Mathematical Modelling of Hall Effect on a steady magneto-convection and radiative heat transfer past a porous plate

ABSTRACT

Heat transfer features significantly in engineering processes. Blowing of fluids is used in designing thrusters and prevention of corrosion. Suction is used in film cooling and coating of wires in chemical engineering. Numerous research done has focused on thermal reduction in processing streams and neglecting the machine efficiency. The paper investigates the effect of hall current on a steady magneto-convection and radiative heat transfer over a porous plate with the motive being improving machine efficiency in industries. The transport model is of an incompressible fluid flowing through a porous plate. The magnetic field is imposed perpendicularly to the plate. Equations governing the flow are formulated then converted to higher order ODE's using the similarity transformation. The resulting ODE's are solved using the fourth order Runge-Kutta method coupled with a shooting technique. The solution are then executed using MAPLE computer programme and the results displayed graphically and in tabular form. The results are then analysed putting into consideration their industrial and engineering applications. It is observed that Increasing Prandtl number and Grashof number leads to an increase in the fluid velocity and a decrease in skin friction. Increase in magnetic field, and radiation results in increased fluid temperature distribution.

1.0 Introduction

MHD flow and heat transfer has wide applications in the fields of power generation, fusion research, medical treatment, material processing, Geophysics and plasma studies [1]. Magnetic effect and Hall current effect on the flow of boundary layer is of great interest to researchers due to its wide application in industrial processes. [2] Investigating the effect of viscous dissipation and joule heating of MHD field convection flow past a semi-infinite vertical plate in the presence of combined effect of Hall and ion-slip current observed that the magnetic field acted as a retarding force on the tangential flow but have a propelling effect on the induced lateral flow. [3] studied the effect of thermal radiation absorption on an unsteady free convective flow past a vertical plate in the presence of a magnetic field and constant wall heat flux. Boundary layer equations were derived, and the resulting approximate nonlinear ordinary differential equations were solved analytically using asymptotic technique. A parametric study of all parameters involved was conducted, and a representative set of numerical results for the velocity and temperature profiles as well as the skin-friction parameter were illustrated graphically to show typical trends of the solutions. [4] Investigated the influence of radiation and temperature-dependent viscosity on the problem of unsteady MHD flow and heat transfer of an electrically conducting fluid past an infinite vertical porous plate taking into account the effect of viscous dissipation. The results show that increasing the Eckert number and decreasing the viscosity of air leads to arise in the velocity, while increasing the magnetic or the radiation parameters is associated with a decrease in the velocity. [5] considered the free convection heat transfer due to the combined action of radiation and a transverse magnetic field with variable suction. Results obtained indicated that increasing the plate velocity increased the flow velocity with this increase being more dramatic for higher values of the free convection. [6] studied the free convection flow of a compressible Boussinesq fluid under the simultaneous action of buoyancy and transverse magnetic field while the Rosseland approximation were invoked to describe the radiative flux in the energy equation. Results obtained which compare favorably well with published data show, that the skin friction for a compressible fluid was lower than that for an incompressible fluid. [7] studied the effects of Joule-heating, chemical reaction and thermal radiation on unsteady MHD natural convection from a heated vertical porous plate in a micro polar fluid. The partial differential equations governing the flow and heat and mass transfer were solved numerically using an implicit finite-difference scheme.

Study conducted by [8] on Hall effects on an unsteady magneto-convection and radiative heat transfer past a porous plate noted that the flow field and temperature distribution were greatly influenced by thermal radiation parameter. Hall currents moderated the flow field significantly, while suction (or injection) impacted the boundary layer thickness with suction reducing thermal boundary layer thickness whereas injection thickening it. The purpose of this study is therefore to critically analyze the effect of hall current on unsteady magneto-convection and radiative heat transfer past a porous medium.

2.0 Mathematical Formulation

In this study we consider a two dimensional steady laminar flow of an incompressible conducting fluid flowing through an infinite vertical porous plate lying parallel to the $y-axis$. The fluid is assumed to have a constant velocity induced by gravity and the pressure gradient. The plate at $y = 0$ is at rest and heated with temperature T_W. A uniform magnetic field of strength B_0 is applied normal to the plate. The plate is assumed to be infinitely long along the $y-axis$ hence the radiative heat flux in the $y-axis$ will be negligible compared to that in the $x-$ direction. See figure 1 below.

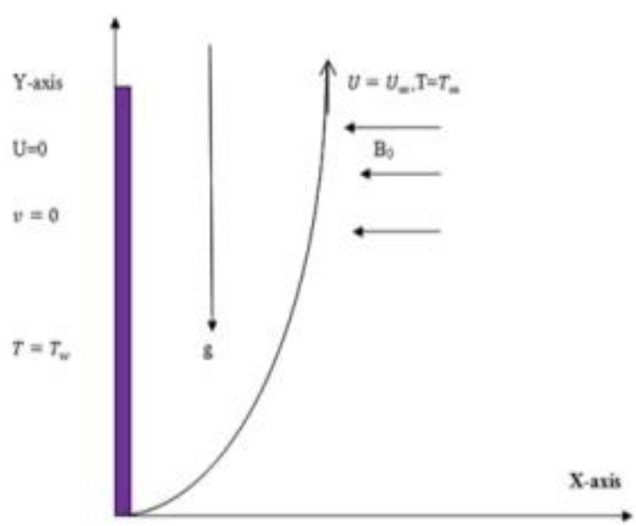

Fig 1: Flow configuration and coordinate system

Assuming the base fluid and the nanofluid are in thermal equilibrium and no slip occurs between them.

The governing equations for the boundary layer flow are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+\beta g(T-T_\infty)-\frac{\sigma B_0^2(u-U_\infty)}{\rho_f} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K_{nf}}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma}{\rho C_p}B_0^2(u-U_\infty)^2-\frac{1}{\rho C_p}\left(\frac{\partial q_r}{\partial y}\right) \tag{3}$$

Where (u,v) are the velocities in(x,y)direction respectively, $\rho C_p$the heat capacitance of the fluid. T is the local temperature,k is the thermal conductivity of the fluid, ρ_f is the density of the fluid, μ is the dynamic viscosity of the nanofluid, σ is the electrical conductivity of the fluid,B_0 is the applied magnetic field and q_r is the radiative flux.

Using Rosseland approximation for the thermal radiative heat flux given by

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{4}$$

Where σ is Stephan Boltzmann constant and k^* is the mass absorption coefficient. The temperature differences within the flow are small enough so that T^4 may be expressed as a linear function of temperature. Using truncated Taylor's series about the free stream temperature T_∞ i.e.

$$T^4\approx 4T^3T-3T_\infty^4 \tag{5}$$

Replacing the radiative flux $q_r(4)$ and (5) we obtain,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma}{\rho C_p}B_0^2(u-U_\infty)^2+\frac{1}{\rho C_p}\left(\frac{16\sigma T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} \tag{6}$$

The variable plate surface porosity is given as;

$$V_w(x)=-\frac{f_w}{2}\sqrt{\frac{Uv_f}{x}} \tag{7}$$

Where $U=U_w+U_{\infty,}\,f_w$ is a constant with $f_w>0$ representing exothermic reactions, $f_w<0$ representing endothermic reactions and f_w=0 for a non-porous surface.

The boundary conditions for the boundary layer flow are:

$$u(x,0)=0\,,\qquad v(x,0)=0\qquad ,T(x,0)=T_w$$

$$u(x,\infty)=U_\infty\qquad ,T(x,\infty)=T_\infty \tag{8}$$

Using the stream function $\psi=\psi(x,y)$, the velocity components of $u, and\ v$ are defined as;

$$u = ax = \frac{\partial \varphi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

And by defining an independent variable η and a dependent variable f in terms of the stream function ψ as:

$$\eta = \left(\frac{a}{v_f}\right)^{\frac{1}{2}} y, \qquad \psi = \left(a v_f\right)^{\frac{1}{2}} x f(\eta) \tag{10}$$

Transforming the governing equations (2) and (3) together with the boundary conditions (8) they give the following ordinary differential equations;

$$f''' + ff'' - (f')^2 - Ha(f' - 1) + \frac{B_f g(T - T_\infty)}{aU_\infty} = 0$$

$$f''' + ff'' - (f')^2 - Ha(f' - 1) + Gr\theta = 0 \tag{11}$$

Where the prime denotes differentiation with respect to η.

Where $Ha = \frac{\sigma_f B_0^2}{\rho_f a}$ is the Hartman number, $Gr = \frac{B_f g(T_w - T_\infty)}{a u_\infty}$ is the Grashof number.

Similarly letting $\Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$

$$\frac{1}{Pr}\left(1 + \frac{4}{3}R\right)\Theta'' + f\Theta' + EC(f'')^2 + HaEC(f' - 1)^2 = 0 \tag{12}$$

The corresponding boundary conditions are;

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1$$

$$f'(\infty) = 1, \qquad \theta(\infty) = 0 \tag{13}$$

Where Ec is the Eckert number number, Ha is Hartman number, $R = \frac{kk^*}{4\sigma^* T_\infty^3}$ is the radiation parameter, $Pr = \frac{(\mu C_p)_f}{k_f}$ is the Prandtl number.

3.0 Numerical Solution

The numerical solutions are obtained by solving (11) and (12) subject to the boundary equations (13) using the fourth order Runge-Kutta Integration scheme with the shooting technique. The computations were done by a MAPLE computer programme which uses the symbolic and computational language maple the method involves transforming the coupled differential equations (11) and (12) which are third order in f and second order in Θ into first order differential equations. The system of first order ordinary equations are obtained by letting,

$$f_1 = f, \qquad f_2 = f', \quad f_3 = f'', \quad f_4 = \theta, \quad f_5 = \theta' \tag{14}$$

Where prime represent differential of f and θ with respect to η. The set of higher order non-linear boundary value problem with their respective boundary conditions are reduced to seven equivalent first order differential equations with appropriate initial conditions, respectively, as given below:

$$f_1' = f_2$$

$$f_2' = f_3$$

$$f_3' = f_1 f_3 - f_2^2 - Haf + Grf_5$$

$$f_4' = f_5$$

$$f_5' = \frac{-Prf_1f_5 - PrECf_3^2 - PrHaEC(f_2-1)^2}{\left(1+\frac{4}{3}R\right)} \qquad (15)$$

Subject to the following initial conditions

$$f_1(0) = f_w\,, \quad f_2(0) = 0, \quad f_3(0) = S_1$$

$$f_4(0) = S_2\,, \qquad f_5(0) = 1 \qquad (16)$$

By applying the shooting method. The unspecified initial conditions $S_1, and\ S_2$ in (16) are assumed and (15) integrated numerically as an initial value problem to give the terminal point. The accuracy of the missing initial conditions was checked and by comparing the calculated value of the dependent variable at terminal point with its value there. The results obtained are represented through graphs and the main features of the problems are discussed.

4.0 Results and Discussion

Numerical computations were performed for various values of the physical parameters involved. Namely; Hartmann number *Ha,* Prandtl number *Pr,* Grashof number *Gr* radiation parameter *R* and Eckert number *Ec.* Detailed discussion on the effect of the governing physical parameters on the velocity profile and temperature profile was done. Table 1 shows effect of various thermophysical parameters on the skin friction$(f''(0))$, and Nusselt number $(-\theta'(0))$. We noticed that the local skin friction increases with an increase in applied magnetic field and increase in Eckert number while local Nusselt number decreases with an increase in magnetic field and thermal radiation, however it increases with increase in Prandtl number and Eckert number. .High Prandtl number and magnetic number has a relatively lower thermal conductivity and thereby increases the heat transfer rate at the surface. During the simulation the injection/suction parameter has very little effect on the velocity profile.

4.1 Effects of parameters variation on the velocity profiles

Figure 2-4 shows the velocity profile $f'(\eta)$ for different values of magnetic field parameter (Ha), Grashof number (Gr), and Eckert number (Ec). Figure 2 shows that the fluid velocity is highest at the plate surface and decreases to zero free stream value satisfying the given boundary conditions. Application of the magnetic field creates a resistant force similar to the drag force that acts in the opposite direction of the fluid motion thus causing the velocity of the fluid to shoot away from the plate. Similar trend is observed in figure 3 and 4 with an increase in viscosity. Since as the fluid viscosity decreases, the boundary layer becomes thick leading to decrease in velocity gradient.

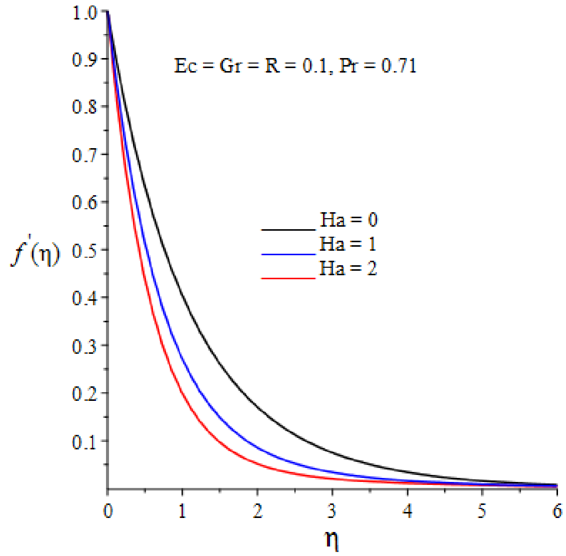

Fig. 2 Velocity profile for different values of Ha

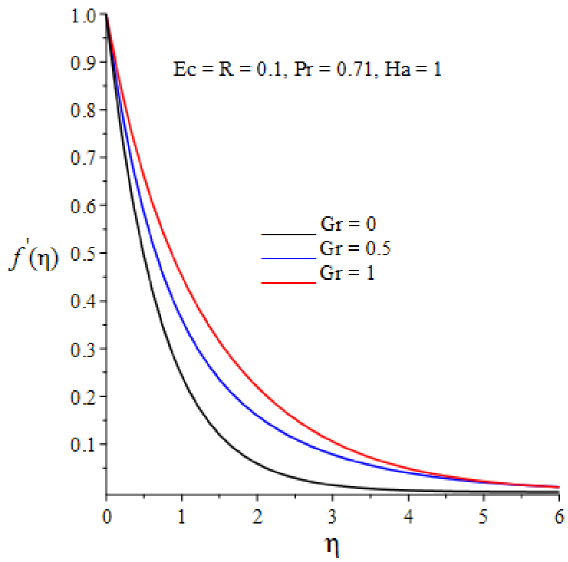

Fig. 3 Velocity profile for different values of Gr

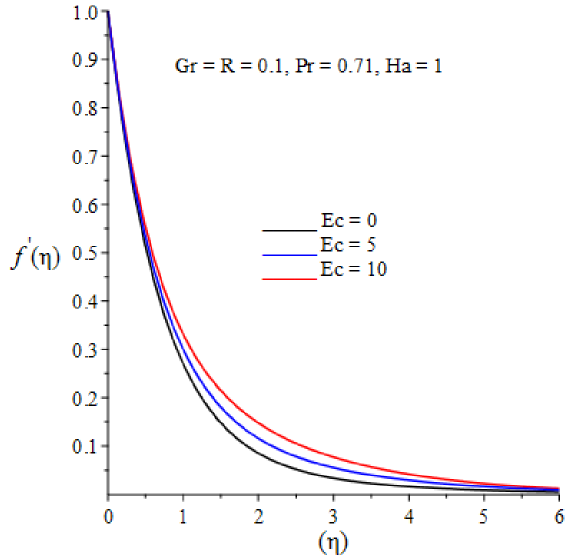

Fig. 4 Velocity profile for different values of Ec

4.2 Effects of parameters variation on the temperature profiles

Figure 5-9 illustrates the effect of various parameters on the temperature profiles. Dimensionless temperature θ is plotted as a function of transverse distance η .Generally, the fluid temperature is highest at the plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary conditions. From figure 5 the temperature increases with an increase in *Ha* accordingly leading to an increase in thermal boundary layer. As explained earlier, the transverse magnetic field gives rise to a resistive force called Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers resulting in increase in temperature. In figure 7 it is evident that the fluid temperature decreases with an increase radiation parameter *R* leading to a decrease in the thermal boundary layer thickness this enables the fluid to release heat energy from the flow region and cause the system to cool. The thermal radiation should be at maximum in order to facilitate the cooling. From the explanation the effect of thermal radiation becomes significant as $R \to \infty$ and can be neglected as $R \to 0$. Figure 9 shows the temperature profiles for different values of the Prandtl number. The fluid temperature decreases with increase in Prandtl number hence decreasing the thermal boundary layer thickness. This is because high Prandtl number has a relative low thermal conductivity which in turn reduces conduction and therefore thermal boundary layer thickness as a result the fluid temperature decreases.

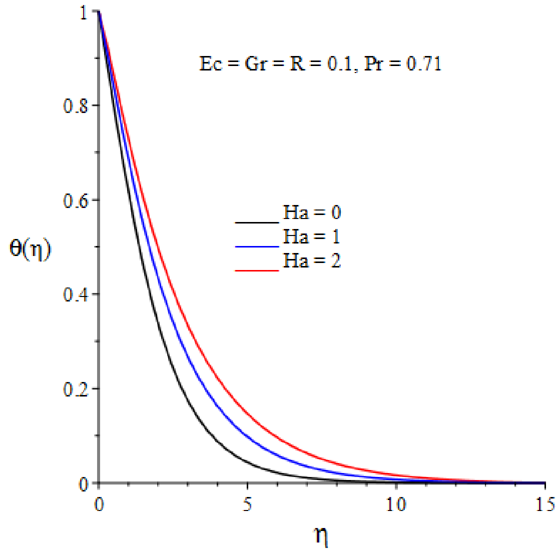

Fig. 5 Temperature profile for different values of Ha

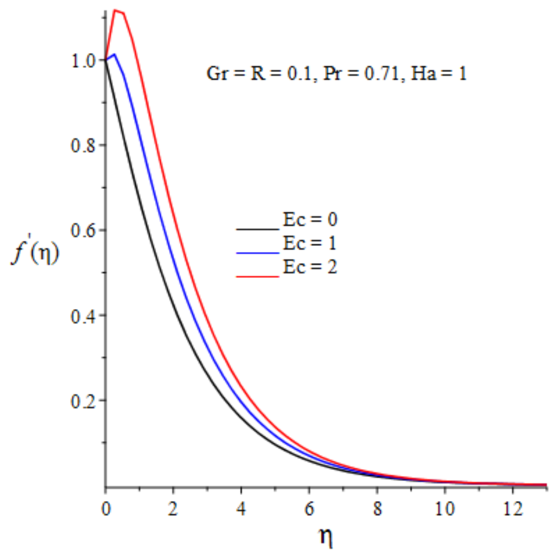

Fig. 6 Temperature profile for different values of Ec

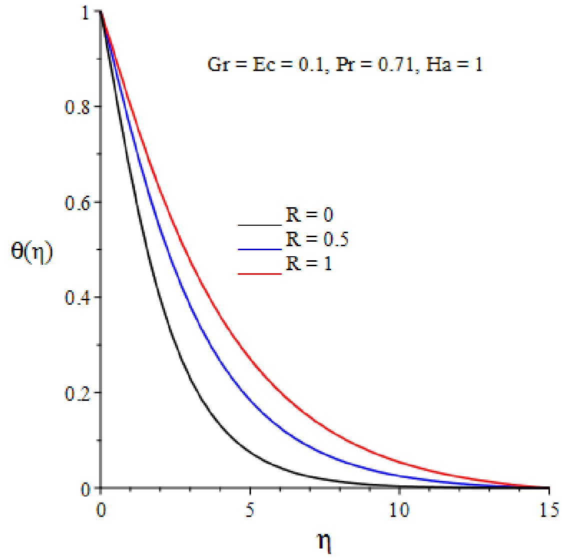

Fig. 7 Temperature profile for different values of R

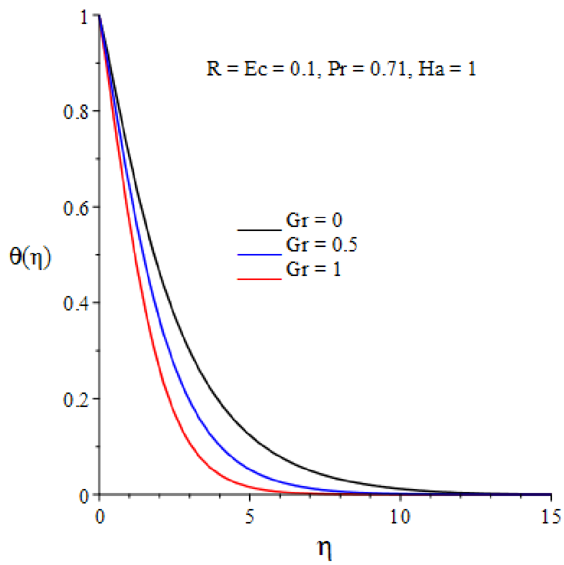

Fig.8 Temperature profile for different values of Gr

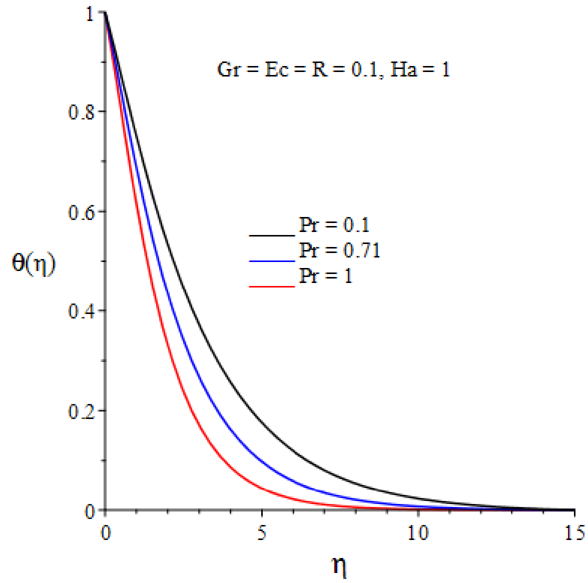

Fig.9 Temperature profile for different values of Pr

Effects of parameters variation on the skin friction C_f, Nusselt number *Nur*

Table 1: Computation showing the values skin friction coefficient $-f''(0)$, and reduced Nusselt number $-\theta'(0)$ for varying governing parameters

M	R	Pr	Ec	$-f''(0)$	$-\theta'(0)$
0	0.1	0.71	0.1	1.1054	0.4391
1				1.5177	0.3610
2				1.8349	0.3109
3				2.1025	0.2736
4				2.3383	0.2439
1	**0**	0.71	0.1	1.5177	0.3888
	1			1.5177	0.2535
	2			1.5177	0.2191
	5			1.5177	0.1903
1	0.1	**0.71**	0.1	1.5177	0.3610
		1		1.5177	0.4443
		1.5		1.5177	0.5707
1	1.0	0.71	**0**	1.3177	0.2216
			0.3	1.4142	0.2876
			0.6	1.5177	0.3610

5.0 Conclusion

The two-dimensional steady laminar flow of an incompressible conducting fluid passing through an infinite vertical porous plate lying parallel to the $y-$ axis was considered. The fluid was assumed to have a constant velocity induced by gravity and the pressure gradient. The plate at $y = 0$ was at rest and was heated with temperature T_W. A uniform magnetic field of strength B_0 was applied normal to the plate. The plate was assumed to be infinitely long along the y −axis hence the radiative heat flux in the y −axis was negligible. The equations governing the flow were formulated then converted to higher order ordinary differential equations using the similarity transformation. The resulting ODE's were solved using the fourth order Runge-Kutta method coupled with a shooting technique. The solutions were executed using MAPLE computer programme and the results presented graphically. The following conclusions can be drawn from the study:

i. The fluid velocity decreased with increase in Hartmann number (Magnetic parameter)

ii. The fluid velocity increased with increase in Grashof number and Eckert number

iii. The fluid temperature increased with increase in Magnetic parameter, Eckert number and Radiation parameter.

iv. The fluid temperature decreased with increase in Grashof number and Prandtl number

v. Increase in Magnetic parameter increases skin friction and reduces Nusselt number

vi. The skin friction and Nusselt number are both enhanced with increase in radiation parameter, Prandtl number and Eckert number.

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