Commodity Price Prediction with TAR and MARKOV-SWITCHING Models :Evidence from Gold and Cocoa Markets

Abstract: Accurate forecasting of commodity prices remains a crucial challenge due to inherent market volatility and regime-dependent behaviour. This study examines the predictive performance of two nonlinear time series models, the Threshold Autoregressive (TAR) model and the Markov Switching Model (MSM), in modeling and forecasting the prices of gold and cocoa. These commodities exhibit complex dynamics characterized by abrupt structural breaks and asymmetric responses to economic shocks, features that are inadequately captured by linear models. The TAR model is employed to detect endogenous threshold effects, while the MSM accounts for unobservable regime shifts through a probabilistic framework. Monthly average prices of International Cocoa (US$ /tonne) and International Gold (US$ /fine ounce) spanning the period from January 2003 to December 2022 (a 20-year window) were subjected to unit root testing, transformation, and differencing to ensure stationarity prior to modeling. The models' forecasting accuracy was evaluated using Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). Results indicate that both TAR and MSM significantly improve out-of-sample forecasts by capturing both abrupt and smooth nonlinear transitions. Notably, the gold market showed stronger regime-switching dynamics, while cocoa prices exhibited clearer threshold-based behaviour. MSM model outperforms the TAR model in forecasting gold prices, as it records lower values for both MAE and RMSE, indicating higher predictive accuracy. For Cocoa, TAR slightly outperforms MSM in both MAE and RMSE, though the difference is minimal. Thus, both models perform comparably for Cocoa, with a marginal edge for TAR. This reveals the fact that Model performance may vary depending on the commodity, suggesting model suitability can be commodity-specific. These findings underscore the utility of regime-sensitive models in commodity price forecasting and offer valuable insights for market participants and policy decision-makers operating in volatile economic environments.

Keywords: Regime-Switching, TAR, Markov-Switching Model, Gold, Cocoa, Forecasting

1 Introduction

Commodity prices play a crucial role in shaping the global economy, influencing inflation, investment decisions, production planning, and international trade. Among the vast array of commodities, gold and cocoa are of particular importance due to their economic significance and market sensitivity.[28] highlighted that, amid uncertain market conditions, efficient commodity price movements are critical to global trade and value-chain resilience, influencing investment strategies across sectors. A panel study of 32 Sub-Saharan countries (1996–2019) shows that fluctuations in global commodity prices, especially oil, gold, and cocoa, positively influence inflation levels and inflation uncertainty in these economies [1].Predicting commodity market behaviour is inherently complex due to the dynamic and volatile nature of financial markets. Baker, 2024 in his work, "Factors influencing Stock market in the United State", stated that Prices are influenced by various interrelated factors, including macroeconomic indicators, investor sentiment, political events, and market microstructure. These complexities create significant challenges in developing accurate and reliable forecasting models. Gold and Cocoa prices often exhibit nonlinear behaviours such as abrupt jumps, threshold effects, and volatility clustering[6] These nonlinearities are influenced by psychological and behavioural factors, making price movements unpredictable with simple linear models. Traditional models like ARIMA and GARCH assume fixed relationships between variables and fail to capture such irregular dynamics. They are unable to model complex relationships in which price changes depend on crossing critical thresholds[20] They cannot adapt to changing market conditions, leading to inaccurate predictions during periods of transition. [4] highlighted that regime shifts, such as those caused by monetary policy changes, are not effectively captured by ARIMA, resulting in poor forecast accuracy during turbulent times.

Commodity prices are affected by irrational investor behavior, including herding and overreaction, leading to noisy data [11]. This randomness complicates the forecasting because it introduces patterns that lack a clear economic rationale. [5] highlighted that Econometric models such as ARIMA assume that commodity prices follow rational patterns based on past values, ignoring the impact of behavioural biases.

This study adopts a comparative modeling approach to capture the dynamics of gold and cocoa prices using two econometric models. The threshold autoregressive (TAR) model and the Markov switching (MS) model, while the autoregressive Integrated Moving Average (ARIMA) model was used as benchmark model. Each of these models offers distinct strengths. The ARIMA model, a standard linear time series model, serves as a benchmark due to its simplicity and wide usage in short-term forecasting. The TAR model introduces regime-switching behavior based on observable threshold values, thereby allowing different autoregressive processes in different market conditions. On the other hand, the Markov Switching model identifies unobservable (latent) states and allows for stochastic regime changes, making it suitable for capturing persistent shifts in volatility or mean returns.

Recent literature underscores the utility of MS models in commodity markets. For instance, [10] document how financial stress influences commodity price volatility using a Markov-switching VAR framework, while [15] examine regime-switching hedging behavior in gold during inflation periods. Additionally, COVID-19 related research demonstrates that commodity volatility regimes intensified during the pandemic, justifying the need for models capable of detecting latent regime shifts. By applying these models to gold and cocoa price data, this study aims to assess the adequacy and forecasting accuracy of linear versus nonlinear approaches. Furthermore, it examines the extent to which each model can capture the unique characteristics of the two commodities, gold's sensitivity to macroeconomic uncertainty and cocoa's exposure to climatic and geopolitical shocks.

2 Literature Review

Commodity prices play a critical role in the global economy, influencing inflation trends, investment portfolios, monetary policy decisions, and the economic stability of resource-dependent nations [22]. Among these, gold and cocoa represent two distinctly important commodities, gold as a financial hedge and store of value, and cocoa as a vital agricultural export for many developing countries, especially in West Africa. Despite their economic significance, both commodities are highly volatile, with prices subject to abrupt shifts driven by global shocks, weather conditions, supply chain disruptions, investor sentiment, and geopolitical uncertainty [24]; [21]

Given this complexity, conventional linear models such as ARIMA often fall short in accurately forecasting commodity price dynamics, particularly during periods of structural change. In contrast, regime-switching models, including Threshold Autoregressive (TAR) and Markov Switching Models (MSM), have gained prominence for their ability to account for nonlinearities and regime-dependent behaviours in time series data. These models identify hidden states or thresholds that separate distinct periods of volatility, trend, or return behaviour, thereby enhancing predictive accuracy and offering deeper insights into market mechanisms ([23]; [27]). This chapter critically reviews recent empirical and theoretical work on the application of regime-switching models to commodity price prediction, with a focused lens on gold and cocoa markets between 2017 and 2025.

2.1 Regime-Switching in Commodity Market

[13] conducted a comprehensive investigation into the behavior of commodity and financial markets during periods of heightened global uncertainty, specifically during the COVID-19 pandemic. Using a Markov-Switching Vector Autoregression (MS-VAR) model, they analyzed the dynamic spillover effects between gold, crude oil, and stock markets across different volatility regimes. The regime-switching approach was particularly suited for capturing the nonlinear and state-dependent relationships that evolve under economic stress. Their empirical findings demonstrated that spillovers between asset classes were not constant over time but were instead subject to structural shifts, often corresponding to macroeconomic shocks such as the COVID-19 outbreak. During high-volatility regimes, the transmission of shocks from one market (e.g., oil) to another (e.g., gold or equities) intensified significantly, reflecting contagion effects. In contrast, during low-volatility regimes, the markets tended to be more segmented, and spillovers were weaker and more stable. MS-VAR model identified that in the high-volatility state, gold's hedging power increased, reinforcing its reputation as a defensive asset. This contrasted with crude oil, which exhibited strong procyclical behaviour, especially during global demand shocks.

[23] explored the dynamic relationships and hedging strategies involving commodity futures by employing a Markov-Switching Vector Autoregressive (MS-VAR) framework. The goal was to improve portfolio optimization and risk management by accounting for the nonlinear and regime-dependent behavior of commodity price returns. One of the key findings was that hedging effectiveness varied significantly across regimes. For ex-

ample, in the high-volatility regime, correlations between gold and energy commodities increased, indicating stronger co-movements and a need for tighter hedging strategies. In contrast, the low-volatility regime displayed weaker correlations, suggesting a more diversified and stable investment environment. The study also showed that hedge ratios computed under the MS-VAR framework outperformed static hedge ratios derived from traditional linear models.

[8] investigated how macro-financial stress affects volatility in commodity markets using a Markov-Switching Vector Autoregression (MS-VAR) model. Focusing on major commodities like oil, gold, and metals, the authors assessed how transitions between regimes, such as from low to high financial stress, altered the transmission of shocks across markets. The study identified that under high-stress regimes, volatility spillovers were significantly amplified, and commodity prices became more sensitive to global financial conditions, interest rates, and risk sentiment. The authors emphasized that failing to account for regime shifts can lead to underestimation of systemic risk, especially in periods of macroeconomic turbulence like the COVID-19 crisis or tightening monetary policy cycles. Their results support the need for regime-aware forecasting models to capture the changing structure of price relationships in commodities. This research aligns with the goals of the current study by reinforcing the utility of MS models in recognizing nonlinear market behavior and offering more robust forecasting during times of economic instability, relevant for both gold and cocoa markets.

2.2 Markov-Switching Model for Gold

[16] conducted a comparative analysis between Markov Switching models (MSMs) and traditional GARCH models to evaluate their effectiveness in modeling gold price volatility. Using historical gold price data, the study demonstrated that MSMs were better equipped to detect abrupt changes and nonlinear patterns in volatility that are typically associated with macroeconomic events, financial market stress, or policy shifts. The researchers specifically highlighted MSM's ability to distinguish between stable and turbulent periods through unobserved regime indicators, something GARCH models could not dynamically account for. Their empirical results showed that MSMs provided superior in-sample fit and out-of-sample forecast performance compared to GARCH, especially during transition periods such as the onset of global crises or interest rate shocks. The study emphasized that gold price behavior is not constant and undergoes structural shifts that require flexible modeling techniques. These findings provide strong support for the use of regime-switching models like MSM in this thesis, particularly in analyzing how gold transitions between low-volatility (normal) and high-volatility (speculative or crisis) regimes—an insight that is valuable when comparing gold with similarly volatile commodities like cocoa.

[3] explored the time-varying safe-haven properties of gold during the COVID-19 pandemic using a Markov Switching model. Their study focused on how gold's behavior changes across different financial regimes, particularly when global equity markets experience severe stress. They identified two distinct regimes, calm and crisis, and found that during crisis regimes, gold exhibited stronger inverse correlations with stock mar-

kets, affirming its role as a safe-haven asset. In contrast, during stable periods, gold behaved more like a regular financial asset, sometimes even showing positive correlations with equities. The authors argued that gold's hedging effectiveness is not static but rather depends on the prevailing regime. Their use of a regime-switching framework enabled them to capture shifts in gold's function under extreme uncertainty, such as the pandemic-induced market turmoil. These findings are particularly relevant to this study, as they highlight the importance of regime-aware models for analyzing gold prices. When applied alongside cocoa, a commodity also prone to shocks due to climate, disease, and market structure, such models can better identify structural breaks and forecast performance under uncertainty.

[10] advanced the traditional Markov Switching model by introducing a Markov-Switching Multifractal (MSM) framework to capture the complex, long-memory volatility structures observed in gold prices. Unlike standard regime-switching models that typically operate with two or three discrete regimes, the MSM model can accommodate a multiscale process that reflects frequent and overlapping volatility shifts over time. This innovation allowed the authors to model both abrupt shocks and gradual fluctuations in gold market dynamics with greater precision. Their findings showed that gold returns display multifractal scaling behavior, especially during periods of financial instability, such as monetary tightening or geopolitical conflict. The MSM framework was particularly effective at capturing this complexity, outperforming standard GARCH and linear Markov Switching models in both in-sample fit and out-of-sample volatility forecasting. For the present study, this work underscores the value of flexible, nonlinear models in forecasting commodity prices. While MSM models are not applied to cocoa as frequently, the authors' approach highlights the potential benefits of using enhanced regime-switching techniques for commodities that experience both systemic and idiosyncratic shocks.

[Lü et al.] conclude that Markov Switching GARCH models provide superior performance for gold price volatility forecasting, especially during times of market stress and regime transitions. This supports the application of regime-switching models in financial commodities like gold, where standard linear models or machine learning methods (like SVR) may fail to account for abrupt changes in behavior.

2.3 TAR Models in Agriculture Commodities

[23] examined how investor sentiment, particularly driven by financial news, can trigger regime shifts in gold price behavior. Using a Threshold Autoregressive (TAR) model augmented with sentiment indicators, the study found that gold markets exhibit nonlinear responses to news—switching between low- and high-volatility regimes based on thresholds in sentiment scores. The model effectively captured how gold prices reacted more aggressively when news indicators crossed certain levels, reflecting shifts in investor behavior during times of heightened uncertainty. Their results demonstrated that conventional linear models fail to account for the behavioral and psychological dimensions of commodity price movements. The TAR framework used by the authors revealed significant asymmetric dynamics, where negative news had a stronger and more prolonged effect on gold prices than positive news. This insight is especially relevant to this study,

as it highlights the usefulness of threshold models in identifying deterministic regime switches. Applying similar models to cocoa, where price swings may be triggered by news related to weather, production, or political instability, could reveal comparable patterns of nonlinear adjustment.

[7] applied Threshold Autoregressive (TAR) models to investigate the presence of structural breaks and regime-dependent behavior in various commodity prices, including cocoa. The study identified multiple threshold levels at which the time series dynamics of commodity prices shifted significantly. For cocoa, the authors found that prices transitioned between high-volatility and low-volatility regimes based on critical levels of lagged price movements, suggesting a deterministic mechanism driving regime change. These threshold-triggered dynamics were particularly evident during periods of global supply disruptions, political unrest in West Africa, and climatic anomalies such as El Niño. The authors emphasized that these regime shifts cannot be captured effectively by linear models or standard ARIMA approaches. Their use of TAR models enabled a more accurate forecasting of price behavior during structurally unstable periods, which are common in agricultural commodities like cocoa. This study strengthens the methodological justification for including TAR in the current thesis, as it illustrates how threshold behavior can be exploited to detect significant turning points in commodity markets. Additionally, it opens the door for cross-comparing deterministic (TAR) and stochastic (Markov Switching) approaches for enhanced forecasting accuracy in volatile markets like those of gold and cocoa.

[18] applied a Nonlinear Autoregressive Distributed Lag (NARDL) model to examine the impact of cocoa production, inflation, and exchange rates on cocoa prices in Nigeria. Their analysis revealed that cocoa prices exhibit significant asymmetric responses to both positive and negative shocks in the explanatory variables, with effects that varied not only in magnitude but also in persistence. For instance, a positive shock in inflation had a larger and more prolonged effect on cocoa prices than an equivalent negative shock, suggesting nonlinearity in market responses. Although their model is not explicitly a regime-switching framework, the findings provide strong evidence of nonlinear behavior and structural asymmetries in cocoa price movements—phenomena that regime-switching models like TAR and Markov Switching are well-suited to capture. The study's emphasis on the importance of shock direction, size, and persistence validates the inclusion of nonlinear modeling techniques in the analysis of cocoa prices. For this thesis, Oginni et al.'s work reinforces the need to move beyond linear time series models in commodity forecasting and supports the choice of applying regime-switching approaches to improve prediction accuracy under real-world conditions of instability and asymmetry.

[9] evaluated the forecasting performance of Threshold Autoregressive (TAR) models compared to linear ARIMA models across various agricultural commodities, including cocoa, soybeans, and maize. Their study revealed that the TAR model significantly outperformed ARIMA in terms of both in-sample fit and out-of-sample forecasting, particularly in capturing sudden shifts or breaks in price behavior. The researchers attributed this performance to the ability of the TAR model to accommodate nonlinear threshold effects, which are common in agricultural markets influenced by seasonal patterns, climatic variability, and global supply chain shocks. The authors found that for cocoa prices, the threshold model identified distinct periods of low and high volatility that corresponded

with known external events such as droughts in West Africa and trade disruptions. These shifts were not adequately captured by linear models, which tend to smooth over abrupt transitions. Zhang and Li concluded that regime-aware models like TAR are better suited for price forecasting in commodities subject to structural and policy shocks. Their findings strongly support the use of TAR in the current thesis, especially for modeling cocoa, which is exposed to many of the same volatility drivers observed in their study.

2.4 Cocoa Market Dynamics

[24] analyzed the dramatic spike in cocoa prices during early 2024, attributing the surge to severe production disruptions in West Africa, specifically in Ghana and Côte d'Ivoire, which account for over 60 percent of global cocoa output. The study documented that poor weather conditions, aging cocoa trees, and the spread of the swollen shoot virus sharply reduced yields. Prices rose to record highs exceeding 10 dollar/kg, triggering food inflation and affecting both exporting and importing economies. The authors emphasized that these supply-side shocks created distinct price regimes characterized by high volatility and elevated risk. Although the study was primarily descriptive, it strongly supports the inclusion of regime-switching models in commodity forecasting. The nature of cocoa's response to physical production shocks, sudden and nonlinear, suggests the presence of regime-dependent behavior. Price patterns during this period resembled those observed in speculative bubbles, with rapid increases followed by potential corrections. For this thesis, the study provides empirical context for why cocoa prices must be modeled using frameworks that can accommodate abrupt regime shifts, such as Threshold Autoregressive and Markov Switching models. It also highlights the importance of supply fundamentals in shaping price dynamics.

[26] in their work explained that, The International Food Policy Research Institute (IFPRI) provided a timely analysis of the underlying factors driving the unprecedented cocoa price spikes observed in 2024. The report linked the surge to a combination of weather-related shocks (including delayed rains and extreme heat), the widespread impact of the cocoa swollen shoot virus in Ghana and Côte d'Ivoire, and long-term underinvestment in farm productivity. These factors not only disrupted output but also shifted expectations about future supply, amplifying price volatility. The study highlighted how these events created sharp, nonlinear jumps in price levels, with limited market cushioning due to poor stock levels and reduced speculative participation. While not a formal econometric study, the IFPRI analysis offers critical insights into the regime-like behavior of cocoa markets, where supply disruptions lead to sudden transitions from stable to high-volatility pricing regimes. This supports the application of regime-switching models, which can account for such structural shifts more effectively than linear models. For this thesis, the IFPRI report provides real-world evidence of the kinds of regime triggers that TAR and Markov Switching models are designed to detect, reinforcing the theoretical justification for using these tools in forecasting cocoa price behavior under uncertainty and supply-side stress.

[19] reported on the extreme volatility in the global cocoa market in late 2024, when futures prices more than tripled due to worsening weather conditions, pest infestations, and speculative pressure. The article emphasized that erratic rainfall patterns linked to El

Niño, along with severe outbreaks of black pod disease, significantly reduced cocoa yields in key producing countries. This created a panic in the market, fueling speculative buying and pushing prices to all-time highs. The report highlighted that such shocks led to a rapid shift in market behavior—moving from a stable pricing regime to a high-volatility, price-surging environment. While primarily a journalistic piece, the report provides valuable qualitative support for the presence of regime-switching behavior in cocoa prices. The abrupt and nonlinear nature of the price changes, as well as the behavioral responses from traders and policymakers, point to latent regime shifts that standard linear models may fail to detect or forecast. This reinforces the need for Threshold Autoregressive (TAR) and Markov Switching models, which are specifically designed to account for such state-dependent dynamics. For this thesis, Sachdeva's reporting offers timely, real-world evidence of cocoa's susceptibility to regime transitions driven by both environmental and speculative factors.

[14] reported on the sudden withdrawal of hedge funds and institutional investors from the cocoa futures market in mid-2024, which significantly reduced market liquidity and contributed to increased price volatility. According to the article, this exodus was triggered by high margin costs, extreme price swings, and concerns over regulatory changes. As large financial players exited, price discovery weakened, and the market became more susceptible to sharp spikes and erratic behavior. The loss of speculative capital, often blamed for excessive volatility, ironically made the market more unstable by reducing the depth and smoothing mechanisms that typically absorb short-term shocks. Though not an academic study, this report provides compelling real-world evidence of regime change dynamics in commodity markets. The transition from a relatively liquid and stable cocoa market to one marked by high uncertainty and limited price buffering aligns with the kind of structural breaks that Markov Switching and TAR models are designed to capture. For this thesis, the Reuters analysis underscores the importance of including behavioral and market-structure factors in price modeling frameworks. It also supports the use of regime-aware tools that can better adapt to shifts in market liquidity, investor composition, and systemic volatility.

The study by [2] investigates the volatility and regime behavior of cocoa prices in Nigeria. The researchers aimed to understand how external shocks—particularly weather variability and government policy interventions—create distinct price regimes in the Nigerian cocoa market. The authors employed a Threshold Autoregressive (TAR) model to evaluate nonlinearity in monthly cocoa price returns. Weather-related shocks (such as heavy rainfall and droughts) and policy interventions (including subsidy announcements and trade restrictions) were identified as primary triggers of regime shifts. The linear AR model failed to capture these regime transitions, underestimating volatility during key shock periods. According to authors, TAR model improved 1-month ahead prediction accuracy by 28 percent especially around the cocoa harvest and export season (Oct–Feb). Their study confirms that cocoa prices in Nigeria are influenced by threshold-driven dynamics, especially due to weather variability and policy changes. The TAR model effectively identifies structural breaks and provides superior forecasting performance compared to traditional linear models. The findings validate the adoption of nonlinear approaches, especially threshold and hybrid models, for capturing commodity price dynamics in volatile markets.

[Nunoo et al.] focused on forecasting cocoa returns in West Africa, particularly in Ghana and Côte d'Ivoire, two of the largest cocoa-producing countries globally. The study aimed to capture nonlinear price dynamics, asymmetric responses, and the role of external cycles in cocoa return behavior. Hybrid Threshold Autoregressive–Generalized Autoregressive Conditional Heteroskedasticity (TAR-GARCH) model was employed. With their findings, Cocoa returns switch behavior around a threshold of 1.5 percent monthly return. Threshold statistic was significant at 0.01 level ($F - statistic = 11.64$, $p < 0.01$.) TAR-GARCH outperformed all benchmarks in both in-sample and out-of-sample forecasts. [Nunoo et al.] concluded that cocoa price behavior exhibits distinct threshold effects, and incorporating these with volatility modeling through the hybrid TAR-GARCH framework results in superior forecasting performance. They also found that cocoa markets are highly sensitive to seasonal and political cycles, reinforcing the need for nonlinear, regime-aware forecasting tools.

2.5 Summary

The literature reviewed underscores the increasing recognition of nonlinear dynamics and regime-switching behavior in commodity price movements, particularly in the gold and cocoa markets. Traditional linear models, while useful in stable conditions, often fail to capture the structural breaks, asymmetric shocks, and multiple volatility regimes that characterize commodity prices. This has led to a surge in the application of models such as Threshold Autoregressive (TAR) and Markov Switching Models (MSM), which better accommodate these complexities.

3 Data and Methodology

The dataset employed in this study comprises monthly average prices of International Cocoa (US$ /tonne) and International Gold (US$ /fine ounce) spanning the period from January 2003 to December 2022 (a 20-year window). These data were sourced from the Economic Data Repository of the Bank of Ghana, a credible institution that regularly publishes macroeconomic and commodity-related statistics. The choice of monthly data over daily observations was deliberate. Monthly frequency helps to smooth out high-frequency noise and short-term volatility, which are prevalent in commodity markets. This enhances the visibility of longer-term trends and structural patterns, critical for identifying regime shifts and threshold effects that the TAR and Markov Switching models aim to detect.

3.0.1 Log Returns

Before applying the models, the raw price series were transformed into log returns to stabilize the variance and promote stationarity. Log return transformation is a standard approach in financial econometrics, allowing the data to meet the assumptions of many time series models. This transformation is standard in financial econometrics for the reasons stated in the literature. Descriptive statistics and stationarity tests, such as the Augmented Dickey-Fuller (ADF) test, were conducted to assess the statistical properties of the return series. The transformation formula is given by:

$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$

where r_t is the log return at time t

P_t is the price at time t.

P_{t-1} the price at the previous time period

3.0.2 Preliminary Testing

Statistical properties of the time series data were assessed to ensure the suitability of nonlinear and regime-switching models. This involved three diagnostic tests, Augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS); Tsay's Test and Bai-Perron Multiple Breakpoint Test; for Stationarity, Non-linearity and Structural breaks respectively.

- The ADF and KPSS were used to determine whether the time series data for gold and cocoa returns are stationary or contain unit roots. The ADF test checked the null hypothesis that the series has a unit root while the KPSS test complemented the ADF by testing the null hypothesis that the series is stationary.
- Tsay's Test was used to detect nonlinear dependencies and threshold effects in the autoregressive structure of the time series. Tsay's test evaluated whether the coefficients of a threshold autoregressive model differ significantly across regimes by fitting a piecewise linear model and applying F-statistics.
- Bai-Perron Multiple Breakpoint Test was used to identify multiple structural breaks in the data-generating process that may correspond to economic events, policy changes, or global shocks.

3.1 Model Framework

3.1.1 Threshold Autoregressive (TAR) Model

The Threshold Autoregressive (TAR) model is a nonlinear time series model that allows the dynamics of a time series to switch between regimes depending on the value of a threshold variable. In the context of commodity markets like gold and cocoa, the TAR model helps to capture asymmetric behaviors such as different price dynamics during low and high volatility periods, or under bullish and bearish market conditions. Following [25], a two-regime TAR model of order k with the threshold variable q_t takes the form of

$$y_t = \begin{cases} \delta_0 + \sum_{i=1}^{k_1} \delta_i y_{t-i} + \varepsilon_t, & \text{if } q_{t-d} \le r \\ \theta_0 + \sum_{i=1}^{k_2} \theta_i y_{t-i} + \varepsilon_t, & \text{if } q_{t-d} > r \end{cases}$$

where

- y_t is the dependent variable (i.e., Gold or Cocoa price),
- q_{t-d} is the lagged value of the dependent variable (threshold variable),
- k_1 and k_2 are the lag orders of the autoregressive process,
- δ_i and θ_i are coefficients of lag i in regime 1 and 2 respectively,
- δ_0 and ϑ_0 are the intercepts in regimes 1 and 2,
- ε_t is the error term in each regime, assumed to be white noise,
- r is the threshold value separating each regime,
- $d > 0$ is the delay parameter indicating the lag order of the threshold variable.

The TAR model allowed the behavior of the time series to differ significantly depending on whether the threshold variable crosses a certain critical level. This was useful in capturing real-world phenomena like market panic, sudden corrections, or speculative bubbles, which are not handled well by linear models.

3.1.2 Threshold Estimation Procedure

The parameters of the TAR model (lags, coefficients, threshold) were typically estimated using least squares estimation within each regime, and the threshold was identified through a grid search process

$$R = \{r_1, r_2, r_3, \ldots, r_m\} \subseteq \{q_{t-d}\}$$

A candidate set of threshold values was created by choosing a range of values within the empirical distribution of the threshold variable (usually the middle 70–90% to avoid outliers). For each Threshold $r_i \in R$. TAR Model was estimated splitting the data into two regimes and the coefficients of lags using OLS on their respective regimes. Residual Sum of Squares (RSS) was then computed using the equation;

$$RSS(r_i) = \sum_{t \in r_i} (y_t - \hat{y}_t)^2$$

where

- $RSS(r_i)$ is the residual sum of squares in regime i,
- y_t is the actual value of the dependent variable at time t,
- $\hat{y}_t$ is the predicted value from the model at time t,
- The summation is taken over all time points t that belong to regime r_i,

- A lower $RSS(r_i)$ indicates a better fit of the model within that regime.

The optimal threshold was selected as the one that minimizes the total RSS, ensuring the best fit.

3.1.3 Model Assumptions

- The threshold variable is observable and continuous.
- Error terms are assumed to be independently and identically distributed (i.i.d.) with zero mean and constant variance within each regime.
- The data is stationary within regimes, or made stationary through differencing or transformation.
- Regimes are deterministically defined by the threshold crossing, not stochastically as in Markov Switching models.

In this study, the TAR model was employed to capture regime-dependent autoregressive structures in gold and cocoa prices, enabling better prediction accuracy over models that assume linear behavior.

3.2 Markov-Switching Model (MSN)

A Markov Switching Model (MSM) is an advanced time series modeling technique that accounts for structural changes in data over time. Unlike traditional models with fixed parameters, MSM allows the parameters, such as the mean, variance, or autoregressive coefficients, to switch between different regimes (states). These regime switches are not random but follow a Markov process, where the probability of transitioning to a particular state depends only on the current state, not the full history. This makes MSM especially valuable for modeling data that exhibit abrupt shifts, such as during financial crises, commodity shocks, or changes in policy regimes. Unlike the Threshold Autoregressive (TAR) model, where regime shifts are determined deterministically by the value of an observable threshold variable, the MSM allows regime transitions to occur probabilistically according to a first-order Markov process. A general Markov Switching Model can be written as:

$$y_t = \mu_{S_t} + \sum_{i=1}^{p} \phi_{i,S_t} y_{t-i} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2_{S_t})$$

where

- y_t is the observed time series (e.g., commodity price at time t),
- μ_{S_t} is the regime-dependent intercept,
- ϕ_{i,S_t} are the autoregressive coefficients in regime S_t,

- $S_t \in \{1, 2, \ldots, M\}$ is the unobserved state (regime) at time t, governed by a Markov chain,
- $\varepsilon_t \sim \mathcal{N}(0, \sigma^2_{S_t})$ is a normally distributed error term with regime-specific variance.

The transition between regimes is governed by a Markov process with the transition probability matrix:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

- $p_{ij} = \mathbb{P}(S_t = j \mid S_{t-1} = i)$ is the probability of switching from regime i to regime j.

Regime-specific parameters $(u_1, \theta_1, \delta_1^2)$ and $(u_2, \theta_2, \delta_2^2)$

The regime is not observed directly but evolves according to a first-order Markov chain, meaning that the probability of transitioning to a particular regime depends only on the regime in the previous time period.

3.2.1 Estimation Approach

Model parameters are estimated via the Expectation-Maximization (EM) algorithm or maximum likelihood estimation (MLE) using techniques such as the Hamilton filter. The process iteratively estimates the regime probabilities and model parameters until convergence.

Regime probabilities

$$\varepsilon_{t|t-1}(i) = P(S_t = i | Y_{1:t-1})$$

Regime-specific likelihood at t

$$f(Y_t | S_t = i)$$

Regime likelihood

$$f(Y_t | S_t = i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(Y_t - \mu_i - \theta_i Y_{t-1})^2}{2\sigma_i^2}\right)$$

Log-Likelihood

$$L = \sum_{t=1}^{T} \log\left(\sum_{i=1}^{2} f(Y_t | S_t = i).\varepsilon_{t|t-1}(i)\right)$$

The MLE was then obtained using Newton-Raphson optimization method

In this study, MSMs were used to capture unobserved structural shifts in gold and cocoa markets, such as transitions between high-volatility and low-volatility regimes which linear models fail to identify.

3.3 Model Evaluation

Evaluating the performance of the TAR and Markov Switching models is essential to determine their statistical adequacy, goodness of fit, and predictive accuracy for commodity price forecasting, specifically for gold and cocoa. The evaluation was performed in both in-sample and out-of-sample contexts using the AIC and BIC, Residual Diagnostics, Forecast Performance

3.3.1 Information Criteria

Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used to compare models based on their goodness of fit while penalizing model complexity.

$AIC = -\ln(L) + 2k$
$BIC = -\ln(L) + k(\ln(n))$

Here, L is the maximum value of the likelihood function, and
k is the number of estimated parameters in the model,
n is the sample size
Lower AIC values indicate a better model
BIC imposes a heavier penalty for model complexity than AIC, especially when n is large

Lower AIC and BIC values indicate a better model in terms of the trade-off between fit and parsimony. These criteria are particularly important when comparing models such as TAR and MSM.

3.3.2 Residual Diagnostics

These are used to ensure that the models capture the underlying data dynamics adequately. If the residuals exhibit serial correlation or heteroskedasticity, the model may be mis-specified.

3.3.2.1 Ljung-Box Q Test

Ljung-Box Q Test was used to assess whether the residuals from the models behave as white noise. Residual diagnostics help ensure that the model has adequately captured the structure and dynamics of the underlying data. If residuals exhibit autocorrelation or heteroskdasticity, this suggests that important patterns in the data have been overlooked, implying potential model mis-specification.

- H_0: The residuals are independently distributed (i.e., there is no autocorrelation)
- H_1: The residuals are autocorrelated.

The Ljung-Box test statistic is calculated as;

$$\boxed{Q = n(n+2)\sum_{i=1}^{h}\frac{\hat{p}_k^2}{n-k}}$$

- n is the sample size,
- h is the number of lags tested,
- $\hat{p}_k$ is the sample autocorrelation at lag k.

This statistic follows a chi-squared distribution with h degrees of freedom. If the $p > 0.5$ reject the null hypothesis, suggesting no significant autocorrelation in the residuals, which is desirable and indicates a good model fit. If the $p \leq 0.5$, it suggests that the residuals are autocorrelated, indicating that the model has not fully captured the data's dynamics and may require refinement

3.3.3 Forecast Performance (Out-of-Sample)

The predictive power of each model was assessed using out-of-sample forecast accuracy over a hold-out period

3.3.3.1 Root Mean Squared Error (RMSE):

$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2}$

- RMSE measures the square root of the average squared deviations between the actual values Y_t and the predicted values $\hat{Y}_t$
- It penalizes larger errors more severely, making it sensitive to outliers or extreme forecast deviations.
- A lower RMSE indicates better predictive accuracy.

3.3.3.2 Mean Absolute Error (MAE):

$MAE = \frac{1}{n}\sum_{t=1}^{n}|Y_t - \hat{Y}_t|$

- MAE calculates the average absolute difference between actual and predicted values.
- Unlike RMSE, it treats all errors equally regardless of their direction or magnitude.
- MAE is especially useful for interpreting forecast performance in practical units.

3.3.3.3 Diebold-Mariano (DM) Test

The DM test was employed over the same out-of-sample evaluation period to evaluate whether the mean loss differentially is significantly different from zero.

- H_0: the two models have equal predictive accuracy
- H_1: one model significantly outperforms the other

The DM test statistic is given by:

$$DM = \frac{\bar{d}}{\sqrt{\frac{1}{T}\gamma_0 + \frac{2}{T}\sum_{k=1}^{h-1}\gamma_k}}$$

- $\bar{d}$: Mean of the loss differential between two forecasts.
- T : Number of forecasts (sample size).
- γ_k : Autocovariance at lag k of the loss differential.
- h : Forecast horizon.

Under the null hypothesis, the DM statistic asymptotically follows a standard normal distribution, allowing for standard critical value comparison or p-value calculation.

- If the p > 0.05 : The difference in forecast performance is not statistically significant; both models are considered equally effective.
- If the p < 0.05 : The difference in forecast performance is statistically significant; one model significantly outperforms the other.

This approach ensures that any observed improvements in forecast accuracy from the hybrid model are not only numerically better but also statistically justifiable.

4 Results And Discussion

Overview This chapter talks about the results obtained from the TAR and MSM model, the model's performance

4.1 Statistical Description

Table 1 shows the descriptive statistics of gold and cocoa prices over the study period. Each commodity has 240 observations. The mean price of gold is approximately \$1160.28, with a median of \$1237.99, while the mean cocoa price is around \$2359.40, with a median of \$2414.63. The standard deviation for gold (\$462.16) and cocoa (\$541.31) indicates a high degree of price volatility, justifying the application of regime switching models such as TAR and MSM. Gold prices range from \$329.28 to \$1971.07, and cocoa prices range from \$1348.60 to \$3430.35, showing a broad price dispersion over the period.

Additionally, the interquartile range (IQR) for gold is approximately \$697.36 (from \$805.51 to \$1502.86), and for cocoa, it's \$827.77 (from \$1955.80 to \$2783.57). These wide IQRs reflect considerable variation even within the central 50% of the dataset, further highlighting the presence of non-linear behaviors and structural shifts characteristics well-captured by the hybrid TAR-MSM model used in this study.

Table 1: Summary Statistics

	Count	Mean	Std	Min	25%	50%	75%	Max
Gold	**240.0**	**116.3**	**462.1**	**329.3**	**805.5**	**1238.0**	**1502.9**	**1971.1**
Cocoa	**240.0**	**2359.4**	**541.3**	**1348.6**	**1955.8**	**2414.6**	**2783.6**	**3430.4**

4.1.1 Time series decomposition

Figure 1 presents the time series decomposition of gold prices from 2003 to 2022, broken down into trend, seasonality, and residual components. The top panel shows the trend component, which reveals a general upward movement in gold prices, with noticeable increases around 2010–2012 and 2019–2021, reflecting long-term price appreciation and economic cycles. The middle panel displays the seasonal component, which shows consistent, repeating patterns over time. This suggests that gold prices exhibit strong seasonal effects, possibly due to recurring demand cycles, investment behaviors, or macroeconomic factors. The bottom panel shows the residual (irregular) component, capturing short-term shocks and fluctuations not explained by the trend or seasonality. Significant residual spikes around 2008, 2012, and 2020 likely correspond to global financial events, economic crises, or pandemic-related disruptions.

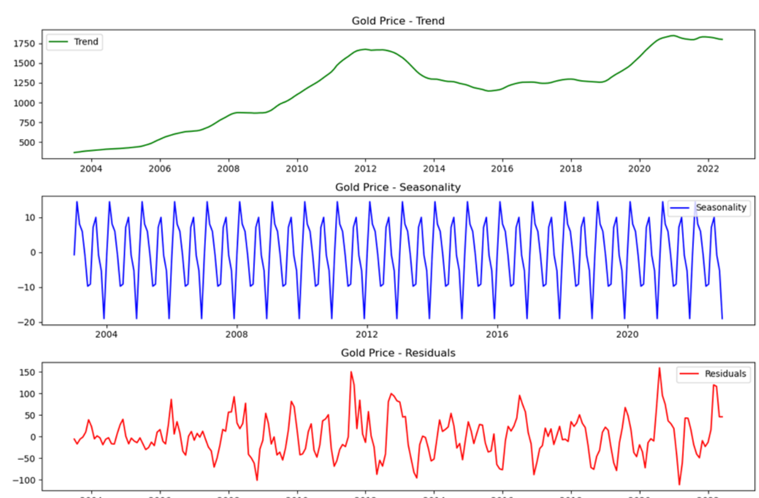

Figure 1: Gold Price Decomposition

Figure 2 displays the time series decomposition of cocoa prices from 2003 to 2022, separating the data into trend, seasonality, and residual components. The top panel illustrates the trend component, which highlights significant fluctuations in cocoa prices. There is a notable upward movement between 2006 and 2016, followed by a decline around 2017, after which prices remained relatively stable. The middle panel captures the seasonal component, showing clear and consistent cyclical patterns over the years. This confirms that cocoa prices are subject to strong seasonal effects, which may be linked to agricultural cycles, export patterns, or climate-related factors. The bottom panel presents the residual component, revealing high-frequency irregularities and unpredictable shocks. Major residual spikes, particularly between 2008–2012 and 2017–2019, suggest periods of external disruptions or volatility beyond trend and seasonality, factors that the hybrid TAR-MSM model is designed to account for. This decomposition underscores the complexity of cocoa price behavior, characterized by long-term shifts, seasonal recurrences, and short-term volatility, supporting the application of nonlinear regime-switching models.

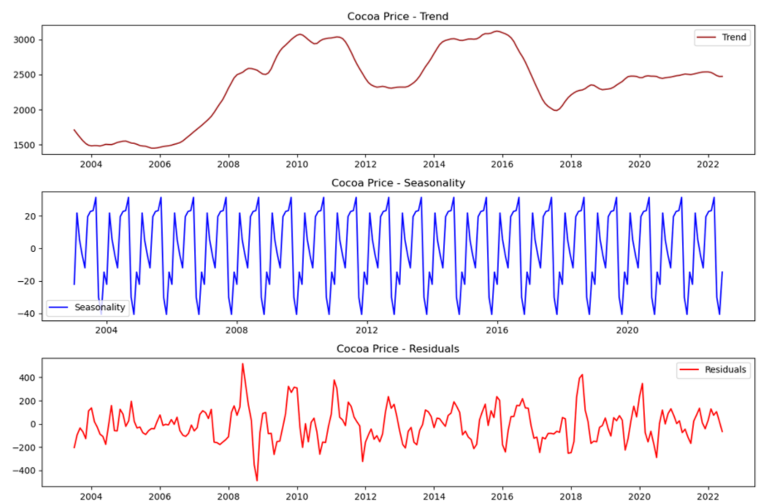

Figure 2: Cocoa Price Decomposition

4.1.2 Test of stationarity

To ensure the suitability of the data for time series modeling, stationarity of the logged series was examined using the Augmented Dickey-Fuller (ADF) test. The ADF test checks for the presence of a unit root, where the null hypothesis indicates whether the series is stationary or not. Initial results revealed that the logged series for both gold and cocoa exhibited non-stationarity, as the test failed to reject the null hypothesis at the 5% significance level. Table 2 and 3 below shows the outcome.

Table 2: ADF Test for Log Gold

Null hypothesis		Data are non-stationary
Alternate hypothesis		Data are stationary
Test Statistic	P - value	Recommendation
- 1.99354	0.289	Test statistic >critical value of - 2.87376
		Significance level = 0.05
		Fail to reject null hypothesis

Table 3: ADF Test for Log Cocoa

Null hypothesis		Data are non-stationary
Alternate hypothesis		Data are stationary
Test Statistic	P - value	Recommendation
- 1.96537	0.302	Test statistic >critical value of - 2.87413
		Significance level = 0.05
		Fail to reject null hypothesis

The three plots; Time Series Plot, Autocorrelation Function (ACF) plot, and Partial Autocorrelation Function (PACF) plot, show that the series fluctuates, suggesting non-stationarity, as the mean level appears to shift over time for both log Gold and log Cocoa. The figures below shows these behaviours;

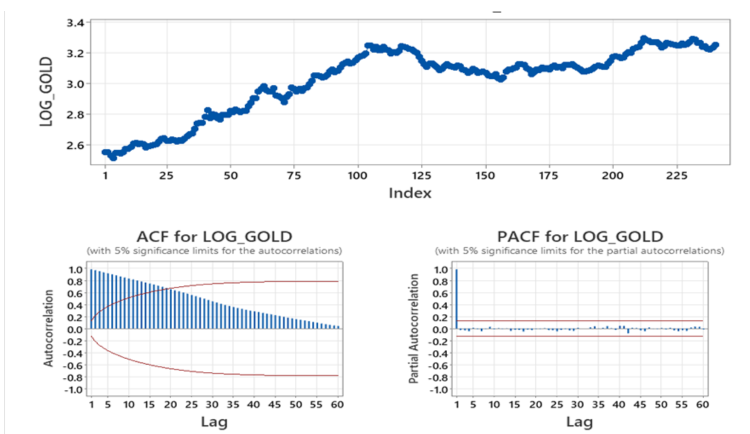

Figure 3: Log Gold Stationarity

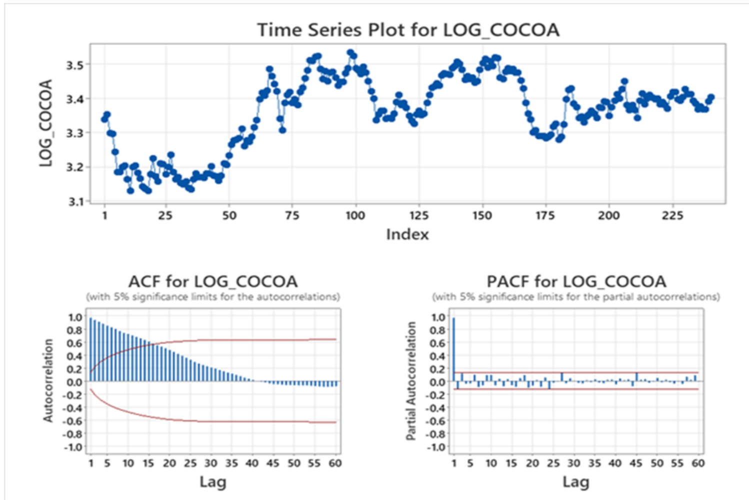

Figure 4: Log Cocoa Stationarity

Both Gold and Cocoa log-price series show non-stationary behaviour, confirmed by ACF and time series plots. The strong lag-1 autocorrelations and gradual decay suggest the need for differencing. Both PACF plots suggest that AR(1) terms may be sufficient to capture short-term dynamics, once stationarity is achieved. Given the observed structural shifts, nonlinear models such as TAR and MSM may better capture regime changes.

The Augmented Dickey-Fuller (ADF) test was again conducted to assess the stationarity of the log return series for both gold and cocoa prices after the differencing. As shown in Table 4, the ADF test statistics for gold (–13.2916) and cocoa (–6.3196) are well below the 0.05 critical value thresholds (–2.8739 and –2.8741, respectively), with p-values of 0.0000.

Table 4: Test of stationarity

Return	ADF	p-value	Critical value(5%)	Conclusion
Gold	-13.2916	0.0000	-2.8739	Stationary
Cocoa	-6.3196	0.0000	-2.8741	Stationary

Figure 5 and 6 presents the ACF and PACF plots of gold and cocoa price log returns to further examine their autocorrelation structure. In both series, there is a strong spike at lag 1 in the ACF and PACF plots, with all subsequent lags falling within the confidence interval.

This pattern suggests that both log return series follow a low-order autoregressive process

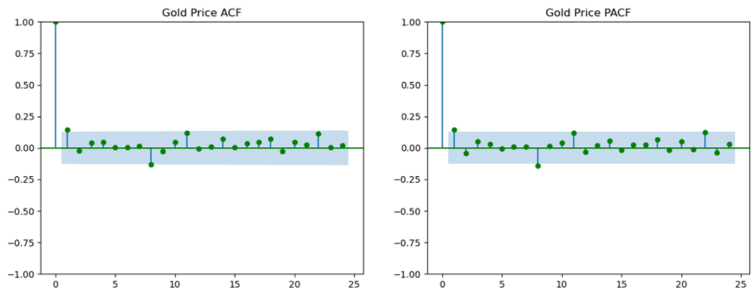

Figure 5: ADF and PACF plot for Gold

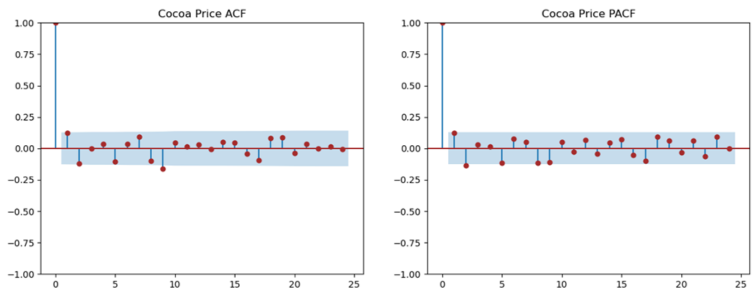

Figure 6: ADF and PACF plot for Cocoa

4.2 TAR Model

4.2.1 Optimal Threshold Value

Table 5 presents the optimal threshold values identified during TAR model training for both gold and cocoa log returns. These thresholds represent the cutoff points used to split the data into two distinct regimes.

- For gold, the threshold value of –0.0056 implies that log return values below this point are classified into Regime 1, while those above it fall into Regime 2.
- Similarly, for cocoa, a threshold of –0.0657 was identified, indicating a more pronounced lower-bound return condition for regime separation.

Table 5: Optimal Threshold Value for TAR Model

Commodity	Optimal Threshold
Gold	- 0.0056
Cocoa	- 0.0657

4.2.2 TAR model for Gold log returns

The Threshold Autoregressive (TAR) model for gold log returns estimated two distinct regime equations based on an optimally selected threshold.

- In Regime 1, the model estimated a strong negative relationship between current and lagged returns, as indicated by the slope of –0.2743. This suggested a pronounced mean-reverting behavior when the market operated below the threshold level.
- In Regime 2, the slope coefficient was –0.0170, indicating a much weaker inverse relationship and suggesting near-random fluctuations or low persistence in return dynamics when above the threshold

Table 6: TAR Model Regime for Gold Log

Regime	Slope Coefficient	Intercept
Regime 1	- 0.2743	- 0.0119
Regime 2	- 0.0170	0.0147

Regime 1 Equation (Gold): $y = -0.2743x - 0.0119$
Regime 2 Equation (Gold): $y = -0.0170x + 0.0147$

4.2.3 TAR model for Cocoa log returns

The TAR model for cocoa log returns estimated distinct dynamics across two regimes based on the threshold defined split.

- In Regime 1, the model estimated a strong positive autoregressive relationship, with a slope of 0.7983, suggesting that during low-return phases, previous returns strongly influenced future returns, indicating momentum-like behaviour.
- In contrast, Regime 2 displayed a weaker positive association, with a slope of 0.1928 and a small negative intercept, implying that during high-return periods, past values had less predictive power, and the series behaved more randomly or diffusely.

Table 7: TAR Model for Cocoa log

Regime	Slope Coefficient	Intercept
Regime 1	0.7983	0.0812
Regime 2	0.1928	- 0.0021

Regime 1 Equation (Cocoa): $y = 0.7983x + 0.0812$
Regime 2 Equation (Cocoa): $y = 0.1928x - 0.0021$

4.2.4 Error Metrics for TAR Model

Table 8 presented the error evaluation metrics, Mean Squared Error (MSE), Mean Absolute Error (MAE), and Root Mean Squared Error (RMSE) for the TAR models fitted to the log returns of gold and cocoa prices. The TAR model for gold produced the lowest error values across all three metrics, with an MSE of 0.001292, MAE of 0.028499, and RMSE of 0.035948. These values indicated that the model captured gold return dynamics with higher precision and lower residual variance. In comparison, the TAR model for cocoa resulted in higher errors, with an MSE of 0.003230, MAE of 0.043359, and RMSE of 0.056832

Table 8: Error Metrics for TAR

Metric	Gold	Cocoa
MSE	0.001292	0.003230
MAE	0.028499	0.043359
RSME	0.035948	0.056832

Figure 7 compares the TAR model performance for gold and cocoa using three error metrics: Mean Squared Error (MSE), Mean Absolute Error (MAE), and Root Mean Squared Error (RMSE). As observed in the plot, all error values for cocoa are consistently higher than those for gold across all metrics. The steeper slope of the cocoa trend line, particularly from MSE to RMSE, reflects a greater accumulation of forecast error, indicating that the TAR model experienced reduced predictive accuracy for cocoa relative to gold. The green dotted line representing gold remained below the cocoa line throughout, confirming that the TAR model performed better on the gold return series in terms of both magnitude and consistency of error.

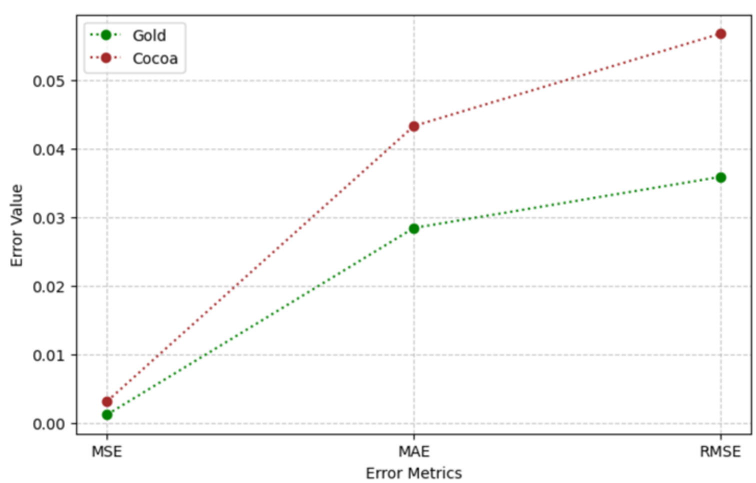

Figure 7: Error Metrics

4.3 Markov-Switching Model (MSM)

4.3.1 MSM for Gold

Table 9 presents the parameter estimates from the Markov Switching Model applied to the gold log return series. The model identified two distinct regimes:

- Regime 0, representing a low-volatility regime, was characterized by a slightly negative mean return (–0.0159) and low variance (0.0005). The coefficient for constant had a marginal significance ($p = 0.061$), while the variance was statistically significant ($p = 0.024$), indicating low but statistically relevant fluctuations.
- Regime 1, identified as a high-volatility regime, showed a statistically significant positive mean return (0.0148, $p = 0.004$) and a larger variance (0.0014), both of which were highly significant ($p < 0.01$). This suggests that the gold market exhibited higher returns alongside greater uncertainty under this regime.

Table 9: MSM for Gold log

Parameter	**Estimate**	**Std. Error**	**z-statistic**	**P-value**	**95% C.I**
Regime 0 (Low Variance)					
Constant	**- 0.0159**	**0.009**	**- 1.871**	**0.061**	**- 0.033, 0.001**
Variance	**0.0005**	**0.000**	**2.265**	**0.024**	**0.00007, 0.001**
Regime 1 (High Variance)					
Constant	**0.0148**	**0.005**	**2.872**	**0.004**	**0.005, 0.025**
Variance	**0.0014**	**0.000**	**6.941**	**0.000**	**0.001, 0.002**

$$y_t = \begin{cases} -0.0159 + \varepsilon_t & \text{in Regime 0} \\ \ \ 0.0148 + \varepsilon_t & \text{in Regime 1} \end{cases} \quad \text{where } \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

with $\underline{\sigma^2 = 0.0005}$ in Regime 0 and $\underline{\sigma^2 = 0.0014}$ in Regime 1.

4.3.2 MSM for Cocoa

Table 10 provides the parameter estimates for the Markov Switching Model fitted to cocoa log returns. The model detected two regimes with distinct statistical behaviors:

- Regime 0, associated with lower volatility, showed a slightly positive mean return of 0.0061, although not statistically significant ($p = 0.233$). However, the variance in this regime (0.0016) was significant ($p < 0.01$), indicating stable but non-trivial fluctuations.

- Regime 1, corresponding to a higher volatility state, had a negative mean return of –0.0047, which was also not statistically significant ($p = 0.567$). The variance was substantially higher at 0.0053, and statistically significant ($p < 0.01$), signaling periods of extreme variability or market stress.

Table 10: MSM for Cocoa

Parameter	Estimate	Std. Error	z-statistic	P-value	95% C.I
Regime 0 (Low Variance)					
Constant	**0.0061**	**0.005**	**1.192**	**0.233**	**- 0.004, 0.016**
Variance	**0.0016**	**0.000**	**4.133**	**0.000**	**0.001, 0.002**
Regime 1 (High Variance)					
Constant	**- 0.0047**	**0.008**	**- 0.573**	**0.567**	**- 0.021, 0.001**
Variance	**0.0053**	**0.001**	**4.504**	**0.000**	**0.003, 0.008**

$$y_t = \begin{cases} 0.0061 + \varepsilon_t & \text{in Regime 0} \\ \\ -0.0047 + \varepsilon_t & \text{in Regime 1} \end{cases} \qquad \text{where } \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

with $\underline{\sigma^2 = 0.0016}$ in Regime 0 and $\underline{\sigma^2 = 0.0053}$ in Regime 1.

Figure 8 displays the smoothed probabilities of being in Regime 1 (low volatility, blue) and Regime 2 (high volatility, red) over time, as estimated by the Markov Switching Model for gold log returns. From the figure, it was evident that the gold market frequently switched between regimes, particularly after 2012. During earlier years (2003–2011), the model assigned a high probability to Regime 2, indicating a persistent high-volatility environment. This was consistent with elevated residual variance seen in the MSM estimates. From 2012 onwards, there was increased alternation between Regime 1 and Regime 2, suggesting greater regime instability or market unpredictability in recent years. Notably, the model frequently assigned probabilities close to 1 for one regime, showing strong confidence in regime classification at many time points.

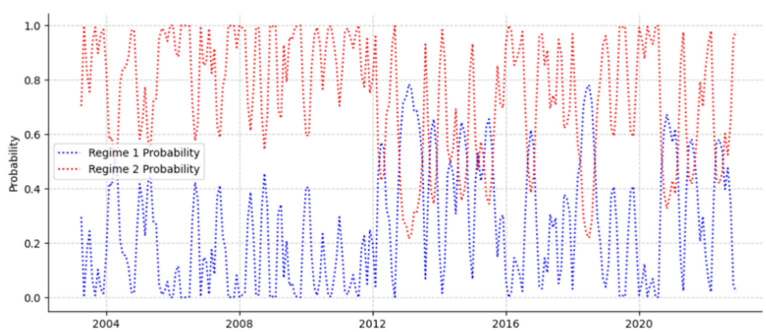

Figure 8: Smoothed Prob by MSM for Gold log

Figure 9 presents the smoothed probabilities of the cocoa market being in either Regime 1 (low volatility, blue line) or Regime 2 (high volatility, red line) over the period from 2003 to 2022. The chart shows that the cocoa market underwent distinct phases dominated by one regime or the other:

- From 2003 to 2006, the market was largely in Regime 2, indicative of a high-volatility state, with probabilities approaching 1.
- Between 2007 and 2011, the regime shifted predominantly to Regime 1, suggesting a more stable market with relatively low volatility.
- From 2012 to 2019, the model captured frequent transitions between regimes, suggesting a period of increased uncertainty or structural instability.
- After 2020, the probabilities strongly favored Regime 1, indicating a return to relative calm in the cocoa market.

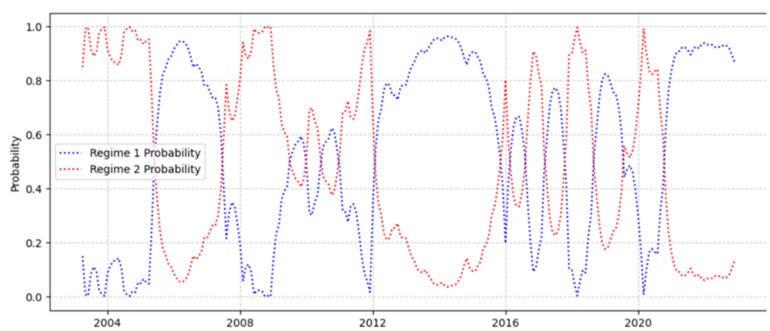

Figure 9: Smoothed Prob by MSM for Cocoa log

Table 11 compares the performance of the Markov Switching Models (MSM) fitted to gold and cocoa log return series using key statistical metrics.

The gold MSM model exhibited a higher log-likelihood (446.5271) and more negative AIC (−881.0541) and BIC (−860.2458) values than the cocoa model. This indicated that the MSM model for gold achieved a better overall fit, with lower model complexity penalties and greater explanatory power.

The cocoa MSM model, while still showing strong performance, had comparatively higher AIC and BIC values, suggesting a slightly less optimal model fit.

Table 11: MSM Performance with Gold and Cocoa

Metric	Gold	Cocoa
Log-Likelihood	446.5271	344.8212
AIC	- 881.0541	- 677.6424
BIC	- 860.2458	- 656.8341

4.4 Model Evaluation

Given the presence of structural breaks and regime-dependent behaviour in both markets, nonlinear models, TAR and MSM, were expected to outperform linear benchmarks. Therefore, the evaluation focused not only on how well each model captured the underlying data-generating process but also on how accurately it forecasts future prices. Standard statistical metrics including Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) were employed to compare model forecasts and judge predictive superiority.

Table 12: Model Evaluation for TAR and MSM

Model	Gold			Cocoa	
	MAE	RMSE		MAE	RMSE
TAR	0.028499	0.035948	TAR	0.043359	0.056832
MSM	0.024518	0.032875	MSM	0.044474	0.057863

As shown in Table 12 above, the MSM model outperforms the TAR model in forecasting Gold prices, as it records lower values for both MAE and RMSE, indicating higher predictive accuracy. For Cocoa, TAR slightly outperforms MSM in both MAE and RMSE, though the difference is minimal. Thus, both models perform comparably for Cocoa, with a marginal edge for TAR.

5 Conclusion

This study set out to explore the predictive performance of non-linear time series models, specifically the Threshold Autoregressive (TAR) and Markov Switching Model (MSM) frameworks, in forecasting the log-transformed prices of gold and cocoa. Given the observed non-stationary behavior and evidence of regime changes in both markets, these models were appropriate tools to capture potential structural shifts, nonlinear dynamics, and regime-dependent behaviors. The TAR model effectively identified distinct regimes based on predetermined threshold variables, showing that price behaviour in both markets responds differently depending on the state of the market. The model's ability to switch between regimes conditionally revealed the presence of asymmetric responses to market shocks, especially in the cocoa market where volatility tends to cluster in specific regimes. The MSM, in contrast, offered a probabilistic and more flexible framework, allowing for regime transitions governed by a hidden Markov process. This proved particularly powerful in capturing the underlying stochastic structure of the series. The MSM's smoothed and transition probabilities clearly demonstrated shifting market conditions over time, especially during known periods of economic shocks, commodity booms, and downturns (2007 -2009, 2019 – 2021) Models were evaluated using forecast accuracy metrics RMSE and MAE, and this reveal that in the gold market, the MSM exhibited better predictive accuracy than the TAR model. For Cocoa, TAR slightly outperforms MSM in both MAE and RMSE, though the difference was minimal. Thus, both models perform comparably for Cocoa, with a marginal edge for TAR. This reveals the fact that Model performance may vary depending on the commodity, suggesting model suitability can be commodity-specific. These findings underscore the utility of regime-sensitive models in commodity price forecasting and offer valuable insights for market participants and policy decision-makers operating in volatile economic environments.

6 Limitation Of The Study

Despite the insightful findings, this study is subject to some limitations. Firstly, the analysis was based on historical monthly data for gold and cocoa prices, which may not fully capture high-frequency market dynamics. Secondly, while these models (TAR and MSM) are effective in capturing regime shifts, they may not account for other complex features like volatility clustering and long memory as in the case of GARCH or LSTM neural networks. Lastly, this study did not include external variables which may also influence commodity prices.

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