3) &> 0 \\
(\frac{nX_0}{\alpha_3} + g_1Q_0 + g_2S_0 - n - c_4) &> 0 \\
(\frac{mY_0}{\alpha_4} + f_1S_0 + f_2Y_0 - m - c_5) &> 0 \\
\rho_2 \parallel \Phi(t,P,S,Q,X,Y) \parallel &\leq \rho_2\chi(t)
\end{aligned} \tag{4}$$

then there exist a constant $K > 0$ *such that every solution of (2) satisfies* $\mid Q \mid < K$, $\mid S \mid < K$, $\mid X \mid < K$, $\mid Y \mid < K$, $\mid P \mid < K$, $\forall t > 0$

Proof :

We shall also defined the Lyapunov function by

$$2L = P^2 + Q_2 + S^2 + X^2 + Y^2$$

Clearly $L \geq 0$, i.e L is positive definite. Time derivative of the Lyapunov function,

$$
\begin{aligned}
\dot{L} &= P\dot{P} + Q\dot{Q} + S\dot{S} + X\dot{X} + Y\dot{Y} \\
\dot{L} &= c_1P^2 + c_1\rho PY + P[mY - \frac{mY^2}{\alpha_4} - f_1SY - f_2XY + c_5Q + \rho_2\Phi(t,P,S,Q,X,Y)] \\
&+ Q[aQ - \frac{aQ^2}{\alpha_1} - b_1SQ - b_2QX + c_2Q,] \\
&+ S[\gamma_1S - \frac{\gamma_1S^2}{\alpha_2} - d_1SX - d_2SY + c_3P] \\
&+ X[nX - \frac{nX^2}{\alpha_3} - g_1QX - g_2SX + c_4P] \\
&+ Y[mY - \frac{mY^2}{\alpha_4} - f_1SY - f_2XY + c_5Q + \rho_2\Phi(t,P,S,Q,X,Y)]
\end{aligned}
$$

$$
\begin{aligned}
\dot{L} &= -[c_1P^2 - \rho P^2Y + aQ^2 - \frac{aQQ^2}{\alpha_1} - b_1E^2S - b_2E^2U + c_2ET \\
&+ \gamma_1S^2 - \frac{\gamma_1SS^2}{\alpha_2} - d_1S^2X - d_2S^2Y + c_3SP \\
&+ nX^2 - \frac{nXX^2}{\alpha_3} - g_1QU^2 - g_2SU^2 \\
&+ c_4TU + mC^2 - \frac{mCC^2}{\alpha_4} - f_1SY^2 - f_2XY^2 + c_5PX + Y\Phi(t,P,S,Q,X,Y)
\end{aligned}
$$

$$
\begin{aligned}
\dot{L} &= -c_1P^2 - \rho P^2C_0 + aE^2 - \frac{aQ_0Q^2}{\alpha_1} - b_1Q^2S - b_2Q^2X_0 + c_2PQ \\
&+ \gamma_1S^2 - \frac{\gamma_1S_0S^2}{\alpha_2} - d_1S^2X_0 - d_2S^2Y_0 + c_3SP \\
&+ nX^2 - \frac{nX_0X^2}{\alpha_3} - g_1Q_0X^2 - g_2S_0X^2 + c_4PX + mY^2 - \frac{mY_0Y^2}{\alpha_4} \\
&- f_1S_0Y^2 - f_2Y_0X^2 + c_5PX + \mid Y \mid \ \parallel \Phi(t,P,S,Q,X,Y) \parallel
\end{aligned}
$$

$$
\begin{aligned}
\dot{L} &\leq -\Bigg[(c_1 + \rho - c_2 - c_3 - c_4 - c_5)P^2 + (\frac{aQ_0}{\alpha_1} + b_1S_0 + b_2X_0 - a - c_2)Q^2 \\
&+ (\frac{\gamma_1S_0}{\alpha_2} + d_1X_0 + d_2Y_0 - \gamma_1 - c_3)S^2 \\
&+ (\frac{nX_0}{\alpha_3} + g_1Q_0 + g_2S_0 - n - c_4)X^2 \\
&+ (\frac{mY_0}{\alpha_4} + f_1S_0 + f_2Y_0 - m - c_5)Y^2 + \mid Y \mid \chi(t)
\end{aligned}
$$

$$
\dot{L} \leq -\Bigg[k_1P^2 + k_2Q^2 + k_3S^2 + k_4X^2 + k_6Y^2\Bigg] + (1 + Y^2)\chi(t)
$$

$$\dot{L} \leq -k_8(P^2 + Q^2 + S^2 + X^2 + Y^2) + k_7\chi(t)$$

$$L \leq K$$

This implies that $P \leq K$, $S \leq K$, $Q \leq K$, $X \leq K$, $Y \leq K$, provided the conditions in (4) are satisfied. For boundedness, $\frac{dL}{dt} \leq 0$, ensures that $L(P, S, Q, X, Y)$ is decreasing, meaning, Economy, Environmental Quality, Social Receptiveness, Tourism Volume, cultural value all remain inside a bounded region over time.

5 Mathematical Simulation

Tourism dynamical models often involve nonlinear interactions between different factors, e.g economic growth leading to environmental degradation which in turns affect social attitudes. Wolfram Mathematica is a powerful symbolic and numerical solvers for accurate solution of these complex equations. Below are simulation results.

```
Stability Analysis
(*Define Parameters*)a = 0.5; b1 = 0.1; b2 = 0.05;
γ1 = 0.4; d1 = 0.07; d2 = 0.03;
n = 0.3; g1 = 0.06; g2 = 0.04;
m = 0.2; f1 = 0.05; f2 = 0.02;
c1 = 0.2; c2 = 0.1; c3 = 0.15; c4 = 0.1;
α = 10; α = 8; α = 6; Kα = 7;

(*Define the System of Nonlinear Differential Equations*)
eqs = {q'[t] == 0.5 * q[t] * (1 - q[t] / 10) - 0.1 * q[t] * s[t] - 0.05 * q[t] * x[t] + 0.1,
   s'[t] == 0.4 * s[t] * (1 - s[t] / 8) - 0.01 * s[t] * v[t] - 0.03 * s[t] * y[t] + 0.1,
   x'[t] == 0.3 * x[t] * (1 - x[t] / 6) - 0.06 * q[t] * v[t] - 0.04 * s[t] * x[t] + 0.15,
   y'[t] == 0.2 * y[t] * (1 - y[t] / 7) - 0.05 * s[t] * y[t] - 0.02 * x[t] * y[t] + 0.1};

(*Initial Conditions*)
initConds = {e[0] == 2, s[0] == 3, v[0] == 1, c[0] == 2};
(*Solve Numerically*)
sol = NDSolve[{eqs, initConds}, {{p, q, s, x, y}, {t, 0, 50}}, t]
```

Figure 1: ***Wolfram Code for stability***

Boundedness of Solutions

```
(*Define Parameters*) a = 0.5; b1 = 0.1; b2 = 0.05;
γ1 = 0.4; d1 = 0.07; d2 = 0.03;
n = 0.3; g1 = 0.06; g2 = 0.04;
m = 0.2; f1 = 0.05; f2 = 0.02;
c1 = 0.2; c2 = 0.1; c3 = 0.15; c4 = 0.1;
α = 10; α = 8; α = 6; Kα = 7;
(*Define nonhomogeneous*)
Φ (t, p, q, s, x, y) = 1/ (t + p[t]^2 + q[t]^2 + s[t]^2 + x[t]^2 + y[t]^2);
(*Define the System of Nonlinear Differential Equations*)
eqs = {q'[t] == 0.5 * q[t] * (1 - q[t] / 10) - 0.1 * q[t] * s[t] - 0.05 * q[t] * x[t] + 0.1,
   s'[t] == 0.4 * s[t] * (1 - s[t] / 8) - 0.01 * s[t] * v[t] - 0.03 * s[t] * y[t] + 0.1,
   x'[t] == 0.3 * x[t] * (1 - x[t] / 6) - 0.06 * q[t] * v[t] - 0.04 * s[t] * x[t] + 0.15,
   y'[t] == 0.2 * y[t] * (1 - y[t] / 7) - 0.05 * s[t] * y[t] - 0.02 * x[t] * y[t]
     + 0.1 + 1/ (t + p[t]^2 + q[t]^2 + s[t]^2 + x[t]^2 + y[t]^2)};

(*Initial Conditions*)
initConds = {e[0] == 2, s[0] == 3, v[0] == 1, c[0] == 2};
(*Solve Numerically*)
sol = NDSolve[{eqs, initConds}, {{p, q, s, x, y}, {t, 0, 50}}, t]
```

Figure 2: ***Wolfram Code for Boundedness***

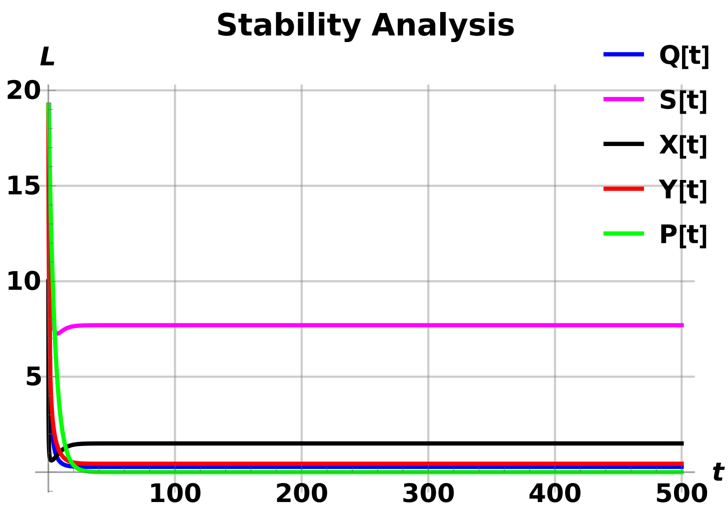


Fig3,*Maintaining the Stability of Q(t), S(t), P(t), Y(t), X(t) at* $d_3 = 0.06, c_3 = 0.08, \delta = 100$

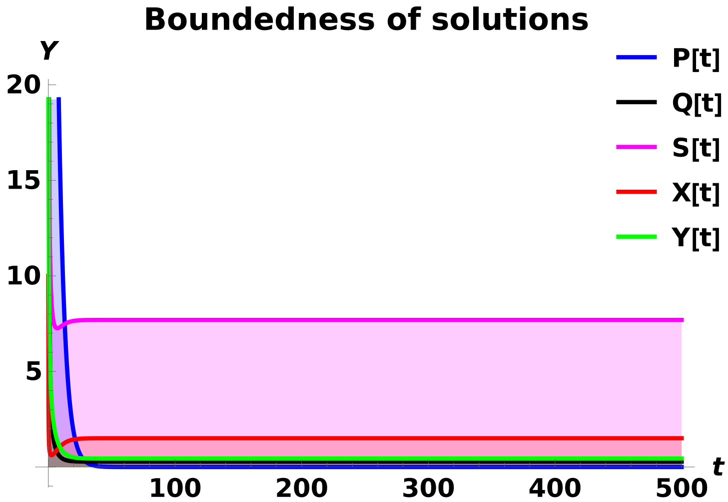


Fig4 *Maintaining the bounds of E(t), S(t), R(t), C(t), V(t) at* $\Phi(t, P, S, Q, X, Y) = \frac{1}{(t^2+P^2+S^2+Q^2+X^2+Y^2)}$

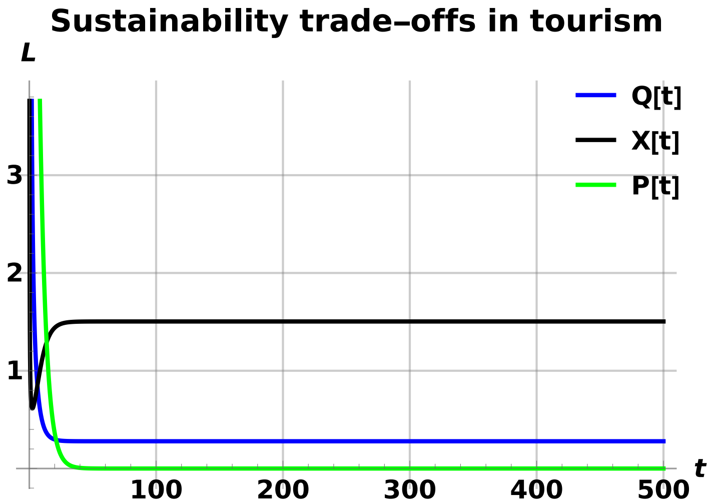


Fig5 *Economic, Environmental and Cultural Conflicts at* $b_1 = 0.1$, $b_2 = 0.5, d_1 = 0.1, d_2 = 0.3 g_1 = 0.6, g_2 = 0.4$

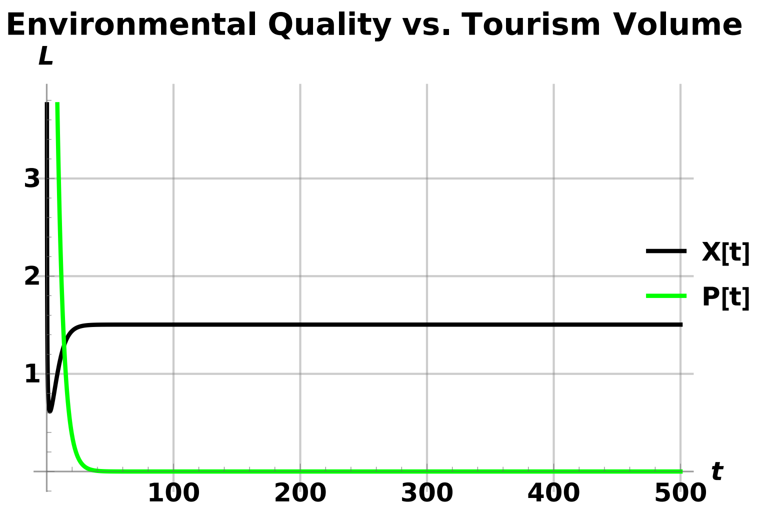
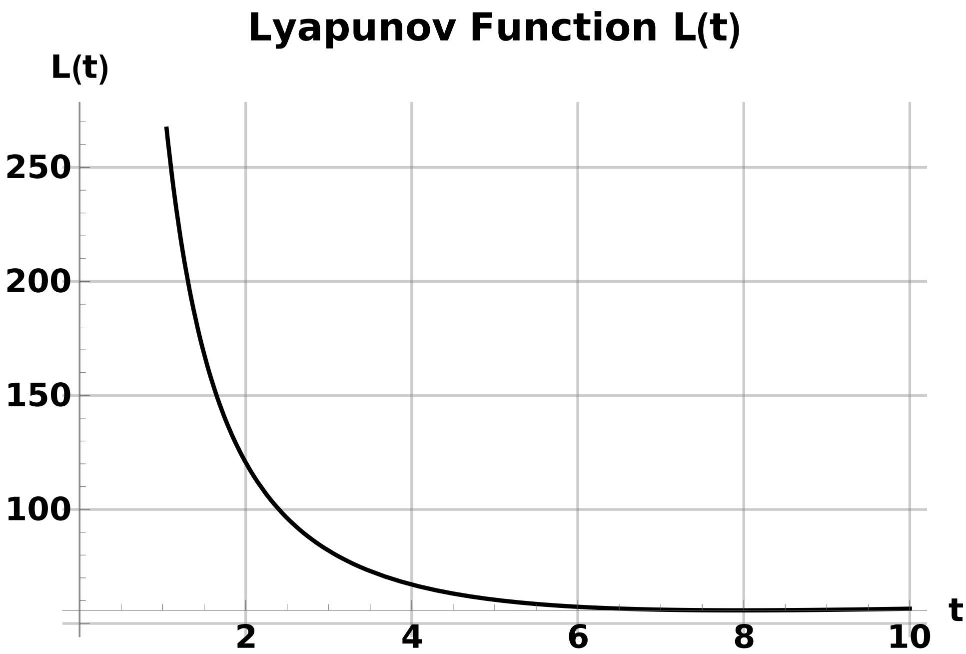


Fig6 *Environmental effects and Tourists at* $b_1 = 0.1$, $b_2 = 0.5, d_1 = 0.1, d_2 = 0.3 g_1 = 0.6, g_2 = 0.4$



, Surface representation of a Lyapunov function used to evaluate the system's resilience.

6 Results and Discussion

Figure 1, 2 explain Wolfram Mathematica simulation (code) to show that $(P(t), Q(t), S(t), X(t), Y(t))$ remain within finite values. Plot inspection confirms the solutions remain within a limited range. If the solutions converge to steady values, the system is stable. If oscillations persist or variables explode, the system might be unstable. Figure 3 shows stability where the system settles over time. Numerical simulations analyze the behavior under large perturbations. Otherwise, if instability occurs, we modify parameters such as b_1, b_2 to reduce negative effects. Figure 4 visualizes boundedness of solutions. If $\frac{dL}{dt} \leq 0$ for all (t, P, S, Q, X, Y), then the solutions are bounded. Bounding effects depend on $\rho_2 \Phi(t, P, S, Q, X, Y)$ to serve as Control Strategies to mitigate negative effects. Negative effects may include the following; Overburdened economy b_1, b_2 which include Local businesses struggle; Social stress d_1, d_2; Environmental degradation g_1, g_2 including Pollution, resource depletion; Cultural erosion and Loss of traditions f_1, f_2. Other form of Control Strategies, in $\rho_2 \Phi(t, P, S, Q, X, Y)$, are sustainable tourism taxation to reduce economic burden and subsidies to small businesses to compete with large hotels.

Figure 5 proved that if the system is unstable and unbounded, we can modify parameters (e.g., reducing b_1, b_2 to lessen economic-environmental conflicts. Tuning interaction parameters $b_1, b_2, d_1, d_2, g_1, g_2, f_1, f_2$ can shift dynamics towards sustainability or collapse. The graph highlights the tension between economic growth and sustainability in tourism systems. While increased capital investment and visitor numbers can enhance short-term economic benefits, they often coincide with declining environmental quality and weakening of cultural identity. However, regions that adopt sustainable tourism models—balancing economic returns with environmental protections and cultural authenticity—show more stable and synergistic outcomes across all three dimensions. This confirms the need for integrated tourism policies that account for complex, multi-directional impacts. Figure 6 depicts as tourism volume increases, environmental quality tends to decrease. This reflects the negative impact of tourism on natural resources due to pollution, land use, and overconsumption. The Lyapunov graph, Figure 7, shows that if the Lyapunov, L(t) decreases over time, it indicates that despite shocks (e.g., sudden tourist surges), the model returns to a stable state. This shows that the tourism system is resilient under the current policy and economic conditions. If L(t) is bounded and oscillating, the system experiences cyclical stress but avoids collapse—possibly due to feedback control like tourism taxes or environmental regulations. If L(t) increases, the system is poorly managed, and long-term sustainability is at risk.

7 Model Limitations

This model is subject to several constraints. It assumes deterministic and continuous behavior, excluding discrete events or stochastic disturbances such as pandemics. Parameter values are considered time-invariant, though in practice they may evolve. Cultural, institutional, and geopolitical factors are abstracted for tractability. These simplifications imply that the model serves as a foundational tool rather than a predictive instrument.

8 Policy Implications

The stability analysis highlights critical levers for maintaining system equilibrium. Sustainable tourism management requires proactive investments in environmental protection and community engagement. Measures such as dynamic pricing, visitor quotas, and reinvestment in local amenities can help stabilize E and S, thereby securing long-term viability. These insights offer concrete guidance for policymakers aiming to harmonize tourism development with socio-environmental sustainability.

9 Conclusion

Lyapunov functions are a powerful tool to analyze the stability of nonlinear systems. A common choice for a Lyapunov function is its is positive definiteness, meaning $L(t) > 0$ for all states except at equilibrium, where $V = 0$ and the negative derivative, $\frac{dL(t)}{dt} < 0$.

The key idea is the construction of the function $L(t, P, S, Q, X, Y)$ that behaves like an energy function, decreasing over time for stable systems. In addition, if boundedness holds, the system is well-behaved and does not exhibit unbounded growth. The system models the nonlinear interactions between tourist influx (P), economic (Q), social (S), environmental (X), and cultural (Y) effects. Growth is limited by carrying capacities $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$. Our models in this context encourage the implementation of Sustainable Economic Growth, setting taxation and investment thresholds to avoid over-tourism. It strikes social balance to adjust visitor quotas to prevent local resentment. Environmental sustainability and Cultural preservation are also introduced to cushion heritage site regulations to limit mass tourism. Findings suggest that effective regulation of tourism flows and capital allocation is essential for maintaining a sustainable balance. Future work could extend this model by incorporating stochastic effects or empirical calibration using real-world data.

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