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| Journal Name: | Asian Journal of Advanced Research and Reports |
| Manuscript Number: | Ms\_AJARR\_140014 |
| Title of the Manuscript: | On Dual Hyperbolic Generalized Edouard Numbers |
| Type of the Article | Research Article |

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PART 1: Comments

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|  | Reviewer’s comment  Artificial Intelligence (AI) generated or assisted review comments are strictly prohibited during peer review. | Author’s Feedback (It is mandatory that authors should write his/her  feedback here) |
| Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part. | This manuscript introduces a novel generalization of the classical Edouard number sequence into the algebraic framework of dual hyperbolic numbers, a relatively recent and evolving structure within hypercomplex number systems. The development of Binet’s formulas, generating functions, and summation identities for the new class of dual hyperbolic generalized Edouard numbers contributes to expanding the known family of hypercomplex recurrence sequences. These extensions hold potential applications in mathematical physics, theoretical computer science, and algebraic coding theory, where such algebraic systems are studied. Furthermore, the incorporation of matrix methods and recurrence relations provides a solid foundation for future generalizations and computational implementations. | Thank you for your comments. |
| Is the title of the article suitable?  (If not please suggest an alternative title) | Partially. While the current title “On Dual Hyperbolic Generalized Edouard Numbers”  reflects the main topic, it could be slightly improved for clarity and academic specificity.  Suggested Alternative Title:  “Dual Hyperbolic Generalized Edouard Numbers: Binet Formulas, Generating  Functions, and Matrix Identities”  This version emphasizes the mathematical content and results more clearly. | We refined the title as A Study On Dual Hyperbolic Generalized Edouard Numbers. |

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| Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here. | The abstract gives a general idea of the manuscript but lacks grammatical precision and omits  key components of a well-structured abstract (such as motivation, methods, and findings). It ends abruptly and includes a fragment: “Moreover, along with matrices associated with these sequences.”  Suggested Revision:  In this study, we introduce dual hyperbolic generalized Edouard numbers, extending the classical Edouard sequence within the framework of dual hyperbolic algebra. We derive their recurrence relations, Binet’s formulas, and generating functions, and examine several special cases such as dual hyperbolic Edouard and Edouard-Lucas numbers. Furthermore, we present associated matrix representations and summation identities that reveal additional algebraic properties of these sequences. | Abstract expended. |
| Is the manuscript scientifically, correct? Please write here. | Yes, overall, it is scientifically correct, with valid definitions, recurrence relations, and logical derivations of Binet-type formulas and generating functions. The definitions are clearly stated, and the use of hypercomplex algebra is consistent with accepted literature. Some identities and matrix theorems (e.g., Lemma 16 and Theorem 17) are well-motivated and carefully constructed. However, a few aspects require better clarity or justification:   Some derivations (especially exponential generating functions) are correct but presented with minimal explanation.   Notational inconsistencies occasionally appear (e.g., typos in superscripts, unexplained variable substitutions).   Proofs rely heavily on earlier results from the author's own previous papers without fully recapitulating them for context.  These issues, however, are technical rather than conceptual and can be fixed during revision. | Some of the proofs in the article were presented concisely due to the already substantial length of the manuscript.  Article fully checked for typos.  Rather than providing lengthy proofs, the desired results were concisely obtained by leveraging established findings available in the literature, As the manuscript is already extensive in length. |
| Are the references sufficient and recent? If you have suggestions of additional references, please mention them in the review form. | Yes, the references are mostly sufficient and include both foundational and recent works  (up to 2023). Key areas like hypercomplex systems, Cayley-Dickson algebras, and special number sequences are well-covered. Some works are self-citations, but relevant.  Suggested Additions (optional, to enhance theoretical breadth):   R. Lounesto, Clifford Algebras and Spinors, Cambridge University Press, 2001 - For hypercomplex algebra structure.   D. Hestenes, New Foundations for Classical Mechanics, Springer, 1999 - For dual numbers in mechanics and geometry.   P. J. Olver, Applications of Lie Groups to Differential Equations, Springer, 2012 - For application-oriented insights into hypercomplex algebra and generating functions. | References added (including these suggestion). |

Created by: DR Checked by: PM Approved by: MBM Version: 3 (07-07-2024)

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| Is the language/English quality of the article suitable for scholarly communications? | Partially. The mathematical content is understandable, but the English throughout the  manuscript requires moderate to major revision for grammar, punctuation, and phrasing. Common issues include:   Missing articles (e.g., “the” before nouns)   Verb agreement errors (e.g., “we gives” instead of “we give”)   Sentence fragments in the abstract and some theorem statements   Repetitive or redundant phrases  Example Fixes:   “We define dual hyperbolic Edouard numbers then using this defination...” → “We define dual hyperbolic Edouard numbers and, using this definition, derive...”   “To proof the above theorem...” → “To prove the above theorem...”  A professional language edit is recommended to meet the standards of scholarly publishing. |  |
| Optional/General comments |  The paper is a solid contribution to the study of generalized recurrence sequences in  hypercomplex systems.   The matrix formulation and Simpson-type identity are particularly interesting and original.   It would enhance readability if a brief discussion on potential applications (e.g., cryptography, signal processing, or theoretical physics) were added.   The authors should consider adding a concluding section to summarize findings and suggest future research directions.   Figures or diagrams to illustrate the algebraic structure of dual hyperbolic numbers or recursive behavior would be helpful, especially for educational clarity. | Conclusion section added. |

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| **PART 2:** | | |
|  | **Reviewer’s comment** | **Author’s Feedback** (It is mandatory that authors should write his/her feedback here) |
| **Are there ethical issues in this manuscript?** | *(If yes, Kindly please write down the ethical issues here in details)* |  |