Study of second-class currents in radiative quasi-elastic neutrino (and antineutrino) scattering on atomic nuclei

Abstract

This paper examines the influence of the F_T form factor, associated with second-class tensor currents (SCCs), in the quasi-elastic radiative scattering of neutrinos on ^{12}C carbon. A general expression for the cross-section is derived via a multipole decomposition of the hadronic currents, taking into account the circular polarization of the emitted photons. The study focuses on the impact of F_T on three sensitive observables : charge asymmetry, angular correlation and triple correlation. Numerical results show that F_T can produce significant effects, particularly at low angles and for an incident energy of 300 MeV. For example, with $F_T = 5 \times 10^{-3}\,\text{MeV}^{-1}$, contributions can reach 67% for charge asymmetry, 31% for angular correlation, and 1% for triple correlation at 10°. These results suggest that precise measurements under these conditions could experimentally reveal the existence of second-class currents.

Keywords : second-class current, charge asymmetry coefficient, correlation coefficient, triple correlation, neutrino scattering, charged current.

1 Introduction

The study of second-class currents (SCCs) via radiative neutrino scattering is a specialized field of neutrino physics, characterized by subtle interactions between neutrinos and matter. These currents, also known as second-class axial currents, are associated with parity-violating transitions and are generally less explored than first-class currents.

CSCs have mainly been studied in low-energy processes, such as β decays or muon capture by nucleons and nuclei [2–7]. On the other hand, few experiments have been carried out at high energies, notably in the context of elastic and quasi-elastic (anti)neutrino scattering, where the study of these currents remains limited [8–10]. However, measurements of angular correlations in β decays seem to have provided clearer indications of the existence of these currents [11, 12].

In their 2018 publication, Fatima et al. pointed out that, thanks to current precision experiments using neutrino and antineutrino beams, it is now conceivable to probe CSCs in quasielastic interactions, particularly in connection with the strangeness component [13]. The work of M. Sajjad Athar et al. on neutrino scattering by nucleon-charged currents has shown that cross sections are sensitive to the presence of CSCs. This highlights the fact that whether or not these currents are taken into account can have a significant impact on the determination of the phenomenological parameter of axial mass (M_A), often evaluated by assuming the absence of CSC [14].

In this study, we specifically examine the influence of the F_T form factor, associated with CSCs, in the quasi-elastic neutrino scattering process, described by the following reaction :

$$\nu(\tilde{\nu}) + (A,Z) \longrightarrow \ell^-(\ell^+) + (A, Z \pm 1)^* \longrightarrow (A, Z \pm 1) + \gamma_{RL} \tag{1}$$

2 Differential cross section for quasi-elastic scattering

In first-order perturbation theory, the square of the matrix element associated with the process (1) is expressed as [15] :

$$\sum_{\mathcal{M}_i\mathcal{M}_f} |\mathcal{M}_{fi}|^2 = \frac{1}{2J_i+1} \sum_{\mathcal{M}_i\mathcal{M}_f} |\sum_{\mathcal{M}_n} \mathcal{M}_{fn}\mathcal{M}_{ni}|^2 \tag{2}$$

In this expression, $\mathcal{M}_{ni}$ and $\mathcal{M}_{fn}$ represent respectively the matrix elements of the weak nuclear transition and the gamma photon emission, defined by :

$$\mathcal{M}_{ni} = -\frac{G_F}{\sqrt{2}} \ell^z_\mu J^z_\mu \tag{3}$$

$$\mathcal{M}_{fn} = -\frac{2\pi}{\sqrt{\omega\Omega}} \sum_{J_1\geq 1} (-i)^{J_1}[J_1] D^{J_1}_{M_1\sigma}(\phi_\gamma, \theta_\gamma, \phi_\gamma) \begin{pmatrix} J_f & J_1 & J_n \\ -M_f & M_1 & M_n \end{pmatrix} \left\langle J_f || \sigma \hat{T}^m_{J_1} - \hat{T}^e_{J_1} || J_n \right\rangle \tag{4}$$

Here $[J_1] = \sqrt{2J_1+1}$.

The magnetic and electric multipole operators $\hat{T}^m_{J_1\sigma}$ and $\hat{T}^e_{J_1\sigma}$ are defined in a coordinate system $X'Y'Z'$ where the z-axis is oriented along the momentum $\vec{k}$ of the photon. In order to define all multipole operators in a coordinate system where the z axis is oriented along the transferred momentum quantity $\vec{q}$, we rotate the coordinate system $X'Y'Z'$ by the angles ϕ_γ and θ_γ. This gives us the relationship [16] :

$$\hat{T}^{m(e)}_{J_1\sigma}(k) = \sum_{M_1} \hat{T}^{m(e)}_{J_1M_1} D^{J_1}_{M_1\sigma}(\phi_\gamma, \theta_\gamma, \phi_\gamma). \tag{5}$$

The function $D^{J_1}_{M_1\sigma}(\phi_\gamma, \theta_\gamma, \phi_\gamma)$ is a Wigner function, and the angles θ_γ and ϕ_γ determine the direction of gamma photon emission.

The constant G_F corresponds to the Fermi constant characterizing the weak interaction. The weak lepton and hadron currents are given by :

$$\ell^z_\mu = \bar{u}_2\gamma_\mu(a_V + a_A\gamma_5)u_1 \tag{6}$$

$$J^z_\mu(q) = \left\langle J_nM_n | \int d\vec{x} exp(-i\vec{q}_\mu\vec{x}) \hat{\mathcal{J}}^{ni}_\mu(\vec{x}) | J_iM_i \right\rangle \tag{7}$$

The constants a_V and a_A represent the vector and axial coupling coefficients of the weak lepton current, their values depending on the physical process under consideration. The Dirac spinners

$u_j (j = 1, 2)$ describe the incoming and outgoing leptons, while $q_\mu = (\mathbf{q}, iq_0)$ is the transferred momentum quadrivector.

The operator $\hat{\mathcal{J}}_\mu^{ni}(\vec{x})$ represents the hadronic current density, which can be decomposed into a vector component and an axial component, taking into account the isotopic structure of the nucleus [16] :

$$\hat{\mathcal{J}}_\mu^{ni} = \beta_V^{(\tau)} (J_\mu)_{\tau M_\tau} + \beta_A^{(\tau)} (J_\mu^5)_{\tau M_\tau} \tag{8}$$

In the case of charged-current transitions, we have $\tau = 1$, $M_\tau = \pm 1$ and the coefficients $\beta_V^{(\tau)} = \beta_A^{(\tau)} = 1$, giving for the hadronic current :

$$J_\mu = (J_\mu)_{1 M_\tau} + (J_\mu^5)_{1 M_\tau} \tag{9}$$

The cross section of the process described by equation (1) occurring in a certain direction and with photon emission can be expressed using the so-called multipole matrix element decomposition method, used in several scientific works [17–19]. This method provides a better understanding of the different contributions to the transition probability.

The following formula gives this differential probability, i.e. the probability per unit solid angle for the neutrino (Ω_ν) and for the emitted photon (Ω_γ) :

$$\begin{aligned}
\frac{d\sigma}{d\Omega_\nu d\Omega_\gamma} = \frac{\Gamma_\gamma^{(n\to f)}}{\Gamma_{total}^{(n\to f)}} \sum_{\tau\tau'} \begin{pmatrix} \tau_n & \tau & \tau_i \\ -M_{\tau_n} & M_\tau & M_{\tau_i} \end{pmatrix} \begin{pmatrix} \tau_n & \tau' & \tau_i \\ -M_{\tau_n} & M_\tau & M_{\tau_i} \end{pmatrix} \\
\times \frac{d\sigma_0}{d\Omega_\nu} \{1 + \frac{1}{K_0^0} \sum_{L\geq 2} f_L^{(n\to f+\gamma)} (P_L(cos\theta_\gamma) K_L^0 \\
+ P_L^1(cos\theta_\gamma) cos\Phi_\gamma K_L^1 + P_L^2(cos\theta_\gamma) cos 2\Phi_\gamma K_L^2) \\
+ \frac{s_\gamma}{K_0^0} \sum_{L\geq 1} f_L^{(n\to f+\gamma)} (P_L(cos\theta_\gamma) \widetilde{K}_L^0 + P_L^1(cos\theta_\gamma) cos\Phi_\gamma \widetilde{K}_L^1) \\
+ \frac{s_\gamma}{K_0^0} \sum_{L\geq 3} f_L^{(n\to f+\gamma)} (P_L^2(cos\theta_\gamma) 2 cos\Phi_\gamma \widetilde{K}_L^2)\}
\end{aligned} \tag{10}$$

In this expression :

- $\Gamma_\gamma^{(n\to f)}$ is the partial probability (or relative width) associated with the emission of a photon during the transition of the nucleus from the n state to the f state,
- $\Gamma_{total}^{(n\to f)}$ is the total transition probability,
- s_γ corresponds to the helicity of the photon, i.e. the direction of its rotation,
- $\frac{d\sigma_0}{d\Omega_\nu}$ is the basic probability for neutrino scattering when the nucleus is not polarized, given by :

$$\frac{d\sigma_0}{d\Omega_\nu} = \frac{16\pi^3 G_F^2}{2\omega\Omega(2J_i + 1)} K_0^0 \tag{11}$$

The quantum number L, which indicates the angular momentum component in the transition, takes integer values from 0 up to $2J_n$, where J_n is the spin of the excited state of the nucleus. The functions $P_L^M(\cos\theta_\gamma)$ are associated Legendre polynomials.

Finally, the functions $f_L^{(n\to f+\gamma)}$ represent the various multipolar contributions to the transition and are defined by :

$$f_L^{(n\to f+\gamma)} = \begin{cases} \dfrac{Y_L^T(q_0)}{Y_0^T(q_0)} \text{ pour } L \text{ paire} \\ \\ \dfrac{Y_L^{T'}(q_0)}{Y_0^T(q_0)} \text{ pour } L \text{ impaire} \end{cases} \tag{12}$$

The function $Y_L^T(q_0)$, is defined by the following expression :

$$\begin{aligned} Y_L^T(q_0) = -(-)^{J_f+J_n} \sum_{J_1' J_1} [J_1'][J_1][L] \begin{pmatrix} J_1 & J_1' & L \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} J_1 & J_1' & L \\ J_n & J_n & J_f \end{Bmatrix} \\ \times\{P^+_{J_1'+J_1}(F_{EJ_1'}F_{EJ_1} + F_{MJ_1'}F_{MJ_1}) + P^-_{J_1'+J_1}(F_{EJ_1'}F_{MJ_1} - F_{MJ_1'}F_{EJ_1})\} \end{aligned} \tag{13}$$

where q_0 is the energy associated with the gamma transition.

The quantities J_1 and J_1' are quantum numbers related to the angular momentum of the system, and they must satisfy the following condition : $|J_n - J_f| \leq J_1(J_1') \leq J_n + J_f$
The symmetry factors $P^{\pm}_{J'+J}$ are defined by :

$P^{\pm}_{J'+J} = \frac{1}{2}(-)^{\frac{1}{2}(J'-J+\eta)}(1 \pm (-)^{J'+J})$ with,

$$\eta = \begin{cases} 0 \text{ pour } P^+_{J'+J} \\ 1 \text{ pour } P^-_{J'+J} \end{cases} \tag{14}$$

The functions K_L^m and $\widetilde{K}_L^m$ that appear in the differential probability expression are given by :

$$\begin{aligned} K_L^0 &= f_1 Y_1^L + f_2 Y_2^L + f_3 Y_3^L + f_4 Y_4^L + f_5 Y_5^L \\ K_L^1 &= f_6 Y_6^L + f_7 Y_7^L + f_8 Y_8^L + f_9 Y_9^L \\ K_L^2 &= f_{10} Y_{10}^L \\ \bar{K}_L^0 &= f_1 \bar{Y}_1^L + f_2 \bar{Y}_2^L + f_3 \bar{Y}_3^L + f_4 \bar{Y}_4^L + f_5 \bar{Y}_5^L \\ \bar{K}_L^1 &= f_6 \bar{Y}_6^L + f_7 \bar{Y}_7^L + f_8 \bar{Y}_8^L + f_9 \bar{Y}_9^L \\ \bar{K}_L^2 &= f_{10} \bar{Y}_{10}^L \\ \bar{K}_1^2 &\equiv 0 \end{aligned} \tag{15}$$

Here, the f_i functions (with $i = 1, 2, \ldots, 10$) represent the leptonic contributions, while Y_i^L and $\bar{Y}_i^L$ designate the corresponding hadronic functions. Detailed expressions for these functions, including the effects of longitudinal electron (or positron) polarization and lepton masses, are given in the appendix.

3 Quasi-elastic scattering of neutrinos (or antineutrinos) on the carbon nucleus ^{12}C

As an example of transition, consider the following diffusion processes :

$$\nu(\tilde{\nu}) + {}^{12}\mathrm{C} \longrightarrow e^-(e^+) + {}^{12}\mathrm{N}^*({}^{12}\mathrm{B}^*) \longrightarrow {}^{12}\mathrm{N}({}^{12}\mathrm{B}) + \gamma_{RL} \tag{16}$$

This is an interaction between a neutrino (or antineutrino) and a carbon nucleus ^{12}C, producing an electron (or positron) and an excited boron or nitrogen nucleus. This excited nucleus then returns to its ground state, emitting a gamma photon.

In this case :

The quantum number L, which describes the orbital angular momentum of the transition, varies between 0 and J_n. As here $J_n = 1$, we have $0 \leq L \leq 2$.

The total angular momentum J and J' satisfy the condition $|J_i - J_n| \leq J(J') \leq J_i + J_n$, giving $J = J' = 1$ in this process.

The differential cross section for this process, deduced from equation (16), is written :

$$\frac{d\sigma}{d\Omega_\nu d\Omega_\gamma} = \frac{\Gamma_\gamma^{(n\to f)}}{\Gamma_{total}^{(n\to f)}} \Sigma_0 \{K_0^0 + f_2^{(n\to f+\gamma)}(P_2(cos\theta_\gamma)K_2^0 + P_2^1(cos\theta_\gamma)cos\phi_\gamma K_2^1 \\ + P_2^2(cos\theta_\gamma)cos2\phi_\gamma K_2^2) + s_\gamma f_1^{(n\to f+\gamma)}(P_1(cos\theta_\gamma)K_1^0 + P_1^1(cos\theta_\gamma)cos\phi_\gamma K_1^1)\}exp(-2y) \tag{17}$$

where, $\Sigma_0 = \dfrac{8\pi^3 G_F^2}{3\omega\Omega}$; $f_1^{(n\to f+\gamma)} = -\dfrac{\sqrt{3}}{\sqrt{2}}$; $f_2^{(n\to f+\gamma)} = \dfrac{1}{\sqrt{2}}$.

The K_L^m functions describing the various multipolar components of the transition are given by :

$$\begin{aligned}
K_0^0 &= \frac{1}{\sqrt{3}}\{v_1H_1 - v_2H_2 + v_3H_3 + v_4H_4 + v_5H_5\}, \\
K_2^0 &= \frac{1}{\sqrt{6}}\{v_1H_1 - v_2H_2 - 2v_3H_3 - 2v_4H_4 - 2v_5H_5\}, \\
K_2^1 &= \frac{1}{\sqrt{6}}\{v_6H_6 - v_7H_7 + v_8H_8 - v_9H_9\}, \\
K_2^2 &= \frac{1}{2\sqrt{6}}v_{10}H_{10}, \\
K_1^0 &= \frac{1}{\sqrt{2}}\{-v_1H_2 + v_2H_1\}, \\
K_1^1 &= \frac{1}{\sqrt{2}}\{-v_6H_7 + v_7H_6 - v_8H_9 + v_9H_8\}.
\end{aligned} \tag{18}$$

4 Angular correlation coefficient

Consider the angular correlation coefficient between the emitted photon and the neutrino (or antineutrino), defined by the following expression :

$$A_{\nu\gamma} = \frac{d\sigma(\vec{p_\gamma} \uparrow\uparrow \vec{p_\nu}) - d\sigma(\vec{p_\gamma} \uparrow\downarrow \vec{p_\nu})}{d\sigma(\vec{p_\gamma} \uparrow\uparrow \vec{p_\nu}) + d\sigma(\vec{p_\gamma} \uparrow\downarrow \vec{p_\nu})} \tag{19}$$

where $\vec{p}_\gamma$ and $\vec{p}_\nu$ denote the photon and neutrino (or antineutrino) impulse vectors respectively. The symbols $\uparrow\uparrow$ and $\uparrow\downarrow$ indicate that the directions of these momenta are respectively parallel or antiparallel.

In the context of radiative emission following a weak interaction in the ^{12}C nucleus, a numerical analysis of the energy dependence of the coefficient $A_{\nu\gamma}(E_\nu, \theta)$, for a scattering angle $\theta = 10°$ and for $F_T = 5.10^{-3} MeV^{-1}$, has been carried out. This study highlights the effect of the second-class current (SCC) on this correlation coefficient. The relative contribution of this current, denoted $\delta_{A_{\nu\gamma}}$, increases significantly with neutrino energy E_ν. More precisely, the following values were obtained :

E_ν (MeV)	$\delta_{A_{\nu\gamma}}$ ($\theta = 10°$)	$\delta_{A_{\nu\gamma}}$ ($\theta = 20°$)
20	1 %	1 %
50	3 %	4 %
200	16 %	27 %
300	31 %	13 %

TABLE 1 – Comparison of the relative contributions $\delta_{A_{\nu\gamma}}$ for different neutrino energies and two angular configurations.

These results show that the angular correlation coefficient $A_{\nu\gamma}$ is a particularly sensitive tool for detecting the possible presence of second-class currents, especially at high energies. This makes it a key parameter in experimental research aimed at testing the limits of the Standard Model of particle physics.

Figure (1) illustrates the variation of this relative contribution as a function of neutrino energy and scattering angle, providing a useful tool for analyzing the presence and potential impact of CSC in nuclear weak interactions.

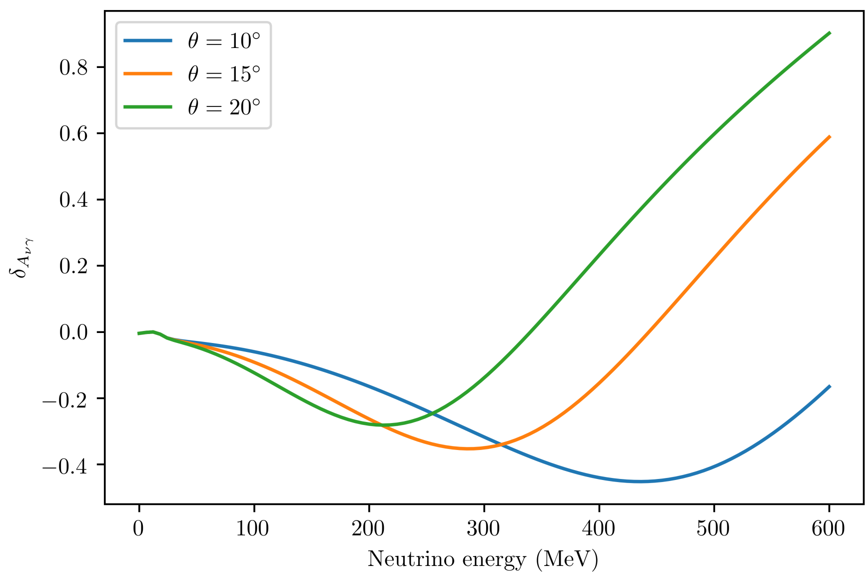

FIGURE 1 – Relative contribution of SCC to the angular correlation coefficient

5 Triple Correlation Coefficient

The triple correlation coefficient $T_{\nu\gamma}$ is a physical quantity used to detect possible asymmetries in particle emission during a decay process. It is defined as follows :

$$T_{\nu\gamma} = \frac{d\sigma(\vec{p_\gamma} \uparrow\uparrow \vec{p_\nu} \times \vec{p_e}) - d\widetilde{\sigma}}{d\widetilde{\sigma}} \tag{20}$$

$d\sigma(\vec{p_\gamma} \uparrow \vec{p_\nu} \times \vec{p_e})$ is the differential cross section associated with a configuration in which the photon pulse is aligned with the vector $\vec{p_\nu} \times \vec{p_e}$, i.e. perpendicular to the plane formed by the neutrino and electron pulses, $d\widetilde{\sigma}$ is the isotropic component of the differential cross section, i.e. the part that does not depend on the photon's emission angles $(\theta_\gamma, \phi_\gamma)$.

This coefficient is sensitive to certain fine properties of the interactions, in particular to the presence of SCC, which are forbidden in certain fundamental symmetries of the Standard Model, but could appear if new interactions were present.

In the particular case of neutrino scattering on a ^{12}C nucleus, with gamma photon emission, the study was carried out for a scattering angle fixed at $\theta = 10°$.

Numerical analysis of the function $T_{\nu\gamma}(E_\nu, \theta)$, which represents the dependence of the angular correlation coefficient on neutrino energy, shows that the relative contribution of the second-class current, denoted $\delta_{T_{\nu\gamma}}$, increases with neutrino energy. More precisely :

$\boldsymbol{E_\nu}$ (MeV)	$\boldsymbol{\delta_{T_{\nu\gamma}}}$ ($\boldsymbol{\theta = 10°}$)	$\boldsymbol{\delta_{T_{\nu\gamma}}}$ ($\boldsymbol{\theta = 20°}$)
20	0.3 %	1 %
50	0.4 %	2 %
200	0.7 %	4 %
300	1 %	7 %

Table 2 – Relative contributions of the second-class current to the function $T_{\nu\gamma}(E_\nu, \theta)$ for different neutrino energies and two angular configurations.

These results suggest that the influence of second-class currents becomes increasingly significant at high energies, making them a good indicator for testing the limits of the Standard Model.

The figure (2) illustrates this growth : it shows how the relative contribution of SCC to the angular triple correlation coefficient varies as a function of neutrino energy and scattering angle.

6 Charge Asymmetry

The charge asymmetry coefficient $A_{\nu\tilde{\nu}}$ quantifies the difference in behavior between a neutrino and an antineutrino during a scattering process. It is defined by the following relationship :

$$A_{\nu\tilde{\nu}} = \frac{d\sigma_\nu - d\sigma_{\tilde{\nu}}}{d\sigma_\nu + d\sigma_{\tilde{\nu}}} \tag{21}$$

where :

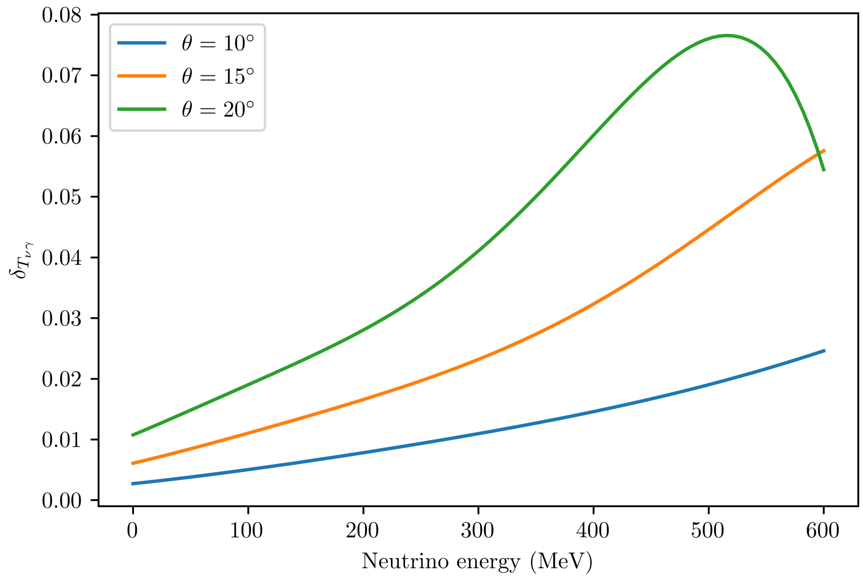

FIGURE 2 – Relative contribution of SCC to the angular correlation coefficient

- $d\sigma_\nu$ represents the differential interaction cross-section for a neutrino,

.

- $d\sigma_{\tilde{\nu}}$ is the same cross-section but for an antineutrino.

This coefficient therefore measures the asymmetry between the two types of particle, and makes it possible to explore effects linked to the violation of certain fundamental symmetries, notably the charge conjugation symmetry(C).

In the case where the photon is emitted in the same direction as the incident neutrino momentum, a numerical study has been carried out to analyze how this coefficient $A_{\nu\tilde{\nu}}$ varies as a function of neutrino energy, at a scattering angle fixed at 10°. This analysis was carried out assuming a value for the form factor $F_T = 5.10^{-3} MeV^{-1}$, which corresponds to an assumption about the strength of the SCC.
The results show that the relative contribution of SCC to the charge asymmetry coefficient, denoted $\delta_{A_{\nu\tilde{\nu}}}$, increases strongly with neutrino energy : These results clearly show that at high

$\boldsymbol{E_\nu}$ (MeV)	$\boldsymbol{\delta_{A_{\nu\tilde{\nu}}}}$ ($\boldsymbol{\theta = 10°}$)	$\boldsymbol{\delta_{A_{\nu\tilde{\nu}}}}$ ($\boldsymbol{\theta = 20°}$)
20	2 %	2 %
50	20 %	19 %
200	58 %	45 %
300	67 %	49 %

TABLE 3 – Relative contributions of the second-class current to the charge asymmetry coefficient $\delta_{A_{\nu\tilde{\nu}}}$ at different neutrino energies and for two angular configurations.

energies, the second-class current plays an increasingly significant role in the difference between neutrino and antineutrino behavior. When the scattering angle is fixed at $\theta = 20°$, a similar trend is observed, though with slightly reduced contributions.

Figure (3) illustrates these results by plotting the evolution of the relative contribution of the second-class current to the charge asymmetry coefficient $\delta_{A_{\nu\tilde{\nu}}}$, as a function of incident neutrino energy, for different scattering angles.

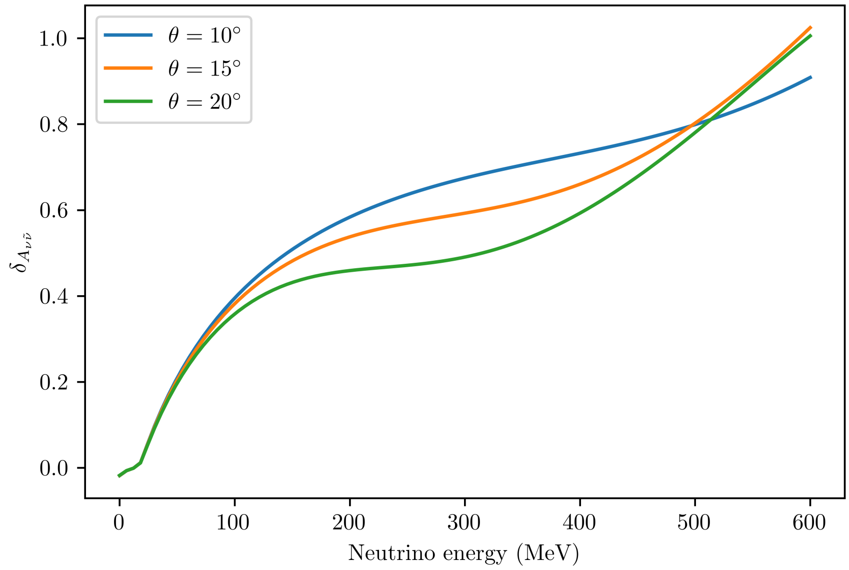

FIGURE 3 – Relative contribution of SCC to the charge asymmetry coefficient

7 Conclusion

In this paper, we have established the general expression of the differential cross section for the neutrino-nucleus radiative scattering process via the charged current. Numerical analysis of the angular correlation coefficient, the triple correlation coefficient and the charge asymmetry coefficient for quasi-elastic neutrino scattering on nuclei, revealed significant sensitivity to the possible presence of second-class currents. In particular, for a typical value of the CSC form factor ($F_T = 510^{-3}MeV^{-1}$), the contribution of these currents can modify the charge asymmetry and angular correlation coefficients by tens of percent, depending on the energy of the incident neutrino (or antineutrino). These results highlight the potential interest of experimental studies of the quasi-elastic neutrino scattering process on nuclei, accompanied by gamma radiation, as a sensitive tool for probing the existence of second-class currents, the detection of which would represent a significant advance beyond the Standard Model.

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8 Appendix

The most general form of the lepton current is :
$\ell_\mu = (\vec{\ell}, \ell_4 = i\ell_0) = \bar{u}_2\gamma_\mu(a_V + a_A\gamma_5)u_1$

In the case of quasi-elastic diffusion we have : $a_V = a_A = 1$ et $\ell_\mu = \bar{u}_2\gamma_\mu u_1$

The leptonic tensor $\ell_\mu \ell_\nu^*$ is given by : $\ell_\mu \ell_\nu^* = \delta_\nu \bar{u}_2\gamma_\mu(a_V + a_A\gamma_5)u_1\bar{u}_1(a_V - a_A\gamma_5)\gamma_\nu u_2$,

$$\delta_\nu = \begin{cases} -1, & \nu = 1,2,3 \\ +1, & \nu = 4 \end{cases}$$

$\ell_\mu \ell_\nu^* = \delta_\nu Tr\left\{\gamma_\mu(a_V + a_A\gamma_5)\Lambda_1(a_V - a_A\gamma_5)\gamma_\nu\Lambda_2\right\}$,
$\Lambda_1 = \frac{1}{4\epsilon E_1}(m_1 - i\epsilon\hat{P}_1)(1 - i\hat{S}_1\gamma_5)$ et $\Lambda_2 = \frac{1}{4\epsilon E_2}(m_2 - i\epsilon\hat{P}_2)(1 - i\hat{S}_2\gamma_5)$
$\epsilon = +1$ for particles, $\epsilon = -1$ for anti-particles.

To evaluate the tensor $\ell_\mu \ell_\nu^*$ it is necessary to calculate the traces of the products of matrices using the known results of the products of gamma matrices :

$$\begin{gathered}
f_1 = \frac{1}{2}(\ell_1\ell_1^* + \ell_2\ell_2^*) \qquad f_2 = Im(\ell_1\ell_2^*) \\
f_3 = \ell_3\ell_3^* \qquad f_4 = -2R_e(\ell_3\ell_0^*) \\
f_5 = \ell_0\ell_0^* \qquad f_6 = -\frac{1}{\sqrt{2}}Re(\ell_1\ell_3^*) \\
f_7 = \frac{1}{\sqrt{2}}Im(\ell_2\ell_3^*) \qquad f_8 = \frac{1}{\sqrt{2}}Re(\ell_1\ell_0^*) \\
f_9 = \frac{1}{\sqrt{2}}Im(\ell_2\ell_0^*) \qquad f_{10} = \frac{1}{2}(\ell_1\ell_1^* - \ell_2\ell_2^*) \\
\widetilde{f}_1 = \frac{1}{\sqrt{2}}Im(\ell_3^*\ell_1) \qquad \widetilde{f}_2 = -\frac{1}{\sqrt{2}}Re(\ell_3^*\ell_2) \\
\widetilde{f}_3 = -\frac{1}{\sqrt{2}}Im(\ell_1\ell_0^*) \qquad \widetilde{f}_4 = \frac{1}{\sqrt{2}}Re(\ell_2\ell_0^*) \\
\widetilde{f}_5 = Re(\ell_1\ell_2^*) \qquad \widetilde{f}_6 = 2Im(\ell_0\ell_3^*)
\end{gathered} \tag{22}$$

In the case of massless neutrinos, the leptonic functions take the form :

$$\begin{aligned}
&v_1 = 2(1 - C_1C_2), \quad v_2 = 2\eta(C_2 - C_1), \quad v_3 = 2(1 + 2C_1C_2 - \cos\theta), \quad v_4 = -4(C_1 + C_2), \\
&v_5 = 2(1 + \cos\theta), \quad v_6 = \frac{-2\sqrt{2}}{\sin\theta}(C_2^2 - C_1^2), \quad v_7 = \frac{2\eta\sqrt{2}}{\sin\theta}(C_1 + C_2)(\cos\theta - 1), \\
&v_8 = \frac{2\sqrt{2}}{\sin\theta}(C_2 - C_1)(1 + \cos\theta), \quad v_9 = \frac{2\eta\sqrt{2}}{\sin\theta}(C_1^2 + C_2^2 - 2C_1C_2\cos\theta), \\
&v_{10} = -\frac{2}{\sin^2\theta}(C_2 - C_1\cos\theta)(C_2\cos\theta - C_1)
\end{aligned} \tag{23}$$

Here, θ is the angle between the neutrino(antineutrino) and electron(positron) pulses, η takes the value +1 for neutrino scattering and −1 for antineutrino and the coefficients C_1, C_2 are given by the relations : $C_1 = (E_\ell cos\theta - E_\nu)/q, \quad C_2 = (E_\ell - E_\nu cos\theta)/q$

The hadronic functions are given by the following formulas :

$$Y_1^L = -\sum_{J'J} A^L_{-1,1}\{P^+_{J'+J}\{F^{(\tau)}_{MJ}F^{(\tau')}_{MJ'} + F^{(\tau)}_{EJ}F^{(\tau')}_{EJ'} + F^{5(\tau)}_{MJ}F^{5(\tau')}_{MJ'} + F^{5(\tau)}_{EJ}F^{5(\tau')}_{EJ'}\} + P^-_{J'+J}\{F^{(\tau)}_{MJ}F^{(\tau')}_{EJ'} - F^{(\tau)}_{EJ}F^{(\tau')}_{MJ'} + F^{5(\tau)}_{EJ}F^{5(\tau')}_{MJ'} - F^{5(\tau)}_{MJ}F^{5(\tau')}_{EJ'}\}\}$$

$$Y_2^L = \sum_{J'J} A^L_{-1,1}\{P^+_{J'+J}\{F^{(\tau)}_{MJ}F^{5(\tau')}_{EJ'} + F^{(\tau)}_{EJ}F^{5(\tau')}_{MJ'} + F^{5(\tau)}_{MJ}F^{(\tau')}_{EJ'} + F^{5(\tau)}_{EJ}F^{(\tau')}_{MJ'}\} + P^-_{J'+J}\{F^{(\tau)}_{MJ}F^{5(\tau')}_{MJ'} - F^{(\tau)}_{EJ}F^{5(\tau')}_{EJ'} + F^{5(\tau)}_{EJ}F^{(\tau')}_{EJ'} - F^{5(\tau)}_{MJ}F^{(\tau')}_{MJ'}\}\}$$

$$Y_3^L = \sum_{J'J} A^L_{0,0}P^+_{J'+J}\{F^{(\tau)}_{LJ}F^{(\tau')}_{LJ'} + F^{5(\tau)}_{LJ}F^{5(\tau')}_{LJ'}\}$$

$$\bar{Y}_3^L = \sum_{J'J} A^L_{0,0}P^-_{J'+J}\{F^{5(\tau)}_{LJ}F^{(\tau')}_{LJ'} - F^{(\tau)}_{LJ}F^{5(\tau')}_{LJ'}\}$$

$$Y_4^L = \sum_{J'J} A^L_{0,0}P^+_{J'+J}\{F^{(\tau)}_{LJ}F^{(\tau')}_{CJ'} + F^{5(\tau)}_{LJ}F^{5(\tau')}_{CJ'}\}$$

$$\bar{Y}_4^L = \sum_{J'J} A^L_{0,0}P^-_{J'+J}\{F^{5(\tau)}_{LJ}F^{(\tau')}_{CJ'} - F^{(\tau)}_{LJ}F^{5(\tau')}_{CJ'}\}$$

$$Y_5^L = \sum_{J'J} A^L_{0,0}P^+_{J'+J}\{F^{(\tau)}_{CJ}F^{(\tau')}_{CJ'} + F^{5(\tau)}_{CJ}F^{5(\tau')}_{C'}\}$$

$$\bar{Y}_5^L = \sum_{J'J} A^L_{0,0}P^-_{J'+J}\{F^{5(\tau)}_{CJ}F^{(\tau')}_{CJ'} - F^{(\tau)}_{CJ}F^{5(\tau')}_{CJ'}\}$$

$$Y_6^L = -\sqrt{2}\sum_{J'J} A^L_{0,1}\{P^+_{J'+J}F^{(\tau)}_{LJ}F^{(\tau')}_{EJ'} + F^{5(\tau)}_{LJ}F^{5(\tau')}_{EJ'}) - P^-_{J'+J}(F^{5(\tau)}_{LJ}F^{5(\tau')}_{MJ'} - F^{(\tau)}_{LJ}F^{(\tau')}_{MJ'})\}$$

$$Y_7^L = \sqrt{2}\sum_{J'J} A^L_{0,1}\{P^+_{J'+J}(F^{(\tau)}_{LJ}F^{5(\tau')}_{MJ'} + F^{5(\tau)}_{LJ}F^{(\tau')}_{MJ'}) - P^-_{J'+J}(F^{5(\tau)}_{LJ}F^{(\tau')}_{EJ'} - F^{(\tau)}_{LJ}F^{5(\tau')}_{EJ'})\}$$

$$Y_8^L = -\sqrt{2}\sum_{J'J} A^L_{0,1}\{P^+_{J'+J}F^{(\tau)}_{CJ}F^{(\tau')}_{EJ'} + F^{5(\tau)}_{CJ}F^{5(\tau')}_{EJ'}) - P^-_{J'+J}(F^{5(\tau)}_{CJ}F^{5(\tau')}_{MJ'} - F^{(\tau)}_{CJ}F^{(\tau')}_{MJ'})\}$$

$$Y_9^L = \sqrt{2}\sum_{J'J} A^L_{0,1}\{P^+_{J'+J}F^{(\tau)}_{CJ}F^{5(\tau')}_{MJ'} + F^{5(\tau)}_{CJ}F^{(\tau')}_{MJ'}) - P^-_{J'+J}(F^{5(\tau)}_{CJ}F^{(\tau')}_{EJ'} - F^{(\tau)}_{CJ}F^{5(\tau')}_{EJ'})\}$$

$$Y_{10}^L = -\sum_{J'J} A^L_{-1,-1}\{P^+_{J'+J}\{F^{(\tau)}_{EJ}F^{(\tau')}_{EJ'} - F^{(\tau)}_{MJ}F^{(\tau')}_{MJ'} + F^{5(\tau)}_{EJ}F^{5(\tau')}_{EJ'} - F^{5(\tau)}_{MJ}F^{5(\tau')}_{MJ'}\} + P^-_{J'+J}\{F^{5(\tau)}_{EJ}F^{5(\tau')}_{MJ'} + F^{5(\tau)}_{MJ}F^{5(\tau')}_{EJ'}) - F^{(\tau)}_{EJ}F^{(\tau')}_{MJ'} + F^{(\tau)}_{MJ}F^{(\tau')}_{EJ'}\}\}$$

$$\bar{Y}_{10}^L = -\sum_{J'J} A^L_{-1,-1}\{P^+_{J'+J}\{F^{(\tau)}_{EJ}F^{5(\tau')}_{MJ'} - F^{(\tau)}_{MJ}F^{5(\tau')}_{EJ'}) + F^{5(\tau)}_{EJ}F^{(\tau')}_{MJ'} - F^{5(\tau)}_{MJ}F^{(\tau')}_{E'}\} + P^-_{J'+J}\{F^{5(\tau)}_{MJ}F^{(\tau')}_{MJ'} + F^{5(\tau)}_{EJ}F^{(\tau')}_{EJ'} - F^{(\tau)}_{MJ}F^{5(\tau')}_{MJ'} + F^{(\tau)}_{EJ}F^{5(\tau')}_{EJ'}\}\}$$

$$\bar{Y}_1^L = Y_2^L, \quad \bar{Y}_2^L = Y_1^L, \quad \bar{Y}_6^L = -Y_7^L, \quad \bar{Y}_7^L = -Y_6^L\ , \quad \bar{Y}_8^L = -Y_9^L, \quad \bar{Y}_9^L = -Y_8^L.$$

F_{MJ},F_{EJ},F_{CJ} and F_{LJ}(F^5_{MJ},F^5_{EJ},F^5_{CJ} and F^5_{LJ}) are matrix elements of the magnetic, electrical, coulombic and longitudinal multipole vector (axial-vector) operators calculated in the core layer model.

$$F_{M1}^{(1)} = \frac{\psi}{6\sqrt{\pi}}\frac{q}{M}e^{-y}(F_1 - \mu(2-y)), \quad F_{C1}^{5(1)} = \frac{\psi}{3\sqrt{2\pi}}\frac{q}{M}e^{-y}(\frac{3}{2}F_A - (1-y)(q_0 F_P - 2\eta M F_T)),$$
$$F_{L1}^{5(1)} = \frac{\sqrt{2}\psi}{3\sqrt{\pi}}e^{-y}(1-y)(F_A - \frac{q^2}{2M}F_P), \quad F_{E1}^{5(1)} = \frac{\psi}{3\sqrt{\pi}}e^{-y}F_A(2-y).$$

where ψ is a nuclear parameter ; $\mu = F_1 + 2MF_2$; q_0 is the transition energy ; $y = (bq/2)^2$ where b is the harmonic oscillator parameter.

The coefficient $A_{m'm}^L$ is given by :

$$A_{m'm}^L = (-)^{J_f + J_n}[J'][J][L]\left(\frac{(L-M)!}{(L+M)!}\right)^{1/2}\begin{pmatrix} J' & J & L \\ m' & m & M \end{pmatrix}\begin{Bmatrix} J' & J & L \\ J_n & J_n & J_i \end{Bmatrix} \tag{24}$$