# *Commentary*

# A Review on BESO Algorithm Research and the Latest Developments

**Abstract:**

As an efficient structural topology optimization method, the BESO algorithm plays a crucial role in enhancing structural performance and achieving rational resource allocation in engineering design, materials science, and other fields. This paper systematically reviews the principle, research status, and existing problems of the BESO algorithm. First, the basic principles and implementation methods of the BESO algorithm are classified, and the applicable scenarios, optimization efficiency, and limitations of each algorithm are summarized. The research progress in multi-physics field optimization, complex structural design, and other directions is deeply discussed. Finally, the conclusion is drawn that the current BESO algorithm still has shortcomings in global optimization capability, calculation efficiency, and multi-constraint condition handling. It is necessary to further improve the algorithm convergence mechanism, integrate intelligent optimization strategies, and construct a more universal and efficient optimization theoretical system to meet the complex optimization needs of engineering practice.

**Keywords:** BESO algorithm; topology; structural performance.

**0 Introduction**

In the fields of engineering design and materials science, how to achieve efficient resource utilization while meeting performance requirements is a key issue that urgently needs to be addressed. With the increasing complexity of engineering structures and the diversification of optimization objectives, traditional optimization methods are gradually unable to meet practical needs. The BESO algorithm (Bidirectional Progressive Structural Optimization Algorithm), as an efficient topology optimization tool [1], can effectively explore the optimal layout of structures by gradually adding and removing materials. It has shown great potential for application in fields such as aerospace, mechanical manufacturing, and civil engineering, and has become a research focus in the field of structural optimization.

The research on BESO algorithm is committed to breaking through the bottleneck of existing structural design, achieving lightweight and high-performance design goals by exploring the synergistic potential of materials and structures, and playing an important role in promoting engineering technology innovation, reducing production costs, and enhancing product competitiveness[2]. This article systematically summarizes the basic principles and extended forms of the BESO algorithm, deeply analyzes its research progress in cutting-edge directions such as multi-objective optimization and dynamic response optimization, and explores the challenges and limitations faced by the algorithm in practical applications. The aim is to provide comprehensive references for the further development and engineering applications of the BESO algorithm.

**1 Basic theory of BESO algorithm**

The Bi-directional Evolutionary Structural Optimization (BESO) first discretizes the continuous structure, establishing element densities of 1 and 0, and identifying solid elements and empty elements, respectively. The setting of sensitivity threshold can be based on binary method, which first extracts and normalizes the sensitivity of all elements in the topology optimization results, and then determines the threshold for refining the element grid based on volume constraints; Alternatively, based on the target volume and evolution rate, calculate the contemporary target volume and the number of reduced units, and determine the threshold by combining the unit recovery ratio; Alternatively, based on strain energy, the unit strain energy can be sorted as sensitivity values and set deletion and recovery thresholds. Based on whether the deformation energy of the unit exceeds the threshold, the retention or removal of the grid is determined, and useless or inefficient materials are gradually removed to achieve topological optimization of the structure. Its mathematical description can be expressed as:

（1-1）

（1-2）



In the formula,  is the objective function, which commonly includes maximizing structural stiffness, minimizing weight, minimizing strain energy, etc; *xi* is the design variable, with a value of 1 for solid units and 0 for empty units; *V*\* is the total structural volume; *Vi* is the volume of a single element.

The three key parameters of the BESO algorithm are sensitivity threshold, evolution rate, and filtering radius. Sensitivity is the derivative of the objective function with respect to the design variable, which measures the sensitivity of the objective function to the influence of the design variable. Taking flexibility C (*X*) as an example, where *X* refers to the design variable, the mathematical expression for sensitivity can be expressed as:

（1-3）

Later, a new method was proposed, namely the unit sensitivity filtering method, which refers to the conversion of unit sensitivity from different degrees to node sensitivity, which can be expressed as:

（1-4）

In the formula, *k* is the number of nodes; ** is the original sensitivity of the nth unit;  is the node sensitivity of the node.

To avoid checkerboard phenomenon and grid dependence, a sensitivity filtering strategy is adopted. Namely, taking the target unit as the center, a spherical area within a filtering radius is determined, and the sensitivity of all units inside the sphere is weighted to obtain a new sensitivity value, which is then reassigned to the target unit. The filtering process can be expressed as:

（1-5）

（1-6）

In the formula,  represents improved sensitivity; *k* is the number of all nodes contained in subdomain ;  is the linear weight factor, and  is the filtering radius.

To solve the problem of non convergence between the objective function and the topology structure during the iteration process, the sensitivity of the previous iteration is averaged with the sensitivity of the current iteration, which can be expressed as:

（1-7）

In the formula, *n* is the current iteration count, and  can be used for the next iteration.

Before adding or deleting units, it is necessary to determine the target volume fraction of for the next iteration. The change in volume fraction before and after the iteration can be expressed as:

（1-8）

In the formula, *ER* is the evolution ratio of the material;  is the material volume fraction at the Kth iteration.

The iterative process stops when the volume constraint and objective function converge, and the convergence criterion is:

（1-9）

In the formula, *error* is the error of the objective function within *N* generations; *k* is the current iteration number;  is the allowable convergence error; *N* is an integer, usually set to 5, indicating that the objective function value has minimal variation over 10 consecutive generations.

**2 The main implementation methods of BESO algorithm**

**2.1 Classic bidirectional progressive structural optimization (BESO) method**

The classic BESO algorithm is based on the progressive addition and removal of materials as its core mechanism, and calculates element sensitivity based on finite element analysis results. During each iteration, by setting a sensitivity threshold, the unit materials that contribute less to the structural performance are gradually removed, and materials are added at key locations to optimize the structural topology [3]. This method optimizes the objective function (such as maximizing structural stiffness and minimizing flexibility) by continuously adjusting the material distribution to satisfy given constraints (such as volume constraints and stress constraints). The classic BESO algorithm is intuitive and easy to implement, demonstrating good optimization results in simple structural topology optimization problems. However, when dealing with complex boundary conditions and multi-objective optimization problems, it is prone to getting stuck in local optimal solutions, and the computational efficiency needs to be improved.

**2.2 Improved bidirectional progressive structural optimization (IBESO) method**

To overcome the limitations of the classical BESO algorithm, the improved BESO (IBESO) method [4] introduces multiple optimization strategies. For example, using adaptive sensitivity filtering technology, dynamically adjusting the filtering radius according to the iterative process, effectively suppressing the checkerboard phenomenon and grid dependence [5], and improving the quality and stability of optimization results; Optimize material addition and subtraction rules by introducing historical information or dynamic threshold adjustment mechanisms to enhance the algorithm's global search capability and avoid getting stuck in local optima. In addition, the IBESO method is often combined with other optimization algorithms such as genetic algorithms [6] and particle swarm optimization algorithms [7] to complement each other's strengths and weaknesses, further improving the efficiency and accuracy of the algorithm in solving complex optimization problems, making it suitable for fields such as aerospace and automotive industries that require strict structural performance[8]. The general trend has dominated the improvement direction of BESO algorithm, and core logic such as bidirectional evolution and multi-objective coupling have become the consensus of various variants; The uniqueness of a single IBESO implementation is usually reflected in customized strategies tailored to specific engineering scenarios, such as constraint conditions and penalty function design.

**2.3 Multi objective bidirectional progressive structural optimization (MO-BESO) method**

With the increasing diversification of engineering requirements, structural optimization often needs to consider multiple conflicting goals simultaneously [9], such as structural lightweighting and strength maximization, stiffness enhancement and vibration suppression. The MO-BESO method has emerged, which transforms multi-objective problems into single objective optimization problems or performs multi-objective parallel searches by introducing concepts such as weight factors and Pareto frontiers. During the iteration process, the MO-BESO algorithm optimizes multiple objective functions simultaneously, generating a set of Pareto optimal solutions [10]. Designers can select the most suitable solution from the solution set according to actual needs. This method provides an effective means for multi-objective optimization of complex engineering systems and has been widely applied in fields such as new energy equipment design and multifunctional integration of building structures.

**2.4 Dynamic response bidirectional progressive structural optimization (DR-BESO) method**

For structures subjected to dynamic loads such as earthquakes, wind loads, and mechanical vibrations [11], the DR-BESO method incorporates the dynamic response characteristics of the structure into the optimization objective. This method is based on the traditional BESO algorithm, combined with dynamic finite element analysis, using the natural frequency, mode shape, and dynamic response amplitude of the structure as optimization indicators, and optimizing the dynamic performance of the structure by adjusting the material distribution [12]. For example, in the design of mechanical components, the DR-BESO method can effectively reduce structural vibration noise; In seismic design of building structures, it can improve the seismic performance of the structure and reduce damage under earthquake action. By considering the dynamic characteristics of the structure, the DR-BESO method makes the optimization results more in line with practical engineering needs, expanding the application scope of the BESO algorithm.

**3 Characteristics and performance indicators of BESO optimization results**

The BESO algorithm achieves structural topology optimization through the gradual increase or decrease of materials, and its optimization results exhibit unique characteristics in structural morphology, material distribution, and other aspects. Performance indicators are the key basis for quantitatively evaluating the optimization effect. Thoroughly analyzing the characteristics and performance indicators of optimization results is of great significance for understanding the advantages of algorithms and guiding engineering applications.

**3.1 Characteristics of optimization results**

Topological structure features: The optimized structure generated by BESO algorithm usually presents a clear force transmission path, with materials concentrated in key stress areas, forming a continuous and efficient load transmission channel. For example, in the optimization of truss structures, the optimization results often exhibit a branching topology similar to biological bones, which can bear the maximum external load with the least amount of material. At the same time, the optimized topology structure has smooth boundary contours, effectively avoiding stress concentration phenomena. Compared with traditional empirical design, it demonstrates a more scientific mechanical rationality[13].

Material distribution characteristics: The material distribution optimized by BESO algorithm follows the principle of "on-demand allocation". In low stress areas, a large amount of material is removed to form hollow or lightweight structures; In areas of high stress concentration, materials are highly concentrated to ensure structural strength and stability. In addition, after being processed by density filtering technology, the material distribution transitions naturally, eliminating unreasonable discrete distribution phenomena such as checkerboard patterns, making the structure more in line with engineering manufacturing requirements[14].

Geometric features: The optimized structural geometry is closely related to the initial design area, boundary conditions, and load conditions. Under complex boundaries and multi-directional loads, the BESO algorithm can generate structures with complex shapes but excellent mechanical properties. For example, in the optimization of high-rise building structures under the combined action of wind and gravity, algorithms can generate geometric shapes with streamlined appearance and reasonable internal support, which not only meet the aesthetic requirements of the building, but also enhance the wind and earthquake resistance performance of the structure.

**3.2 Performance indicators**

Structural stiffness index: Structural stiffness is one of the core indicators for measuring the optimization effect of BESO, usually expressed as the reciprocal of structural flexibility. In the optimization process, the algorithm aims to improve the structural stiffness by adjusting the material distribution reasonably and enhancing the bearing capacity of key parts. The increase in stiffness of the optimized structure can be quantitatively evaluated by comparing displacement response, strain energy, and other parameters before and after optimization. For example, in the optimization of mechanical components, increasing stiffness can effectively reduce the deformation of the components under working loads and improve the accuracy of equipment operation.

Material efficiency index: Material efficiency reflects the degree of utilization of materials by optimization results. It is commonly measured by the ratio of the material volume of the optimized structure to the initial design area volume, or the ratio of structural performance to material usage[15]. The BESO algorithm significantly reduces material usage while ensuring structural performance by accurately removing redundant materials. For example, in aerospace structural design, the improvement of material efficiency means a reduction in structural weight, which in turn improves the fuel economy and load capacity of the aircraft.

Stability and reliability indicators: The stability and reliability of structures are key considerations in engineering applications. The optimization results of BESO can be evaluated by calculating the critical instability load, safety factor, fatigue life and other indicators of the structure. For example, in bridge structure optimization, it is necessary to ensure that the optimized bridge does not experience instability and failure under extreme loads (such as strong winds and earthquakes), and has sufficient fatigue life to ensure long-term safe use.

Dynamic performance indicators: For structures that bear dynamic loads, such as buildings under earthquake action and mechanical components in vibration environments, dynamic performance indicators are particularly important. Common dynamic performance indicators include natural frequency, mode shape, vibration amplitude, etc. In dynamic response optimization, the BESO algorithm adjusts the material distribution to change the mass and stiffness distribution of the structure, thereby optimizing the dynamic characteristics of the structure, reducing resonance risk, and improving the stability of the structure under dynamic conditions.

The characteristics and performance indicators of BESO optimization results are interrelated and jointly reflect the optimization effectiveness of the algorithm. In practical engineering applications, it is necessary to comprehensively consider these characteristics and indicators to obtain the optimal design solution that meets various performance requirements.

**4 Optimization process of classic BESO algorithm**

(1) Initialization: First, determine the design area, boundary conditions, load conditions, and initial distribution of design variables (material density) for the optimization problem. At the same time, key parameters of the BESO algorithm are set, such as evolution rate, convergence criterion, filtering radius, etc.

(2) Finite element analysis: Conduct finite element analysis on the structure based on the current material density distribution, and calculate the displacement, stress, and other responses of the structure[16].

(3) Sensitivity calculation: Using the response results obtained from finite element analysis, calculate the sensitivity index of each finite element element, reflecting the contribution of the element to the optimization objectives (such as structural compliance, stress, etc.).

(4) Density update: Based on sensitivity indicators, update the material density of each unit according to specific rules of the BESO algorithm. Usually, the high-density parts are retained and the low-density parts are removed, but a certain proportion of intermediate density units are retained for the next iteration optimization[17].

(5) Convergence judgment: Check whether the convergence criteria are met, such as whether the changes in structural response, material volume, etc. are within the set convergence range.

(6) Iteration: If the convergence criterion is not met, return to the finite element analysis step and proceed to the next iteration; If satisfied, output the optimized material density distribution and obtain the topological form of the structure [18].

The above is the standard process of the classic BESO algorithm, covering the entire process from initialization to final convergence and output optimization results. It is the foundation for subsequent improvements and extensions of the BESO algorithm.

**5 Research status of BESO algorithm**

Since its proposal, the BESO algorithm has sparked a widespread research boom worldwide. Compared to developed countries such as Europe and America, China started systematic research on the BESO algorithm slightly later. In the early days, foreign scholars were the first to explore the basic theory and framework of algorithms, while China was in the stage of learning and verifying classical algorithms. But with the rapid development of computing technology and the increasing demand for engineering optimization, China has been catching up in this field in recent years, and research results at home and abroad have shown a trend of multi-point flowering and rapid iteration, mainly reflected in the following aspects:

(1) Deepening and Innovation of Algorithm Theory: At the core mechanism level of algorithms, scholars at home and abroad have conducted extensive improvement research on the classic BESO algorithm, which is prone to falling into local optima and has low computational efficiency. For example, in order to enhance global search capabilities, adaptive threshold adjustment strategies are widely used[19]. By dynamically changing the threshold for adding or removing materials, the algorithm can quickly remove redundant materials in the early stages of iteration and finely adjust the structural topology in the later stages; In terms of computational efficiency optimization, the combination of fast finite element solving techniques and parallel computing methods significantly reduces the computation time for complex structural optimization. Tang Pengshan et al. established an improved bidirectional asymptotic structural optimization analysis under the topology optimization model of steel nodes, introduced sensitivity filtering radius and threshold segment, combined with convergence parameters, and through weighted processing and iterative optimization, found that SJ-BESO effectively improved computational efficiency and overcame adverse phenomena compared to BESO. Further comparison with SIMP through bifurcation nodes shows that the material utilization efficiency is higher, and the results indicate that SJ-BESO provides good computational efficiency and optimization effect for topology optimization of such steel nodes[20]. Nanbo et al. conducted topology optimization analysis of structures under stress constraints by establishing a model combining continuum structures with improved bidirectional asymptotic structure optimization methods. By comparing and verifying the topology optimization methods with engineering examples, the effectiveness of this method in solving stress constrained topology optimization problems was obtained. The results showed that it exhibited good optimization effects for such structures[21]. Zhang Manzhe et al. established a bidirectional asymptotic structural topology optimization model under the modified even stress elasticity theory, and obtained the results of the influence of size effects and related parameters on the optimization design by combining sensitivity analysis and interpolation functions of design variable iteration updates. The results showed that this type of structure has potential in the direction of size effect related topology optimization design[22]. Li Wenzheng et al. analyzed the sensitivity penalty and clustering algorithm under the optimization model of multi-layer fiber composite materials, and obtained the optimized fiber orientation distribution and laying path by using the bidirectional progressive optimization method and finite element theory. The results showed that this method effectively solved the numerical problem, improved the optimization efficiency and structural stiffness, and reduced the structural volume fraction[23]. Wang Linjun et al. conducted bidirectional asymptotic structural optimization analysis by establishing a reliability topology optimization model for continuum structures. With the help of reliability index method and moving asymptote method, the optimized structural topology configuration was obtained. The results showed that this method can efficiently find the limit state points and the obtained topology configuration is more reliable than traditional methods[24].

(2) Expansion of Multi Domain Engineering Applications: The BESO algorithm, with its powerful topology optimization capabilities, has achieved breakthroughs in numerous engineering fields. The BESO algorithm plays a crucial role in the fields of aerospace, civil engineering and construction, as well as mechanical manufacturing. Huang Chenglei et al. established a bidirectional progressive structural optimization analysis under the topology optimization model of solar unmanned aerial vehicle wings, with the minimum strain energy as the objective and the wing rib volume fraction as the constraint. By using sensitivity analysis to add, delete and redesign the internal materials of the wing ribs, the results showed that this method effectively improved material utilization and provided strong reference for lightweight research of solar unmanned aerial vehicles[25]. Cai Qi et al. proposed the T-BESO method by establishing a bidirectional progressive structural optimization analysis model for truss structures, deriving optimization formulas using energy principles and full stress design criteria. This method takes the cross-sectional area of the members as the design variable, the structural strain energy as the objective function, and stress constraints and full stress design criteria as the constraint conditions. The results show that this method is more efficient in optimizing truss structures and can overcome the shortcomings of the traditional BESO method[26]. Li Mingzhen et al. established a topology optimization model for the roller of the oil tea seed huller, and used the bidirectional asymptotic structural optimization method (BESO) and simulation experiments to write an optimization program for structural optimization. The design of the new roller was obtained, and the results showed that the stress distribution of the new roller was more uniform, the volume and mass were reduced by 37.2%, and the moment of inertia was reduced by 31.8%, effectively reducing costs and energy consumption[27]. Jing Haiquan et al. conducted an interactive analysis between finite element and topology program under the joint topology optimization model of wind turbine tower, and achieved the optimization design of tower structure with the help of optimization program. The results showed that the optimized tower structure had reduced mass and improved performance[28].Salman Khayoon Aldriasawi et al. designed experiments based on the Taguchi method and used artificial neural networks (ANN) to predict the mechanical properties such as hardness, grain size, and residual stress of AISI 1035 steel after nanofluid quenching. This study optimizes material properties through data-driven methods and complements the BESO algorithm in achieving structural performance improvement based on material distribution adjustment, reflecting the different paths of material performance optimization and structural topology optimization in material structure collaborative design [29]. Salah Amrone et al. proposed an algorithm for calculating the distribution of turbine blades based on electronic scale data to address the issue of turbine rotor balance. By simulating blade assembly using MATLAB, the algorithm optimized the static balance of the rotor. This method focuses on adjusting the quality distribution, in contrast to the BESO algorithm that achieves structural lightweighting through material topology optimization. The two approach mechanical structural performance optimization from different perspectives, jointly expanding the application scope of optimization techniques in the field of mechanical engineering [30].

(3) Interdisciplinary integration and integration of new technologies: With the deepening of interdisciplinary research, the BESO algorithm is deeply integrated with disciplines such as fluid mechanics, thermodynamics, and electromagnetics. In multi physics field coupling optimization, scholars establish a multi field collaborative objective function and use the BESO algorithm to achieve synchronous optimization of structure and performance. Duan Xianbao et al. conducted a bidirectional asymptotic structural optimization analysis under a fluid dynamics topology optimization model, and combined the artificial permeability term and conjugate method with the BESO method to obtain the optimized fluid flow path. The results showed that this method has stability and effectiveness in dealing with fluid dynamics topology optimization problems[31]. Xia Xuyu analyzed anisotropic materials and thermoelastic structures by establishing a bidirectional asymptotic structural topology optimization model. With the help of improved BESO method and Lagrange number multiplication, the optimized structural configuration and performance improvement results were obtained. The results showed that the proposed method is effective in topology optimization of anisotropic material structures, thermoelastic structures, and dual scale designs[32]. Lin Fengchun et al. conducted multi-objective topology optimization analysis under the establishment of a multifunctional microstructure material optimization model, and obtained optimized multifunctional structures using an improved bidirectional evolutionary structure optimization method (BESO). The results showed that this method can effectively design multifunctional materials that meet various needs, providing an effective approach for the functional design of microstructure materials[33].Barhm Mohamad et al. combined CFD analysis with SolidWorks and Creo 4.0 software to conduct frequency domain analysis on the intake system of racing cars to optimize acoustic performance. This study achieved component optimization through multidisciplinary coupling modeling, and compared it with the performance improvement path of BESO algorithm through topology adjustment in multi physics field coupling optimization, highlighting the unique potential and technical challenges of BESO algorithm in interdisciplinary applications for structural topology optimization [34].

Although BESO algorithm research has achieved fruitful results, there are still urgent problems that need to be solved. In the optimization of complex engineering systems, algorithms face exponential growth in computational complexity when dealing with high-dimensional and multi constraint problems, which limits their application efficiency; The conversion standards of existing research results in different fields have not been unified, and there is a lack of systematic engineering application standards; In addition, the theory and practice of algorithms in cutting-edge directions such as microscale material design and biomimetic structure optimization are still in the exploratory stage. In the future, it is necessary to further deepen the research on algorithm theory, strengthen interdisciplinary collaborative innovation, and promote the efficient application of BESO algorithm in more engineering scenarios.

**6 Conclusion**

In recent years, with the continuous advancement of research related to the BESO algorithm, scholars at home and abroad have achieved fruitful results in algorithm improvement, application expansion, and interdisciplinary integration. However, at present, there is still a problem of fragmentation in the research results of BESO algorithm, and there is a lack of systematic integration of research in different fields. A standardized theoretical and application system with strong universality and direct guidance for complex engineering practices has not yet been formed. On the basis of existing research, it is urgent to further explore the optimization mechanism of algorithms in high-dimensional complex constraint scenarios, effectively balancing computational efficiency and optimization accuracy; At the same time, it is necessary to strengthen the quantitative analysis of various uncertain factors in cutting-edge directions such as multi physics field coupling and dynamic real-time optimization, and establish a more comprehensive theoretical framework for BESO algorithm. To promote the deep application and innovative development of BESO algorithm in the engineering field, it still requires joint exploration and long-term efforts from the academic and engineering communities.

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