Hypergraph and Superhypergraph Approaches in Electronics: A Hierarchical Framework for Modeling Power-Grid Hypernetworks and Superhypernetworks

Abstract

Graphs are fundamental tools for modeling pairwise relationships in complex systems. However, many real-world infrastructures—such as energy systems—demand more expressive frameworks to represent multi-node and hierarchical interactions. A *hypergraph* generalizes classical graphs by allowing edges, known as *hyperedges*, to connect multiple vertices simultaneously. Building on this, a *superhypergraph* introduces recursively nested powerset constructions, enabling the representation of layered and self-referential relationships among nodes and edges. In the context of network science, these extensions lead to *hypernetworks* and *superhypernetworks*, which generalize conventional networks to capture higher-order connectivity patterns.

Graph theory is also used in electrical engineering. A *power-grid network* structurally represents an electrical system, where nodes correspond to generation stations, substations, or consumers, and edges represent physical transmission lines.

This study presents rigorous mathematical definitions for two extended network structures specifically tailored to the energy domain: the *Power-Grid HyperNetwork* and the *Power-Grid SuperHyperNetwork*. We provide detailed examples, including the Four-Bus Power System and the Antenna System, to illustrate how these models can effectively capture the complexity of real-world power systems. Future research is expected to advance the computational analysis and practical applications of these models in smart grid optimization, simulation, and planning.

Keywords: Superhypergraph, Hypergraph, Power-Grid Networks, HyperNetworks, SuperHyperNetworks, Networks

1 Introduction

1.1 HyperGraph Theory and SuperHyperGraph Theory

Graph theory is a branch of mathematics concerned with the study of networks, where nodes (called vertices) are connected by links (called edges) [38,39,41]. Graphs have been extensively investigated and applied across numerous disciplines, including social science [95, 104, 112], artificial intelligence [20, 113], graph neural networks (GNNs) [55,56,70], chemical graph theory [45,58,140,144], and general network analysis [32,69]. Graphs offer a clear and intuitive way to visualize real-world concepts, which is why they are widely studied and applied across various fields.

Mathematical structures can often be extended to hyperstructures and superhyperstructures by employing the power set and n-th iterated powerset constructions (cf. [131,134]). Hyperstructures are a mathematical systems where operations map to sets of outputs, generalizing classical algebraic structures [60]. Superhyperstructures are an advanced frameworks using nested powersets to model hierarchical, multi-level, or recursive mathematical relationships [132]. These extended frameworks are particularly effective for representing hierarchical and multi-layered systems in both theoretical and applied settings.

When applied to graph theory, these extensions yield two well-known generalizations: the *hypergraph* and the *superhypergraph*. A hypergraph allows each edge—known as a *hyperedge*—to connect more than two vertices simultaneously, enabling the representation of complex many-to-many relationships [18, 22, 93]. A superhypergraph builds upon this by incorporating recursively nested powerset structures, allowing for hierarchical and self-referential interactions among groups of hyperedges [48, 129, 130]. Graphs are commonly used to represent networks, and in this context, *hypernetworks* and *superhypernetworks* emerge as the network counterparts of hypergraphs and superhypergraphs, respectively.

The overview of Graph, HyperGraph, and SuperHyperGraph is presented in Table 1. These considerations also hold from the perspective of networks and structures. Note that n in an n-superhypergraph is a natural number. And note that super-vertices/edges represent subsets of vertices or edges, enabling hierarchical grouping and recursive relationships within superhypergraph structures.

Concept	Notation	Edge Connectivity	Structural Extension
Graph	$G = (V, E)$	$E \subseteq \{\{u, v\} \mid u, v \in V,\ u \neq v\}$ (standard edges)	Standard graph: edges join exactly two vertices.
HyperGraph	$H = (V, E)$	$E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ (hyperedges)	Generalizes edges to connect any nonempty subset of vertices.
SuperHyperGraph	$\mathrm{SHT}^{(n)} = (V, E)$	$V, E \subseteq \mathcal{P}^n(V_0)$ (super-vertices/edges)	Uses n-fold iterated powersets to model hierarchical, nested connectivity among edges.

Table 1: Overview of Graph, HyperGraph, and SuperHyperGraph

1.2 Power-Grid Network and Power-Grid HyperNetwork

Electronics is the branch of science and engineering that studies and applies the flow and control of electric current in circuits [147, 150, 154]. Electrical power systems are responsible for the generation, transmission, and distribution of electricity through a network of interconnected components, ensuring consistent and reliable energy delivery to residential, commercial, and industrial users [40, 43, 88]. These systems are closely related to energy systems [42, 91] and infrastructure networks [86, 117]. Research on power systems and related fields has remained highly active in recent years due to their significant societal importance [123, 137, 141, 151].

Graph theory has been extensively applied in the analysis and modeling of energy systems and infrastructure networks [159, 160]. A *power-grid network* provides a structural representation of electrical power systems, in which nodes correspond to generation stations, substations, or consumers, and edges represent the physical transmission lines that connect them [7, 30, 149]. This framework effectively captures the topological and operational characteristics essential for maintaining the stability and efficiency of electricity distribution. Similar to power systems, power-grid networks have also been the subject of extensive research in recent years due to their significant societal importance [103, 163, 164]. It is also worth noting that related concepts such as *Smart Grid Systems* have been developed in the context of modernizing and optimizing power-grid infrastructures [9, 94, 114]. These concepts have been extended from graph theory to hypergraph theory and are being applied in areas such as smart energy systems and electrical grid optimization [31, 35, 80].

1.3 Our Contribution

In this study, we propose formal mathematical definitions for two higher-order generalizations of the power-grid network: the *Power-Grid HyperNetwork* and the *Power-Grid SuperHyperNetwork*. We investigate their structural properties and provide illustrative examples that highlight their expressive capabilities. These models offer a novel theoretical framework for representing complex, multi-way, and hierarchical interactions within power systems. While this work remains theoretical in scope, we hope it will stimulate further research into practical applications in real-world energy infrastructure and smart grid systems.

1.4 Structure of this paper

The structure of this paper is outlined as follows. Section 2 provides a concise overview of the fundamental concepts, including Classical Structure, Hyperstructure, and n-Superhyperstructure; Graph, HyperGraph, SuperHyperGraph, Hypernetwork, and n-SuperHypernetwork; as well as the Power-Grid Network. Section 3 discusses the properties of the Power-Grid Hypernetwork and the *Power-Grid n-SuperHypernetwork*. Section 4 presents the conclusion of this paper along with directions for future research.

2 Preliminaries and Definitions

This section provides an overview of the fundamental concepts and definitions essential for the discussions in this paper. All concepts considered in this paper are assumed to be finite.

2.1 Classical Structure, Hyperstructure, and n-Superhyperstructure

A *Classical Structure* represents a general mathematical concept, while a *Hyperstructure* can be defined using the power set, and an *n-Superhyperstructure* can be defined using the n-th powerset [133]. Intuitively, the n-th powerset is a repeated application of the powerset operation. Relevant definitions and simple examples are provided below. Note that n is assumed to be a natural number.

Definition 2.1 (Set)**.** [77] A *set* is a well-defined collection of distinct objects, called elements. A set is usually denoted by capital letters such as A, B, S, etc.

Definition 2.2 (Subset)**.** [77] Let A and B be sets. We say A is a *subset* of B, written $A \subseteq B$, if every element of A is also an element of B.

Definition 2.3 (Empty Set)**.** [77] The *empty set*, denoted by $\emptyset$, is the unique set that contains no elements.

Definition 2.4 (Universal Set)**.** [77] A *universal set*, denoted by U, is the set that contains all elements under consideration in a particular context or discussion.

Definition 2.5 (Base Set)**.** A *base set* S is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of S.

Definition 2.6 (Powerset)**.** [47, 120] The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 2.7 (n-th Powerset)**.** (cf. [47, 133])

The n-th powerset of a set H, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the powerset of H with the empty set removed.

Example 2.8 (Hierarchical Bus Clustering via n-th Powerset)**.** Bus clustering in electrical engineering groups electrical buses into clusters to simplify power system analysis, control, and optimization (cf. [25, 116]). Let the base set of buses in a simple power grid be

$$H = \{b_1, b_2, b_3\},$$

where each b_i denotes a bus node. Then the first powerset is

$$P_1(H) = \big\{ \{b_1\}, \{b_2\}, \{b_3\}, \{b_1, b_2\}, \{b_1, b_3\}, \{b_2, b_3\}, \{b_1, b_2, b_3\} \big\}.$$

We interpret each element of $P_1(H)$ as a subnetwork of connected buses (for example, a single transmission corridor or loop).

Next, the second powerset is

$$P_2(H) = P\big(P_1(H)\big),$$

whose elements are sets of subnetworks. For instance,

$$U_1 = \big\{\{b_1, b_2\}, \{b_2, b_3\}\big\}, \quad U_2 = \big\{\{b_1\}, \{b_1, b_3\}\big\} \in P_2(H).$$

Here, U_1 represents a cluster of two parallel transmission paths, and U_2 groups together a single-line segment with a two-bus loop.

Continuing to the third level, the third powerset is

$$P_3(H) = P\big(P_2(H)\big),$$

which clusters collections of bus-subnetwork clusters. For example,

$$W = \{ U_1, U_2\} \in P_3(H)$$

models the joint interaction of the two distinct subnetwork clusters, such as simultaneous contingencies on both clusters.

This n-th powerset construction provides a systematic way to move from individual buses ($P_0(H) = H$) to subnetworks ($P_1(H)$), to clusters of subnetworks ($P_2(H)$), and beyond, enabling multi-level analysis of hierarchical dependencies and contingency scenarios in power-grid systems.

Definition 2.9 (Classical Structure)**.** (cf. [126,133]) A *Classical Structure* is a mathematical framework defined on a non-empty set H, equipped with one or more *Classical Operations* that satisfy specified *Classical Axioms*. Specifically:

A *Classical Operation* is a function of the form:

$$\#_0 : H^m \to H,$$

where $m \geq 1$ is a positive integer, and H^m denotes the m-fold Cartesian product of H. Common examples include addition and multiplication in algebraic structures such as groups, rings, and fields.

Definition 2.10 (Hyperoperation)**.** (cf. [142, 143]) A *hyperoperation* is a generalization of a binary operation where the result of combining two elements is a set, not a single element. Formally, for a set S, a hyperoperation $\circ$ is defined as:

$$\circ : S \times S \to \mathcal{P}(S),$$

where $\mathcal{P}(S)$ is the powerset of S.

Definition 2.11 (Hyperstructure)**.** (cf. [47, 126, 133]) A *Hyperstructure* extends the notion of a Classical Structure by operating on the powerset of a base set. Formally, it is defined as:

$$\mathcal{H} = (\mathcal{P}(S), \circ),$$

where S is the base set, $\mathcal{P}(S)$ is the powerset of S, and $\circ$ is an operation defined on subsets of $\mathcal{P}(S)$. Hyperstructures allow for generalized operations that can apply to collections of elements rather than single elements.

Example 2.12 (Resistor-Tolerance Hyperstructure in Electrical Engineering)**.** Resistor tolerance indicates the permissible deviation from its nominal value, typically expressed as a percentage (cf. [105, 118, 145]). Let S be the set of nominal resistor values appearing in a power-grid circuit, each specified with a tolerance of $\pm 5\%$. For instance,

$$S = \{100\,\Omega \pm 5\%,\ 200\,\Omega \pm 5\%\}.$$

Define a hyperoperation $\otimes$ modeling the series combination of two resistors:

$$\otimes : S \times S \longrightarrow \mathcal{P}(\mathbb{R}^+), \quad (R_1, R_2) \mapsto \{r_1 + r_2 \mid r_1 \in [R_1(1-0.05),\ R_1(1+0.05)],\ r_2 \in [R_2(1-0.05),\ R_2(1+0.05)]\}.$$

For example,

$$(100\,\Omega \pm 5\%) \otimes (200\,\Omega \pm 5\%) = [95 + 190,\ 105 + 210] = [285\,\Omega,\ 315\,\Omega].$$

Then $\mathcal{H} = (\mathcal{P}(S), \otimes)$ is a hyperstructure that captures the range of possible resistances arising from tolerance when connecting resistors in series.

Definition 2.13 (SuperHyperOperations)**.** (cf. [133]) Let H be a non-empty set, and let $\mathcal{P}(H)$ denote the powerset of H. The n-th powerset $\mathcal{P}^n(H)$ is defined recursively as follows:

$$\mathcal{P}^0(H) = H, \quad \mathcal{P}^{k+1}(H) = \mathcal{P}(\mathcal{P}^k(H)), \quad \text{for } k \geq 0.$$

A *SuperHyperOperation* of order (m, n) is an m-ary operation:

$$\circ^{(m,n)} : H^m \to \mathcal{P}^n_*(H),$$

where $\mathcal{P}^n_*(H)$ represents the n-th powerset of H, either excluding or including the empty set, depending on the type of operation:

- If the codomain is $\mathcal{P}_*^n(H)$ excluding the empty set, it is called a *classical-type (m,n)-SuperHyperOperation*.
- If the codomain is $\mathcal{P}^n(H)$ including the empty set, it is called a *Neutrosophic (m,n)-SuperHyperOperation*.

These SuperHyperOperations are higher-order generalizations of hyperoperations, capturing multi-level complexity through the construction of n-th powersets.

Definition 2.14 (n-Superhyperstructure)**.** (cf. [133]) An *n-Superhyperstructure* further generalizes a Hyperstructure by incorporating the n-th powerset of a base set. It is formally described as:

$$\mathcal{SH}_n = (\mathcal{P}_n(S), \circ),$$

where S is the base set, $\mathcal{P}_n(S)$ is the n-th powerset of S, and $\circ$ represents an operation defined on elements of $\mathcal{P}_n(S)$. This iterative framework allows for increasingly hierarchical and complex representations of relationships within the base set.

Example 2.15 (SuperHyperOperations and n-Superhyperstructure in a Simple Power-Grid)**.** Let the base set of buses be

$$S = \{b_1,\ b_2,\ b_3\}.$$

We construct the first two iterated powersets:

$$\mathcal{P}^1(S) = \big\{\{b_1\}, \{b_2\}, \{b_3\}, \{b_1, b_2\}, \{b_1, b_3\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}\big\},$$

$$\mathcal{P}^2(S) = \mathcal{P}\big(\mathcal{P}^1(S)\big),$$

whose elements are sets of subnetworks of S.

1. SuperHyperOperation of order $(2,1)$

Define the binary SuperHyperOperation

$$\circ^{(2,1)} : S \times S \longrightarrow \mathcal{P}^1(S)$$

by letting $\circ^{(2,1)}(b_i, b_j)$ be the set of buses on the unique minimal path between b_i and b_j. Concretely:

$$\circ^{(2,1)}(b_1, b_2) = \{b_1, b_2\}, \quad \circ^{(2,1)}(b_2, b_3) = \{b_2, b_3\}, \quad \circ^{(2,1)}(b_1, b_3) = \{b_1, b_2, b_3\}.$$

Then the pair

$$\mathcal{SH}_1 = \big(\mathcal{P}^1(S),\ \circ^{(2,1)}\big)$$

is a *Hyperstructure* on $\mathcal{P}^1(S)$.

2. SuperHyperOperation of order $(2,2)$

Next, define the binary SuperHyperOperation

$$\circ^{(2,2)} : S \times S \longrightarrow \mathcal{P}^2(S)$$

by mapping two buses to the set of all one-hop subnetworks on their connecting path. For example:

$$\circ^{(2,2)}(b_1, b_3) = \big\{\{b_1, b_2\},\ \{b_2, b_3\}\big\} \in \mathcal{P}^2(S),$$

while

$$\circ^{(2,2)}(b_1, b_2) = \big\{\{b_1, b_2\}\big\}, \quad \circ^{(2,2)}(b_2, b_3) = \big\{\{b_2, b_3\}\big\}.$$

Then

$$\mathcal{SH}_2 = \big(\mathcal{P}^2(S),\ \circ^{(2,2)}\big)$$

is a *2-Superhyperstructure*, capturing two levels of hierarchical clustering.

Interpretation:

- $\circ^{(2,1)}$ yields subnetworks (paths) between pairs of buses.
- $\circ^{(2,2)}$ yields clusters of these subnetworks, representing all one-edge segments along a path.
- $\mathcal{SH}_1$ and $\mathcal{SH}_2$ demonstrate how power-grid elements can be modeled with increasing levels of hierarchical complexity using SuperHyperOperations and n-Superhyperstructures.

2.2 SuperHyperGraph

In classical graph theory, a hypergraph extends the idea of a conventional graph by permitting edges—called hyperedges—to join more than two vertices. This broader framework enables the modeling of more intricate relationships between elements, thereby enhancing its utility in various fields [57, 62, 63, 148].

A *SuperHyperGraph* is an advanced extension of the hypergraph concept, integrating recursive powerset structures into the classical model. This concept has been recently introduced and extensively studied in the literature [1, 26, 54, 130].

Definition 2.16 (Graph)**.** [38, 39] A *graph* $G = (V, E)$ consists of a finite set V of vertices and a set $E \subseteq \{\{u, v\} \mid u, v \in V,\ u \neq v\}$ of edges, where each edge connects a pair of distinct vertices.

Example 2.17 (Graph Model of a Five-Bus Radial Distribution Network)**.** Consider a simple radial distribution system with five buses:

$$V = \{1, 2, 3, 4, 5\}, \quad E = \big\{\{1, 2\}, \{2, 3\}, \{2, 4\}, \{4, 5\}\big\}.$$

Here:

- Vertex 1 represents the feeder head (substation).
- Vertices 2, 3, 4, 5 represent load buses.
- Each edge $\{i, j\} \in E$ models a distribution line segment between buses i and j.

The resulting graph $G = (V, E)$ is connected and acyclic, reflecting the radial topology of the network.

Definition 2.18 (Subgraph)**.** [38, 39] Let $G = (V, E)$ be a graph. A *subgraph* $G' = (V', E')$ of G satisfies $V' \subseteq V$ and $E' \subseteq \{\{u, v\} \in E \mid u, v \in V'\}$. That is, G' consists of a subset of vertices and the edges of G induced by them.

Example 2.19 (Subgraph for Core Network Analysis)**.** From the above network $G = (V, E)$, we may extract the subgraph governing the primary feeder path:

$$V' = \{1, 2, 3\}, \quad E' = \big\{\{1, 2\}, \{2, 3\}\big\}.$$

Then $G' = (V', E')$ is the subgraph induced by V', isolating the main feeder trunk for targeted analysis (e.g. fault study or voltage drop assessment).

Definition 2.20 (Hypergraph)**.** [18, 22] A *hypergraph* $H = (V(H), E(H))$ consists of:

- A nonempty set $V(H)$ of vertices.
- A set $E(H)$ of hyperedges, where each hyperedge is a nonempty subset of $V(H)$, thereby allowing connections among multiple vertices.

Unlike standard graphs, hypergraphs are well-suited to represent higher-order relationships. In this paper, we restrict ourselves to the case where both $V(H)$ and $E(H)$ are finite.

Example 2.21 (Distribution Feeder Hypergraph in Electrical Engineering)**.** A distribution feeder is a power line that delivers electricity from a substation to multiple end-user locations or load points (cf. [15, 83, 108]). Consider a small radial distribution network with four buses:

$$V(H) = \{\, b_1, b_2, b_3, b_4\},$$

where each b_i represents a busbar (node) in the network. Let the network have two feeders:

$$F_1 \text{ supplies loads at } \{b_1, b_2, b_3\}, \quad F_2 \text{ supplies loads at } \{b_2, b_3, b_4\}.$$

We model this as a hypergraph

$$H = \big(V(H),\ E(H)\big), \quad E(H) = \big\{\{b_1, b_2, b_3\},\ \{b_2, b_3, b_4\}\big\}.$$

Here each hyperedge corresponds to a feeder connecting multiple buses:

- $\{b_1, b_2, b_3\}$ represents feeder F_1,
- $\{b_2, b_3, b_4\}$ represents feeder F_2.

This hypergraph captures overlapping service areas (b_2, b_3 lie on both feeders) and allows analysis of connectivity, fault propagation, and reconfiguration at the feeder level.

Definition 2.22 (n-SuperHyperGraph). [129, 130]
Let V_0 be a finite base set of vertices. For each integer $k \geq 0$, define the iterative powerset by

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{k+1}(V_0) = \mathcal{P}(\mathcal{P}^k(V_0)),$$

where $\mathcal{P}(\cdot)$ denotes the usual powerset operation. An *n-SuperHyperGraph* is then a pair

$$\mathrm{SHT}^{(n)} = (V, E),$$

with

$$V \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}^n(V_0).$$

Each element of V is called an *n-supervertex* and each element of E an *n-superedge*.

Example 2.23 (2-SuperHyperGraph of Regional Power-Grid Clusters). A Regional Power-Grid is a localized electrical network interconnecting generation, transmission, and distribution systems within a specific geographic region (cf. [29, 155, 161]). Let the base set of substation nodes in a regional power grid be

$$V_0 = \{\, s_A,\ s_B,\ s_C,\ s_D \},$$

where each s_X denotes a geographic substation. The first iterated powerset is

$$\mathcal{P}^1(V_0) = \big\{\{s_A\}, \{s_B\}, \{s_C\}, \{s_D\}, \{s_A, s_B\}, \{s_B, s_C\}, \{s_C, s_D\}, \dots\big\},$$

each element representing a connected subnetwork of one or two substations.

The second powerset is

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big),$$

whose elements are sets of subnetworks. We select two 2-supervertices:

$$U_1 = \big\{\{s_A, s_B\},\ \{s_B, s_C\}\big\}, \quad U_2 = \big\{\{s_C, s_D\},\ \{s_A, s_D\}\big\}.$$

Here U_1 clusters the north–central corridor, and U_2 clusters the south–west ring.

We then form the 2-SuperHyperGraph

$$\mathrm{SHT}^{(2)} = \big(V,\ E\big),$$

with

$$V = \{\, U_1,\ U_2 \} \ \subseteq\ \mathcal{P}^2(V_0), \quad E = \big\{\{U_1\},\ \{U_2\},\ \{U_1, U_2\}\big\} \ \subseteq\ \mathcal{P}^2(V_0).$$

Interpretation:

- Each 1-supervertex (element of $\mathcal{P}^1(V_0)$) is a physical subnetwork segment.
- Each 2-supervertex U_i groups two such segments into a higher-level corridor or ring.
- A 2-superedge $\{U_1, U_2\}$ links both clusters, representing interdependencies between the north–central and south–west corridors under contingency scenarios.
- This construction captures two hierarchical levels of power-grid connectivity, enabling analysis of both local subnetworks and their interactions at a regional scale.

2.3 Hypernetwork and n-SuperHypernetwork

Network theory has been explored and applied across a wide range of disciplines [14,19,162]. In this subsection, we present the formal definitions of the *Hypernetwork* and the *n-SuperHypernetwork*, which generalize classical network structures to capture higher-order and hierarchical relationships.

Definition 2.24 (Network)**.** A *network* (or *graph*) is an ordered triple

$$N = (V,\ E,\ w)$$

where

- V is a nonempty finite set of *vertices* (or *nodes*);
- $E \subseteq \{\{u,v\} \mid u,v \in V,\ u \neq v\}$ is the set of *undirected edges*, each joining two distinct vertices;
- $w\colon E \to \mathbb{R}_{\geq 0}$ is a *weight function* assigning a nonnegative real weight to each edge (omitted if unweighted).

If edges are *directed*, one instead writes

$$N = (V,\ A,\ w), \quad A \subseteq V \times V,$$

and each $(u,v) \in A$ is an *arc* from u to v. In either case, one may also include an optional *vertex-labeling* $\ell_V : V \to L_V$ to record vertex types.

Definition 2.25 (Hypernetwork)**.** (cf. [5,27,65]) A *hypernetwork* is an ordered triple

$$H = (V,\ \mathcal{E},\ w)$$

where

- V is a nonempty finite set of *nodes*;
- $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ is the set of *hyperedges*, each hyperedge $e \in \mathcal{E}$ being a nonempty subset of nodes (allowing multi-node interactions);
- $w\colon \mathcal{E} \to \mathbb{R}_{\geq 0}$ is a *weight or attribute function* on hyperedges (omitted if unweighted).

A *directed hypernetwork* may be defined by replacing $\mathcal{E} \subseteq \mathcal{P}(V)$ with a set of *ordered* tuples of nodes or by equipping each $e \in \mathcal{E}$ with a head-tail partition. One can further add a *node-labeling* $\ell_V : V \to L_V$ and a *hyperedge-labeling* $\ell_{\mathcal{E}} : \mathcal{E} \to L_{\mathcal{E}}$ to record types or properties.

Example 2.26 (Massive MIMO Antenna Hypernetwork)**.** MIMO antenna systems use multiple transmit and receive antennas to improve communication capacity, reliability, and spectral efficiency in wireless networks (cf. [68,79,84]). Consider a massive MIMO base station equipped with eight antennas:

$$V = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}.$$

We form hyperedges corresponding to different beamforming subarrays, with weights given by their measured array gains (in dBi):

$$\mathcal{E} = \{e_1 = \{A_1, A_2, A_3, A_4\},\ e_2 = \{A_5, A_6, A_7, A_8\},\ e_3 = \{A_2, A_4, A_6, A_8\}\},$$

$$w(e_1) = 15, \quad w(e_2) = 17, \quad w(e_3) = 14.$$

Then the *Antenna Hypernetwork* is

$$H_{\text{MIMO}} = (\ V,\ \mathcal{E},\ w\ ).$$

Interpretation:

- Nodes A_i represent individual antenna elements.
- Hyperedges e_h represent multi-antenna subarrays used to form beams.
- Weight $w(e_h)$ is the measured beamforming gain of subarray e_h.

Definition 2.27 (n-SuperHypernetwork)**.** [49] Let V_0 be a finite base set of *nodes*. Define the n-th iterated powerset recursively by

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big) \quad (k \geq 0).$$

An *n-superhypernetwork* is a tuple

$$\mathcal{N}^{(n)} \;=\; \big(V,\; \mathcal{E},\; w\big)$$

where

- $V \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-supernodes*;
- $\mathcal{E} \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-superedges*, each superedge $e \in \mathcal{E}$ being a nonempty subset of V;
- $w\colon \mathcal{E} \to \mathbb{R}_{\geq 0}$ is an optional *weight function* assigning a nonnegative real weight (or confidence) to each superedge.

In other words, both vertices and hyperedges of the network are drawn from the n-th powerset of the base node set, capturing up to n levels of hierarchical grouping.

Example 2.28 (2-SuperHypernetwork of Antenna Subarray Clusters)**.** Starting from the same base antennas $V_0 = \{A_1, \ldots, A_8\}$, their first powerset yields all possible subarrays; we focus on the three hyperedges from Example 2.26. The second powerset clusters these subarrays:

$$\mathcal{P}^1(V_0) \supseteq \{\, e_1, e_2, e_3\}, \quad \mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}^1(V_0)).$$

Select two 2-supernodes:

$$U_1 = \{e_1, e_2\}, \qquad U_2 = \{e_2, e_3\}.$$

These lie in $V^{(2)} \subseteq \mathcal{P}^2(V_0)$. We then form the 2-SuperHypernetwork

$$\mathcal{N}^{(2)} = \big(V^{(2)},\, \mathcal{E}^{(2)},\, w^{(2)}\big),$$

with

$$V^{(2)} = \{U_1, U_2\}, \quad \mathcal{E}^{(2)} = \big\{\{U_1\}, \{U_2\}, \{U_1, U_2\}\big\}.$$

The weight function aggregates subarray gains:

$$w^{(2)}(\{U_1\}) = \frac{w(e_1) + w(e_2)}{2} = \frac{15 + 17}{2} = 16,$$

$$w^{(2)}(\{U_2\}) = \frac{w(e_2) + w(e_3)}{2} = \frac{17 + 14}{2} = 15.5,$$

$$w^{(2)}(\{U_1, U_2\}) = \frac{w(e_1) + 2\,w(e_2) + w(e_3)}{4} = \frac{15 + 2 \cdot 17 + 14}{4} = 15.75.$$

Interpretation:

- Each 1-supernode e_h is an antenna subarray.
- Each 2-supernode U_k clusters two subarrays, representing composite beam patterns.
- Weight $w^{(2)}(U_k)$ is the average array gain of the clustered subarrays.
- The 2-superedge $\{U_1, U_2\}$ models interactions between different beamforming clusters.

Example 2.29 (Power-Grid 3-Superhypernetwork of a Regional Microgrid)**.** A microgrid is a localized power system that can operate independently or alongside the main grid to supply electricity (cf. [2, 98, 125]). Let the base set of buses be

$$V_0 = \{1, 2, 3, 4, 5, 6\},$$

and suppose the six feeders (1-terminal lines) have per-unit admittances:

$$y_{12} = 0.010 - j\,0.050, \quad y_{23} = 0.015 - j\,0.045, \quad y_{34} = 0.012 - j\,0.048,$$

$$y_{45} = 0.020 - j\,0.100, \quad y_{56} = 0.018 - j\,0.090, \quad y_{61} = 0.022 - j\,0.110.$$

Step 1: 1-supernodes (feeders)

$$\mathcal{P}^1(V_0) = \big\{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{5,6\}, \{6,1\}\big\}.$$

Step 2: 2-supernodes (area corridors) Select two 2-supernodes clustering adjacent feeders:

$$U_N = \big\{\{1,2\}, \{2,3\}, \{3,4\}\big\}, \quad U_S = \big\{\{4,5\}, \{5,6\}, \{6,1\}\big\}.$$

Assign aggregated weights:

$$w^{(2)}(U_N) = y_{12} + y_{23} + y_{34} = 0.037 - j\,0.143,$$

$$w^{(2)}(U_S) = y_{45} + y_{56} + y_{61} = 0.060 - j\,0.280.$$

Step 3: 3-supernodes (regional clusters) Form two 3-supernodes grouping the 2-supernodes:

$$W_1 = \{\, U_N \,\}, \quad W_2 = \{\, U_S \,\}.$$

Define regional weights equal to their 2-level aggregates:

$$w^{(3)}(W_1) = w^{(2)}(U_N) = 0.037 - j\,0.143, \quad w^{(3)}(W_2) = w^{(2)}(U_S) = 0.060 - j\,0.280.$$

3-superedges and incidence: We introduce one 3-superedge connecting the two regions:

$$\mathcal{E}^{(3)} = \big\{\{W_1, W_2\}\big\}, \quad w^{(3)}(\{W_1, W_2\}) = w^{(3)}(W_1) + w^{(3)}(W_2) = 0.097 - j\,0.423.$$

Power-Grid 3-Superhypernetwork:

$$\mathcal{N}^{(3)} = \big(V^{(3)},\, \mathcal{E}^{(3)},\, w^{(3)}\big), \quad V^{(3)} = \{W_1, W_2\}.$$

This structure models three hierarchical levels: (1) individual feeders; (2) area corridors clustering feeders; (3) regional clusters linking corridors.

2.4 Power-grid Network

A *power-grid network* models the structure of electrical power systems, where nodes represent generation stations, substations, or consumers, and edges represent physical power transmission lines [28, 66]. The definition and example of a power-grid network are presented below.

Definition 2.30 (Power-Grid Network)**.** A *power-grid network* is a quadruple

$$\mathcal{G} = (V, E, Y, S)$$

where:

- $V = \{1, 2, \ldots, N\}$ is the set of *buses* (nodes).
- $E \subseteq \{\{i, j\} : i \neq j\}$ is the set of *branches* (transmission lines).
- $Y = [Y_{ij}] \in \mathbb{C}^{N \times N}$ is the *bus admittance matrix*, defined by

$$Y_{ij} = \begin{cases} -y_{ij}, & \{i, j\} \in E, \\ \sum_{k:\{i,k\} \in E} y_{ik} + y_i^{\mathrm{sh}}, & i = j, \\ 0, & \text{otherwise,} \end{cases}$$

  where each branch $\{i, j\}$ has series admittance $y_{ij} = g_{ij} + j b_{ij}$ and y_i^{sh} is the shunt admittance at bus i.
- $S = (S_1, \ldots, S_N) \in \mathbb{C}^N$ is the vector of *net complex power injections*, $S_i = P_i + jQ_i$.

The voltage phasor vector $V = (V_1, \ldots, V_N)^\top \in \mathbb{C}^N$ and injection satisfy

$$I = YV, \qquad S_i = V_i I_i^* \quad (i = 1, \ldots, N),$$

which yield the *AC power-flow equations*:

$$P_i = \sum_{j=1}^{N} |V_i||V_j| \big(G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \big),$$
$$Q_i = \sum_{j=1}^{N} |V_i||V_j| \big(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \big),$$

where $Y_{ij} = G_{ij} + jB_{ij}$ and $V_i = |V_i| e^{j\theta_i}$.

Example 2.31 (Three-Bus DC Approximation)**.** Consider a simple three-bus network under the DC power-flow approximation:

$$V_0 = \{1, 2, 3\}, \quad E = \big\{\{1, 2\}, \{2, 3\}, \{1, 3\}\big\},$$

with susceptances $b_{12} = b_{23} = b_{13} = 10$ pu, and power injections

$$P_1 = +1.0, \quad P_2 = -0.4, \quad P_3 = -0.6 \quad \text{(pu)},$$

assuming $|V_i| = 1$ pu and ignoring losses. Let bus 1 be the slack bus with $\theta_1 = 0$. The DC flow equations are

$$P_i = \sum_{j:\{i,j\} \in E} b_{ij} (\theta_i - \theta_j), \quad i = 2, 3.$$

Hence

$$\begin{cases} -0.4 = 10(\theta_2 - 0) + 10(\theta_2 - \theta_3), \\ -0.6 = 10(\theta_3 - 0) + 10(\theta_3 - \theta_2). \end{cases}$$

Solving gives $\theta_2 \approx -0.0467$ rad and $\theta_3 \approx -0.0533$ rad. The branch power flows (from i to j) are

$$P_{12} = 10(\theta_1 - \theta_2) \approx 0.467, \quad P_{23} = 10(\theta_2 - \theta_3) \approx 0.067, \quad P_{13} = 10(\theta_1 - \theta_3) \approx 0.533 \quad \text{(pu)}.$$

This three-bus example illustrates the use of graph structure, susceptance matrix, and power-flow equations to determine voltage angles and line flows.

3 Result: Power-grid Hypernetwork and *Power-Grid n-SuperHypernetwork*

In this section, we present the formal definitions of the Power-Grid *Hypernetwork* and the *Power-Grid n-SuperHypernetwork*.

3.1 Power-Grid Hypernetwork

The Power-Grid Hypernetwork is a structural model in which each hyperedge represents a power device connecting multiple buses, allowing for the representation of complex multi-terminal interactions within a power system. The definition and illustrative examples of the Power-Grid Hypernetwork are presented below.

Definition 3.1 (Power-Grid HyperNetwork)**.** Let $V = \{1, 2, \ldots, N\}$ be the set of *buses*, and let

$$\mathcal{E} = \{e_1, e_2, \ldots, e_M\} \subseteq \mathcal{P}(V)$$

be a collection of *hyperedges*, each $e_h \subseteq V$ representing a network element connecting one or more buses (e.g. a two-terminal line, a three-winding transformer, or a multi-bus substation). Define the *incidence set*

$$I = \{(v, e) \in V \times \mathcal{E} : v \in e\},$$

with attachment maps $\pi(v, e) = v$ and $e(v, e) = e$. Assign to each hyperedge e_h a *device admittance* $y_h \in \mathbb{C}$ (or a shunt admittance vector). Then the *Power-Grid HyperNetwork* is the tuple

$$\mathcal{H} = (V, \mathcal{E}, I, \pi, e, \{y_h\}_{h=1}^{M}).$$

Example 3.2 (Three-Bus HyperNetwork with a Three-Winding Transformer)**.** Three-winding transformers have three sets of windings, enabling power transfer between three voltage levels or systems with improved flexibility (cf. [99, 119, 124]). Let the set of buses be

$$V = \{1, 2, 3\}.$$

We model two transmission lines and one three-winding transformer as hyperedges:

$$\mathcal{E} = \left\{ e_{12},\ e_{23},\ e_{123} \right\} \ \subseteq\ \mathcal{P}(V),$$

where

$$e_{12} = \{1, 2\}, \quad e_{23} = \{2, 3\}, \quad e_{123} = \{1, 2, 3\}.$$

Device admittances:

- Line e_{12} has admittance $y_{12} = 0.01 - j\,0.05$ (pu).
- Line e_{23} has admittance $y_{23} = 0.015 - j\,0.045$ (pu).
- Three-winding transformer e_{123} is represented by star-equivalent admittances to a fictitious neutral:

  $$y_{1T} = 0.02 - j\,0.10, \quad y_{2T} = 0.018 - j\,0.09, \quad y_{3T} = 0.016 - j\,0.08 \quad \text{(pu)}.$$

  We collect these into the vector $y_{123} = (y_{1T},\ y_{2T},\ y_{3T}) \in \mathbb{C}^3$.

Incidence set and attachment maps:

$$I = \big\{(1, e_{12}),\ (2, e_{12}),\ (2, e_{23}),\ (3, e_{23}),\ (1, e_{123}),\ (2, e_{123}),\ (3, e_{123})\big\},$$

with $\pi(v, e) = v$ and $\epsilon(v, e) = e$.

Power-Grid HyperNetwork:

$$\mathcal{H} = \big(V,\ \mathcal{E},\ I,\ \pi,\ \epsilon,\ \{y_{12}, y_{23}, y_{123}\}\big).$$

This hypernetwork captures:

- Two-terminal devices as 2-bus hyperedges e_{12}, e_{23}.
- One multi-terminal device as the 3-bus hyperedge e_{123} with vector admittance y_{123}.

Restricting $\mathcal{H}$ to the two-bus hyperedges $\{e_{12}, e_{23}\}$ recovers the standard three-bus power-grid network $G = (V, \{e_{12}, e_{23}\})$.

Example 3.3 (Five-Bus Microgrid HyperNetwork)**.** Consider a small microgrid with five buses:

$$V = \{1, 2, 3, 4, 5\},$$

where bus 1 is the utility interconnection, buses 2–4 host distributed energy resources (DERs), and bus 5 is a critical load center. We model the network with four hyperedges:

$$\mathcal{E} = \left\{ e_{12},\ e_{23},\ e_{135},\ e_{\mathrm{DER}} \right\} \ \subseteq\ \mathcal{P}(V),$$

with

$$e_{12} = \{1, 2\}, \quad e_{23} = \{2, 3\}, \quad e_{135} = \{1, 3, 5\}, \quad e_{\mathrm{DER}} = \{2, 3, 4\}.$$

Device admittances:

- Line e_{12} has admittance $y_{12} = 0.02 - j\,0.06$ (pu).
- Line e_{23} has admittance $y_{23} = 0.025 - j\,0.055$ (pu).
- Three-winding transformer e_{135} uses star-equivalent admittances:

  $$y_{1T} = 0.03 - j\,0.12, \quad y_{3T} = 0.028 - j\,0.11, \quad y_{5T} = 0.026 - j\,0.10 \quad \text{(pu)},$$

  collected as $y_{135} = (y_{1T}, y_{3T}, y_{5T}) \in \mathbb{C}^3$.
- DER cluster e_{DER} aggregates three resources with nodal shunt admittances:

  $$y_{2D} = 0.01 - j\,0.03, \quad y_{3D} = 0.012 - j\,0.032, \quad y_{4D} = 0.011 - j\,0.031,$$

  forming the vector $y_{\mathrm{DER}} = (y_{2D}, y_{3D}, y_{4D}) \in \mathbb{C}^3$.

Incidence set:

$$I = \big\{(1, e_{12}), (2, e_{12}),\ (2, e_{23}), (3, e_{23}),\ (1, e_{135}), (3, e_{135}), (5, e_{135}),\ (2, e_{\mathrm{DER}}), (3, e_{\mathrm{DER}}), (4, e_{\mathrm{DER}})\big\}.$$

Power-Grid HyperNetwork:

$$\mathcal{H} = \big(V,\ \mathcal{E},\ I,\ \pi,\ \epsilon,\ \{y_{12}, y_{23}, y_{135}, y_{\mathrm{DER}}\}\big).$$

This hypernetwork captures:

- Two-terminal lines as 2-bus hyperedges e_{12}, e_{23}.
- A three-terminal transformer as hyperedge e_{135}.
- A DER aggregation device connecting buses 2, 3, 4 as hyperedge e_{DER}.

Theorem 3.4 (HyperNetwork Structure)**.** *The Power-Grid HyperNetwork $\mathcal{H}$ is a finite hypernetwork: its vertex set V and hyperedge set $\mathcal{E}$ satisfy*

$$V \subseteq V, \quad \mathcal{E} \subseteq \mathcal{P}(V),$$

and the incidence relation I is exactly the membership relation of vertices in hyperedges.

Proof. By construction, each $e_h \in \mathcal{E}$ is a subset of V. The incidence set I pairs each bus $v \in V$ with exactly those hyperedges e_h that contain it. These definitions coincide with the axioms of a hypernetwork (or hypergraph with incidence), proving that $\mathcal{H}$ indeed carries the full hypernetwork structure. □

Theorem 3.5 (Reduction to Power-Grid Network)**.** *If every hyperedge $e_h \in \mathcal{E}$ has cardinality $|e_h| = 2$, then $\mathcal{H}$ reduces to a classical Power-Grid Network $\mathcal{G} = (V, E, Y, S)$ by letting*

$$E = \big\{\{i, j\} : e_h = \{i, j\}\big\}, \qquad Y_{ij} = -y_h \text{ for } e_h = \{i, j\},$$

and setting shunt injections accordingly. Conversely, any Power-Grid Network can be represented as the special case of $\mathcal{H}$ with only two-bus hyperedges.

Proof. When $|e_h| = 2$, write $e_h = \{i, j\}$. The two-terminal device admittance y_h becomes the off-diagonal bus admittance entry $Y_{ij} = -y_h$, and the diagonal entries Y_{ii}, Y_{jj} are formed by summing adjacent admittances plus shunts. All AC power-flow equations and injection definitions coincide with those of the classical $\mathcal{G} = (V, E, Y, S)$. Conversely, each branch in $\mathcal{G}$ is a two-bus hyperedge in $\mathcal{H}$. Thus $\mathcal{H}$ generalizes $\mathcal{G}$. □

Theorem 3.6 (Bus Admittance Matrix from HyperNetwork)**.** *Let $\mathcal{H} = (V, \mathcal{E}, I, \pi, \epsilon, \{y_h\})$ be a Power-Grid HyperNetwork with $N = |V|$ buses and $M = |\mathcal{E}|$ hyperedges, each carrying a complex admittance y_h. Define the incidence matrix $H \in \{0, 1\}^{N \times M}$ by*

$$H_{ih} = \begin{cases} 1, & \text{if bus } i \in e_h, \\ 0, & \text{otherwise}. \end{cases}$$

Then the nodal admittance matrix $Y \in \mathbb{C}^{N \times N}$ is given by

$$Y \;=\; H \operatorname{diag}(y_1, \dots, y_M)\, H^T.$$

Proof. By nodal analysis, each hyperedge e_h contributes admittance y_h equally between every pair of buses in e_h and adds y_h to each diagonal entry for buses in e_h. Algebraically, letting $h_h \in \{0, 1\}^N$ be column h of H, the contribution to Y from e_h is

$$y_h\, h_h\, h_h^T,$$

so summing over all hyperedges yields

$$Y \;=\; \sum_{h=1}^{M} y_h\, h_h\, h_h^T \;=\; H \operatorname{diag}(y_1, \dots, y_M)\, H^T.$$

□

Theorem 3.7 (Positive Semidefiniteness and Connectivity)**.** *Let Y be as in Theorem 3.6. Then:*

1. *Y is Hermitian and positive semidefinite.*
2. $\operatorname{rank}(Y) = N - c$*, where c is the number of connected components of the hypernetwork (viewed as a bipartite graph on $V \cup \mathcal{E}$).*

Proof.

1. Since $\operatorname{diag}(y_1, \dots, y_M)$ is Hermitian for real or complex admittances and H is real, $Y = H \operatorname{diag}(y)\, H^T$ is Hermitian. For any $x \in \mathbb{C}^N$,

$$x^* Y x = x^* H \operatorname{diag}(y)\, H^T x = \sum_{h=1}^{M} y_h \left|h_h^T x\right|^2 \;\geq\; 0$$

   whenever $\Re(y_h) \geq 0$, proving positive semidefiniteness.

2. The nullspace of Y is $\{x : Yx = 0\} = \{x : h_h^T x = 0 \;\forall\, h\}$. Hence x must be constant on each connected component of the incidence bipartite graph. Therefore $\dim\ker(Y) = c$, and by the rank–nullity theorem, $\operatorname{rank}(Y) = N - \dim\ker(Y) = N - c$.

□

Theorem 3.8 (Equivalent Two-Terminal Reduction)**.** *Any Power-Grid HyperNetwork $\mathcal{H}$ can be transformed to an equivalent classical network $\mathcal{G} = (V, E, Y')$ by replacing each hyperedge e_h of size $|e_h| = k$ with a complete subgraph on the same buses, assigning each branch admittance*

$$y'_{ij} = \frac{y_h}{k-1}, \quad \textit{for all distinct } i, j \in e_h.$$

The resulting nodal admittance matrix Y' coincides with Y from Theorem 3.6.

Proof. In the complete subgraph representation, each bus $i \in e_h$ connects to each other bus $j \in e_h$ with admittance $y_h/(k-1)$. The total off-diagonal entry contributed by e_h to Y'_{ij} is $-y_h/(k-1)$, and summing contributions from all pairs within e_h yields the same net off-diagonal term as in $Y = H \operatorname{diag}(y) H^T$. Diagonal entries also match because each bus i accumulates $\sum_{j \neq i} y_h/(k-1) = y_h$. Thus $Y' = Y$. □

3.2 Power-Grid n-SuperHypernetwork

In contrast, the Power-Grid n-SuperHypernetwork is a higher-order extension that employs n-level nested superedges to capture hierarchical, recursive, and multi-scale interconnections, making it suitable for modeling layered structures and subsystems in advanced electrical networks. The definition and illustrative examples of the Power-Grid n-SuperHypernetwork are presented below.

Definition 3.9 (Power-Grid n-Superhypernetwork)**.** Let $V_0 = \{1, 2, \ldots, N\}$ be the set of *buses*. For each $k \geq 0$, define the iterated powerset

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big).$$

A *Power-Grid n-Superhypernetwork* is a tuple

$$\mathcal{N}^{(n)} \; = \; \big(V^{(n)}, \, \mathcal{E}^{(n)}, \, w\big),$$

where

- $V^{(n)} \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-supernodes*;
- $\mathcal{E}^{(n)} \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-superedges*, each $e \in \mathcal{E}^{(n)}$ a nonempty subset of $V^{(n)}$;
- $w : \mathcal{E}^{(n)} \to \mathbb{C}$ assigns to each superedge e a *generalized admittance y_e*.

Intuitively, each n-superedge models a device or subnetwork connecting multiple $(n-1)$-supernodes, capturing up to n hierarchical levels of interconnection.

Example 3.10 (2-SuperHyperNetwork of a Four-Bus Power System)**.** Let the base set of buses be

$$V_0 = \{1, 2, 3, 4\},$$

and suppose the following line admittances (in pu):

$$y_{12} = 0.01 - j0.05, \quad y_{23} = 0.015 - j0.045, \quad y_{34} = 0.012 - j0.048, \quad y_{14} = 0.02 - j0.10.$$

Iterated powersets:

$$\mathcal{P}^1(V_0) = \big\{\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,4\}, \ldots\big\},$$

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big).$$

2-supernodes (elements of $V^{(2)} \subseteq \mathcal{P}^2(V_0)$):

$$U_1 = \big\{\{1,2\}, \, \{2,3\}\big\}, \qquad U_2 = \big\{\{3,4\}, \, \{1,4\}\big\}.$$

Here, U_1 clusters the north-east line corridor and U_2 clusters the south-west loop.

2-superedges (elements of $\mathcal{E}^{(2)}$):

$$\mathcal{E}^{(2)} = \big\{\{U_1\},\ \{U_2\},\ \{U_1, U_2\}\big\}.$$

Weight function $w : \mathcal{E}^{(2)} \to \mathbb{C}$:

$$w(\{U_1\}) = y_{12} + y_{23} = (0.01 - j0.05) + (0.015 - j0.045) = 0.025 - j0.095,$$

$$w(\{U_2\}) = y_{34} + y_{14} = (0.012 - j0.048) + (0.02 - j0.10) = 0.032 - j0.148,$$

$$w(\{U_1, U_2\}) = (y_{12} + y_{23}) + (y_{34} + y_{14}) = (0.025 - j0.095) + (0.032 - j0.148) = 0.057 - j0.243.$$

Power-Grid 2-SuperHyperNetwork:

$$\mathcal{N}^{(2)} = \big(V^{(2)},\ \mathcal{E}^{(2)},\ w\big), \quad V^{(2)} = \{U_1, U_2\}.$$

Interpretation:

- Each 1-supernode $\{i, j\}$ corresponds to a physical transmission line between buses i and j.
- Each 2-supernode U_k groups two line segments into a higher-level corridor or loop.
- The weight $w(\{U_k\})$ aggregates admittances of the underlying lines, modeling the equivalent admittance of that corridor.
- The 2-superedge $\{U_1, U_2\}$ captures interactions between the north-east corridor and the south-west loop, representing a multi-level cluster in the grid topology.

Example 3.11 (2-SuperHyperNetwork of a Five-Bus Ring System)**.** A ring network in electrical engineering connects nodes in a closed loop, ensuring redundancy and continuous power flow (cf. [121, 146]). Consider a simple five-bus ring network with buses

$$V_0 = \{1, 2, 3, 4, 5\}.$$

The physical transmission lines and their per-unit admittances are:

$$y_{12} = 0.020 - j\,0.100, \quad y_{23} = 0.025 - j\,0.125, \quad y_{34} = 0.018 - j\,0.090,$$

$$y_{45} = 0.015 - j\,0.085, \quad y_{51} = 0.022 - j\,0.110.$$

First powerset (1-supernodes):

$$\mathcal{P}^1(V_0) = \big\{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 1\}, \dots\big\}.$$

Second powerset (2-supernodes):

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big).$$

Select two 2-supernodes:

$$U_1 = \big\{\{1, 2\}, \{2, 3\}, \{3, 4\}\big\}, \qquad U_2 = \big\{\{3, 4\}, \{4, 5\}, \{5, 1\}\big\}.$$

Here, U_1 clusters the north-east corridor (buses 1–4) and U_2 clusters the south-west loop (buses 3–1).

2-superedges:

$$\mathcal{E}^{(2)} = \big\{\{U_1\},\ \{U_2\},\ \{U_1, U_2\}\big\}.$$

Weight function $w : \mathcal{E}^{(2)} \to \mathbb{C}$**:**

$$w(\{U_1\}) = y_{12} + y_{23} + y_{34} = (0.020 + 0.025 + 0.018) - j\,(0.100 + 0.125 + 0.090) = 0.063 - j\,0.315,$$

$$w(\{U_2\}) = y_{34} + y_{45} + y_{51} = 0.055 - j\,0.285,$$

$$w(\{U_1, U_2\}) = \sum_{(i,j)\in\{12,23,34,45,51\}} y_{ij} = 0.100 - j\,0.510.$$

Resulting 2-SuperHyperNetwork:

$$\mathcal{N}^{(2)} = \big(V^{(2)}, \mathcal{E}^{(2)}, w\big), \quad V^{(2)} = \{U_1, U_2\}.$$

Interpretation:

- Each 1-supernode $\{i, j\}$ corresponds to a single transmission line between buses i and j.
- Each 2-supernode U_k groups contiguous line segments into a higher-level corridor or loop.
- The weight $w(\{U_k\})$ is the aggregate admittance of that corridor.
- The hyperedge $\{U_1, U_2\}$ represents the interaction between the two corridors, capturing multi-scale connectivity in the ring topology.

Example 3.12 (Power-Grid 3-SuperHypernetwork of a Six-Bus Ring System)**.** Consider a six-bus ring network with buses

$$V_0 = \{1, 2, 3, 4, 5, 6\},$$

and per-unit line admittances:

$$y_{12} = 0.010 - j\,0.050, \quad y_{23} = 0.015 - j\,0.045, \quad y_{34} = 0.012 - j\,0.048,$$

$$y_{45} = 0.020 - j\,0.100, \quad y_{56} = 0.018 - j\,0.090, \quad y_{61} = 0.022 - j\,0.110.$$

First powerset (1**-supernodes):**

$$\mathcal{P}^1(V_0) = \big\{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 6\}, \{6, 1\}\big\}.$$

Second powerset (2**-supernodes):** Select three 2-supernodes:

$$U_1 = \big\{\{1, 2\}, \{2, 3\}\big\}, \quad U_2 = \big\{\{3, 4\}, \{4, 5\}\big\}, \quad U_3 = \big\{\{5, 6\}, \{6, 1\}\big\}.$$

Assign weights

$$w^{(2)}(U_1) = y_{12} + y_{23} = 0.025 - j\,0.095,$$

$$w^{(2)}(U_2) = y_{34} + y_{45} = 0.032 - j\,0.148,$$

$$w^{(2)}(U_3) = y_{56} + y_{61} = 0.040 - j\,0.200.$$

Third powerset (3**-supernodes):** Now form two 3-supernodes from the 2-supernodes:

$$W_1 = \{U_1, U_2\}, \qquad W_2 = \{U_2, U_3\}.$$

Define weights

$$w^{(3)}(W_1) = w^{(2)}(U_1) + w^{(2)}(U_2) = (0.025 - j0.095) + (0.032 - j0.148) = 0.057 - j\,0.243,$$

$$w^{(3)}(W_2) = w^{(2)}(U_2) + w^{(2)}(U_3) = (0.032 - j0.148) + (0.040 - j0.200) = 0.072 - j\,0.348.$$

3-superedges and incidence:

$$\mathcal{E}^{(3)} = \{\{W_1\}, \{W_2\}, \{W_1, W_2\}\},$$

with incidence relation inherited from membership of W_i.

Power-Grid 3-SuperHypernetwork:

$$\mathcal{N}^{(3)} = (V^{(3)}, \mathcal{E}^{(3)}, w^{(3)}), \quad V^{(3)} = \{W_1, W_2\}.$$

This structure captures three hierarchical levels: • individual lines (1-supernodes), • corridors of two lines (2-supernodes), • interactions between adjacent corridors (3-supernodes and superedges).

Example 3.13 (Power-Grid 4-Superhypernetwork of a Four-Bus Microgrid)**.** Let the base set of buses be

$$V_0 = \{1, 2, 3, 4\},$$

with per-unit line admittances:

$$y_{12} = 0.01 - j\,0.05, \quad y_{23} = 0.015 - j\,0.045, \quad y_{34} = 0.012 - j\,0.048, \quad y_{14} = 0.02 - j\,0.10.$$

Step 1: 1-supernodes (feeders)

$$U_1 = \{1, 2\}, \quad U_2 = \{2, 3\}, \quad U_3 = \{3, 4\}, \quad U_4 = \{1, 4\}.$$

Step 2: 2-supernodes (corridors)

$$C_1 = \{U_1, U_2\}, \quad C_2 = \{U_2, U_3\}, \quad C_3 = \{U_3, U_4\}, \quad C_4 = \{U_4, U_1\}.$$

Assign weights:

$$\begin{aligned} w^{(2)}(C_1) &= y_{12} + y_{23} = 0.025 - j\,0.095, \\ w^{(2)}(C_2) &= y_{23} + y_{34} = 0.027 - j\,0.093, \\ w^{(2)}(C_3) &= y_{34} + y_{14} = 0.032 - j\,0.148, \\ w^{(2)}(C_4) &= y_{14} + y_{12} = 0.030 - j\,0.150. \end{aligned}$$

Step 3: 3-supernodes (regions)

$$R_1 = \{C_1, C_2\}, \quad R_2 = \{C_3, C_4\}.$$

Assign weights:

$$\begin{aligned} w^{(3)}(R_1) &= w^{(2)}(C_1) + w^{(2)}(C_2) = 0.052 - j\,0.188, \\ w^{(3)}(R_2) &= w^{(2)}(C_3) + w^{(2)}(C_4) = 0.062 - j\,0.298. \end{aligned}$$

Step 4: 4-supernodes (microgrid clusters)

$$X_1 = \{R_1\}, \quad X_2 = \{R_2\}.$$

Assign weights:

$$\begin{aligned} w^{(4)}(X_1) &= w^{(3)}(R_1) = 0.052 - j\,0.188, \\ w^{(4)}(X_2) &= w^{(3)}(R_2) = 0.062 - j\,0.298. \end{aligned}$$

4-superedges and incidence: We connect the two clusters with one 4-superedge:

$$\mathcal{E}^{(4)} = \{\{X_1, X_2\}\}, \quad w^{(4)}(\{X_1, X_2\}) = w^{(4)}(X_1) + w^{(4)}(X_2) = 0.114 - j\,0.486.$$

Power-Grid 4-Superhypernetwork:

$$\mathcal{N}^{(4)} = (V^{(4)}, \mathcal{E}^{(4)}, w^{(4)}), \quad V^{(4)} = \{X_1, X_2\}.$$

This network captures four hierarchical levels:

1. Individual feeders (1-supernodes).
2. Line corridors (2-supernodes).
3. Regional groupings (3-supernodes).
4. Microgrid clusters (4-supernodes and 4-superedge).

Theorem 3.14 (Intrinsic n-Superhypernetwork Structure)**.** *The Power-Grid n-Superhypernetwork $\mathcal{N}^{(n)}$ is an n-superhypernetwork on base set V_0: its node set $V^{(n)}$ and hyperedge set $\mathcal{E}^{(n)}$ satisfy*

$$V^{(n)} \subseteq \mathcal{P}^n(V_0), \qquad \mathcal{E}^{(n)} \subseteq \mathcal{P}^n(V_0),$$

and the natural membership relation provides the incidence structure.

Proof. By definition $V^{(n)} \subseteq \mathcal{P}^n(V_0)$ and $\mathcal{E}^{(n)} \subseteq \mathcal{P}^n(V_0)$. Each superedge $e \in \mathcal{E}^{(n)}$ is by construction a nonempty subset of $V^{(n)}$. The incidence relation $I = \{(v, e) : v \in V^{(n)}, e \in \mathcal{E}^{(n)}, v \in e\}$ then exactly matches the membership relation of an n-superhypernetwork. Hence $\mathcal{N}^{(n)}$ satisfies all axioms of an n-superhypernetwork. □

Theorem 3.15 (Reduction to Power-Grid HyperNetwork)**.** *If $n = 1$, then $\mathcal{N}^{(1)} = (V^{(1)}, \mathcal{E}^{(1)}, w)$ coincides with the Power-Grid HyperNetwork defined by*

$$V^{(1)} = V_0, \qquad \mathcal{E}^{(1)} \subseteq \mathcal{P}(V_0),$$

where each hyperedge e connects one or more buses and carries weight $w(e)$ = device admittance. Conversely, any Power-Grid HyperNetwork is obtained as $\mathcal{N}^{(1)}$ for appropriate choice of $\mathcal{E}^{(1)}$ and w.

Proof. For $n = 1$, $\mathcal{P}^1(V_0) = \mathcal{P}(V_0)$. Setting $V^{(1)} = V_0$ and choosing $\mathcal{E}^{(1)} \subseteq \mathcal{P}(V_0)$ as the collection of device-connection sets yields precisely the Power-Grid HyperNetwork structure. The weight function w assigns to each hyperedge its corresponding admittance or transformer ratio. Conversely, any hypernetwork on buses with hyperedges representing multi-terminal devices matches this form for $\mathcal{N}^{(1)}$. Thus $\mathcal{N}^{(1)}$ and the Power-Grid HyperNetwork are equivalent. □

Theorem 3.16 (Skeleton Consistency)**.** *Let $\mathcal{N}^{(n)} = (V^{(n)}, \mathcal{E}^{(n)}, w)$ be a Power-Grid n-Superhypernetwork on base buses V_0. Define recursively for $k = n-1, n-2, \ldots, 0$:*

$$V^{(k)} = \bigcup_{S \in V^{(k+1)}} S, \qquad \mathcal{E}^{(k)} = \{ F \subseteq V^{(k)} : F \subseteq e \text{ for some } e \in \mathcal{E}^{(k+1)} \}.$$

Then for each $0 \le k \le n$, the tuple $\mathcal{N}^{(k)} = (V^{(k)}, \mathcal{E}^{(k)}, w|_{\mathcal{E}^{(k)}})$ is itself a Power-Grid k-Superhypernetwork. In particular:

- *$\mathcal{N}^{(1)}$ coincides with the Power-Grid HyperNetwork.*
- *$\mathcal{N}^{(0)}$ reduces to the classical Power-Grid Network.*

Proof. We prove by downward induction on k. For $k = n$, $\mathcal{N}^{(n)}$ is given. Assume $\mathcal{N}^{(k+1)} = (V^{(k+1)}, \mathcal{E}^{(k+1)}, w)$ satisfies $V^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$, $\mathcal{E}^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$. Then

$$V^{(k)} = \bigcup_{S \in V^{(k+1)}} S \subseteq \mathcal{P}^k(V_0),$$

and each $F \in \mathcal{E}^{(k)}$ sits inside some $e \in \mathcal{E}^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$, so $F \subseteq \mathcal{P}^k(V_0)$. The restriction of w to $\mathcal{E}^{(k)}$ endows $\mathcal{N}^{(k)}$ with the same weight structure. Thus $\mathcal{N}^{(k)}$ is a valid Power-Grid k-Superhypernetwork. For $k = 1$ and $k = 0$, this recovers the hypernetwork and the classical network, respectively. □

Theorem 3.17 (Connectivity Inheritance)**.** *If the underlying Power-Grid Network $\mathcal{N}^{(0)}$ is connected, then for every $1 \le k \le n$, the 2-section graph of $\mathcal{N}^{(k)}$ is connected.*

Proof. The 2-section of a hypernetwork $(V^{(k)}, \mathcal{E}^{(k)})$ has vertices $V^{(k)}$ with an edge between two supernodes S, T whenever $S \cap T \neq \emptyset$. For $k = 0$, this is the original network, which is connected by hypothesis. Assume the 2-section at level $k-1$ is connected. At level k, any two k-supernodes $S, T \in V^{(k)}$ that share a $(k-1)$-supernode become adjacent in the 2-section. Since the $(k-1)$ 2-section is connected, there is a sequence of overlaps linking S to T. Hence the 2-section at level k remains connected. By induction, all skeletons inherit connectivity. □

Theorem 3.18 (Subedge-Induced Subnetwork Connectivity)**.** *In $\mathcal{N}^{(n)} = (V^{(n)}, \mathcal{E}^{(n)}, w)$, each n-superedge $e \in \mathcal{E}^{(n)}$ induces a connected subnetwork in the classical Power-Grid Network on the union of its constituent base buses.*

Proof. Let $e \in \mathcal{E}^{(n)}$. By Skeleton Consistency (Theorem 3.16), e corresponds at level $n-1$ to a collection of $(n-1)$-supernodes whose union forms a connected 2-section at level $n-1$. Recursively descending through levels, this ensures that the union of base-level buses in e is connected in the 0-skeleton network. Thus the induced classical subnetwork on these buses is connected. □

4 Conclusion and Future Works

In this study, we introduced formal mathematical definitions for two higher-order generalizations of power-grid systems: the *Power-Grid HyperNetwork* and the *Power-Grid SuperHyperNetwork*. These structures extend classical power-grid network models by capturing multi-node interactions and hierarchical relationships, which are critical for analyzing complex and interconnected energy infrastructures. In the future, conducting computational and various numerical experiments may lead to more precise definitions and modeling approaches for the *Power-Grid HyperNetwork* and the *Power-Grid SuperHyperNetwork*. We hope that such research will continue to progress.

And as future work, we aim to further enhance these models by incorporating advanced uncertainty-handling frameworks. Specifically, we plan to investigate fuzzy and neutrosophic extensions of power-grid networks using: Fuzzy Sets [37,157,158], Intuitionistic Fuzzy Sets [12,13], Vague Sets [4,6,59], shadowed Sets [24,111], Rough Sets [23,109,110], Soft Sets [96,100], Bipolar Fuzzy Sets [3,122], HyperFuzzy Sets [78,136], Picture Fuzzy Sets [33, 67], Hesitant Fuzzy Sets [138, 139], Neutrosophic Sets [76, 127, 135], Quadripartitioned Neutrosophic Sets [85, 156], Pentapartitioned Neutrosophic Sets [8, 36], HyperPlithogenic Sets [50–52], and Plithogenic Sets [48, 53, 128]. Such extensions are expected to significantly increase the expressiveness and practical relevance of hypernetwork-based models by enabling robust representation of ambiguity, variability, and multi-level uncertainty within power-grid systems.

We also aim to explore potential applications in electrical engineering by extending the current framework to encompass graph neural networks (GNNs) [55, 64], artificial intelligence (AI) [10, 11, 71], linear programming [72–74], Power Grid resilience studies [90, 92, 102], smart grid modeling [89, 153], graph-based energy system analysis [34], and decision-making [44, 75, 97]. Furthermore, as a future direction, we intend to investigate generalized structures such as directed graphs [46], dynamic graphs [16, 17, 82], SemiGraphs [101, 115], bidirected graphs [21, 61, 152], and multidirected graphs [106, 107]. We also aim to consider possible extensions, including the study of graph parameters (cf. [87]) and graph algorithms (cf. [81]).

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Considerations

This work does not involve any experiments or studies involving human participants or animals, and therefore no ethical approvals were required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Research Integrity

The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Disclaimer (Note on Computational Tools)

No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

Disclaimer (Limitations and Claims)

The theoretical concepts presented in this paper have not yet been subject to practical implementation or empirical validation. Future researchers are invited to explore these ideas in applied or experimental settings. Although every effort has been made to ensure the accuracy of the content and the proper citation of sources, unintentional errors or omissions may persist. Readers should independently verify any referenced materials.

To the best of the authors' knowledge, all mathematical statements and proofs contained herein are correct and have been thoroughly vetted. Should you identify any potential errors or ambiguities, please feel free to contact the authors for clarification.

The results presented are valid only under the specific assumptions and conditions detailed in the manuscript. Extending these findings to broader mathematical structures may require additional research. The opinions and conclusions expressed in this work are those of the authors alone and do not necessarily reflect the official positions of their affiliated institutions.

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