**Monitoring Dispersion in the Compressive Strength of Parts Using Group Runs Control Chart**

**Abstract**

Since the Shewhart type control charts are slower in detecting smaller shifts, synthetic control charts were introduced as an efficient alternative. Group Runs (GR) control chart is a further extension of synthetic chart, which performs better than the synthetic chart. In this paper, we propose GR control chart based on range (GR-R chart) to monitor the process dispersion for normally distributed process. With the help of extensive simulation, the proposed GR control chart (GR-R) is compared with synthetic R chart using run length characteristics. The proposed chart is found to be superior to the synthetic chart in detecting shifts in process standard deviation. An illustrative example is provided to demonstrate the application of proposed GR-R chart.

**Keywords:** R chart, average run length, group runs, process dispersion

1. **Introduction**

A continuous scrutiny of the industrial manufacturing processes is always necessary to ensure that the process is subjected to chance causes alone and no assignable cause is present. The variation in the process parameters on account of assignable causes is always a cause of worry as it deteriorates the quality of the product or process (Iqbal et al., 2020). The early detection of assignable causes is always encouraged to save wastage of time and other resources (Wu & Liang, 2024). The monitoring of any process output demands an early detection of the shifts in the process parameters. The shifts may be in the process mean or the process standard deviation or in both. Control charts are widely used to detect shifts in the parameters of monitoring process. Shewhart  chart is most widely used to monitor process average, while R and S control charts are used to monitor process variability (Malindzakova et al., 2023). All these control charts are based on the fundamental assumption that the underlying distribution of the quality characteristic is normal. The R chart is based on the sample range (R) whereas S chart is based on sample standard deviation (S). Both R and S charts are easy to implement and are effective in the detection of large shifts in process standard deviation but become less effective for small shifts because they are based on only the most recent observation (Raza et al., 2024).

Though Shewhart-type charts are simple to implement, they are not good enough to detect comparatively smaller shifts in the process parameters. The performance improvement of Shewhart type control charts has attracted continuous research interest. Several approaches have been introduced in the literature for improving the performance of the chart to detect shifts in the process parameters (Park et al., 2023). One such approach is the development of synthetic control charts. To enhance the performance of  chart, Wu and Spedding (2000) proposed a syntheticcontrol chart, as a combination of Shewhart chart and Conforming Run Length (CRL) chart. They considered a sample or group as a unit and identified it as conforming or non-conforming. In this setup, the decision about process is taken based on run length between two successive non-conforming groups, including the end group. It was shown that proposed synthetic  chart has better performance than the Shewhart chart in detecting shifts in the process mean of normally distributed process. Chen and Huang (2005) applied this technique for monitoring process dispersion by considering sample range as charting statistic. They showed that the performance of synthetic R chart is better than Shewhart R chart. Huang and Chen (2005) also dealt with the problem of monitoring the process dispersion by developing synthetic chart based on sample standard deviation, which improved the performance of Shewhart S chart. Many authors since then have developed various synthetic control charts for monitoring dispersion, such as Costa and Rahim (2006), Rajmanya and Ghute (2014), Ghute and Shirke (2007), Ghute and Shirke (2008). An overview of synthetic control charts is presented by Rakitzis et al. (2019).

In the literature, Group Runs (GR) chart is suggested as an improved version of the synthetic control chart. Gadre and Rattihalli (2004) first proposed GR control chart to improve the performance of Shewhart  chart and synthetic  chart for detecting shifts in the process mean of normally distributed process. The proposed GR control chart is a combination of the Shewhart chart and extended version of CRL chart (Ghute & Ghadge, 2024). It was shown that GR control chart shows better performance than the Shewhart  chart and synthetic  chart for monitoring the process mean of normally distributed process. In many practical situations, it is important to monitor variability change for variability reduction purposes. Thus, it is highly desirable to develop GR control chart specifically designed for variability change detection in univariate process. More related work on GR charts can be seen in Gadre et al. (2005), Gadre and Kakade (2014, 2016), Chong et al. (2017), Khilare and Shirke (2023), Ghadge and Ghute (2023), Ghute and Gardi (2023), Ghute and Ghadge (2024).

The objective of the present paper is to enhance the performance of Shewhart R and synthetic R chart, using the concept of GR chart. This paper extends the work of Gadre and Rattihalli (2004) by proposing a GR-R chart for improved monitoring of the process dispersion of normally distributed process. The proposed chart is used for monitoring increasing and decreasing shifts in the process standard deviation of normally distributed process. The remainder of the paper is structured as follows. Section 2 presents briefing on the traditional Shewhart R chart and CRL chart which are the basis of proposed GR-R control chart. The design and operation of proposed GR-R control chart is provided in section 3. In section 4, we evaluate the performance of proposed chart for increasing and decreasing shifts in the process standard deviations. The performance of the proposed chart is compared with its synthetic and traditional counterparts. A demonstration of GR-R chart to a real dataset is given in section 5. Concluding remarks are summarized in section 6.

**2. Shewhart R Chart for Process Dispersion**

Consider a process where quality characteristic of interest is normally distributed with mean  and standard deviation. Let  and  be the in-control values of and  respectively. In practice, in-control process parameters are usually unknown and are estimated from the samples taken when process is assumed to be in-control. When a shift in process standard deviation occurs, we have a change from the in-control value  to the out-of-control value . Thus  is the ratio between the out-of-control and the in-control process standard deviation. Therefore, when R chart is employed, the process shifts are measured through. For an increase (decrease) in process standard deviation occurs, while for , the process standard deviation is considered to be in-control. Further, we assume that the mean of the quality characteristic remains unaffected as its in-control level  by the presence of assignable causes in the production line.

*2.1 Shewhart R chart*

The monitoring of process dispersion with R chart is based on the successive values of sample ranges defined by

 (1)

which are plotted on the control chart against the sample numbers, where denotes the  order statistic from sample. When the process goes out-of-control,  indicates an increase in the process standard deviation and upper control limit,of R chart is required, and a signal is issued if. Whenindicates decrease in  and lower control limit,of the R chart is required. The average run length (ARL) of the R chart for and  can be calculated, respectively, as follows:

When ,the probability of R statistic falling outside its control limit is given as



 (2)

When ,the probability of R statistic falling outside its control limit is given as



  (3)

where denotes the relative range and denotes its cumulative distribution function.

The average run length of the R chart to detect a change in  can be calculated as

 (4)

*2.2 Conforming Run Length chart*

Bourke (1991) introduced the conforming run length (CRL) chart, which is an attribute control chart used to detect shift in the fraction nonconforming. The random variable for this chart is the number of conforming units between two successive nonconforming units including ending nonconforming unit. The charting statistic follows a geometric distribution with cumulative distribution function (c.d.f.) as

 (5)

whereis the probability of obtaining a nonconforming unit. The lower control limit  of the CRL chart is given by

 (6)

whereis the type-I error probability for the CRL chart and is the in-control fraction nonconforming. If it indicates an increase in nonconforming fraction and generates out-of-control signal.

The ARL of the CRL chart to detect the increase in the fraction nonconforming is given by

 (7)

Chen and Huang (2005) developed synthetic R chart by combining Shewhart chart and  chart. In this paper, we have developed GR-R control chart, as a further improvement to synthetic chart.

**3. GR-R Control Chart for Process Dispersion**

In this section, we present a general structure of a GR-R control chart for monitoring process dispersion. Group Runs (GR) control chart consists of two sub charts: usual Shewhart-type chart and extended version of CRL chart. Shewhart-type chart marks a subgroup as non-conforming, if subgroup statistic falls outside the specified limit. The extended CRL chart decides whether the process is in-control or not, based on two successive conforming run lengths.

Following the work of Gadre and Rattihalli (2004), in order to increase the detection ability of Shewhart-R chart, we integrate the Shewhart-R sub-chart and extended version of CRL sub-chart to produce GR-R chart. The Shewhart-R sub-chart has lower control limit (LCL) and upper control limit (UCL) and CRL sub-chart has lower control limit, where. The values of and are used to determine whether a group (sample) is conforming or nonconforming, whereas, the value of is used to determine if the process is in-control or out-of-control. The process is considered to be out-of-control when the group is nonconforming and  or two successiveand, for the first time.

 Following Gadre and Rattihalli (2004), the operation of the GR-R chart is prescribed as below:

1. Fix the lower control limit of the CRL chart and the control limits of the Shewhart-R chart,, when  and , when .
2. Draw a sample (group) of  items at each inspection pointand compute the chart statistic, the sample range, .
3. If  when  or when , this group is regarded as a conforming group in CRL chart, and the control flow goes back to step 2. Otherwise, the group is called a nonconforming group, and the control flow goes to the next step.
4. Check the number of samples (groups) between the current and previous nonconforming groups. This number is taken as  sample for the CRL chart.
5. If , the process is said to be under control, and control flow moves back to step 2. If or two successiveand , for for the first time, the process is said to be out-of-control, and control flow proceeds to the next step.
6. Signal the out-of-control state.
7. An assignable cause should be traced and removed from the process.

The ARL of the GR-R chart, say, is defined as average number of samples that are required to detect a shift of magnitude in the process standard deviation. Following Gadre and Rattihalli (2004), the can be calculated as follows:

 (8)

For in-control process, the in-control ARL of GR-R chart is given by

 (9)

where denotes the probability of nonconforming group will occur at given shift .

Based on above general structure, we design GR-R control chart for monitoring increase or decrease in the process standard deviation when parameters of the process are known. The proposed GR chart is based on combination of each of Shewhart-R sub-chart with the extended version of CRL sub-chart.

The average run length ARL, which denotes the average number of R samples required to detect a change in  of the GR-R chart can be respectively calculated for and  by replacing in Eq. (8) with Eqs. (2) and (3):

When 

 (10)

When

  (11)

*Optimal Design Procedure*

Gadre and Rattihalli (2004) provided an algorithm to obtain the optimal design parameters, that is, search for the parameters  such that specified in-control ARL value is attained for the GR chart. Based on this algorithm, the optimal design parameters and for the GR-R chart those result in minimum out-of-control ARL value, subject to in-control ARL which at least equal to  are obtained. The optimal parameters  and for GR-R chart are obtained using the following algorithm.

*Algorithm to obtain optimal design parameters of GR-R chart*

The optimal design procedure of GR-R chart is as follows:

1. Specify group size , shift size  and in-control ARL value as 
2. Initialize as 2.
3. If , obtain the value of by solving Eq. (10) (use ) taking as  If , obtain  by solving Eq. (11) (use ) taking  as 
4. When, evaluate from the current values of  andusing Eq. (10) (use ). When , evaluate  from current values of  and using Eq. (11) (use ).
5. If  is not equal to 2, proceed to step (7); else, increaseby one and go back to step (3).
6. If the present ARL for is greater than the preceding one, go to the step (8); otherwise, increase  by one and go back to step (3).
7. Take the preceding  and (or ) as the optimal design parameters for the GR-R chart.

In order to illustrate design of GR-R chart, consider the case when , and in control . Table I shows that at each set of results in different values of . The ARL first decreases and then increases. The value reaches its minimum at 8.7383 when and . So, in this case with ,and , the optimal design parameters for GR-R chart are and.

Table I: Different pairs of  and for , and specified 

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
| 2 | 3.8686 | 12.2883 |
| 3 | 4.0046 | 10.7250 |
| 4 | 4.0970 | 9.9128 |
| 5 | 4.1669 | 9.4447 |
| 6 | 4.2226 | 9.1524 |
| 7 | 4.2689 | 8.9688 |
| 8 | 4.3082 | 8.8507 |
| 9 | 4.3425 | 8.7859 |
| 10 | 4.3728 | 8.7501 |
| **11** | **4.3998** | **8.7383** |
| 12 | 4.4242 | 8.7473 |
| 13 | 4.4465 | 8.7718 |
|  |  |  |

1. **Performance Evaluation and Comparison**

In this section, the performance of the proposed GR-R control chart is investigated using ARL as a performance measure through extensive simulation. In-control observations are generated from distribution. A shift of magnitude is applied to the standard deviation, so that resultant standard deviation becomesThe out-of-control observations are generated from distribution with shifted dispersion value. For simulation purpose, we have takenand the in-control ARL value as By performing 50000 simulations, the out-of-control ARL values along with their standard errors are determined. For positive shift in the standard deviation, we have considered shift sizes as and for negative shift, size of shifts considered are The sample sizes considered are. The simulated ARL values of the proposed GR-R charts along with their standard errors (shown in parenthesis) are presented in Table II and Table III for increases and decreases in shifts respectively. The ARL values of GR-R chart are compared with that of Shewhart-R as well as synthetic R chart.

Table II: ARL comparison of GR-R and synthetic R chart for positive shift in dispersion

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| Shift  |  |  |  |
| She-R | Syn-R | GR-R | She-R | Syn-R | GR-R | She-R | Syn-R | GR-R |
| 1.0 | 200 | 200 | 200.8 (1.260) | 200 | 200 | 200.47 (1.218) | 201 | 201 | 200.08 (1.211) |
| 1.1 | 68.47 | 45.38 | 35.24 (0.241) | 59.59 | 37.01 | 27.84 (0.19) | 57.36 | 33.47 | 24.73 (0.169) |
| 1.2 | 30.47 | 16.85 | 12.14 (0.079) | 24.71 | 12.47 | 8.68 (0.055) | 22.53 | 10.98 | 7.51 (0.047) |
| 1.3 | 16.56 | 8.76 | 6.49 (0.036) | 12.49 | 6.14 | 4.52 (0.023) | 10.93 | 5.42 | 3.83 (0.019) |
| 1.4 | 10.47 | 5.64 | 4.33 (0.020) | 7.27 | 3.9 | 3.03 (0.013) | 6.30 | 3.47 | 2.62 (0.01) |
| 1.5 | 6.94 | 4.12 | 3.27 (0.013) | 5.04 | 2.91 | 2.33 (0.008) | 4.22 | 2.50 | 2.02 (0.007) |
| 2.0 | 2.41 | 1.85 | 1.66 (0.005) | 1.72 | 1.38 | 1.29 (0.003) | 1.52 | 1.28 | 1.19 (0.002) |
|  | 4.488--- | 4.33718 | 4.07015 | 5.265--- | 4.68513 | 4.39911 | 5.434--- | 4.86213 | 4.55810 |
|  |  |  |  |  |  |  |  |  |  |

Table III: ARL comparison of GR-R and synthetic R chart for negative shift in dispersion

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| Shift  |  |  |  |
| She-R | Syn-R | GR-R | She-R | Syn-R | GR-R | She-R | Syn-R | GR-R |
| 1.0 | 200 | 200 | 200.51 (1.14) | 201 | 200 | 200.48 (1.159) | 201 | 201 | 199.85 (1.146) |
| 0.9 | 133.99 | 96.90 | 76.1 (0.461) | 106.37 | 68.40 | 49.07 (0.31) | 93.65 | 57.35 | 40.06 (0.255) |
| 0.8 | 85.25 | 44.75 | 28.02 (0.181) | 54.23 | 23.41 | 13.54 (0.091) | 41.12 | 16.79 | 9.77 (0.064) |
| 0.7 | 51.68 | 19.81 | 10.67 (0.071) | 15.28 | 8.59 | 4.55 (0.028) | 17.24 | 5.62 | 3.31 (0.018) |
| 0.6 | 29.37 | 8.58 | 4.43 (0.027) | 11.31 | 3.51 | 2.13 (0.009) | 7.26 | 2.56 | 1.71 (0.006) |
| 0.5 | 15.41 | 3.82 | 2.15 (0.01) | 4.98 | 1.86 | 1.34 (0.003) | 3.08 | 1.47 | 1.19 (0.002) |
| 0.1 | 1.00 | 1.00 | 1 (0) | 1.00 | 1.00 | 1 (0) | 1.00 | 1.00 | 1 (0) |
|  | 0.555--- | 0.9425 | 1.14874 | 1.073--- | 1.4307 | 1.6765 | 1.333--- | 1.7007 | 1.9375 |
|  |  |  |  |  |  |  |  |  |  |

Figure 1 graphically displays the ARL profiles of Shewhart-R, synthetic R and GR-R charts for positive and negative shifts in the standard deviation. It can be seen from Table II and Table III as well as from Figure 1, that the ARL performance of GR-R chart is not only better than Shewhart-R chart, but it is also better than synthetic R chart for all sizes of shift as well as for all sizes of the subgroups considered in this study.



Figure 1: *ARL comparison of Shewhart R, synthetic R and GR-R charts*

1. **Application**

In this section we consider a real dataset for demonstrating the proposed GR-R chart. Consider the Strength Data, as in Montgomery (2009), which is obtained as a result of compressive strength test on parts manufactured by an injection molding process, given in Table IV. First twenty samples, each of size 5, are phase-I samples and are used for estimating process parameters. The Shapiro-Wilk test performed on these sample observations reveal that the data can be considered to be normally distributed. The estimated parameters of this normal process are and  As phase-II samples, total 15 samples of size 5 each are provided. Table IV gives phase-I as well as phase-II samples. On the phase-II data, three charts, Shewhart-R, synthetic R and GR-R are applied.

Both the Shewhart-R chart and synthetic R chart gives out-of-control signal at sample no. 5, whereas GR-R chart gives the signal at sample no. 4. It underlines the early detection ability of GR-R chart as compared to corresponding synthetic and Shewhart chart.

Table IV: Compressive strength data

|  |  |
| --- | --- |
|  |  |
| Phase-I | Phase-II |
| No. | Observations | No. | Observations |
| 1 | 83 | 81.2 | 78.7 | 75.7 | 77 | 1 | 68.9 | 81.5 | 78.2 | 80.8 | 81.5 |
| 2 | 88.6 | 78.3 | 78.8 | 71 | 84.2 | 2 | 69.8 | 68.6 | 80.4 | 84.3 | 83.9 |
| 3 | 85.7 | 75.8 | 84.3 | 75.2 | 81 | 3 | 78.5 | 85.2 | 78.4 | 80.3 | 81.7 |
| 4 | 80.8 | 74.4 | 82.5 | 74.1 | 75.7 | 4 | 76.9 | 86.1 | 86.9 | 94.4 | 83.9 |
| 5 | 83.4 | 78.4 | 82.6 | 78.2 | 78.9 | 5 | 93.6 | 81.6 | 87.8 | 79.6 | 71 |
| 6 | 75.3 | 79.9 | 87.3 | 89.7 | 81.8 | 6 | 65.5 | 86.8 | 72.4 | 82.6 | 71.4 |
| 7 | 74.5 | 78 | 80.8 | 73.4 | 79.7 | 7 | 78.1 | 65.7 | 83.7 | 93.7 | 93.4 |
| 8 | 79.2 | 84.4 | 81.5 | 86 | 74.5 | 8 | 74.9 | 72.6 | 81.6 | 87.2 | 72.7 |
| 9 | 80.5 | 86.2 | 76.2 | 64.1 | 80.2 | 9 | 78.1 | 77.1 | 67 | 75.7 | 76.8 |
| 10 | 75.7 | 75.2 | 71.1 | 82.1 | 74.3 | 10 | 78.7 | 85.4 | 77.7 | 90.7 | 76.7 |
| 11 | 80 | 81.5 | 78.4 | 73.8 | 78.1 | 11 | 85 | 60.2 | 68.5 | 71.1 | 82.4 |
| 12 | 80.6 | 81.8 | 79.3 | 73.8 | 81.7 | 12 | 86.4 | 79.2 | 79.8 | 86 | 75.4 |
| 13 | 82.7 | 81.3 | 79.1 | 82 | 79.5 | 13 | 78.5 | 99 | 78.3 | 71.4 | 81.8 |
| 14 | 79.2 | 74.9 | 78.6 | 77.7 | 75.3 | 14 | 68.8 | 62 | 82 | 77.5 | 76.1 |
| 15 | 85.5 | 82.1 | 82.8 | 73.4 | 71.7 | 15 | 83 | 83.7 | 73.1 | 82.2 | 95.3 |
| 16 | 78.8 | 79.6 | 80.2 | 79.1 | 80.8 |  |  |  |  |  |  |
| 17 | 82.1 | 78.2 | 75.5 | 78.2 | 82.1 |  |  |  |  |  |  |
| 18 | 84.5 | 76.9 | 83.5 | 81.2 | 79.2 |  |  |  |  |  |  |
| 19 | 79 | 77.8 | 81.2 | 84.4 | 81.6 |  |  |  |  |  |  |
| 20 | 84.5 | 73.1 | 78.6 | 78.7 | 80.6 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

1. **Conclusions**

In the present paper, a control chart based on group runs namely GR-R chart is proposed for improved monitoring of the changes in the process dispersion of normally distributed process. The proposed GR-R chart is a combination of the Shewhart R chart with the extended version of CRL chart. The performance of the proposed chart is evaluated using Monte Carlo simulations and is compared with the Shewhart-R and synthetic-R charts. It is found that the proposed GR-R chart is very effective for monitoring increases and decreases in process dispersion of normally distributed process. The results of comparison showed that GR-R chart performs better than synthetic R chart.

**Statements and Declarations**

* The authors have no competing interests to declare that are relevant to the content of this article.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

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