Mathematical Modeling: Gateway to Technological Advancements for Sustainable Development.

Abstract

With pressing global challenges such as climate change, diseases and resource depletion there is a dire need for innovative solutions. Over the years, mathematical modeling has proven to be an essential tool that enables researchers across various sectors to represent complex systems by breaking down the intricate phenomena into manageable frameworks. Given that mathematics is a concise language with well-defined rules for manipulation, the mathematical models formulated can be used to test changes in a system using numerical simulation. Researchers are therefore able to predict outcomes, optimize processes, and innovate solutions that are both efficient and environmentally friendly. The integration of mathematical models into technological development fosters a deeper understanding of dynamic systems, enabling the design of adaptive strategies that promote sustainability. The results are essential in decision and policy making across various sectors such as in health, education, environment, wildlife conservation, finance, marketing and urban planning. Mathematical modeling is therefore a fundamental tool in bridging theoretical concepts with practical applications, driving technological advancements crucial for sustainable development. With increase in computational power and data availability, the sophistication of the mathematical models has evolved. This paper highlights case studies where mathematical modeling has successfully contributed to advancements in technology, illustrating its potential to enhance sustainability efforts across various sectors. Ultimately, mathematical modeling serves as a gateway to realizing the goals of sustainable development by providing the necessary tools to address pressing global issues and spur growth in all sectors of any country.

Keywords: Mathematical modeling, technology, sustainable development.

1 Introduction

The effects of human economic and industrial activities on the environment has spurred a great interest on sustainable development [1]. According to the World Commission on Economic Development (WCED), sustainable development is the development that meets the needs of the present without compromising the ability of the future generations to meet their own needs. In 2015, all United Nations members adopted the 2030 Agenda for sustainable development with 17 world Sustainable Development Goals (SDGs). The aim of these global goals is peace and prosperity for people and the planet [2]. Mathematical modeling plays an important role towards sustainable development in aiding the understanding, prediction and control of development process.

Mathematical modeling is the process in which real world problems are described in mathematical terms, usually in the form of equations. Using these equations, the original problem is well understood and new features about the problem are discovered. According to [3], the modelling process begins consciously or sub-consciously whenever mathematics is applied to another science or sector of life. Hence, mathematical modeling is the primary testing and development ground for the power of mathematics as applied to real life problems.

According to [4], the concept of mathematical modeling can be represented in three main parts as shown in Figure 1. The first part is the extra-mathematical domain (D), which is another subject or discipline, an area of practice or a sphere of private or social life, normally referred to as the 'real world'. The extra-mathematical world is an essential way of representing the part of the wider 'real world' that is relevant to particular problem [5]. The second part is some mathematical domain (M). Within M, mathematical deliberations, manipulations and inferences are made. The use of technology is fundamental to enhance the mathematical process. The third part is the mapping from extra-mathematical domain (D) to the mathematical domain (M), resulting in outcomes that are translated back to the extra-mathematical domain (D). These outcomes are interpreted as conclusions regarding that domain.

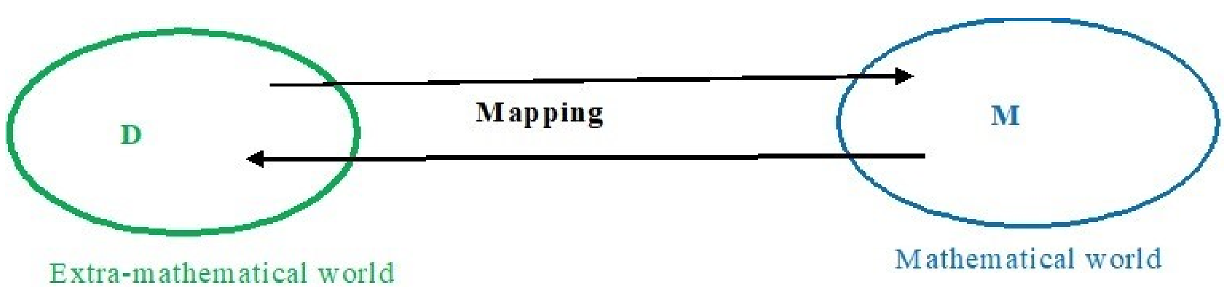

Figure 1: Concept of mathematical modeling.

This modelling cycle may be iterated several times for the purposes of validation and evaluation of the model in relation to the domain, until the resulting conclusions concerning D fulfil the purpose of the model construction. Modelling refers to the entire process, and everything involved in it - from structuring D, to formulating a suitable mathematical domain M and a suitable mapping from D to M, to working mathematically within M, to interpreting and evaluating conclusions with regard to D, and to repeating the cycle several times if necessary [5].

The mathematical modeling framework as illustrated in Figure 2, involves several sequential stages [4]. The first step involves describing the real-world problem. The practical aspects of the situation should be identified and well understood. Next, the mathematical problem is specified. In this stage, real-world scenario is framed as an appropriate, related mathematical question(s). In the next step, the mathematical model is formulated. Simplifying assumptions are made, variables are chosen and magnitudes of inputs are estimated. It is important to have justifications for the decisions made in this stage. The mathematics are then solved and solution(s) interpreted. The mathematical results should be considered in terms of their real-world meanings. In the next step the model is evaluated. Judgment is made as to the adequacy of the solution to the original question(s). If necessary, the model is modified and the cycle repeated until an adequate solution has been found. The last stage involves reporting the success of the research as well as documenting how further research could make adjustments and try for a better solution. The interpretation and evaluation stages indicate the cyclic nature of mathematical modelling. If the proposed first solution is not an adequate solution to the original question, the problem needs to be readdressed by repeating of earlier stages (stages 3 to 6) in sequence, and this may need to be carried out several times before an adequate solution is found.

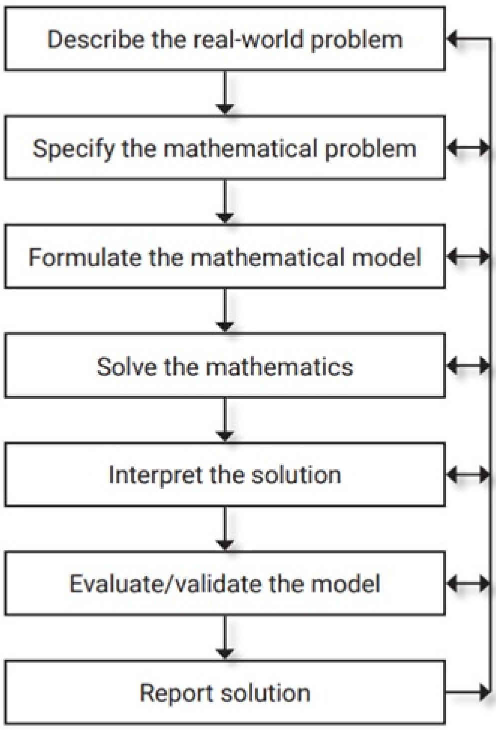

Figure 2: Mathematical modeling framework [4].

Figure 2 shows a mathematical modelling framework. The central vertical arrows portray a logical flow in the progression of a solution through a series of stages. For instance, a model cannot be solved before it is formulated nor can the model be validated before the mathematics are solved. The horizontal arrows on the right indicate that iterative backtracking may occur between any phases of the modelling cycle, as and when a need is identified.

There are various types of mathematical models classified in different ways depending on their developmental programming context, style, and scope. For instance, optimization models can be either linear or non-linear, application models can either be descriptive or prescriptive, time representation models can be static (steady-state) or dynamic while time continuity models can be discrete or continuous [6]. Other types of models include deterministic or stochastic depending on calculation mode. Depending on the modelling goal, a given type of model is used to best represent the real world problem and hence enable the researcher to simulate, predict, optimize outcomes and provide unique insights into system behaviors. The following sections of this paper present the pivotal role that mathematical modeling has played in research and informed decision-making in various sectors resulting to advancement in technology for sustainable development.

2 Mathematical Modeling in the Health Sector

Given that public health is core to humanity, there is a long history of research in the health sector to understand and improve it. Mathematical modeling has been used over the years to demystify, predict and improve health outcomes in areas such as epidemiology, health operations, genomic and personalized medicine, pharmacokinetics and pharmacodynamics, chronic disease management, among others. The first mathematical model of infectious disease dates back in 1760 by Bernoulli on the spread of smallpox [7]. Over the years, mathematical modeling in the health sector has had significant improvements and frequently used to inform WHO guideline recommendations for clinical and public health practices [8].

2.1 Mathematical Modeling of Infectious Diseases Within a Population

Infectious diseases continue to cause havoc in the human population over the years. Given that one of the sustainable development goals is to ensure healthy lives and promote well-being for all, there has been consolidated efforts to curb the spread of these diseases. Through mathematical modeling, various mathematical frameworks have been formulated to describe how different diseases spread through the population. The models help researchers and public health officials understand the dynamics of disease transmission, evaluate intervention strategies, and predict future outbreaks. For instance, several mathematical models were formulated to help understand the transmission dynamics of Corona Virus when it broke out in December 2019. Several researchers including [9], [10] , [11], [12], [13], [14] among others provided useful insights on how to curb the spread of the virus. In collaboration with various stakeholders such as CDC, WHO and various ministries of health, measures such as vaccination, social distancing and frequent hand washing were put in place. In May 2023, the WHO declared that corona virus was no longer a global health emergency [15]. Several other diseases have been studied through mathematical modelling such as HIV/Aids, Tuberculosis, Pneumonia, Malaria, Influenza, Cholera, among others. Results from their respective researches have continued to inform various stake holders and policy makers on different control strategies that can be employed to curb their spread. One of the most notable and profound successes in public health is the complete eradication of smallpox virus. In fact, the first mathematical model of infectious disease transmission was constructed by Bernoulli in 1760 to determine the effectiveness of smallpox vaccination [7] . In 1980, The World Health Organization (WHO) declared smallpox eradicated [16]. Before its eradication, smallpox was one of the most devastating diseases causing millions of deaths in the human population. Its eradication remains as a legacy of hope for other diseases.

2.2 Mathematical modeling of disease-causing agents within-host

In this type of modeling, researchers formulate models which represent the spread of pathogens inside an individual (host). These models aim at investigating the dynamics of pathogen replication, host immune response and the interactions between them. They enable better understanding of the progression of an infection at an individual level, which can differ from person to person depending on specific immunology and can change according to age and co-morbidity status [17]. Using these models, better and more personalized treatments are developed. In addition, within-host models facilitate better understanding and representation of changing infectiousness, and hence the transmission process inherent to between-host models.

Using various models, several researchers have investigated the mechanistic interactions governing chronic infections with pathogens such as HIV ([18], [19], [20], [21], [22]), tuberculosis ([23], [24], [25], [26]), malaria ([27], [28], [29]), among others. Analytical Results from these models have helped quantify the in-host basic reproduction numbers (R_0), which estimate the number of secondary infections that arise from one infected cell over the course of its life-span at the beginning of infection when cells susceptible to infection are not depleted [30] . Through data fitting and numerical simulation, significant biological parameters such as pathogen and infected cell half-lives and the daily pathogen production have been realized. A key achievement of such models is that researchers are able to estimate the efficacy of different drug therapies, the strength of the immune responses (innate and/or adaptive immune responses), and to eventually predict disease outcome.

3 Mathematical Modeling in Agriculture

To enhance global food security, agricultural engineering plays a vital role in ensuring long-term quality and productivity of agricultural produce. Various mathematical models continue to be used to improve various aspects in the agricultural sector such as crop production, management of resources, design of innovative machinery, pest and disease management among others. Mathematical modelling has played a crucial role to help navigate several factors influencing agriculture such as abrupt change in environmental and technological dynamics [31] thus enhancing informed decision making. Researchers such as [32], [33], [34] have used mathematical models to investigate crop growth. The models aid in predicting crop produce potential and optimize planting strategies by simulating crop development over time having considered factors like sunlight, temperature, water availability, and nutrient uptake.

Due to climate change, the intensity and frequency of floods and droughts have been gradually increasing worldwide [35], [36], [37]. Given that agriculture is highly vulnerable to changing meteorological conditions, there is a dire need to effectively manage the available water resources. The Water Evaluation and Planning (WEAP) model invented by the Stockholm Environment Institute (SEI) has been used to assist in planning and management of water resources [38]. The model is used to tackle problems encompassing water demand analyses, water saving and optimizing available water distribution. A research study by [38] to investigate the effect of different agricultural irrigation scenarios on MRQ76 rice variety having incooporated the WEAP model found out the model is suitable for providing the best irrigation scheduling strategy for optimum rice yield and efficient water management.

In order to develop effective control strategies to curb the growth and spread of pests and diseases affecting agricultural produce, researchers have developed different mathematical models. For instance, a research study by [39] on Maize Streak Virus (MSV) showed that spread of the disease mainly depends on the infection and predation rates, hence, efforts should be made to minimize the contact of infected maize and susceptible leafhopper while MSV infected maize should be treated promptly. Various mathematical models have also been used to investigate agricultural aspects such as irrigation management , soil erosion, fertilizer applications, among other subjects [31]. Results from these studies have given insights on planting schedules, best-suited crop variety under given climatic conditions, different irrigation techniques and how much water to apply, pest and disease control strategies, optimum fertilizer application and efficient techniques to manage soil erosion.

4 Mathematical Modeling of Transport Systems

To achieve sustainable development, it is crucial to have efficient transport systems which enhance economic opportunities and promote social equity. Transportation networks provide the foundation for the functioning of economies and societies through the movement of people, goods and services [40]. Mathematical modeling of the various modes of transport- road, air, rail and waterway networks, helps to improve efficiency, reduce congestion and enhance safety. Different models have been used to investigate traffic management ([41], [42], [43], [44]), public transportation and optimization of routes ([45], [46], [47]), infrastructure planning and environmental impact. Results from these models have enabled various stake holders and policy makers to make informed decisions on infrastructure investments, traffic management and policy formulation.

5 Mathematical Modeling in Wildlife Conservation

In order to understand the complex ecological systems, predict wildlife population trends and evaluate conservation strategies, mathematical modelling has proved to be beneficial. Researchers have used mathematical equations to represent relationship between different variables thereby shedding more light on wildlife population dynamics, zoonotic diseases, impact of environmental changes, human-wildlife interactions, effective management plans [48] , among other aspects. A study by [49] on conservation of biodiversity taking into account environmental factors such as climate change, natural resource use, and the impact of anthropogenic factors showed that taking into account these factors allows for more accurate predictions of changes in ecosystems. The model demonstrated its effectiveness in predicting changes in plant and insect populations which is key in developing adaptation strategies. According to [50], zoonoses are a global public health concern, accounting for about 75% of human infectious diseases. In a bid to mitigate the adverse effects of zoonotic diseases, researchers such as [51], [52], [53], [54], among others have developed mathematical models to study these diseases and identify areas where further modeling efforts are needed. Mathematical models, whether deterministic or stochastic, are effecive tools which when used in together with laboratory, field data and geographic information and remote sensing tools, they lead to a better understanding of infection and transmission dynamics [50], thus enabling more effective and sustainable conservation efforts.

6 Conclusion

Mathematical models sustain majority of human activity and are widely used to solve real life situations. From the case studies discussed above and many others that are not mentioned in this paper, it is without a doubt that mathematical modeling is a crucial enabler of technological advancements aimed at achieving sustainable development. Through mathematical modeling, a framework for understanding complex systems is established which is a gateway for guiding innovative solutions. Therefore, there is need for continuous interdisciplinary collaboration and local knowledge integration in order to enhance the efficacy of the models.

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