**MODELLING CURRENCY IN CIRCULATION IN GHANA: AN APPLICATION OF EXTREME VALUE THEORY**

**Abstract**

The Bank of Ghana is dedicated to achieving and maintaining price stability, formulating and implementing monetary policy instruments to influence interest rates and manage liquidity. Among the challenges confronting local economies worldwide, currency circulation is essential for economic development. Currency in circulation (CiC) serves as a critical indicator of economic health, influencing inflation, monetary policy, and overall economic stability. The objective here is to identify an appropriate trend model; to ascertain the volatility characteristics; evaluate the risk; and predict the future volatility of CiC. Leveraging 415 data points spanning from 1990 to 2024, the analysis reveals that the CiC data exhibit heavy-tailed characteristics, positive skewness, and high kurtosis. These features indicate the presence of significant outliers and extreme events that may not be adequately captured by conventional models. The analysis employs both the Generalized Pareto Distribution (GPD) and the Generalized Extreme Value (GEV) models through block maxima and threshold-based methodologies. These models, grounded in extreme value theory (EVT), enable robust estimation of the probabilities and magnitudes of extreme events. This approach effectively estimates tail risk measures, particularly Value-at-Risk (VaR) and Expected Shortfall (ES). The CiC exhibits high volatility, as evidenced by the elevated standard deviations of the scale parameters, specifically 0.08296 and 1.8263. The fitted GPD models yield insights into the likelihood and severity of extreme events, underscoring the risks associated with extreme but impactful occurrences in both directions (gains and losses). The VaR estimates, computed at the 99th percentile, are 54.81% for monthly positive returns and 44.17% for negative returns. The Expected Shortfall estimates are 676.79% for monthly positive returns and 194.24% for negative returns.

**Keywords:** extreme events, risk management, value at risk, heavy tail, peak over threshold and forecasting

Subject classification number: E51

1. **Introduction**

The major goal of the Central Bank of the Republic of Ghana (CBRG) is to attain and preserve price stability. To this end, CBRG controls interest rates and money market liquidity by actively deciding on and implementing a range of monetary policy tools. That is, a central bank must control the circumstances that balance supply and demand in the market for bank reserves in order to successfully influence interest rates. Controlling short-term interest rates in accordance with the primary objective of attaining price stability is thus greatly aided by liquidity management that is founded on precise liquidity projections. Of the numerous challenges facing the local economies all over the world, currency in circulation can be singled out as one of the most important areas, which is germane to any meaningful economic development and therefore needs immediate attention. There has been a great deal of uncertainty in Ghana's and the global economy's growth in recent years. This has prompted several critiques of the risk management system and motivated researchers to look for a better approach to deal with infrequent occurrences that have significant repercussions. The challenge is how to represent the extreme phenomena that fall outside the scope of current observation. Depending on the data, these extreme observations can provide significant economic issues regardless of their magnitude. One must rely on a theory that can be applied to statistical models that describe uncommon occurrences to address such fears. One will enquire. How much worse may things go if they go wrong? (WYNN, 2006).

In Ghana, the dynamics of CiC play a crucial role in the stability of the national economy, influencing inflation, liquidity management, and monetary policy. Understanding the extreme fluctuations in currency supply is essential for forecasting economic risks and improving financial decision-making. Traditional models often fail to accurately capture the behavior of extreme events, such as sudden surges or shortages in currency circulation, which can have significant consequences on the financial system. Drawing on previous research that emphasizes the importance of accurate currency modeling such as Kulatunge, (2019), this study filled the gap in applying EVT to the Ghanaian financial context, where limited research has been conducted in this area.

This study proposes the application of EVT to model and analyze the tail behavior of CiC in Ghana. EVT provides a robust framework for studying rare and extreme events, allowing for a more precise risk assessment in currency supply management. By identifying the factors that drive these extremes, this research aims to: identify an appropriate trend for the CiC in Ghana, identify an appropriate volatility of the CiC in Ghana, find the risk associated with the CiC in Ghana, and predict the volatility of the CiC in Ghana. The findings will contribute to a better understanding of extreme fluctuations in currency circulation and offer a valuable tool for managing financial risk in Ghana.

According to Luguterah et al. (2013), the key determinant of the Currency in Circulation (CiC) is the cash demand of both the public and the banking system and the variations in the CiC are vital indicators for monetization and demonetization of the economy. They claimed that the fluctuation in the CIC may reduce the amount of loan money available for investments and put inflationary pressure on the economy. That said, the share of the CiC money supply and its ratio to nominal Gross Domestic Product (GDP) reveals its relative importance in any economy, (Simwaka, 2006).

According to the External Statistics Division of the European Central Bank (ECB) (Darracq et al., 2021), recent empirical evidence on CiC has shown a significant inconsistency between the total CiC and the estimates of holdings in various statistical domains. Some of the evidence points to the ECB estimate of euro currency circulating outside the euro area as a prominent cause of this inconsistency. As pricing changes demonstrate, Ghana's growth trajectory is closely linked to the global economy, as is to be expected when sectors are intertwined. These phenomena might also apply to the volatility during the period and shocks that frequently occur among commodities, Bhattacharya and Joshi (2001) stated that to model CiC in India, understanding the elements that influence its movement is crucial, particularly its pattern and seasonality. Nasiru et al. (2013) stated that the Currency in Circulation in Ghana is volatile and subject to several unobservable developments in the economy therefore continuous monitoring of the forecasting performance of the models, review of market conditions, and necessary adjustments are required to make the use of the models more realistic. This assertion was earlier opined by Avdulaj and Baruník (2009) which stated in general that the historical simulation method overestimates the Value at Risk (VaR) for extreme events while the variance-covariance method underestimates it. According to Bonam (2024) downward price rigidities (DPR) stylized two-sector. Four key insights are offered by the new Keynesian model. The first is that a relative demand shock with DPR causes inflationary pressures to accumulate that are very non-linear in shock magnitude. Secondly, because they induce both a rise in inflation and a fall in measured output, such inflationary relative demand shocks appear to be "supply shocks" when seen through the prism of a basic Phillips curve framework. The third is that by temporarily permitting higher inflation, the central bank mitigates the real-side effects of the inflationary relative demand shock. Nominal rigidities along the equilibrium path hinder the quick adjustment of relative prices, a process facilitated by inflation. Furthermore, there is no other plausible explanation for why price pressures rise after relative demand shocks apart from "demand-shift inflation." Instead, this mechanism amplifies the inflationary effects of other proposed theories, making it a valuable complement to existing ideas. EVT has been applied to extreme events measurements on many occasions, such as prediction of natural disasters, yet its history as a regular tool in financial risk management is short especially in CiC in Ghana (Lin et al., 2014).

Assessing an extreme event is crucial in financial risk management. All risk managers and financial institutions will want to know the risk and volatility of their portfolio under rare event scenarios. Estimating CiC is a crucial part of the reserve money forecast which guides daily liquidity management and any observed volatility in the interbank call rate and other money market interest rates suggests that improvements could be made to the liquidity forecasting and management process (Ikoku, 2014). On the other hand, more proactive liquidity management will promote price stability and lessen money market interest rate volatility (Ikoku, 2014). According to Bhattacharya and Joshi (2001), net injection or absorption of liquidity in an economy is crucially dependent on the public's demand for currency, and yet this is one variable on which neither the central bank nor the banking system has full control. It was argued that the "Public's preference for a given amount of currency holding is not constant but is influenced by a host of factors such as payment habits in the economy, availability of banking services, statutory requirements regarding the mode of specific payments, desire for precautionary balances, apart from economic factors such as the rate of inflation, level of income and seasonal factors”, (Ramirez-Rojas, 1985). Many studies have recently examined the sharp fluctuations that occur in financial markets, primarily as a result of currency crises, stock market meltdowns, and significant loan defaults. Since financial institutions with high trading activity have shown themselves to be extremely susceptible to sharp market swings, regulators and internal risk control have made measuring market risk a top priority. Balli and Elsamadisy, (2012), however, argued that money market liquidity is inﬂuenced by several autonomous factors that are beyond the control of the central banks, and a considerable change of these factors increases or decreases liquidity, thereby leading to ﬂuctuations in the money supply and most importantly these ﬂuctuations in money supply led to volatility in daily interest rates. For instance, market risk rules apply to Ghanaian banks and bank holding companies that have a sizable trading portfolio, at some point, the complexity surpasses human naive expectations and can lead to several financial catastrophes. The 2008 crisis and the 1990s bubble burst are two instances where shocks happened and the unanticipated market decline caused both public and private sectors to collapse, many of them in a single day.

The rest of the article is organized as follows: Section one deals with introduction whilst section two covers statistical methods and tools employed in the study. Section three duals on the results obtained and Section four summarizes the findings, and Section five covers conclusions made from the study

1. **Methodology**

This section deals with the source of data for the research and the innumerable statistical methods used in the data analysis.

**2.1 Source of data**

The Central Bank of Ghana (BoG) database provided secondary statistics on monthly currency in circulation, expressed in billions of Ghana cedi. From January 1990 to July 2024, 415 monthly Currency in Circulation data points make up the data. The statistical program R was used to analyze the data.

**2.2 Stationarity test**

Stationarity in time series analysis is grouped into two types; strong stationarity and weak stationarity. If the mean and auto-covariance of the series' first two moments are finite, then the variable is said to be covariant or weakly stationary and are independent of time and the auto-covariance function, stay constant over time at any lag, say k. The model's estimated parameters' standard qualities are guaranteed by the stationarity requirement. The estimated model can be used for predicting once this requirement is met. If the mean, variance, auto-covariance, and all other higher moments at any lag, say k, stay constant throughout time, the series is said to be firmly stationary. This can be mathematically stated as:

 and

where is constant and is independent of time. Weak stationarity is always taken into consideration for the study of time series data, nevertheless, because the postulation of strong stationarity is not always acceptable for practical applications.

**2.3 Unit root test**

The unit root test ascertains the presence of either a stochastic or deterministic trend within a time series. A series is stationary when the roots of its distinctive equation reside outside the unit circle. To test for a unit root in a specified series, its characteristics must be thoroughly understood. If the series exhibits both seasonal and non-seasonal behaviors, it is essential to perform stationarity tests on both components. Identifying unit roots is crucial for understanding the processes generating the time series data and examining the order of integration of the series. Various methodologies have been developed to assess the stationarity of time series data, including both graphical and quantitative techniques. Visual examination of the Autocorrelation Function (ACF) plots is part of the graphical technique. If the ACF decays quickly after a few lags, the series is said to be stationary; if it decays strongly and gradually over several lags, it is said to be non-stationary. To check for unit roots in this study, the ACF method was combined with two quantitative techniques.

**2.4 Augmented Dickey-Fuller (ADF) Test**

The ADF test was employed in the study to determine whether the data involved contains a unit root (non-stationary) or has stationary covariance. The Dickey-Fuller (DF) test was expanded upon by Dickey and Fuller's ADF test, which was predicated on the idea that the series follows a random walk. Given an order one autoregressive process, AR (1):

where represents a white noise sequence that is serially uncorrelated, has a constant variance, and a zero mean. Equation (1) transforms into a random walk model without drift, or a non-stationary process, if . To determine whether or not the estimated ϕ is statistically equal to one, the basic idea behind the ADF test is to simply regress on its lagged value . When is subtracted from both sides of equation (1), the result is:

where and

To test the null hypothesis, contrast δ=0 with the alternative δ≠0. If δ=0, then ϕ=1, indicating that the series has a unit root. If δ=0, the t-value of the estimated coefficient of does not have an asymptotic normal distribution (Erdogdu, 2007).

Rejecting the null hypothesis or anything else is based on the DF critical values of the τ-statistic and the fact that DF test errors usually show serial correlation, rather than presuming that the error components may be uncorrelated. By adding the delays of the first difference series to the regression equation, the ADF test circumvents this problem by making the error term white noise.

adding an intercept and time trend t, to equation (3), gives:

Where is a constant, the coefficient of the series, is the total of the dependent variable's lag values and p is the autoregressive process's lag order. In the ADF test, δ is the parameter of interest. The initial augmentation order is determined by the frequency of the data, the importance of white noise residuals and estimates. To experience more efficient estimations, non-significant parameter augmentation can be removed after preliminary estimation. The ADF test's test statistic is provided by

where the standard error of the least square estimate of is denoted by . If the test statistic exceeds the crucial value, the null hypothesis is rejected.

**2.5 Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test**

The KPSS is another harmonizing test for determining the order of integration of a series by testing the null hypothesis that, the data-generating process is stationary () against the alternative that it is non-stationary (. The test assumes that a data-generating process of the following type is the point of departure in the absence of a linear trend.

Where is a random walk, , and is a white noise. Given the foregoing, the pair of assumptions described above is identical to the pair;

.

.

If H\_0 is true, then Y\_t is stationary since it is made up of a constant and the stationary process, ε\_t.

If is true, is stationary since it is made up of a constant and the stationary process, . The test statistic is given by

where is the number of observations, with and is an estimator of

That is, is an estimator of the long-run variance of the process is integrated of order one (I (1)) if is a stationary process, and the quantity in the denominator of the KPSS statistic is an estimator of its variance with a stochastic limit. The term in the denominator guarantees that there are no unknown nuisance parameters in the overall limiting distribution. However, for large sample sizes, an unbounded increase in the numerator will produce a big statistic if is integrated of order one (I (1)), and the null hypothesis of stationarity is rejected for high KPSS values.

**2.6 Criterion for Model Selection**

The most appropriate model that best describes a time series of data must be produced by meeting the model selection criteria. This is because there is a chance that two or more models will compete for the title of best model. To determine the optimal model, the Bayesian Information Criterion (BIC), the Akaike Information Criterion corrected (AICc), and the Akaike Information Criterion (AIC) were employed. The information requirement includes a penalty that rises in proportion to the number of parameters. Increasing the number of parameters almost always improves the goodness of fit since the penalty prevents overfitting. The best model among a group of candidate models is the one with the lowest AIC, AICc, or BIC values. The AIC, AICc, and BIC are normally supplied by;

where the model’s parameter count is denoted by *k,* *L* stands for the likelihood function’s maximised value. The number of observations in the data is denoted by n and the error variance is denoted by .

**2.7 Extreme Value Theory (EVT) models**

Extreme Value Theory (EVT) is a probabilistic framework for analyzing rare events, specifically those at the tails of a probability distribution. It is used for parametric modeling of these tails, enabling the estimation of extreme quantiles without specific assumptions about the overall distribution shape. EVT relies solely on information about extreme events for tail modeling, allowing application without comprehensive distributional assumptions. Its parametric nature also facilitates out-of-sample extrapolation for quantile estimation. EVT employs two primary methodologies for examining extreme events. First, the block maxima method, characterizes the distribution of maximum or minimum intuitions within a process. Using this method, the data sample is divided into discrete blocks, and the highest value from each block is identified as an extreme event. A Generalized Extreme Value (GEV) distribution is then fitted to these maximum values to model them separately. The distribution of exceedances over a predetermined threshold is modelled using the second strategy, the peak-over-threshold method. This approach creates a cutoff point beyond which all realizations are deemed excessive. Once the threshold is set, observations that exceed it are extracted, and exceedances are computed as the difference between the extreme value and the threshold. These exceedances are then fitted to a generalized Pareto distribution to characterize the extreme events. Both methodologies within EVT incorporate a parameter essential for identifying the extreme values of the analyzed process.

**2.8 Generalized Pareto Distribution**

The Generalized Pareto Distribution (GPD) was introduced in Embrechts et al. (1997) about the theory of extreme values. The following function defines the typical generalized Pareto distribution:

Where if and otherwise. The distribution function is generally denoted by

To obtain the full generalized Pareto distribution, the scale parameter and a location parameter are introduced:

where ξ is the shape parameter that dictates the distribution's behaviour and tail type, μ is the location parameter that determines the distribution's center, and δ is the scale parameter that measures the distribution's spread and δ >0.

Where lies in and respectively. This is denoted by which is simply . It is to be noted that when , it gives Pareto II distribution. when , it yileds a shifted exponential distribution and when gives a generalized beta I distribution.

From this expression the cumulative density function for the generalized Pareto distribution is given as:

**2.9 Value-At-Risk**

Value at Risk (VaR) calculates the maximum loss that can be anticipated from a portfolio over a certain time horizon at a specified confidence level, (Morgan, 1996). By using the appropriate quantile, we can estimate Value at Risk if we indicate profits with a negative sign and losses with a positive sign qα of the conditional distribution.

While VaR provides an easy-to-understand method for assessing risk, Artzner et al., (1998) have questioned it as a danger indicator for two primary reasons. First, they demonstrated that VaR is not always sub-additive, making it an incoherent indicator of risk. Secondly, this metric provides a maximum loss amount for a given confidence level, but it provides no insight into the possible magnitude of the loss should this maximum be exceeded.

**Expected Shortfall and Return Level**

The Expected Shortfall (ES) also known as the Conditional VaR (CVaR) or tail Value at Risk (TVaR) of a portfolio or asset is the average loss when VaR is surpassed, (Artzner et al., 1999).

where rt is the return at time t.

Despite being a logical indicator of risk, ES's accuracy also hinges on the capacity to determine the portfolio's actual loss distribution (Artzner et al., 1999).

The return level is provided as follows, assuming that H is the distribution of the maxima seen across consecutive nonoverlapping periods of equal length:

Accordingly, the threshold predicted to be surpassed in one out of k periods of length n is represented by equation (17). A more cautious metric than Value-at-Risk, the return level can be used to gauge a portfolio's maximum loss.

**2.10 Mean Excess Plot**

A graphical tool for selecting the threshold is offered by the mean excess plot. The GDP's mean excess function is linear and reaches infinity. According to Iii (1975), for high threshold, the excess over a threshold for a given series converges to a GPD. By identifying a region on the graph with a linear shape, one can determine the threshold at which a GPD approximation makes sense.

**2.11 Mean Excess Function**

This is a statistical instrument used to describe the expected value of excess losses or exceedances over a certain threshold. It plays a significant role in risk management, EVT, and reliability analysis. The excess function is given below:

Or

Where X is a random variable representing losses or exceedance, μ is the threshold and

E[X-μ|X>μ] is the expected excess given that X > μ.

**2.12 Generalized Extreme Value (GEV) Distribution**

The Generalized Extreme Value (GEV) distribution constitutes a family of continuous probability distributions that emerges within the framework of extreme value theory, particularly in the context of modeling the maxima of large data blocks. This distribution encompasses three distinct types of extreme value distributions: the Gumbel, Fréchet, and Weibull distributions, each of which characterizes different tail behaviors.

The probability density function of the GEV distribution is given by:

 When ξ=0, the GEV reduces to the Gumbel distribution.

The GEV distribution is defined by the cumulative distribution function (CDF) given by:

But , since it must always be positive.

It ensures that x lies within the valid domain of the GEV distribution.

The shape parameter ξ controls the type of distribution that the GEV represents:

When ξ=0: The distribution is of the Gumbel type, which models light-tailed distributions (exponential-like decay). The Gumbel distribution has no specific limit for the maxima.

This can be expresses as:

where,

1. When ξ>0: The distribution is of the Frechet type, which models heavy-tailed distributions (long tails, often seen in financial risks and natural disasters).
2. Frechet distribution models cases where the maxima can grow without bounds.
3. When ξ<0: The distribution is of the Weibull type, which models short-tailed distributions (finite upper limit for maxima).

**2.12 Return Levels**

Derived from the inverse of the CDF, return levels estimate the value x predicted to be exceeded once every T years, where T is the return period. The return level for a return period T is given by:

If ξ=0, the formula simplifies to:

**2.13 Peak Over Threshold (POT)**

The Generalized Pareto Distribution (GPD) model analyzes exceedances over a specified threshold, fitting these values to a generalized Pareto distribution. Point process characterization is more reliable for examining extreme values, as it is consistent with a Poisson process for exceedances above a high threshold and the GPD for excesses over this threshold. This approach offers a comprehensive interpretation of extreme values that integrates all previously examined models. For instance, parameters from the point process model can be translated into those of the Generalized Extreme Value (GEV) parameterization. Furthermore, compared to the block maxima method, the point process approach uses more information about the upper tail of the distribution and can be viewed as an indirect way to fit data to the GEV distribution.

1. **Results and Discussions**

In this section, the monthly currency in circulation (CiC) volatility is analyzed by modelling the distribution's tails using the EVT ideas. Included as well are the exploratory methods and preliminary tests performed on the data, threshold determination, GPD fitting, and tail modelling analysis.

**3.1 Preliminary Analysis**

The data for this thesis had a total of 415 observations from 1990 to 2024 from Central Bank.

**Table 1: Summary Statistics for Currency in Circulation.**

|  |  |  |  |
| --- | --- | --- | --- |
| **Mean**  | 6262.57 | **Std. Deviation** | 10012.48 |
| **Median**  | 1048.62 | **C.V** | 159.88 |
| **Minimum**  | 53.50 | **Skewness**  | 2.24 |
| **Maximum**  | 50365.44 | **Ex. Kurtosis** | 4.77 |

The mean of 6262.57 is elevated, indicating that the data is biased towards larger values.

A standard deviation of 10,012.48, which exceeds the mean, suggests considerable variability, with many values dispersed widely from the mean. This is characteristic of datasets influenced by extreme events or outliers. The coefficient of variation of 159.88% is notably high. In the context of EVT, a high coefficient of variation often signifies significant fluctuations relative to the mean, indicating the presence of extreme values or tail events. The dataset exhibits a lack of homogeneity, with extremes playing a pivotal role. The skewness of 2.24 denotes a positive skew, indicating that the data distribution has a long right tail. In EVT, this skewness implies the existence of infrequent yet substantial extreme values, which EVT seeks to model. The positive skewness suggests that most data points are concentrated towards the lower end, with a limited number of large, positive outliers extending the distribution. An excess kurtosis of 4.77 indicates a leptokurtic distribution (kurtosis greater than 3), suggesting that the dataset possesses heavy tails and a sharp peak compared to a normal distribution. Elevated kurtosis in EVT implies that extreme events (outliers) are more plausible than expected under a normal distribution.

This characteristic suggests that the dataset has an increased probability of extreme deviations from the mean, aligning with EVT's objectives to model and comprehend such rare occurrences.

EVT focuses on the behavior of the tails of the distribution, specifically the maximum (or minimum) observations that represent rare or extreme values. Here, the maximum value of 50,365.44 is significantly larger than both the mean and median, reinforcing the concept of extreme events. Conversely, the minimum value (53.50) is considerably below the mean but does not contribute significantly to the tail behavior as the extreme high values do. EVT can be employed to model the extreme upper tail of this data, given its right skew and high kurtosis, to predict the likelihood and impact of further extreme values.

The elevated skewness and kurtosis values indicate that the dataset contains substantial outliers on the high end, which could be effectively modeled using EVT. The presence of extreme values, particularly in the upper tail, supports the application of EVT to estimate the probability and magnitude of future extreme events.



**Figure 1: Density for GEV Distribution of CiC**

In Figure 1 above, a histogram is produced for the standardized returns, and it was noted that the data in the tails was significantly out of the ordinary.

An analysis on the trend on the CiC was carried out using the log-linear, log-quadratic trend, linear and quadratic models. This indicates that the log-quadratic model was best since it has the lease AIC and BIC as shown in the Table 2 below.

**Table 2: Trend analysis of Currency in Circulation**

|  |  |  |  |
| --- | --- | --- | --- |
| **Model** | **Adjusted R-squared** | **AIC** | **BIC** |
| **Linear**  | 0.614205 | 1789.57 | 1794.86 |
| **Quadratic**  | 0.881332 | 1667.94 | 1675.87 |
| **Log-linear** | 0.802722 | 193.893 | 199.182 |
| **Log-quadratic** | 0.814672 | 188.37\* | 196.303\* |

**\*: indicates best according to the selection criteria.**

Table 3 below also shows the parameters of the log-quadratic model. At the 5% level, every parameter was extremely significant. The model gives an upwards trend of linearity in logarithm in the CiC. The adjusted R-squared was about 81.47% which indicates that large part of the variations in the CiC is caused by the trend. Thus, the model can be written as

**Table 3: Estimated Log-Quadratic Trend Model Parameters**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Variable** | **Coefficient** | **Standard error** | **T -Statistics** | **P -Values** |
| **Constant** | 4.77524 | 0.176974 | 26.9827 |

|  |  |
| --- | --- |
| <0.00001 | \*\*\* |

 |
| **Time**  | 0.0200091 | 0.00778025 | 2.5718 |

|  |  |
| --- | --- |
| 0.01157 | \*\* |

 |
| **Square time** | 0.000197604 | 7.17881e-05 | 2.7526 |

|  |  |
| --- | --- |
| 0.00701 | \*\*\* |

 |
| AIC=188.37 | BIC=196.303 | R-squared=0.81827 | Adj.R-squared= 0.814672 | Durbin Watson=0.549888 |

**\*\*\* and \*\*: means at 0.1 and 1 percent respectively**

 Figure 2 below shows a correlogram of the residual of the log-quadratic trend model which clearly shows a seasonality in the residuals and the Durbin Watson value of 0.549888 shows serial correlation in the errors. The residual ACF has significant spike seasonal displacement of 2, 3, 12, and 29 and the PACF showed a substantial spear at lag 5 and 13.



**Figure 2: Residual Log-Linear Trend Model Correlogram**

A unit root test was used to confirm stationarity. Using the KPSS test, we investigated the null hypothesis, which states that the original series is stationary at the non-seasonal level. Based on the test results, which are shown in Table 4, we reject the null hypothesis that the series is stationary because the calculated value falls within the critical zone at the 5% level of significance.

|  |  |  |
| --- | --- | --- |
| **Test** | **Test Statistic** | **Critical value** |
| KPSS | 1.49432 | 0.463 |

**Table 4: KPSS test of Currency in Circulation**

We reject the null hypothesis because the test statistic (1.49432) is greater than the crucial value (0.463). This result implies that the data is non-stationary and probably shows seasonality, trends, or fluctuating variance across time. The ADF test was used to confirm the existence of the unit root when either a constant or a constant with a linear trend was incorporated into the test. The results are shown in Table 5, where the P-values of the test statistic, constant, and trend are added to yield one (1.00).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Test** |  | **Constant** |  |  **Constant+ Trend** |  |
|  | **Test Statistic** | ***P*-value** | **Test Statistic** |  | ***P*-value** |
| **ADF** | 3.68799 |   | 1.0000 | 1.66389 |   | 1.0000 |

**Table 5: ADF test of Currency in Circulation**

The regression results in Table 5 show that the series exhibits signs of seasonality. The series was then subjected to the logarithmic transformation to stabilize the variance. Stationarity was tested and seasonal differences were applied to the modified dataset. The converted seasonal differenced series was shown to be stationary by the KPSS and ADF tests, which are displayed in Tables 6 and 7, respectively.

|  |  |  |
| --- | --- | --- |
| **Test** | **Test Statistic** | **Critical value** |
| KPSS | 0.0353839 | 0.463 |

**Table 6: KPSS of Seasonal Differenced**

The test statistic of 0.0353839 is well below the critical value of 0.463. The test has a null hypothesis that the data is stationary. Since the test statistic is significantly lower than the critical value, we fail to reject the null hypothesis, indicating that the data is stationary. The results suggest that, based on this test alone, the CiC in Ghana seems to be stationary. This means that fluctuations in the CiC revert to a deterministic trend or constant mean over time, rather than experiencing permanent shock. This stationary behavior implies that temporary shocks to the currency supply would eventually dissipate, and the currency level would return to a long-run equilibrium.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Test** |  | **Constant** |  | **Constant+ Trend** |  |
|  | **Test Statistic** | ***P*-value** | **Test Statistic** |  | ***P*-value** |
| **ADF** | -5.79993 |  | 3.574e-007 | -5.77227 |  | 4.046e-006 |

**Table 7: ADF Analysis of Seasonal Variations**

The results from both the constant-only and constant-plus-trend models show that the CiC in Ghana is stationary. This means that the series reverts to a mean or trend over time, and any shocks or fluctuations in the series are temporary. The fact that the p-values are almost zero further strengthens the confidence in rejecting the null hypothesis of a unit root, indicating the robustness of the stationarity finding. The test statistic for the model with only a constant is -5.79993, and the corresponding p-value is 3.574e-007 (essentially zero). Since the test statistic is significantly lower (more negative) than the critical value for standard significance levels (e.g., 1%, 5%, or 10%), we can reject the null hypothesis of a unit root. This suggests that the data, without considering any trend component, is stationary.

The test statistic when accounting for both a constant and a trend is -5.77227, with a p-value of 4.046e-006. Similarly, the test statistic is lower than the critical values, meaning we also reject the null hypothesis in this case. This indicates that the series is stationary even when considering a deterministic trend.

**3.2 Distribution of Gains and Losses**

The lower and upper tails act differently and should be considered independently when calculating risk measures. The empirical study's findings indicate that there is little variation in the risk statistics of the two tails, suggesting that their thicknesses are probably comparable.

The empirical distribution of the distribution's two tails is displayed in Figure 4.3 below. Compared to the losses distribution (bottom), the gains distribution (top) displays a higher extremal observation. In contrast to the distribution of gains, the losses exhibit a greater number of extreme findings.



**Figure 3: Distribution of Gains and Losses of CiC**

Figure 3 shows a curve (top) that starts low, rises steeply, and then levels off. The few points at the far-right end indicate rare but significant gains. This suggests that while most gains are moderate, there are occasional extreme gains.

The bottom figure is the loss graph. It starts high, drops steeply, and then levels out with a few distinct points at the far-left end. This indicates that while most losses are moderate, there are occasional extreme losses.

These distributions help in understanding the frequency and impact of extreme events, which can be crucial for strategic planning.

**3.3 GEV or Block Maximum Approach**

A block maxima approach was employed to analyse the monthly return data for the gains (right tail) and losses (left tail) distributions. A critical aspect of this method is the suitable selection of periods that define the blocks. The recommended block size is six observations, justified by the volatile nature of the returns and the necessity for a sufficiently large dataset. The study utilized a total of 415 observations, divided into six non-overlapping sub-samples to identify the maximum observed value within each block, resulting in a total sample size of 69 observations.



**Figure 4: A Fitted POT Model's Graphic Diagnostic (Univariate Case)**

The fitted model's graphic diagnostics are displayed in Figure 4 above. The maximum likelihood estimates of the fitted model seem appropriate. Diagnostic plots for a fitted Peak Over Threshold (POT) model typically include Quantile-Quantile (QQ) plots, Probability-Probability (PP) plots, return level plots, and residual plots. Each of these graphical representations evaluates the adequacy of the POT model fit and identifies potential issues regarding model assumptions.

A detailed examination of the QQ plot reveals a close alignment of data points along a straight line, suggesting that the model effectively captures the extremes of the data.

The return level plot illustrates the relationship between return periods (or recurrence intervals) and the corresponding return levels, conveying the expected magnitude of extreme values anticipated to be exceeded within a specified return period (e.g., once every 10 years or once every 50 years).

A comprehensive understanding of the frequency and magnitude of such extreme values is essential for informing decisions related to economic stability.

The use of diagnostic plots is a critical step in validating the fit of the POT model and its relevance to the analysis of CiC. A satisfactory fit, as indicated by these diagnostic tools, would imply that the Generalized Pareto Distribution (GPD) accurately represents the extreme behaviour observed in the data, offering reliable insights into the risks associated with unusually high or low currency levels. In contrast, poor alignment in these diagnostic assessments may suggest the necessity to adjust model parameters, such as the threshold, to enhance the model's accuracy.

**3.4 Return Level Plot with GEV Fit for Positive and Negative Returns**

The return level plot in Figure 5 shows the return level together with an estimated 95% confidence interval. We anticipate that the return level will only be surpassed once every k (time period). On average, it is anticipated that this level (in %) will be surpassed once every m time points (in months). The amount of time anticipated to pass before a specific return level is exceeded is known as the return period.



**Figure 5: Plots of Profile Likelihood for a Monthly Return LEVEL with 95% Confidence (Top Positive Returns) and Bottom Negative Returns.**

From Figure 5 the top graph shows the line trends downward, indicating a decreasing return level over time. The markers at 90%, 95%, and 99% show the return levels for these confidence intervals, with the line flattening out as it approaches higher percentages. Whiles the bottom graph shows a steeper decline, suggesting a more significant decrease in return levels. Like the top graph, the markers indicate return levels at different confidence intervals, with a sharper drop.

These graphs help in understanding the frequency and impact of extreme events, which is crucial for risk assessment and decision-making.

**Table 8: Return Levels for Negative and Positive Returns for GEV Fit**

|  |  |  |  |
| --- | --- | --- | --- |
|  | **CI** | **RL** | **CI** |
| **Positive Returns** | 11.385 | 16.720 | 33.230 |
| **Negative Returns** | 19.791 | 24.238 | 31.115 |

In Figure 5, the upper graph illustrates the profile likelihood plots for the five-quarter return level, with a 95% confidence interval for the return value of 16.720 (11.385, 33.230) for the right tail. The lower graph depicts the left tail, showing a return level value of 24.238 (19.791, 31.115). A return value of 16.72 indicates that the maximum gain (positive returns) observed over a one-year period is expected to exceed 16.72% (19.79%, 31.11%) in one out of four months, on average, with the 95% confidence interval provided in parentheses. For the left tail, a return value of 24.238 indicates that the maximum loss observed over a four-month period is anticipated to surpass 24.24% (19.79%, 31.11%) in one out of four months, on average, with the 95% confidence interval also noted in parentheses.

The distribution's fat-tail is indicated in Table 9 by the shape parameter's point estimate being bigger than zero. Additionally, the heavy-tail distribution at the right tail is confirmed by the confidence interval's exclusion of zero. One (1) indication of extreme events is not included in the scale parameter.

The location parameter of the GEV distribution shifts the distribution along the horizontal axis and indicates the central tendency of extreme values. In this analysis, the location parameter is estimated at 1.6415, with a confidence interval of [1.2795, 2.0032]. This estimate implies that extreme gains are, on average, centered around 1.6415. The confidence interval shows the series within which the true location of the extreme gains is expected to lie 95% of the time, suggesting that extreme gains are likely to fluctuate between 1.2795 and 2.0032.

The location parameter is pivotal in determining the overall level of extreme events within a dataset. In financial contexts, this suggests that extreme gains related to currency circulation or financial markets in Ghana tend to cluster around this value. EVT emphasizes rare but significant events; thus, understanding the central tendency of these gains helps policymakers and economists anticipate the magnitude or volatility of such occurrences. As noted by Ghosh and Resnick, (2010), the location parameter plays a crucial role in defining the typical size of extreme values in economic datasets.

The scale parameter in the GEV distribution measures the variability or spread of the extreme values. The estimate here is 1.7003, with a confidence interval of [1.3110, 2.0895], suggesting that the extreme gains have a relatively widespread. This means the size of the gains during extreme events fluctuates substantially, with the actual extreme values potentially deviating significantly from the central estimate (location parameter). The confidence interval indicates that this variability could be as small as 1.3110 or as large as 2.0895.

In the context of modeling extreme gains in currency circulation in Ghana, the scale parameter’s relatively large value implies that extreme gains are not tightly clustered but are distributed over a broad range of values. This highlights a degree of unpredictability in the extreme gains that could result from economic shocks, policy changes, or other exogenous factors. In practical terms, higher variability suggests that the gains during extreme market conditions could range significantly, emphasizing the need for careful risk management in these scenarios (Coles, 2001).

Perhaps the most critical parameter in the GEV distribution is the shape parameter, which determines the behaviour of the distribution’s tail. In this case, the shape parameter is 0.6919, with a confidence interval of [0.4698, 0.9141]. The positive value of the shape parameter indicates that the extreme gains follow a Fréchet distribution, which is characterized by a heavy right tail. This suggests that the extreme gains in the dataset are not capped at a certain level and can grow to substantial magnitudes, with higher-than-expected extreme values occurring more frequently than in light-tailed distributions such as the normal distribution.

The heavy-tailed nature of the gains indicates that extreme events (i.e., significant gains) are more likely than one might expect based on traditional statistical models. This could be particularly important in financial and economic planning for Ghana, as it suggests the potential for substantial gains during extreme conditions, though such gains are rare. The confidence interval confirms this positive shape parameter, reinforcing the conclusion that the tail of the distribution is heavy, meaning the risk of extreme, unexpected gains should not be underestimated (Beirlant et al., 2004).

**Table 9: Estimates of Generalised Extreme Value Parameters for Gains**

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Point estimate** | **Confidence Interval** |
| **Location** | 1.6415 | [1.2795 2.0032] |
| **Scale**  | 1.7003 | [1.3110 2.0895] |
| **Shape**  | 0.6919  |  [0.4698 0.9141] |

The distribution's fat-tail is indicated in Table 10 by the shape parameter's point estimate being bigger than zero. Additionally, the heavy-tail distribution at the right tail is confirmed by the confidence interval's exclusion of zero. One (1) indication of extreme events is not included in the scale parameter.

The location parameter represents the central tendency of the extreme losses. The point estimate for this parameter is 1.9816, with a 95% confidence interval of [1.6024, 2.3608]. This indicates that the extreme losses are typically centred around 1.9816. In the context of losses, this value suggests that the average or typical extreme loss is around this magnitude, although the true location could vary within the confidence interval.

The importance of the location parameter in modeling extreme losses lies in its ability to identify the "average" size of these extreme events. For example, in financial markets or currency circulation in Ghana, a location parameter of 1.9816 indicates that when extreme losses occur, they are expected to hover around this value. According to Coles (2001), the location parameter is critical in EVT because it helps anchor the distribution and provides a reference point for analysing the extremities of the data. Policymakers or risk managers could use this estimate to prepare for significant, yet typical, loss events in the future.

The scale parameter represents the spread or variability of extreme losses. The point estimate for the scale parameter is 2.2438, with a 95% confidence interval of [1.8157, 2.6719]. This relatively high value indicates that there is a wide range of possible extreme loss values, suggesting substantial variability in how extreme the losses can be.

The high value of the scale parameter highlights the uncertainty and unpredictability surrounding extreme losses. This is particularly important in financial or economic systems where the magnitude of potential losses can vary greatly depending on various market forces or external shocks. The confidence interval also indicates that this spread could be as low as 1.8157 or as high as 2.6719, reinforcing the notion that extreme losses are subject to considerable fluctuation. This level of variability suggests that stakeholders should be prepared for a wide range of outcomes when extreme losses occur, as emphasized in the work of Beirlant et al. (2004) on the statistical modeling of extremes.

The shape parameter is one of the crucial components of the GEV distribution, as it determines the tail behaviour of the distribution. The point estimate for the shape parameter is 0.7798, with a confidence interval of [0.5836, 0.9759]. This positive shape parameter indicates that the extreme losses follow a Fréchet distribution, which is characterized by a heavy right tail. A heavy-tailed distribution means that extreme losses can be very large, with the potential for even larger losses as one moves further into the tail of the distribution.

The positive shape parameter (close to 0.8) suggests that the distribution of extreme losses does not decay quickly, meaning that substantial losses could occur more frequently than one would expect in a light-tailed distribution (such as the normal distribution). This is critical for risk management, as it implies that rare but severe losses may happen with higher probability, thus necessitating robust risk mitigation strategies. As discussed by Ghosh and Resnick (2010), a positive shape parameter in the GEV distribution points to the need for careful planning around extreme events, as the potential for large outliers is significantly higher. This is particularly relevant in the context of extreme financial losses or economic downturns, where extreme value theory can help predict and manage rare but catastrophic events.

**Table 10: Estimates of Generalised Extreme Value Parameters for Losses**

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Point Estimate** | **Confidence Interval** |
| **Location**  | 1.9816  |  [1.6024, 2.3608] |
| **Scale** | 2.2438 |  [1.8157, 2.6719] |
| **Shape** | 0.7798 |  [0.5836, 0.9759] |

**3.5 The Selection of the Threshold**

Adopting the POT approach to represent the tails of the monthly return distribution requires selecting an appropriate threshold. As of right now, there isn't an automatic method that performs well enough to choose the threshold u. Determining the percentage of data that belongs to the tail is a problem that is addressed by Daníelsson et al., (2001), Danfelsson and De Vries, (1997), and Dupuis, (1998) Among other works.

But the references that are now available don't offer a clear solution as to which approach is best. Finding a threshold that is both high enough to guarantee the theoretical development's validity and low enough to produce sufficient data for precise fitting is the goal while an overly high criterion could result in significant variance in these estimates, a threshold that is too low could provide biased parameter estimations. A technique that involves fitting the data to a generalized Pareto distribution (GPD) several times using various thresholds and the mean excess plot, sometimes called the mean residual life plot, are useful tools for threshold selection. The quantile that the Pareto connection holds above can be visually identified with the help of the mean excess plot.

Since a GPD is the underlying distribution of the sample data, the empirical mean excess plot is described as being approximately linear in the threshold u by the estimated parameter of the log-linear trend model in Table 4. Precisely, light and heavy-tailed models can be distinguished using the data's mean excess plot. A medium-tailed distribution plots a horizontal line, a heavy-tailed distribution plots an upward trend, and light-tailed data plots a downward slope. The mean excess plots of negative and positive returns have a common feature in our sample data: an upward-trending section at the far end is followed by an irregular region. Up to u ≈ 0 to 5.5, the first and small portion of the gain plot slopes downward. After that, it slopes upward, where it varies dramatically. In the event of losses, the threshold exhibits a minor rising trend from u≈ 1 to u≈ 5.5, indicating approximate linearity. Accordingly, there is some support for selecting thresholds between 4.5 and 6 for the right tail and between 4.5 and 5.5 for the left tail using the linearity criterion in the mean excess plots displayed in Figure 6 below.

****

**Figure 6: A Display of the Mean Excess Plot for Negative and Positive Returns.**

Figure 7 shows the empirical distribution function on a double logarithmic scale; it draws attention to the tail area. In this case, a linear plot is equivalent to an accurate Pareto distribution.



**Figure 7: Log-Log Distribution of Empirical Data**

The sample mean excess function should be linear since it is an approximation of the mean excess function e(u), which is defined in equation (19). One criterion for choosing u could be this characteristic. The sample mean excess plots for the CiC data are displayed in Figure 8. A deeper look at the plots suggests that the right tail's threshold should be set at u = 5.0. This figure appears at the start of a relatively linear section of the sample mean excess plot.

The GPD is indicated as a straight line with a positive slope over a specified threshold, u = 5.0.



**Figure 8: Mean Excess Plot**



**Figure 9: Mean Residual Plot**

Figure 9 displays the mean residual plot for the CiC data, based on all exceedances over thresholds u = 5.0, along with confidence bands. The dotted lines below and above indicate the estimated ranges of confidence. In contrast, the solid line in the middle represents the computed theoretical mean excess, given the GPD at threshold u. The graph seems to show a more consistent departure from a straight line, reinforcing the notion that threshold 5.0 is preferable, even if it is placed almost anywhere within the confidence bands.

The results can be regarded as preliminary conclusions, and it may be challenging to interpret the mean excess plots as a threshold selection method. Applying GPD fitting and searching for stability in shape parameter estimates is an additional step. a fit to the monthly returns in each tail of the GPD that surpass the corresponding threshold. We may compute the corresponding approximate standard errors and create confidence intervals for the shape parameter as its maximum likelihood estimator is asymptotically normal. Figure 10 for gains and Figure 11 for losses display the shape parameter estimate plotted against various threshold values. Confidence intervals at a 95% level are represented by the upper and lower dashed lines.



**Figure 10: Shape Parameter Estimates (Positive Returns)**

The Shape Parameter (ξ) helps determine the heaviness of the tail of the distribution. A higher ξ indicates a heavier tail, meaning more extreme values are likely.

The different thresholds (0.00209, 0.87000, 1.56000, 2.97000, 5.08000) affect the number of exceedances and the shape parameter estimates. Higher thresholds typically result in fewer exceedances but more extreme values.

The risk estimation here is that the risk of extreme positive returns is higher if the shape parameter is high. This means that while extreme positive returns are rare, they can be very large when they do occur.

The dashed red lines show the range within which the true value of ξ is expected to lie. Wider intervals indicate more uncertainty in the risk estimates and the implication is that a higher ξ suggests a higher risk of extreme market movements, which could impact investment strategies and risk management practices.

 

**Figure 11: Estimates for Shape Parameter (Negative Returns)**

The horizontal axis represents the number of exceedances, which are the data points below a certain threshold.

The dashed line represents the confidence intervals around the shape parameter estimate (solid line), indicating the range within which the true value of ξ is expected to lie with a certain probability. The values at the top (0.0296, 0.7400, 1.7600, 2.7500, 5.3200, 9.3500) indicate different thresholds are used for the analysis, affecting the number of exceedances and the shape parameter estimates. This is crucial for assessing the risk of extreme negative returns in CiC in Ghana and suggests that as fewer exceedances are considered (i.e., more extreme values). The estimated shape parameter increases, indicating a higher risk of extreme negative returns.

Table 11 lists the estimated shape parameter and scale parameter for both tails along with the standard errors that correspond to them under various thresholds.

**Table 11:** **Estimation of Maximum Likelihood Parameter under various Gains Thresholds**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **U1** | **U2** | **U3** | **U4** | **U5** | **U6** |
| **Threshold** | 4.694  | 5.081  | 5.3629  | 6.0459  | 6.5339  | 7.0073 |
| **Shape** | 0.857  | 0.907  | 0.9359713  | 1.0667603  | 1.2208020  | 1.3423249 |
| **S.E** | 0.309  | 0.337  | 0.3595008  | 0.4133521  | 0.4734766  | 0.5520913 |
| **Exceedances** | 36 | 32 | 29 | 25 | 22 | 18 |

The threshold for using EVT must be high enough to allow for the analysis of only the distribution's tail. Too many observations are included when the threshold is near zero. Based on real-world experience, it makes sense to include observations for both negative and positive returns up to about one-fifth of the overall sum of observations. Although a little random, this offers a fair compromise. The threshold range is determine to be between 4.5 and 5.5 for positive returns and between 4.6 and 5.4 for negative returns by combining this restriction with the findings from the mean excess plots and the shape parameter plots.

**Table 12: Estimation of Maximum Likelihood Parameter under various Losses Thresholds**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **U1** | **U2** | **U3** | **U4** | **U5** | **U6** |
| **Threshold** | 6.425 | 7.923  | 8.701  | 9.355 | 11.422 | 12.433 |
| **Shape** | 0.331 | 0.424  | 0.447  | 0.433  |  0.622 |  0.659 |
| **S.E** | 0.173  | 0.197  | 0.213  | 0.226  |  0.282 |  0.319 |
| **Exceedances** | 59 | 52 | 46 | 40 | 33 | 27 |

**3.6 Generalized Pareto Distribution (GPD)**

If a lot of the data is ignored, utilizing only block maximum can occasionally be wasteful. Examining exceedances over a specified threshold rather than just the data's maximum (or minimum) is frequently more helpful. EVT offers threshold selection and data fitting to GPD models.



**Figure 12: Diagnostic for the GPD Fit Plots of CiC Returns Using a Threshold of 5.5 (Gains)**



**Figure 13: Diagnostic for the GPD Fit Plots of CiC Returns Using a Threshold of 4.5 (Losses)**

The actual monthly CiC thresholds are displayed in circles (on a logarithmic scale) in Figure 13 (upper left), while the predicted GPD model for the excess CiC is plotted as a curve. With threshold of *u* = 5.5, the estimated parameters are and. From the CiC data sets, there are 20 exceedances. The data vector's empirical distribution is plotted on logarithmic scaled axes in the upper right corner. A scatter plot of the residuals showing the time of exceedance is displayed in the lower left corner of figure 13. Here, the goal is to look for any potential time trends in the observations.

To facilitate the evaluation of this analysis, a fitted curve (utilizing the Extremes package in R) is overlaid, revealing indication of a consistent trend. The compact line represents the smoothed residuals obtained through a spline method. The QQ-plot of the residuals is displayed in the bottom right corner. Based on the Generalized Pareto Distribution (GPD) fitted to all exceedances above the threshold of 5.5, the plot of the fitted model exhibits minimal deviation from the straight line. The data are consistent with the fitted GPD, which is characterised by an exceptionally fat-tailed distribution, even if there are reasons to regard the largest observations as outliers because they do not deviate considerably from the straight line. To initiate this procedure, threshold values of u = 5.0 and u = 4.5 must be selected for the right and left tails, respectively, as the estimates are contingent upon the excesses over these thresholds. Figures 12 and 13 display the estimates for the primary shape parameter and the corresponding standard deviations for both tails as a function of u (or the number of order statistics employed).

**Table 13: Positive Tail-Index GPD Estimation (Gains)**

|  |  |  |
| --- | --- | --- |
| **PARAMETER** | **GPD ESTIMATE** | **STANDARD DEVIATION** |
| **Shape parameter** |  0.9225 | 0.3000 |
| **Scale parameter** | 2.6840  | 0.8296 |

In this case, the shape parameter is positive and relatively high (0.9225), suggesting a heavy-tailed distribution. This indicates that large gains (extreme positive values) are expected with a non-negligible probability. The standard deviation for the shape parameter is 0.3000, indicating estimation uncertainty. The scale parameter, denoted by σ, measures dispersion for excesses over a high threshold. Larger values indicate a broader distribution of extreme values.

Here, the scale parameter of 2.6840 implies a relatively high spread of extreme gains, meaning the magnitude of extreme positive tail values is considerable. The standard deviation for the scale parameter is 0.8296, reflecting variability in this estimate. Given these estimates, the positive tail of the distribution is heavy-tailed with large expected extreme gains. This suggests a high potential for occasional large gains, with significant uncertainty in both parameters due to the standard deviations provided. The model indicates that gains beyond a certain threshold can vary widely, with an increasing probability of very large gains.

**Table 14: Positive Tail-Index GPD Estimation (Losses)**

|  |  |  |
| --- | --- | --- |
| **Parameter** | **GPD Estimate** | **Standard Deviation** |
| **Shape parameter** | 0.8105 | 0.3979 |
| **Scale parameter** | 4.3599 | 1.8263  |

In this analysis, the shape parameter is 0.8105, indicating that the losses in the negative tail exhibit heavy-tailed characteristics and a higher probability of substantial extreme losses. The standard deviation of 0.3979 reflects considerable uncertainty in this estimate, suggesting moderate variability in the shape parameter. The scale parameter, measured at 4.3599, indicates a significant spread of

extreme losses, implying that losses in the negative tail can be considerable. The standard deviation for the scale parameter, 1.8263, is relatively high and underscores the variability in this estimate, indicating substantial uncertainty regarding the exact magnitude of the losses. This Generalized Pareto Distribution (GPD) model for the negative tail suggests that losses conform to a heavy-tailed distribution, wherein extreme losses are relatively frequent and potentially large. Both parameters exhibit moderate to high variability, indicating uncertainty in estimating the precise heaviness and spread of the tail. Overall, the analysis highlights the potential for significant losses in the negative tail, with non-trivial probabilities associated with extreme loss values, which is a crucial consideration for risk management.

According to the findings in Table 15, the future gain will not surpass 54.81% with a chance of 0.01 or a 99% confidence interval. The average gain in situations where gains surpass 54.81% is known as the expected gain, and it is 676.78%. Likewise, Table 16's findings indicate that the future loss on a short position will not surpass 44.17% with a chance of 0.01 or a 99% confidence range. The expected loss, representing the average loss in scenarios where losses exceed 44.17%, is 194.23%.

For greater quantiles, specifically a 99.99% confidence interval, the results in Table 15 indicate that the future gain on a long position will exceed 3658.11%. The expected gain in scenarios where gains surpass 3658.11% is 47194.209%.

**Table 15: Right-Tail Distribution Risk Measures (GPD-Fit)**

|  |  |  |
| --- | --- | --- |
| **Probability** | **Value-At-Risk(VaR)** |  **Expected Shortfall(ES)** |
| 0.9500 | 14.423 | 155.334 |
| **0.9900** | **54.814** | **676.788** |
| 0.9950 | 101.579 | 1280.497 |
| 0.9990 | 439.518 | 5643.199 |
| 0.9995 | 830.765 | 10694.072 |
| 0.9999 | 3658.105 | 47194.209 |

 Likewise, the findings in Table 16 show that with a 99.99% confidence interval, the future loss on a short position will not exceed 1474.67%, and the expected loss, representing the average loss in circumstances where losses exceed 1474.67%, is 7749.31%.

**Table 16: Left-Tail Distribution Risk Measures (GPD-Fit)**

|  |  |  |
| --- | --- | --- |
| **Probability** | **Value-At-Risk(VaR)** |  **Expected Shortfall (ES)** |
| 0.9500 | 18.631 | 59.336 |
| **0.9900** | **44.174** | **194.237** |
| 0.9950 | 70.602 | 333.815 |
| 0.9990 | 235.768 | 1206.126 |
| 0.9995 | 406.660 | 2108.677 |
| 0.9999 | 1474.677 | 7749.313 |

Figure 14 shows the shape parameter's upper bound on the confidence interval which guarantees that the first-order moment is finite (1/0.35 > 1). This condition verifies that the projected expected shortfall, or conditional first moment, exists for both tails.



**Figure 14: The Shape Parameter's Profile Log-Likelihood and Confidence Intervals. Profits (Upper) and Losses (Lower)**

Figure 14 illustrates the fitted Generalized Pareto Distribution (GPD) for both the positive (right) tail, corresponding to gains, and the negative (left) tail, corresponding to losses. The GPD is instrumental in modeling extreme values, highlighting the probability and magnitude of unusually large gains or losses that exceed a specified threshold. The tails depicted in Figure 14 exhibit a slow decay, suggesting that they remain "heavy" and do not converge to zero rapidly. This characteristic indicates that the distribution is heavy-tailed, signifying that substantial gains and losses are relatively probable and that extreme values are likely to occur. Consequently, this suggests the potential for significant outliers in financial or risk datasets.

For the positive tail (gains), the parameters are as follows: Shape = 0.9225, Scale = 2.6840. For the negative tail (losses), the parameters are: Shape = 0.8105, Scale = 4.3599. These parameters indicate that both tails exhibit heavy-tailed characteristics, with losses potentially exhibiting a wider spread, as evidenced by the higher scale parameter for losses. The fitted GPD models yield insights into the likelihood and severity of extreme events, underscoring the risks associated with rare but impactful occurrences in both directions (gains and losses). This model is particularly valuable in risk management for assessing the probability of extreme financial events.

Typically, a threshold is established beyond which data points are regarded as "extreme" and fitted by the GPD. Figure 14 represents data points that exceed the selected threshold, illustrating the tail behaviour and the distribution of extreme values in both directions. It visualizes the extreme gains and losses fitted using the GPD model, indicating a heavy-tailed distribution for both sides and suggesting the potential for significant outliers. This analysis is essential for evaluating extreme risks within the context of financial modeling.

**Table 17: Summary of fit for Gains**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Parameter** | **Estimate**  | **Std. Error** | **z value**  | **Pr(>|z|)**  | **Confidence level** |
| **Scale (Intercept)**  | 2.68376  | 1.01273  | 2.6500 | 0.0080485 | \*\*(99%) |
| **Shape (Intercept)**  | 0.92235  | 0.36996  | 2.4931 | 0.0126620  | \*(95%) |

As shown in Table 17 the threshold selected is significant at 0.01 and 0.05 levels for the scale and shape parameters respectively.

A higher scale estimate indicates a broader distribution, suggesting that larger values (extremes) are more widely dispersed. The standard error of 1.01273 reflects the uncertainty in the estimation process. A lower standard error relative to the estimate implies greater confidence in the estimated value. The z-value of 2.6500 denotes the number of standard deviations the estimate is from zero (or the null hypothesis). A larger absolute z-value typically suggests that the estimate is statistically significant. The p-value represents the probability of observing a z-value as extreme as 2.6500 under the null hypothesis. Given that this p-value is low (0.008 < 0.01), the estimate is statistically significant at the 99% confidence level, providing strong evidence that the scale parameter is non-zero.

The shape parameter (intercept) estimate of 0.92235 (ξ>0) indicates a heavy tail, suggesting that the distribution has high probabilities for extreme values. The standard error associated with the shape parameter reflects the variability of this estimate. A smaller standard error relative to the estimate indicates higher reliability. The z-value of 2.4931 for the shape parameter also suggests statistical significance. The associated p-value of 0.0126620, which is below the 0.05 threshold, indicates that the estimate is statistically significant at the 95% confidence level. This finding suggests that the shape parameter is likely non-zero, supporting the existence of a heavy tail in the data. Confidence levels convey the probability that the interval includes the true parameter value.

The scale parameter is significant at the 99% confidence level (\*\*), indicating a strong level of confidence in the estimate. Conversely, the shape parameter is significant at the 95% confidence level (\*), suggesting a reasonable level of confidence in the presence of a heavy tail.

The positive and significant shape parameter (0.92235) suggests a heavy-tailed distribution, indicating the likelihood of extreme currency values in circulation. The scale parameter (2.68376) implies a relatively large spread of extreme values, suggesting that extreme currency circulation values are expected to vary widely. The low p-values for both parameters indicate that these estimates are statistically significant, providing strong evidence that the data exhibit heavy tails and significant variability in extreme values.

**Table 18: Summary of fit for Losses**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Parameter** | **Estimate**  | **Std. Error** | **z value**  | **Pr(>|z|)**  | **Confidence level** |
| **Scale (Intercept)**  | 4.35909  | 1.85485  | 2.3501  | 0.018768  | \*(95%) |
| **Shape (Intercept)**  | 0.81062  | 0.40487  | 2.0022 | 0.045264 | \*(95%) |

As shown in Table 18, the threshold selected is significant at 0.05 level for both the scale and shape parameters. A higher scale parameter (4.35909) indicates a wider distribution, suggesting that extreme values reflecting significant currency circulation are more dispersed and may be considerable in size. Furthermore, a higher standard error (1.85485) implies greater variability in the estimation of the parameter, indicating uncertainty regarding the precise dispersion of extreme currency values. The z-value (2.3501) denotes the number of standard deviations the estimate is from zero. A larger absolute z-value (exceeding 1.96 for a 95% confidence level) signifies statistical significance. The p-value of 0.018768 is below 0.05, indicating that the scale parameter is statistically significant at the 95% confidence level. This provides evidence that the scale parameter differs from zero, implying significant dispersion in the extreme values of currency circulation. The shape parameter estimate of 0.81062 is positive, suggesting a heavy-tailed distribution. In extreme value theory (EVT), a positive shape parameter indicates that extreme values in the data exhibit a heavy tail, signifying that large, rare events (such as extreme levels of currency circulation) are more likely to occur. A higher standard error (0.40487) denotes moderate uncertainty; however, the estimate remains statistically significant. The z-value is approximately 2.0022, which is close to the threshold for significance at the 95% confidence level (1.96), indicating that the estimate is marginally statistically significant. The p-value of 0.045264 is below 0.05, rendering the shape parameter statistically significant at the 95% confidence level. This supports the existence of a heavy tail in the distribution of extreme currency levels.

Both parameters are statistically significant at the 95% confidence level, suggesting confidence in the presence of a heavy tail and significant dispersion in extreme currency values. The positive and significant shape parameter (0.81062) indicates a heavy-tailed distribution, suggesting that extreme values (unusually high or low currency circulation) have a relatively high probability of occurrence. The scale parameter (4.35909) indicates that extreme currency values are widely spread, suggesting that when extremes manifest, they can be substantial in magnitude. The low p-values for both parameters provide evidence that these estimates are statistically significant, supporting the model's validity in describing the extreme values of currency circulation in Ghana

1. **Summary of findings**

Value-at-Risk calculates the best- and worst-case scenarios for the CiC's central bank value over a given period. Using the statistics in Tables 11 and 12, we first examine the situations where point estimates fall below the thresholds for the two tails (u=5.5 for the right tail and 4.5 for the left). For the right tail, for instance, we compute VaR as u = 5.5 at the 99th percentile. In other words, we anticipate that a monthly variation in the rate of the currency in circulation won't rise by more than 54.81% under normal circumstances. In other words, the central bank will post a value, with a probability of 1%, which would be anticipated to increase/gain by 54.81% of Gh¢1 billion or more if we have an investment of Gh¢1 billion in the vault. However, for the left tail, VaR is calculated to be u=4.5 at the first percentile. This suggests that the worst monthly loss in the CiC value could be higher than the anticipated 44.174% for the lowest 1% negative monthly returns. In other words, we have 99% confidence that, even if we post Gh¢1 billion in CiC, our monthly loss would not surpass Gh¢441,740 in a single month. Similarly, the calculated VaR is 18.631% for losses and 14.423% for profits at a lower quantile of 95-level. With 95% certainty, we can say that, in a month, the projected Central Bank value of CiC would not rise or fall by more than 155.334% in the best-case scenario or 59.336% in the worst-case scenario. The VaR estimates may or may not be higher than those under a lower threshold, and they are quite similar to their corresponding values under the lower threshold under the higher criterion for both tails (5.5 for the right tail and 5.8 for the left). There are various applications for these estimates, that is, the VaR results in Tables 15 and 16 suggest that an investment in the Central Bank has a comparatively smaller chance of losing money than of making money, even with the same amount of capital. Furthermore, there is a greater disparity between the VaR and ES for negative returns than for positive returns. This indicates that the projected gain over the VaR in a gain scenario is greater than the expected loss over the VaR in a loss scenario. Investors may also be able to predict how much money they will need to invest over a given period with the aid of risk assessments.

1. **Conclusion**

Effective risk management techniques must be put in place due to the volatility of capital investment in Ghanaian markets. This study applies EVT to the monthly returns of Ghana's currency in circulation in the Ghanaian market, demonstrating how it may be used to simulate tail-related risk indicators like Value-at-Risk and Expected Shortfall. The asymmetric features of distributions and the thin-tailed behaviour in monthly returns are captured by the analysis, suggesting that positive and negative returns should be handled differently. Since there is no indication of conditional heteroskedasticity in our sample data, an unconditional approach is recommended. When using EVT, the peak-over-threshold approach works well for estimating parameters and choosing thresholds. We use theoretical distribution simulations, empirical excess distribution, and survival functions to evaluate the goodness of fit in tail modeling. The positive monthly return series also fits a Generalized Pareto Distribution (GPD) somewhat better than the negative series at either a lower or higher threshold level; additionally, the fits for both gains and losses become less accurate as the threshold is raised. This study's Value-at-Risk methodology, which is based on Extreme Value Theory (EVT), offers quantitative insights for examining possible extreme hazards connected to currency in circulation, especially concerning the Central Bank of Ghana.

In conclusion, EVT is useful for estimating how big extreme events are. Various approaches can be used to address this problem, depending on the frequency and availability of data, the intended time horizon, and the level of complexity one is willing to include in the model. In practical applications, the POT method demonstrated superiority as it more effectively utilizes the information contained within the data. Given our interest in long-term behavior rather than short-term forecasting, we favored an unconditional approach. In conclusion, the regression analysis reveals that both time and its square are significant predictors of currency circulation in Ghana, indicating a strong and accelerating upward trend over time. The model performs well regarding goodness-of-fit, as evidenced by the high R-squared values and low AIC/BIC scores. However, the low Durbin-Watson statistic suggests potential issues with autocorrelation, which may need to be addressed in future modeling efforts. Overall, this model provides valuable insights into the time-based trends in currency demand, consistent with broader economic theories on monetary growth and inflationary pressures.

**COMPETING INTERESTS DISCLAIMER:**

Authors have declared that they have no known competing financial interests OR non-financial interests OR personal relationships that could have appeared to influence the work reported in this paper.

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