Application of Markov chain model to analysis and prediction of green gram price transitions in Kitui County, Kenya

Original Research Article

Abstract

This study investigated the stochastic price dynamics of green grams in Kitui County, Kenya, aiming to analyze and predict price movements using a Markov chain model. Employing monthly price data from January 2012 to December 2024, sourced from the Ministry of Agriculture and Livestock Development and the Kenya National Bureau of Statistics, the research addressed the limitations of traditional time series models in capturing agricultural price volatility and mean recurrent times. A three-state Markov process (price increase, decrease, or no change) was constructed. The study estimated the transition probability matrix, determined the long-run price distribution, and calculated mean recurrent times, revealing rapid price state transitions and a dynamic market equilibrium. Notably, mean recurrent times ranged from 1.35 to 2.5 months between price increase and decrease states. Data analysis, conducted using R software, included descriptive statistics, price variability analysis, and Markov chain model fitting. The findings provide crucial insights into green gram price dynamics, offering a robust forecasting approach for farmers, traders, and consumers. This research highlights the significance of Markov chain models as a practical and effective tool for predicting and managing price volatility in emerging agricultural markets, such as Kitui County, thereby enhancing profitability and market stability.

Keywords: Markov chain models, Mean recurrent time, Green gram prices, Analysis and Forecasting.

1 Introduction

According to Mutwiri, 2019, agriculture played a major role in the Kenyan economy, contributing about 30% of the GDP and accounting for about 80 % of employments. The agriculture sector employed more than 40 percent of the total population and 70 % of the rural population. Smallholder farmers *and agricultural enterprises continued to face price fluctuation challenges, impeding business growth. Lusike et al., 2023 described agricultural development as one of the most powerful tools to end extreme poverty, boost shared prosperity, and feed a projected 9.7 billion people by 2050. Growth in the agricultural sector was two to four times more effective in raising incomes among the poorest compared to other sectors. This growth correlated to the existing price of agricultural produce. Lusike et al., 2023 analyzed and found that 65 % of poor working adults made a living through agriculture. Agriculture is also crucial to global economic growth. In 2018, it accounted for about 4% of the global gross domestic product (GDP) and in some least developed countries, it accounted for more than 25 % of GDP.*

Lusike et al., 2023 reported a bourgeoning trend in green gram production in East African countries. Despite this growth, small-scale farmers in rural East Africa faced significant constraints, particularly price fluctuations. As reported by **?***, Kenya accounted for approximately 96% of the 143,552 metric tonnes of green grams consumed in 2014, with Tanzania contributing 3.78 %. Consumption in Uganda, Burundi, and Rwanda was negligible. Only 30 % of total green gram production was consumed by farmers, while about 70% was sold commercially. The demand for green grams is driven by population growth, market importance, expanding external markets, and prices of substitute legumes.*

Green gram (Vigna radiata), also known as mung bean, is a drought-tolerant legume increasingly cultivated in Kenya's arid and semi-arid regions (ASALs), particularly in Kitui County, due to its short maturity period, adaptability to erratic rainfall, and growing domestic and export demand (Maluvu et al., 2024; Muchomba et al., 2023; Mugo et al., 2023). It serves as a critical cash crop and food resource, especially for smallholder farmers seeking income diversification and improved household nutrition (Maluvu et al., 2024). Over the past decade, green gram production has expanded significantly in lower eastern Kenya, driven by government programs like the National Accelerated Agricultural Inputs Access Program (NAAIAP) and development initiatives promoting drought-resilient crops Kirimi et al., 2023. Major production areas in Kenya included Kitui, Tharaka Nithi, Machakos, Mbeere, and Mwingi districts, with a specific focus on Kitui County in this study. However, price volatility remains a persistent challenge. Prices fluctuate seasonally, typically declining sharply during harvest months and spiking during lean periods, undermining the income stability of producers (Pani et al., 2019). Several factors contribute to this instability, including market inefficiencies, inadequate storage infrastructure, poor access to real-time price information, and the lack of localized forecasting tools. These conditions often force farmers to sell their produce at sub-optimal prices, limiting their profitability and investment in subsequent production cycles (Mugo et al., 2023).

Statistical models provided an important tool for predicting future prices of agricultural commodities, offering accurate information relevant to economic planning and decision-making. Agricultural commodity price behavior referred to the response evaluation of prevailing prices to economic factors over a specified time interval.Traditional forecasting methods often relied on historical data and expert opinions, which did not always capture the complex dynamics of agricultural price volatility.

Over the years traditional forecasting approaches, such as ARIMA and linear regression models, have been widely used to predict agricultural prices in Kenya (Gathondu, 2014; Okello, 2023; Tomek et al., 2014). While these models are effective for short-term forecasting and linear trends, they fail to capture discrete state-based price transitions such as moving from a low to a medium or high-price regime which is critical for long-term strategic planning.According to Tomek et al., 2014, knowledge about the future prices of agricultural products was crucial to the economic growth of a country, and price prediction had a significant impact on the stability of the market economy. The prediction of future prices of agricultural commodities provide vital information to farmers to aid in decision-making, especially concerning selling in relation to production costs. Price information is also vital to the government to enable the application of monetary and fiscal policies in the agricultural sector.

Zhu et al., 2013 applied Markov Chain Models to vegetable price fluctuation in China. In contrast, the Markov Chain model provides a probabilistic framework for understanding how prices move between different states over time. It is particularly suited for modeling systems where future outcomes depend

on current conditions rather than the full sequence of historical events (Anderson & Goodman, 1957). Markov Chain Models offered a potential solution by combining long-run price analysis with the estimation of mean recurrent times, enabling farmers to plan optimal selling times. This makes it ideal for agricultural markets with discrete price categories and observable seasonal behavior.
Despite its potential, few studies have applied Markov Chain models to price forecasting in Kenya's agricultural sector, and even fewer have focused on pulses like green gram. This gap is especially significant in Kitui County, where reliable and localized price forecasting tools could enhance decision-making for both farmers and policymakers. The aim of this study is to apply the Markov chain analysis to predict green gram price transitions, identify key price transition patterns, determine the average time spent in different price states, and forecast future price movements.

2 Materials and Methods

2.1 Research Design

The researcher employed a longitudinal research design, which allowed for the tracking of changes and developments over time, thus providing a more in-depth understanding of the phenomena under investigation. Specifically, by adopting a case study longitudinal approach, the researcher was able to identify green gram price transition trends and offer a more nuanced understanding of the specific circumstances surrounding these price transitions.

2.2 Data and Data Source

The study utilized secondary monthly data from January 2012 to December 2024 from and Ministry of Agriculture and Livestock Development and Kenya National Bureau of Statistics Kitui County covering all sub-counties. The data was extracted from agricultural produce market survey data, cleaned and recorded for analysis. The findings of this sample research could be extended to make inference and generalization of the population of green gram markets in Kenya.

2.3 The Markov Process

A Markov chain is a stochastic process where any sequence of outcomes satisfies the Markov property. The Markov property states that the future occurrence of any event depends only on present state and no additional information about previous states is required to make possible future predictions. If the present is known the past and future are unrelated, with probability constant over time and the outcomes at any stage dependent on chance. The set of all possible outcomes is finite. A Markov process can either be discrete or continuous process.
Let $X_t : (t) \in T$ *and* $X_t : t \in T$ *denote a continuous and discrete parameter space respectively. Let* S *be a discrete state space which can be assume a range of values* X_t*. A stochastic process* $X_t : t \in T$ *taking values in* S *express a Markov dependence if:*

$$\begin{aligned} P_r[X(t) \leq x \mid X(t_n) = x_n] &= P_r[X(t) \leq x \mid X(t_n) = x_n, X(t_{n-1}) \\ &= x_{n-1}, \cdots\cdots, X(t_o) = x_o] \\ &= F(x_n, x, t_n, t) \end{aligned} \tag{2.1}$$

where $0 \leq x_i < t_n$ *denotes the "history" of the process up to, but not including, time* $X(t)$ *for* $n = 0, 1, 2 \cdots\cdots \in T$ *and* $i, j \in S$*.*
The Markov chain allowed reduction in the number of parameters to be estimated. Markov chains were used to calculate probabilities in state transition though intermediate state and back. By developing transition matrices and diagrams state transition probabilities were analyzed.

2.4 Probability Distribution

A stochastic process is a collection of random variables indexed by time, represented as X_t, where $t \in T$. Here, T denotes the index set, which can be either continuous or discrete. X_t is a random variable, t is an index parameter (often representing time), and T is the index set (e.g., discrete time: $T = \{0, 1, 2, \ldots\}$; continuous time: $T = [0, \infty]$). Key elements include: the probability space $(\Omega, \mathcal{F}, P)$, comprising the sample space Ω, the sigma-algebra $\mathcal{F}$, and the probability measure P; the state space S, which encompasses all possible values of X_t; the index set T, representing the time points; and the random variables X_t themselves. Formally, the stochastic process $X = \{X_t : t \in T\}$ is a collection of random variables, where each X_t is a function $X_t : \Omega \to S$, meaning that for a given outcome $\omega \in \Omega$, $X_t(\omega)$ is the state of the process at time t. It can also be described as a function of two variables: $X : \Omega \times T \to S$. Important considerations for characterizing this stochastic process include the finite-dimensional distributions, which describe the joint probabilities of $(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$ for any finite set of time points $t_1, t_2, \ldots, t_n$, and the necessary measurability conditions for the process to be well-defined.

Lemma 1: *Suppose a discrete random variable X has a range $R_X = \{x_1, x_2, x_3, ..., x_n\}$. The function $P_X(x_t) = P(X = x_t)$, for $t = 1, 2, 3, ..., n$ is its probability mass function (PMF) of X. Further, the PMF of X for all real numbers is given by:*

$$P_X(x) = \begin{cases} P(X = x) & : \textit{if } x \in R_X \\ 0 & : \textit{otherwise} \end{cases}$$

where $0 \leq P_X(x) \leq 1$ for all x, and $\sum P_X(x) = 1$.

The equation above defines the probability mass function (PMF) of a discrete random variable and states that: (i) the probability of the variable taking a valid value is given by the PMF at that value, (ii) the probability of the variable taking an invalid value is zero, and (iii) the sum of all valid probabilities must equal one.

2.5 Conditional Probability and Stochastic Processes

2.5.1 Lemma 2: Conditional Probability Mass Function

The conditional probability mass function of a discrete random variable is given by:

$$P_{X|Y=y}(x) = \frac{P_{XY}(xy)}{P_Y(y)} \quad \textit{and} \quad P_{Y|X=x}(y) = \frac{P_{XY}(xy)}{P_X(x)} \tag{2.2}$$

$P_{X|Y=y}(x)$ represents the probability that the discrete random variable X takes the value x, given that another discrete random variable Y has taken the value y. $P_{XY}(xy)$ is the joint probability mass function of X and Y, representing the probability that $X = x$ and $Y = y$. $P_Y(y)$ is the marginal probability mass function of Y, representing the probability that $Y = y$. Essentially, it calculates the probability of $X = x$ given $Y = y$ by dividing the joint probability of $X = x$ and $Y = y$ by the probability of $Y = y$.

2.5.2 Conditional Probability Mass Function of a Stochastic Process

Given that $t_0 < t_1 < t_2 < \cdots < t_n$, the function

$$F(x_0, x_1, \ldots, x_n, t_0, t_1, \ldots, t_n) = P_r(X(t_n) \leq x_n \mid X(t_{n-1}) \leq x_{n-1}) \tag{2.3}$$

is the conditional probability mass function of a stochastic process.This description outlines the conditional probability within a stochastic process, specifying that the probability of the process X at time t_n is conditioned on its state at time t_{n-1}. Specifically, it denotes the probability that the state *of the process at time t_n is less than or equal to x_n, given that its state at time t_{n-1} was less than or equal to x_{n-1}. In essence, this conditional probability captures how the past state of the stochastic process influences its future state.*

2.6 Transition Probability and Stationary Transition Probabilities

Let

$$P_{ij}^{(n,n-1)} = P_r(X_n = j \mid X_{n-1} = i) \tag{2.4}$$

where $n = 0, 1, 2, \cdots \in T$ and $i, j \in S$ be the transition probability from state i to state j. Equations (3.4) and (3.5) denote the transition probability from state i to j. The Markov chain is said to have stationary transition (time homogeneity) probabilities if:

$$P_{ij}^{(n+1,n)} = P_r(X_1 = j \mid X_0 = i) \tag{2.5}$$

$$= P_r(X_n = j \mid X_{n-1} = i) \tag{2.6}$$

The term $P_{ij}^{(n,n-1)}$ represents the transition probability, specifically defining the likelihood of the stochastic process moving from state i at time $n-1$ to state j at time n, essentially the probability of a one-step transition. Stationary transition probabilities indicate that this transition probability from state i to state j remains constant across all time steps, implying that the transition probabilities are time-invariant or exhibit time homogeneity.

Lemma 2 introduces the concept of conditional probability in the context of discrete random variables and stochastic processes. It then defines transition probabilities, which are essential for understanding how a stochastic process evolves over time, and introduces the concept of stationary transition probabilities, which is a key property of Markov chains.

2.7 Probability and Markov Chains

*Consider a stochastic process $\{X_t : t \in T\}$ and let P_{ij} be the transition probability from state i in n^{th} trial to state j in $n+1^{th}$ trial. By association, the transition matrix **P** below with entries P_{ij} is derived with its rows or columns been probability vectors.*

$$P = \begin{pmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,n} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n,1} & P_{n,2} & \cdots & P_{n,n} \end{pmatrix} \text{ where} \sum P_{ij} = 1 \quad \forall \quad n_i, \quad 0 < P_{ij} < 1 \tag{2.7}$$

$$P_{ij}^{(n,n-1)} = P_r(X_n = j \mid X_{n-1} = i) \tag{2.8}$$

We denote the transition probability that a process currently in state i will be in state j after n additional transitions by;

$$P_{ij}^{(n)} = P_r(X_n = j \mid X_o = i) n, i, j \geq 0 \tag{2.9}$$

Note that ${P_{ij}}^{(1)} = P_{ij}$ and ${P_{ij}}^{(0)} = \begin{cases} 1 & \text{if } i = j \\ 0 & otherwise \end{cases}$

Equally the k step transition probability of state i text to state j text in k text steps is defined as;

$$P_{ij}^{(k)} = P_r(X_{n+k} = j \mid X_n = i) \forall k > 0, n \geq 0, i, j \in S \tag{2.10}$$

In the matrix form the k step transition probability of state i to state j in k steps is represented as $P^k = P_{ij}^{(k)} \quad i, j \in S, \quad \forall \quad k > 0$ and P_{ij}^k is the $(i,j)^{th}$ element of the matrix $P^{(k)}$.

2.8 Markov Chain Stationary Distribution

A stationary distribution is a probability distribution that remains unchanged after n steps of the Markov chain. A limiting distribution is a special case of a stationary distribution.

2.8.1 Markov Chain Limiting Distribution

For any time homogeneous Markov chain that is aperiodic and irreducible $\lim_{n\to\infty} P^{(n)}$ converges to a matrix with all rows identical and equal to π. This π may be dependent or independent of initial distribution $\pi^{(0)}$. If π exists and is independent of initial distribution then the sequence of matrices $P^(n), n = 1, 2\dots$ must converges to a matrix P^{∞}, in which all rows are equal to π. That is

$$\lim_{n\to\infty} P^{(n)} = \pi = \begin{bmatrix} \pi_0, \pi_1, \dots, \pi_n \\ \pi_0, \pi_1, \dots, \pi_n \\ \vdots \\ \pi_0, \pi_1, \dots\ \pi_n \end{bmatrix} = \begin{bmatrix} \pi \\ \pi \\ \vdots \\ \pi \end{bmatrix} \tag{2.11}$$

where π = $\begin{bmatrix} \pi_0, & \pi_1, & \dots, & \pi_n \end{bmatrix}$ with $0 < \pi_j < 1$. The elements of π are the elements of unique solution of $\pi = \pi P$ and $\sum_{i=0}^{n} \pi_i = 1$.
The limiting distribution provides information about the long-term steady-state behavior of the Markov chain. A state with a high limiting probability is visited with a high frequency in the long run

2.9 The Chapman-Kolmogorov Equations

Consider $w > wf > w_f$. We want to find $P_{ij}^{wf,w}$.

$$\begin{aligned}
P_{ij}^{wf,w} &= P_r(X_w = j \mid X_{wf} = i) \\
&= \sum_{h\in S} P_r(X_w = j, X_{wf} = h \mid X_{wf} = i) \\
&= \sum_{h\in S} P_r(X_w = j \mid X_{wf} = h, X_{wf} = i) P_r(X_{wf} = h \mid X_{wf} = i) \\
&= \sum_{h\in S} P_r(X_w = j \mid X_{wf} = h) P_r(X_{wf} = h \mid X_{wf} = i) \\
&= \sum_{h=1}^{\infty} P_r(X_w = j \mid X_{wf} = h) P_r(X_{wf} = h \mid X_{wf} = i) \\
&= \sum_{h=1}^{\infty} P_{ih}^{wf,wf} P_{hj}^{w,wf} \qquad (2.12)
\end{aligned}$$

Equation (3.17) is the discrete form of the Chapman-Kolmogorov equation.

2.10 Markov State and Chain classification

A Markov chain is said to be regular if some power of the transition matrix contains only positive elements. Non-regular transition matrices have no unique eigenvector corresponding to eigenvalue 1. Consider a Markov chain $X_n, n \geq 0$ with state space S. State j is said to be accessible from state i if $P_{i,j}^{n} > 0$ for some $n \geq 0$. If i and j communicate (i.e., are accessible from each other), then $P_{i,j}^{n} > 0$ and $P_{j,i}^{m} > 0$ for some $n, m \geq 0$, and we write $i \leftrightarrow j$. A Markov chain is closed if it is impossible to leave once entered, that is, $P_{ij}^{n} = 0$ for all $j \neq i$ and all $n \geq 0$. A state i is absorbing if it is a closed class. Irreducible Markov chains have all states reachable from every other state (i.e., all *states communicate). For any state i, f_{ii} denotes the probability that starting in state i, the process will re-enter i.*

$$f_{ii} = P(X_n = i \text{ for some finite } n \mid X_0 = i) \tag{2.13}$$

$$= P\left(\bigcup_{n=1}^{\infty}\{X_n = i \mid X_0 = i\}\right) \tag{2.14}$$

A state is known as recurrent or transient depending upon whether or not the Markov chain will eventually return to it. State i is recurrent if $f_{ii} = 1$ and transient if $f_{ii} < 1$. A state is known as recurrent or transient depending upon whether or not the Markov chain will eventually return to it.
State i is recurrent if

$$f_{ii} = \Sigma_{(n=1)}^{\infty} f_{ij}^{(n)} = 1 \quad or \quad \Sigma_{n=1}^{\infty} P_{ii}^{(n)} = \infty. \tag{2.15}$$

Any recurrent state is revisited infinitely. A recurrent state is known as positive recurrent if it is expected to return within a finite number of steps and null recurrent otherwise.
State i is transient if

$$f_{ii} = \Sigma_{(n=1)}^{\infty} f_{ij}^{(n)} < 1 \quad or \quad \Sigma_{n=1}^{\infty} P_{ii}^{(n)} < \infty. \tag{2.16}$$

if X_n is a Markov chain defined with a state $j \in S$ is accessible from a state $i \in S$, then the probability of the first transition at n steps is given by;

$$f_{ij}^{n} = P_r\left[X_{n+1} = j \mid X_n = i_n, X_{n-1} = i_{n-1}, ..., X_1 = i_1, X_0 = i_0\right] \tag{2.17}$$

The expected first passage time is given by

$$\begin{aligned}\mu_{ij} &= \mathbb{E}\left[X_{n+1} = j \mid X_n = i_n, X_{n-1} = i_{n-1}, ..., X_1 = i_1, X_0 = i_0\right] \\ &= nP_r\left[X_{n+1} = j \mid X_n = i_n, X_{n-1} = i_{n-1}, ..., X_1 = i_1, X_0 = i_0\right] \\ &= \sum_{n=1}^{\infty} n f_{ij}^{n} \end{aligned}\tag{2.18}$$

If $i = j$ then,

$$f_{ij}^{n} = P_r\left[X_{n+1} = i_{n+1} \mid X_n = i_n, X_{n-1} = i_{n-1}, ..., X_1 = i_1, X_0 = i_0\right] \tag{2.19}$$

and

$$\mu_{ii} = \mu_i = \Sigma_{n=1}^{\infty} n f_{ii}^{n} \tag{2.20}$$

is the mean recurrence from state i if state i is recurrent.
Let P be the transition matrix of Markov chain $X = X_n; n = 0, 1, 2 \ldots$. A state i has period $k \geq 1$ if any chain starting at state i through j and returning to state i with positive probability must take a number of steps divisible by k. If $k = 1$, then the state is known as aperiodic, and if $k > 1$,the state is known as periodic. If all states are aperiodic then the Markov chain is known as aperiodic.

2.11 Model Formulation and Specification

Let Y_t be the monthly price of green grams in Kitui County at time t, where $t = 1, 2, \ldots, n$. Let $S_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ be the percentage change in the price of green grams at time t. We define a random variable X_t representing green gram price transition states, taking values -1, 0, and 1, where:

$$f_X(x_t) = \begin{cases} -1 & : S_t < Q_1 \quad \text{(Price decrease)} \\ 0 & : Q_1 \leq S_t \leq Q_3 \quad \text{(No change in price)} \\ 1 & : S_t > Q_3 \quad \text{(Price increase)} \end{cases} \tag{2.21}$$

Here, Q_1 and Q_3 represent the first and third quartiles of the percentage price changes S_t.

2.12 Markov Chain State Development

The study applied the concept of quartiles as thresholds to determine movement between the price states of the modeled Markov chain.

[label=()]

1. ***Price Decrease (-1):*** *The green gram price percentage change is less than the first quartile ($S_t < Q_1$).*
2. ***No Change (0):*** *The green gram price percentage change is within or equal to the first and third quartiles ($Q_1 \leq S_t \leq Q_3$).*
3. ***Price Increase (1):*** *The green gram price percentage change is greater than the third quartile ($S_t > Q_3$).*

2.13 Indicator Vectors and State Counts

We define an indicator vector $\gamma_{(i,t)}$. The indicator vector $\gamma_{(i,t)}$ is a function that takes two inputs:

- *i: The specific state we are interested in (e.g., -1, 0, or 1 in our green gram price model).*
- *t: The time point at which we are observing the state.*

It outputs either 1 or 0:

- *1: If the random variable X_t (the state of the process at time t) is equal to i.*
- *0: If X_t is not equal to i.*

Mathematically, this can be represented as:

$$\gamma_{(i,t)} = \begin{cases} 1 & : X_t = i \\ 0 & : X_t \neq i \end{cases} \tag{2.22}$$

where $i \in \{-1, 0, 1\}$ and $t = 1, \ldots, n$. The total number of times the process is in state i is given by:

$$n_i = \sum_{t=1}^{n} \gamma_{(i,t)} \tag{2.23}$$

$$= \sum_{t=1}^{n} \left\{ \begin{array}{ll} 1 & : X_t = i \\ 0 & : X_t \neq i \end{array} \right\} \tag{2.24}$$

$$= \sum_{j=-1}^{1} n_{ij} \quad \forall i \in \{-1, 0, 1\} \tag{2.25}$$

The total number of time steps is:

$$n = \sum_{i=-1}^{1} n_i \tag{2.26}$$

2.14 Initial Transition Probabilities

The initial probabilities of the green gram price change states are given by:

$$\widehat{P}_i = \frac{n_i}{n} \quad \text{for } i \in \{-1, 0, 1\} \tag{2.27}$$

2.15 Transition Probabilities

For the stochastic process $\{X_t, t = 1, \ldots, n\}$, we estimate the transition probabilities $P_{ij}^{(t,t-1)} = P(X_t = j \mid X_{t-1} = i)$ for $i, j \in \{-1, 0, 1\}$ and $t = 2, \ldots, n$.
When calculating transition probabilities for a time series data Markov chain summations begins from $t = 2$. Transition probabilities, by definition, describe the movement from one state at a given time to another state at a subsequent time. To observe a transition, you need at least two consecutive time points: $t - 1$ and t.
Initial State
X_{t-1} represents the state at the previous time point, and X_t represents the state at the current time point. When $t = 1$, $t - 1$ would be 0. If data starts at $t = 1$, there's no data point for $t = 0$ to serve as the "previous" state.
First Transition
The first possible transition occurs from $t = 1$ to $t = 2$. Therefore, the summation for n_{ij} (the number of transitions from state i to state j) must start from $t = 2$. Two consecutive data points are required to observe a transition. Since data starts at $t = 1$, the first transition observable is from $t = 1$ to $t = 2$, hence the summation starts at $t = 2$.
We define an indicator variable $\omega_t^{(i,j)}$ as:

$$\omega_t^{(i,j)} = \begin{cases} 1 & : X_{t-1} = i \text{ and } X_t = j \\ 0 & : \text{otherwise} \end{cases} \tag{2.28}$$

The number of transitions from state i to state j is given by:

$$n_{ij} = \sum_{t=2}^{n} \omega_t^{(i,j)} \tag{2.29}$$

$$= \sum_{t=2}^{n} \left\{ \begin{array}{ll} 1 & : X_{t-1} = i \text{ and } X_t = j \\ 0 & : \text{otherwise} \end{array} \right\} \quad \text{for } i, j \in \{-1, 0, 1\} \tag{2.30}$$

Finally, the transition probabilities are estimated as:

$$P_{ij} = \frac{n_{ij}}{n_i} \tag{2.31}$$

$$= \frac{\sum_{t=2}^{n} \omega_t^{(i,j)}}{\sum_{t=1}^{n} \gamma_{(i,t)}} \quad \text{for } i, j \in \{-1, 0, 1\} \tag{2.32}$$

2.16 Maximum Likelihood Estimation of Transition Probabilities

Thie study details the estimation of Markov chain transition probabilities using the Maximum Likelihood Estimation (MLE) method.

2.16.1 Likelihood Function for a Markov Chain

Let $X_1, X_2, \ldots, X_n$ be a sequence of observed states from a discrete Markov chain with q states. We want to estimate the transition probabilities P_{ij}, which represent the probability of transitioning from state i to state j.The likelihood function, representing the probability of observing the sequence given the parameters, is:

$$L(\mathbf{P}) = P(X_1 = x_1) \prod_{t=2}^{n} P(X_t = x_t \mid X_{t-1} = x_{t-1}) \tag{2.33}$$

where $\mathbf{P}$ is the transition matrix. By applying the Markov property, we simplify this to:

$$L(\mathbf{P}) = P(X_1 = x_1) \prod_{t=2}^{n} P_{x_{t-1},x_t} \tag{2.34}$$

where P_{x_{t-1},x_t} is the transition probability from state x_{t-1} to state x_t. Let n_{ij} be the number of transitions from state i to state j. Then, the likelihood function can be expressed as:

$$L(\mathbf{P}) = P(X_1 = x_1) \prod_{i=1}^{q} \prod_{j=1}^{q} P_{ij}^{n_{ij}} \tag{2.35}$$

2.16.2 Maximizing the Log-Likelihood

To find the MLE estimates the likelihood function is maximized. It is often easier to maximize the log-likelihood:

$$\log L(\mathbf{P}) = \log P(X_1 = x_1) + \sum_{i=1}^{q} \sum_{j=1}^{q} n_{ij} \log P_{ij} \tag{2.36}$$

The study further found the values of P_{ij} that maximize $\log L(\mathbf{P})$. We also have the constraint that $\sum_{j=1}^{q} P_{ij} = 1$ for all i. To account for this constraint, we use Lagrange multipliers. We define the Lagrangian as:

$$\mathcal{L}(\mathbf{P}, \lambda) = \log P(X_1 = x_1) + \sum_{i=1}^{q} \sum_{j=1}^{q} n_{ij} \log P_{ij} + \sum_{i=1}^{q} \lambda_i \left(1 - \sum_{j=1}^{q} P_{ij}\right) \tag{2.37}$$

where λ_i are Lagrange multipliers. To find the maximum, we take the partial derivatives with respect to P_{ij} and λ_i and set them to zero:

$$\frac{\partial \mathcal{L}}{\partial P_{ij}} = \frac{n_{ij}}{P_{ij}} - \lambda_i = 0 \implies P_{ij} = \frac{n_{ij}}{\lambda_i} \tag{2.38}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = 1 - \sum_{j=1}^{q} P_{ij} = 0 \implies \sum_{j=1}^{q} P_{ij} = 1 \tag{2.39}$$

Substituting $P_{ij} = \frac{n_{ij}}{\lambda_i}$ into $\sum_{j=1}^{q} P_{ij} = 1$, we get:

$$\sum_{j=1}^{q} \frac{n_{ij}}{\lambda_i} = 1 \implies \frac{1}{\lambda_i} \sum_{j=1}^{q} n_{ij} = 1 \implies \lambda_i = \sum_{j=1}^{q} n_{ij} = n_i \tag{2.40}$$

Therefore, the MLE estimate for P_{ij} is:

$$\widehat{P}_{ij} = \frac{n_{ij}}{n_i} \tag{2.41}$$

2.16.3 Transition Matrix Estimate

For a Markov chain with three states (-1, 0, 1), the estimated transition matrix is:

$$\widehat{\mathbf{P}} = \begin{pmatrix} \widehat{P}_{-1,-1} & \widehat{P}_{-1,0} & \widehat{P}_{-1,1} \\ \widehat{P}_{0,-1} & \widehat{P}_{0,0} & \widehat{P}_{0,1} \\ \widehat{P}_{1,-1} & \widehat{P}_{1,0} & \widehat{P}_{1,1} \end{pmatrix} \tag{2.42}$$

where $\widehat{P}_{ij} = \frac{n_{ij}}{n_i}$.
[label=()]

1. *The sum of each row in the transition matrix must equal 1: $\sum_{j=-1}^{1} \widehat{P}_{ij} = 1$ for all i.*
2. *The value of each transition probability must be between 0 and 1: $0 \leq \widehat{P}_{ij} \leq 1$.*
3. *$\widehat{P}_{ij}$ is the estimated probability of transitioning from state i at time t to state j at time $t+1$.*

2.17 Transition Prediction

Transition prediction in Markov chains involved estimating the probability distribution of moving from one state to another over a specific time period. This process was feasible after the analysis of historical data and the construction of a transition matrix. Given the initial state vector, the future evolution of the Markov chain process can be forecasted. To predict the state vector at time n given the initial state vector V_0, the formula below was used:

$$V_n = V_0 P^n \tag{2.43}$$

where P^n represents the one-step transition matrix P raised to the power of n. This effectively multiplies the transition matrix by itself n times. To predict the state vector i steps into the future, given that we are currently at step n, we use the formula:

$$V_{n+i} = V_n P^i \tag{2.44}$$

or, if we want to predict i steps into the future from the initial state:

$$V_i = V_0 P^i \tag{2.45}$$

The resulting vector V_n (or V_{n+i}) provides the probabilities of the Markov chain occupying each state at time n (or $n+i$), enabling the estimation of future state likelihoods based on historical transition patterns encapsulated within the transition matrix P. However, several important considerations must be taken into account during predictions. Firstly, the formulas presuppose a stationary transition matrix P, implying time-invariant transition probabilities; time-dependent transitions would necessitate model adjustments. Secondly, the initial vector V_0 (or V_n) is critical for prediction accuracy, serving as the starting point for the prediction process. Thirdly, prediction accuracy tends to diminish as the prediction horizon i extends. Finally, it's crucial to acknowledge that predictions are probabilities, and actual outcomes may diverge, as the model does not account for inherent randomness or external factors influencing the Markov chain.

2.18 Model Performance

In this research, the goodness of fit test (χ^2) was used to verify the Markov Property. At the $\alpha \in (0,1)$ level of significance, a Null hypothesis H_0 was created and the alternative hypothesis H_1. To verify the hypothesis H_0, the test statistics was calculated.

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \tag{2.46}$$

$$\chi^2 = \mu_{i=1}^{k} \sum_{j=1}^{k} \frac{(n_{ijv} - n_{ij}P_{jv})^2}{n_{ij}P_{jv}} \tag{2.47}$$

which has a χ^2 distribution with k^3 degrees of freedom and n_{ijv} = the number of transition from state s_i to state s_j and next to state s_v for $1 \leq i, j, v \leq k$.

Chi-square test determined if the Markov property held for the chain $\{X_t\}_{t\in N}$.If the calculated chi-square value is less than the critical value, we fail to reject the null hypothesis H_0 at the α significance level, indicating that the Markov property is likely satisfied. However, if the calculated chi-square value is greater than or equal to the critical value, we reject H_0 in favor of the alternative hypothesis, suggesting that the Markov property does not hold for the chain.

2.19 Data Processing and Exploratory Analysis

Time series plots of the monthly prices (Y_t) were generated to visually inspect trends and seasonal patterns.The percentage change in prices ($S_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$) was computed for each time step to normalize the data and focus on relative changes.The first (Q_1) and third (Q_3) quartiles of the percentage changes (S_t) were calculated to define the Markov chain states. State Definition: The price transition states (X_t) were defined as:

- $X_t = -1$ *if* $S_t < Q_1$ *(Price Decrease)*
- $X_t = 0$ *if* $Q_1 \leq S_t \leq Q_3$ *(No Change)*
- $X_t = 1$ *if* $S_t > Q_3$ *(Price Increase)*

Auto correlation plots of the percentage changes (S_t) were used to assess the stationarity of the time series data.

2.20 Markov Chain Analysis

Indicator vectors $\gamma_{(i,t)}$ were computed to track the occurrences of each state ($i \in \{-1, 0, 1\}$) at each time point (t). The number of times each state occurred (n_i) and the number of transitions between states (n_{ij}) were calculated using the indicator vectors.The initial probabilities of each state ($\widehat{P}_i = \frac{n_i}{n}$) were estimated. The transition probability matrix ($\widehat{P}$) was estimated using the Maximum Likelihood Estimation (MLE) method.

$$\widehat{P}_{ij} = \frac{n_{ij}}{n_i}$$

Future state distributions were predicted using the formula:

$$V_{n+i} = V_n \widehat{P}^i \tag{2.48}$$

where V_n is the state vector at time n and $\widehat{P}^i$ is the transition matrix raised to the power of i. Dendrograms of the transition probability matrix were plotted to visualize the relationships between states.goodness-of-fit test was conducted to assess whether the data satisfies the Markov property.

3 Results and discussion

3.1 Descriptive Analysis

The data was distributed with a range from KES 6656 to KES 15617 for a ninety kilogram bag. The median (KES 9178) and mean (KES 9275) were relatively close, suggesting a somewhat symmetrical distribution. The maximum value being so much higher than the third quartile indicated that there was a right skew to the data. The difference between the third quartile and the maximum value was much larger than the difference between the minimum value and the first quartile. This indicated a right skew.The quartiles provided a sense of the data spread and concentration. KES 6656 for a ninety kilogram bag was the Minimum. This was the smallest value in the dataset. The first quartile was KES 7872 for a ninety kilogram bag. This is the value below which 25% of the data falls.The median or second quartile was KES 9178. This is the middle value of the dataset, where 50% of the data is *below and 50% is above. The mean or average was KES 9275 for a ninety kilogram bag. This is the arithmetic average of all values in the dataset.The third quartile was KES 10179. This is the value below which 75% of the data falls.The Maximum was KES 15617 for a ninety kilogram bag. This was the largest value in the dataset. The time series plot (Figure 1) illustrates the price fluctuations from*

Table 1: Summary Statistics of Green Gram Prices

Minimum Price	6656
First Quartile Price	7872
Median Price	9178
Mean Price	9275
Third Quartile Price	10179
Maximum Price	15617

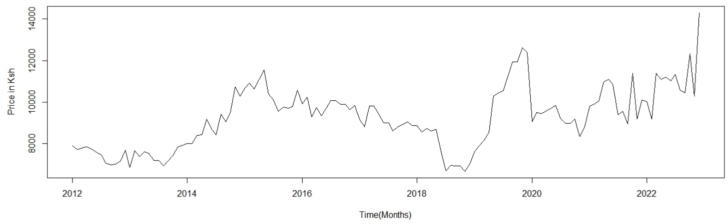

Figure 1: Time series plot

2012 to 2024, revealing a dynamic and volatile market. The price, represented by 'yts' (KES per 90 kg bag), exhibits significant variations throughout the period, lacking a consistent trend. Starting with a relatively low price in 2012, a notable increase occurs, peaking around 2015, followed by a decline to a low point in 2019. Subsequently, a sharp price surge is observed in 2020. Most strikingly, the period from 2022 to 2024 is characterized by heightened volatility, with significant price spikes and drops, suggesting substantial market instability and potential risks for market participants. This volatility necessitates a thorough investigation into the underlying factors driving these changes, particularly the extreme volatility in the later years, to develop strategies for price stabilization and risk mitigation.
The monthly distribution of green gram prices per kg in Kitui County, as depicted in the boxplot (Figure 2), reveals significant price variability throughout the year. While the median price fluctuates, indicating no consistent seasonal pattern, notable price spikes, represented by outliers, occur in months such as April, January, and May, suggesting potential supply disruptions or localized market anomalies. The varying widths of the boxplots demonstrate inconsistent price ranges, with December exhibiting the widest range and February the narrowest, underscoring the inherent price instability. This highlights the need for a deeper investigation into factors influencing green gram prices in Kitui County, such as weather patterns, transportation logistics, and market demand, to inform strategies for price stabilization.
The histogram (Figure 3) reveals a distinct right-skewed distribution of green gram prices per kg in Kitui County, with the majority of prices clustered within the 80 to 120 KES range and a modal price peaking around 100-110 KES. This concentration suggests a typical price bracket for consumers. However, the extended tail towards higher prices, as shown by the density curve, indicates occasional significantly elevated prices, implying market volatility or supply constraints. The low frequency of

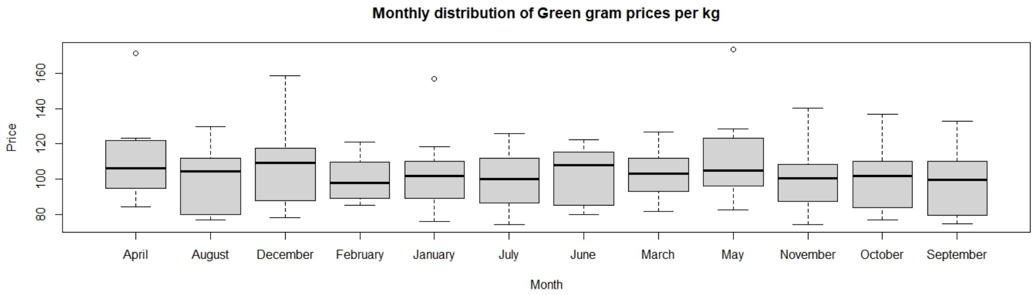

Figure 2: The Green Gram Price Box plot

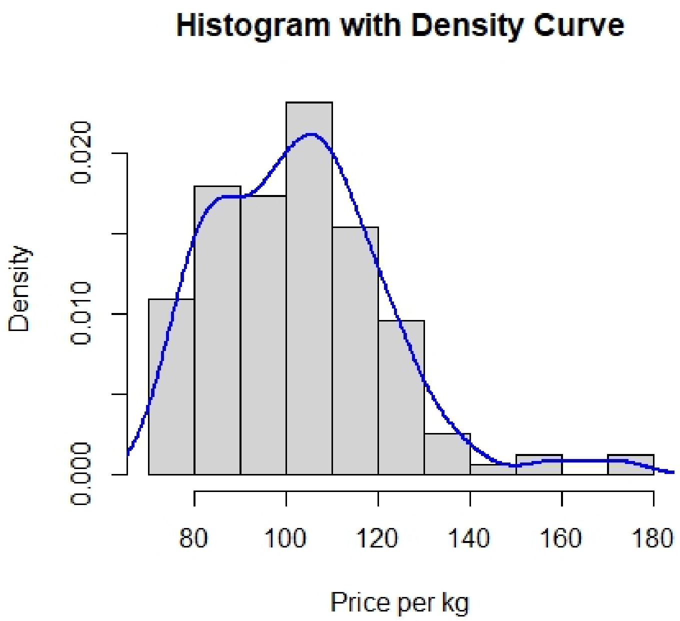

Figure 3: The Green Gram Price Histogram

prices exceeding 140 KES highlights the rarity of these high-price events. This distributional pattern underscores the need for a thorough investigation into factors contributing to the observed skewness and price outliers, including supply-demand dynamics, climatic influences, and logistical challenges, to develop strategies for price stabilization.

3.2 Transition Probability Matrix

$$\begin{array}{c} \\ \textit{Decrease(D)} \\ \textit{Increase(I)} \\ \textit{No change(N)} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Decrease(D)} & \textit{Increase (I)} & \textit{No change(N)} \end{array} \\ \begin{pmatrix} 0.3676471 & 0.5882353 & 0.04411765 \\ 0.4625000 & 0.5000000 & 0.03750000 \\ 0.8333333 & 0.1666667 & 0.00000000 \end{pmatrix} \end{array} \tag{3.1}$$

The given transition matrix represents the maximum likelihood estimate (MLE) of a Markov chain, where each element signifies the transition probability from one state to another in a single step. Each row sums to 1. For the "Decrease (D)" row, the probability of prices decreasing in the next period given a current decrease is 36.76%, increasing is 58.82%, and remaining unchanged is 4.41%. For the "Increase (I)" row, if prices increase currently, the probability of a decrease next period is 46.25%, an increase is 50%, and no change is 3.75%. Lastly, for the "No Change (N)" row, if prices remain unchanged currently, the probability of a decrease next period is 83.33%, an increase is 16.67%, and no change is 0%. These probabilities illustrate the likelihood of green gram price movements between decrease, increase, and no change states. The Green Gram price state transitions are visually represented in the transition diagram depicted in Figure 4.

3.3 Test of Markovian Property

A chi-squared goodness-of-fit test was conducted to evaluate the null hypothesis $\mathbb{H}_0$ *that the data sequence follows a Markov process. The test statistic was calculated as:*
$\chi^2 = 11.61143$*, with* 13 *degrees of freedom, the resulting p-value was* 0.5597343 *and significance level* α *was set at* 0.05.
The null and alternative hypotheses were defined as follows:
$\mathbb{H}_0$*: The future state does not depend on past state (The data sequence follows a Markov process).*
$\mathbb{H}_1$*: The future state depends on past state (The data sequence does not follow a Markov process).*
The decision rule for the test was:

- *If* $p \geq \alpha$*, fail to reject* $\mathbb{H}_0$.
- *If* $p < \alpha$*, reject* $\mathbb{H}_0$.

Since $p = 0.5597343 > \alpha = 0.05$*, we fail to reject the null hypothesis (*$\mathbb{H}_0$*). Therefore, there is insufficient evidence to conclude that the data sequence does not exhibit Markovian properties. In summary, the statistical results are presented in Table 2. The chi-squared goodness-of-fit test, with* $\chi^2 = 11.61143$*,* $df = 13$*, and* $p = 0.5597343$*, failed to reject the null hypothesis that the data follows a Markov process (*$\alpha = 0.05$*). This suggests the data's future states depend primarily on the present state. However, the interpretation is subject to limitations, including data quality, the choice of significance level, the need for alternative tests (e.g., transition matrices), and the consideration of external factors. A robust assessment requires addressing these factors to ensure reliable conclusions about the data's Markovian nature.*

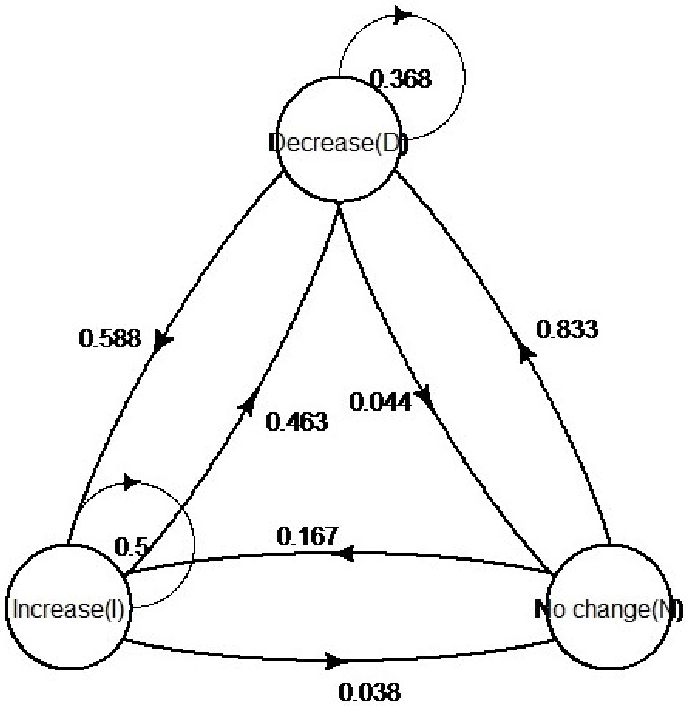

Figure 4: Green Gram Price Transition Diagram

3.4 Limiting Probability for the Green Gram Prices

This analysis aims to determine the long-term behavior of the Markov chain, specifically the distribution of states that the chain will eventually reach, irrespective of its initial state. The steady-state distribution is given by the row vector:

$$\begin{array}{cc} & \textit{Decrease(D)} \quad \textit{Increase (I)} \quad \textit{No change(N)} \\ [1,] & \begin{pmatrix} 0.4356147 & 0.5254622 & 0.03892312 \end{pmatrix} \end{array} \tag{3.2}$$

This vector represents the probabilities of the chain being in each state (Decrease, Increase, No change) in the long run. The relationship can be expressed as:

$$\begin{bmatrix} P(D) \\ P(I) \\ P(N) \end{bmatrix} = \begin{bmatrix} 0.4356147 & 0.5254622 & 0.03892312 \end{bmatrix} \begin{bmatrix} P(D) \\ P(I) \\ P(N) \end{bmatrix} \tag{3.3}$$

The left-hand side represents the steady-state vector, where $P(D)$, $P(I)$, and $P(N)$ are the probabilities of being in the Decrease, Increase, and No change states, respectively, in the long run. The right-hand side shows the steady-state vector being multiplied by the given row vector. It is noted that this representation is unusual, as steady states are typically represented by a column vector multiplied by the transition matrix. In steady state, the probabilities remain constant over time, meaning the probabilities at time $t + 1$ are the same as those at time t.

Table 2: Green Gram Chi-Squared Test Results

Statistic	**Value**
Chi-Squared (χ^2)	11.61143
Degrees of Freedom (df)	13
P-value (p)	0.5597343
Significance Level (α)	0.05
Decision	Fail to reject $\mathcal{H}_0$

3.5 Predicting State Probabilities Using the Transition Matrix

To predict the probabilities of green gram price state transitions in the next time period, a vector representing the current state probabilities is multiplied by the transition probability matrix. Let $P(t)$ denote the row vector representing the probabilities of being in each state (Decrease, Increase, No change) at time t:

$$P(t) = \begin{bmatrix} \textit{Probability(Decrease)} & \textit{Probability(Increase)} & \textit{Probability(No change)} \end{bmatrix} \tag{3.4}$$

The transition probability matrix T is given by:

$$T = \begin{pmatrix} 0.3676 & 0.5882 & 0.0441 \\ 0.4625 & 0.5000 & 0.0375 \\ 0.8333 & 0.1667 & 0.0000 \end{pmatrix} \tag{3.5}$$

The probabilities of the states in the next time period, $P(t+1)$, are obtained by matrix multiplication:

$$P(t+1) = P(t) \times T \tag{3.6}$$

For example, if at time t, the price is definitively in the "Increase (I)" state, then $P(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$. The probabilities for the next period are:

$$P(t+1) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \times \begin{pmatrix} 0.3676 & 0.5882 & 0.0441 \\ 0.4625 & 0.5000 & 0.0375 \\ 0.8333 & 0.1667 & 0.0000 \end{pmatrix} \tag{3.7}$$

$$P(t+1) = \begin{bmatrix} 0.4625 & 0.5000 & 0.0375 \end{bmatrix} \tag{3.8}$$

These predicted probabilities can inform stakeholders about the likelihood of future price changes, aiding in decisions related to buying, selling, or storing green grams. To predict probabilities for n time steps into the future, the current state vector $P(t)$ is multiplied by the transition matrix raised to the power of n, i.e., $P(t+n) = P(t) \times T^n$.

To further explore the long-term behavior of the green gram price states, we can predict the state probabilities over multiple time periods. For instance, to predict the probabilities two time steps into the future, $P(t+2)$, we can multiply $P(t+1)$ by the transition matrix T:

$$P(t+2) = P(t+1) \times T \tag{3.9}$$

Using the result from our previous example, $P(t+1) = \begin{bmatrix} 0.4625 & 0.5000 & 0.0375 \end{bmatrix}$, we can calculate $P(t+2)$:

$$P(t+2) = \begin{bmatrix} 0.4625 & 0.5000 & 0.0375 \end{bmatrix} \times \begin{pmatrix} 0.3676 & 0.5882 & 0.0441 \\ 0.4625 & 0.5000 & 0.0375 \\ 0.8333 & 0.1667 & 0.0000 \end{pmatrix} \tag{3.10}$$

$$P(t+2) = \begin{bmatrix} 0.4373 & 0.5281 & 0.0346 \end{bmatrix} \quad (3.11)$$

By repeatedly multiplying the resulting probability vector by the transition matrix T, we can observe how the state probabilities evolve over time. This process can help us understand the long-term stability and convergence of the price states, providing valuable insights for long-term planning and risk management. Furthermore, analyzing the convergence of these probabilities towards a steady-state distribution can validate the long-run distribution obtained through other methods, reinforcing the robustness of the Markov model.

4 Conclusions

*This study successfully applied a three-state Markov model to analyze green gram price transitions in Kitui County, Kenya, revealing significant market volatility and dynamic price movements. High transition probabilities between "Decrease" and "Increase" states, indicating frequent price fluctuations. The transient nature of the "No change" state, highlighting the market's inherent volatility. The dominance of the transition probability matrix (**P**) in determining long-term price trends. Short Mean First Passage Times (MFPTs) between price states, indicating rapid market adjustments.The recurrent and irreducible nature of the Markov chain, signifying a dynamic equilibrium. It is important to note that the model assumed constant transition probabilities, and future research could explore time-varying probabilities to further enhance the model's applicability and capture the dynamic nature of agricultural markets.*

5 Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

6 Competing Interests

Authors have declared that no competing interests exist.

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