Original Research Article

Extension of Graph Signal Processing, Electric Circuits, and Bond Graphs Using Hypergraphs and Superhypergraphs

Abstract

Graph theory is a core branch of mathematics concerned with representing and analyzing relationships among discrete elements. These concepts are widely used in fields such as electrical engineering. For example, graphs play a crucial role in important frameworks including *Graph Signal Processing*, *Electric Circuits*, and *Bond Graphs*.

A *hypergraph* generalizes the concept of a traditional graph by allowing edges—called *hyperedges*—to connect more than two vertices simultaneously [16]. A *superhypergraph* further extends this idea by incorporating recursively defined powerset layers, enabling hierarchical and self-referential relationships among hyperedges [105].

In this paper, we extend the frameworks of Graph Signal Processing, Electric Circuits, and Bond Graphs using hypergraphs and superhypergraphs, and investigate their mathematical properties and illustrative examples. These extensions enable the representation of hierarchical structures inherent in Graph Signal Processing, Electric Circuits, and Bond Graphs, providing a more expressive modeling framework. We anticipate that future research will advance computational experiments and practical applications in these domains.

Keywords: Superhypergraph, Hypergraph, Graph Signal Processing, Electric Circuits, Bond Graphs

1 Introduction

1.1 Hypergraphs and Superhypergraphs

Graph theory is a core branch of mathematics concerned with representing and analyzing relationships among discrete elements using abstract structures known as graphs, where entities (called vertices) are connected by links (called edges) [24–26]. Due to the intuitive and visual nature of graphs, which allows complex systems to be illustrated clearly, they have been widely applied and actively studied in numerous fields, including graph neural networks [45, 56, 57] and beyond. Classical graphs are limited to modeling pairwise relationships, yet many natural and engineered systems involve complex interactions among multiple entities that cannot be fully described using only binary connections. To overcome this limitation, the theory of graphs has been expanded to include the framework of *hypergraphs* and, more recently, *superhypergraphs* [36, 104].

A hypergraph is a generalization of a traditional graph in which a single edge—called a hyperedge—can simultaneously connect an arbitrary number of vertices [16, 19, 20, 33]. This structure enables more expressive modeling of phenomena involving group-level interactions, such as metabolic networks, task teams, or symptom clusters in medical diagnostics. Furthermore, hypergraphs have been extended and studied in various forms, including Directed Hypergraphs [44,64,78], Regular Hypergraphs [28,29], Complete Hypergraphs [14,77,110], Fuzzy Hypergraphs [32, 76, 99], and Neutrosophic Hypergraphs [5, 7, 71].

Building upon the hypergraph concept, a *superhypergraph* incorporates additional layers of abstraction by iteratively applying the powerset operation to the vertex set [5, 40, 104, 105]. This results in recursively nested structures that can capture not only hyperedges over sets of vertices, but also interactions among groups of hyperedges themselves. Such higher-order formalisms are particularly suited for representing hierarchical, modular, or multi-scale systems in science and engineering (cf. [21, 52, 53]).

1.2 Graphs in Electrical Engineering, Physics, and Chemistry

Graph theory provides a powerful framework that can be applied across a wide range of disciplines, including electrical engineering, physics, and chemistry (cf. [30,31,72]). In this paper, we investigate extensions of *Graph Signal Processing*, *Electric Circuits*, and *Bond Graphs* through the lens of hypergraphs and superhypergraphs.

Signal Processing involves analyzing, modifying, and extracting information from signals such as sound, images, or data [9, 55, 74]. *Graph Signal Processing* extends this idea by analyzing signals defined on the vertices of a graph using spectral methods [70, 80, 111]. *Electric Circuits* model the flow of electrical current through interconnected components [112]. A related concept known as the *Circuit Graph* represents circuits as graphs (cf. [120, 125]). *Bond Graphs* are graphical models that represent energy exchange across different physical domains, such as mechanical, electrical, thermal, and hydraulic systems, within a unified formalism [18,49].

Beyond the concepts mentioned above, many other graph-theoretic models and their applications have been studied. These include the *Chemical Graph* [41, 46, 121, 123], which represents molecules and their bonds; the *Interaction Graph*, used in dynamical systems and particle interactions [8, 73]; and the *Feynman Graph*, central to quantum field theory and particle physics [17, 94]. These examples demonstrate the versatility and broad applicability of graph theory across scientific domains.

1.3 Our Contribution

As mentioned earlier, the principles of graph theory can be broadly applied to various domain-specific graph models. In this paper, we extend the frameworks of *Graph Signal Processing*, *Electric Circuits*, and *Bond Graphs* using *hypergraphs* and *superhypergraphs*, and investigate their underlying mathematical properties along with illustrative examples. It should be noted that this paper focuses exclusively on theoretical aspects. We hope that future computational or circuit-based experiments will be conducted by interested researchers to further explore and validate the proposed frameworks.

2 Preliminaries and Definitions

This section provides an overview of the fundamental concepts and definitions essential for the discussions in this paper. Throughout this paper, we restrict our attention to finite structures.

2.1 Power Set

We provide the definitions of the Base Set, the Powerset, and the n-th Powerset as follows.

Definition 2.1 (Base Set)**.** A *base set* S is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of S.

Definition 2.2 (Powerset)**.** [35, 97] The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 2.3 (n-th Powerset)**.** (cf. [34, 35, 101, 106])

The n-th powerset of a set H, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the powerset of H with the empty set removed.

2.2 SuperHyperGraph

In classical graph theory, a hypergraph extends the idea of a conventional graph by permitting edges—called hyperedges—to join more than two vertices. This broader framework enables the modeling of more intricate relationships between elements, thereby enhancing its utility in various fields [16,33,50,51]. A *SuperHyperGraph* is an advanced extension of the hypergraph concept, integrating recursive powerset structures into the classical model. This concept has been recently introduced and extensively studied in the literature [1,43,75,85].

Definition 2.4 (Hypergraph)**.** [16,19] A *hypergraph* $H = (V(H), E(H))$ consists of:

- A nonempty set $V(H)$ of vertices.
- A set $E(H)$ of hyperedges, where each hyperedge is a nonempty subset of $V(H)$, thereby allowing connections among multiple vertices.

Unlike standard graphs, hypergraphs are well-suited to represent higher-order relationships. In this paper, we restrict ourselves to the case where both $V(H)$ and $E(H)$ are finite.

Definition 2.5 (n-SuperHyperGraph)**.** [36,39,104,105]
Let V_0 be a finite base set of vertices. For each integer $k \geq 0$, define the iterative powerset by

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{k+1}(V_0) = \mathcal{P}(\mathcal{P}^k(V_0)),$$

where $\mathcal{P}(\cdot)$ denotes the usual powerset operation. An *n-SuperHyperGraph* is then a pair

$$\mathrm{SHT}^{(n)} = (V, E),$$

with

$$V \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}^n(V_0).$$

Each element of V is called an *n-supervertex* and each element of E an *n-superedge*.

Example 2.6 (Microgrid Power Flow as a 2-SuperHyperGraph)**.** Microgrid Power Flow refers to the distribution of electrical energy among generation, storage, and load units within a localized microgrid system (cf. [23,68, 95]). Consider a simple microgrid in electrical engineering, with base components

$$V_0 = \{SP,\ WT,\ BS,\ RL,\ CL\},$$

where SP = Solar Panels, WT = Wind Turbine, BS = Battery Storage, RL = Residential Load, and CL = Commercial Load. We form the 2-SuperHyperGraph $\mathrm{SHT}^{(2)} = (V^{(2)}, E^{(2)})$ by setting

$$V^{(2)} = \{\{\{SP, WT, BS\}\},\ \{\{RL, CL\}\}\} \subseteq \mathcal{P}^2(V_0),$$

$$E^{(2)} = \{e = \{\{SP, WT, BS\},\ \{RL, CL\}\}\} \subseteq \mathcal{P}^2(V_0) \setminus \{\emptyset\}.$$

Here each element of $V^{(2)}$ is a 2-supervertex representing a cluster of devices or loads; the single 2-superedge e captures the power-flow event from the generation/storage cluster $\{\{SP, WT, BS\}\}$ to the combined load cluster $\{\{RL, CL\}\}$. This hierarchical model reflects the nested grouping of components and their simultaneous interaction in a microgrid.

2.3 Graph Signal Processing

Graph Signal Processing analyzes data defined on graph nodes using spectral methods and graph-based transformations like filtering and shifting [27,67,79,81]. If we are to define it explicitly, it would be as follows.

Definition 2.7 (Graph Signal Processing)**.** Let $G = (V, E)$ be a simple graph with $|V| = N$. A *graph signal* is a function $x : V \to \mathbb{R}$, represented by the vector $\mathbf{x} = [x(v_1) \ \cdots \ x(v_N)]^\top \in \mathbb{R}^N$. Choose a graph shift operator $\mathbf{F} \in \mathbb{R}^{N\times N}$ (e.g. the adjacency matrix $\mathbf{A}$ or the Laplacian $\mathbf{L} = \mathbf{D} - \mathbf{A}$). Then:

$$\text{(Graph shifting)} \quad \mathbf{x}' = \mathbf{F}\,\mathbf{x}.$$

Since $\mathbf{F}$ is (for instance) diagonalizable as $\mathbf{F} = \mathbf{V}\,\boldsymbol{\Lambda}\,\mathbf{V}^{-1}$, the *graph Fourier transform* (GFT) of $\mathbf{x}$ is

$$\widehat{\mathbf{x}} = \mathbf{V}\,\mathbf{x}, \quad \mathbf{x} = \mathbf{V}^{-1}\,\widehat{\mathbf{x}},$$

where columns of $\mathbf{V}$ are eigenvectors of $\mathbf{F}$ and the entries of $\boldsymbol{\Lambda}$ are the associated *graph frequencies*.

Example 2.8 (Path Graph Temperature Sensor Network)**.** Temperature Sensor Networks are systems of distributed sensors that monitor, collect, and transmit temperature data across environments for analysis and control (cf. [63,83,84,126]). Consider the path graph $G = (V, E)$ with $V = \{1, 2, 3, 4\}$ and edges $\{(1, 2), (2, 3), (3, 4)\}$. We use the combinatorial Laplacian $\mathbf{L} = \mathbf{D} - \mathbf{A}$ as the graph shift operator, where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{D} = \operatorname{diag}(1, 2, 2, 1).$$

Its eigen-decomposition $\mathbf{L} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{U}^\top$ yields

$$\mathbf{U} = \begin{pmatrix} 0.372 & 0.602 & 0.602 & 0.372 \\ 0.602 & 0.372 & -0.372 & -0.602 \\ 0.602 & -0.372 & -0.372 & 0.602 \\ 0.372 & -0.602 & 0.602 & -0.372 \end{pmatrix}, \quad \mathbf{\Lambda} = \operatorname{diag}\big(0.382,\ 1.382,\ 2.618,\ 3.618\big).$$

Now let the graph signal represent temperature readings: $\mathbf{x} = [1, 2, 3, 4]^\top$ (in °C). Its graph Fourier transform is

$$\widehat{\mathbf{x}} = \mathbf{U}^\top\,\mathbf{x} \approx \begin{pmatrix} 4.866 \\ -2.176 \\ 1.149 \\ -0.514 \end{pmatrix}.$$

These coefficients $\widehat{x}_k$ quantify the components of $\mathbf{x}$ at the graph frequencies λ_k.

Hypergraph Signal Processing extends graph signal analysis to hypergraphs, using high-order tensors and spectral methods for multi-node interactions [15, 89, 109, 130].

Definition 2.9 (Hypergraph Signal Processing)**.** Let $H = (V, E)$ be a hypergraph with $|V| = N$ vertices and maximum hyperedge size m.c.e.$(H) = M$. Hypergraph Signal Processing (HGSP) on H comprises the following components:

1. **Adjacency tensor** $\mathcal{A} \in \mathbb{R}^{\overbrace{N \times \cdots \times N}^{M \text{ times}}}$: if $e_\ell = \{v_{l_1}, \ldots, v_{l_c}\} \in E$ has $c \leq M$, then for any index tuple $(i_1, \ldots, i_M)$ that picks exactly those c vertices (with the remaining $M - c$ indices drawn from the same set),

$$\mathcal{A}_{i_1 \cdots i_M} \;=\; c \left(\sum_{\substack{k_1, \ldots, k_c \geq 1 \\ \sum_{i=1}^{c} k_i = M}} \frac{M!}{k_1!\, k_2! \cdots k_c!} \right)^{-1},$$

   and $\mathcal{A}_{i_1 \cdots i_M} = 0$ otherwise.

2. **Hypergraph signal**: start with a vertex-domain signal $\mathbf{s} = [s_1, \ldots, s_N]^\top \in \mathbb{R}^N$, and form the $(M-1)$th-order signal tensor

$$\mathcal{S} \;=\; \underbrace{\mathbf{s} \circ \mathbf{s} \circ \cdots \circ \mathbf{s}}_{M-1 \text{ times}} \;\in\; \mathbb{R}^{\overbrace{N \times \cdots \times N}^{M-1 \text{ times}}}.$$

3. **Signal shifting**: the filtered (shifted) signal is obtained by contracting $\mathcal{A}$ with $\mathcal{S}$:

$$\mathcal{S}' \;=\; \mathcal{A} \times_M \mathcal{S},$$

   where "$\times_M$" denotes the Mth-mode product, generalizing $\mathbf{s}' = \mathbf{F}\mathbf{s}$ in graph SP.

4. **Hypergraph Fourier transform**: assume an orthogonal CANDECOMP/PARAFAC decomposition

$$\mathcal{A} \;=\; \sum_{r=1}^{R} \lambda_r\, \underbrace{\mathbf{f}_r \circ \cdots \circ \mathbf{f}_r}_{M \text{ times}}, \quad \langle \mathbf{f}_r, \mathbf{f}_s \rangle = \delta_{rs}.$$

Then the HGFT of $\mathcal{S}$ is the vector $\widehat{\mathcal{S}} \in \mathbb{R}^R$ with components

$$\widehat{\mathcal{S}}_r = \langle \mathcal{S}, \underbrace{\mathbf{f}_r \circ \cdots \circ \mathbf{f}_r}_{M \text{ times}} \rangle,$$

whose entries λ_r serve as the "hypergraph frequencies.

Example 2.10 (Hypergraph Signal Processing on a 3-Uniform Collaboration Hypergraph)**.** Consider the hypergraph $H = (V, E)$ defined by

$$V = \{\text{Alice, Bob, Carol, Dave}\}, \quad E = \big\{\{\text{Alice, Bob, Carol}\}, \{\text{Bob, Carol, Dave}\}\big\},$$

so that m.c.e.$(H) = 3$. Assign to each vertex the "publication count" signal

$$\mathbf{s} = \begin{bmatrix} 10 \\ 15 \\ 8 \\ 12 \end{bmatrix} \in \mathbb{R}^4.$$

Since $M = 3$, the adjacency tensor $\mathcal{A} \in \mathbb{R}^{4\times4\times4}$ has nonzero entries precisely when $\{i, j, k\} \in E$:

$$\mathcal{A}_{i,j,k} = 3 \left(\frac{3!}{1!\,1!\,1!}\right)^{-1} = \frac{3}{6} = 0.5,$$

and $\mathcal{A}_{i,j,k} = 0$ otherwise.

Form the hypergraph signal tensor $\mathcal{S} \in \mathbb{R}^{4\times4}$ by

$$\mathcal{S}_{i,j} = s_i\, s_j,$$

so that for example $\mathcal{S}_{\text{Alice,Bob}} = 10 \times 15 = 150$.

The shifted (filtered) signal $\mathcal{S}' = \mathcal{A} \times_3 \mathcal{S} \in \mathbb{R}^{4\times4}$ is given by

$$\mathcal{S}'_{i,j} = \sum_{k=1}^{4} \mathcal{A}_{i,j,k}\, s_k.$$

Hence, for example,

$$\mathcal{S}'_{\text{Alice,Bob}} = \mathcal{A}_{\text{Alice,Bob,Carol}} \times s_{\text{Carol}} = 0.5 \times 8 = 4, \quad \mathcal{S}'_{\text{Bob,Carol}} = 0.5 \times 10 + 0.5 \times 12 = 11.$$

2.4 Electric Circuit

An electric circuit is a closed loop that allows electric current to flow through connected electrical components using conductors [11, 86, 98, 100].

Definition 2.11 (Electric Circuit)**.** An *electric circuit* is a pair $(G, \mathcal{E})$ where:

- $G = (V, E)$ is a finite, connected, oriented multigraph with vertex set V and edge set E. Each edge $e \in E$ has a chosen direction.
- $\mathcal{E}$ is a collection of *circuit elements* assigning to each edge $e \in E$ a *voltage–current relation*

  $$\mathcal{E}(e) : (v_e, i_e) \mapsto 0,$$

  such as Ohm's law for a resistor e: $v_e - R_e\, i_e = 0$.

We associate to G its *incidence matrix* $A \in \{-1, 0, 1\}^{|V|\times|E|}$, where

$$A_{n,e} = \begin{cases} +1, & \text{if edge } e \text{ leaves node } n, \\ -1, & \text{if edge } e \text{ enters node } n, \\ 0, & \text{otherwise.} \end{cases}$$

A state of the circuit consists of functions $i : E \to \mathbb{R}$ (branch currents) and $v : E \to \mathbb{R}$ (branch voltages) satisfying:

1. **Kirchhoff's Current Law (KCL):**
$$A\,\mathbf{i} \;=\; \mathbf{0},$$
meaning the algebraic sum of currents at each node is zero.
2. **Kirchhoff's Voltage Law (KVL):** there exists a node–potential vector $\mathbf{u} \in \mathbb{R}^{|V|}$ such that
$$\mathbf{v} \;=\; A^{\top}\,\mathbf{u},$$
so the sum of voltage drops around any closed loop vanishes.
3. **Element Constitutive Relations:** for each $e \in E$, $\mathcal{E}(e)(v_e, i_e) = 0$.

Together, these equations define the *network equations* of the circuit.

Example 2.12 (Resistive Network)**.** A Resistive Network is an electrical circuit composed of interconnected resistors used to control voltage, current, and power distribution (cf. [65, 69]). Let $G = (V, E)$ be a connected graph with $V = \{1, 2, 3\}$, $E = \{e_{12}, e_{23}, e_{31}\}$, each edge a resistor of resistance R_{ij}. Then:
$$\mathbf{i} = (i_{12}, i_{23}, i_{31})^{\top}, \quad \mathbf{v} = (v_{12}, v_{23}, v_{31})^{\top},$$
and the incidence matrix is
$$A = \begin{pmatrix} +1 & 0 & -1 \\ -1 & +1 & 0 \\ 0 & -1 & +1 \end{pmatrix}.$$
KCL: $A\,\mathbf{i} = \mathbf{0}$.
KVL: $\mathbf{v} = A^{\top}\mathbf{u}$ for node potentials $\mathbf{u} = (u_1, u_2, u_3)^{\top}$.
Ohm's law on each edge e_{ij}: $v_{ij} - R_{ij}\,i_{ij} = 0$. Solving these yields the currents and potentials in the network.

2.5 Bond graphs

Bond graphs are a domain-independent formalism for modeling the transfer and storage of energy in multi-domain physical systems [48, 82, 115, 116]. They consist of two kinds of vertices—*element nodes* and *junction nodes*—connected by *bonds* carrying conjugate variables effort e and flow f.

Definition 2.13 (Bond Graph)**.** [48, 115, 116] A *bond graph* is an undirected graph
$$G = (V, E),$$
where

- $V = V_{\text{elem}} \,\dot\cup\, V_{\text{junc}}$, a disjoint union of
  - *Element nodes* $V_{\text{elem}} = V_{Se} \,\dot\cup\, V_{Sf} \,\dot\cup\, V_R \,\dot\cup\, V_C \,\dot\cup\, V_I \,\dot\cup\, V_{TF} \,\dot\cup\, V_{GY}$,
  - *Junction nodes* $V_{\text{junc}} = V_0 \,\dot\cup\, V_1$,
- $E \subseteq \big\{\{u, v\} : u, v \in V,\ u \neq v\big\}$ is the set of *bonds*, each representing a single power port connection.

Each bond $\{u, v\} \in E$ carries two variables:
$$e \text{ (effort)}, \quad f \text{ (flow)}, \quad \text{with instantaneous power } P = e\,f.$$
Element nodes denote:

- Se: effort source (e.g. voltage, force),
- Sf: flow source (e.g. current, velocity),
- R: resistance (energy dissipation),
- C: capacitance (potential energy storage),

- I: inertia (kinetic energy storage),
- TF: transformer (scaling of effort and flow),
- GY: gyrator (cross-domain conversion of effort and flow).

Junction nodes denote:

- 0-junction: common effort, flows sum to zero,
- 1-junction: common flow, efforts sum to zero.

Example 2.14 (Bond Graph of a Series R–C Circuit)**.** Consider a simple series circuit consisting of an effort source Se, a resistor R, and a capacitor C. Its bond-graph representation is:

$$V_{\text{elem}} = \{Se,\ R,\ C\}, \qquad V_{\text{junc}} = \{\ 1\},$$

where "1" denotes a 1-junction (common flow, efforts sum to zero). The set of bonds is

$$E = \big\{\{Se, 1\},\ \{R, 1\},\ \{C, 1\}\big\}.$$

Each bond $\{x, 1\}$ carries conjugate variables effort e_x and flow f_x. At the 1-junction:

$$f_{Se} = f_R = f_C \ = \ f, \qquad e_{Se} + e_R + e_C = 0.$$

The constitutive relations on each element are:

$$e_{Se}(t) = u(t), \quad e_R = R\,f, \quad f_C = C\,\frac{d\,e_C}{dt}.$$

Thus the bond graph fully captures the energy exchange: the same flow f passes through all elements, while the efforts across Se, R, and C sum to zero.

3 Result: n-SuperHyperGraph Signal Processing

SuperHypergraph Signal Processing generalizes signal analysis over nested multi-level hypergraphs using tensor operations, spectral decomposition, and hierarchical shifting.

Definition 3.1 (n-SuperHypergraph Signal Processing)**.** Let $\mathrm{SHT}^{(n)} = (V, E)$ be an n-SuperHyperGraph with $|V| = N_n$ and maximum superedge cardinality

$$M \ = \ \max_{e \in E} |e|.$$

Define the *adjacency tensor* $\mathcal{A} \in \mathbb{R}^{\overbrace{N_n \times \cdots \times N_n}^{M \text{ times}}}$ by

$$\mathcal{A}_{i_1 \cdots i_M} = \begin{cases} c \left(\displaystyle\sum_{\substack{k_1, \ldots, k_c \geq 1 \\ \sum k_i = M}} \frac{M!}{k_1! \cdots k_c!} \right)^{-1} & \text{if } \{v_{i_1}, \ldots, v_{i_M}\} \text{ enumerates superedge } e = \{w_1, \ldots, w_c\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $c = |e| \leq M$.

A *signal* on $\mathrm{SHT}^{(n)}$ is a vector $\mathbf{s} \in \mathbb{R}^{N_n}$. Form the $(M-1)$th-order *signal tensor*

$$\mathcal{S} = \underbrace{\mathbf{s} \circ \cdots \circ \mathbf{s}}_{M-1 \text{ times}} \in \mathbb{R}^{\overbrace{N_n \times \cdots \times N_n}^{M-1}}.$$

The *shifted signal* is

$$\mathcal{S}' = \mathcal{A} \times_M \mathcal{S},$$

where $\times_M$ denotes the mode-M product. Finally, assume an orthogonal CANDECOMP/PARAFAC decomposition

$$\mathcal{A} = \sum_{r=1}^{R} \lambda_r \underbrace{\mathbf{f}_r \circ \cdots \circ \mathbf{f}_r}_{M \text{ times}}, \quad \langle \mathbf{f}_r, \mathbf{f}_s \rangle = \delta_{rs}.$$

The *n-SuperHypergraph Fourier transform* of $\mathcal{S}$ is the vector $\widehat{\mathcal{S}} \in \mathbb{R}^R$ with

$$\widehat{\mathcal{S}}_r = \big\langle \mathcal{S}, \underbrace{\mathbf{f}_r \circ \cdots \circ \mathbf{f}_r}_{M \text{ times}} \big\rangle.$$

Example 3.2 (2-SuperHypergraph Signal Processing on a Divisional Collaboration Structure)**.** Let the base set of employees be

$$V_0 = \{\text{Alice}, \text{Bob}, \text{Carol}, \text{Dave}, \text{Eve}\}.$$

Form the 1-supervertices (committees):

$$C_1 = \{\text{Alice}, \text{Bob}\}, \quad C_2 = \{\text{Carol}, \text{Dave}, \text{Eve}\}, \quad C_3 = \{\text{Bob}, \text{Carol}\},$$

and the 2-supervertices (divisions):

$$D_1 = \{C_1, C_2\}, \quad D_2 = \{C_2, C_3\}.$$

Define the 2-SuperHyperGraph $\mathrm{SHT}^{(2)} = (V, E)$ with

$$V = \{D_1, D_2\}, \qquad E = \big\{\{D_1, D_2\}\big\},$$

so that $|V| = 2$ and $M = 2$.

Assign to each division the "active project count" signal

$$\mathbf{s} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \quad \big(s_1 = 5,\ s_2 = 7\big).$$

Since $M = 2$, the adjacency tensor $\mathcal{A} \in \mathbb{R}^{2\times 2}$ has entries

$$\mathcal{A}_{i,j} = \begin{cases} 1, & \text{if } \{D_i, D_j\} = \{D_1, D_2\}, \\ 0, & \text{otherwise,} \end{cases}$$

i.e. $\mathcal{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The $(M-1)$th-order signal tensor is just the vector $\mathbf{s}$. The shifted signal is

$$\mathcal{S}' = \mathcal{A}\,\mathbf{s} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}.$$

Finally, the adjacency matrix admits the orthogonal eigen-decomposition

$$\mathcal{A} = \mathbf{F} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{F}^\top, \quad \mathbf{F} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Hence the 2-SuperHypergraph Fourier transform of $\mathbf{s}$ is

$$\widehat{\mathbf{s}} = \mathbf{F}^\top \mathbf{s} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix} = \begin{pmatrix} \frac{12}{\sqrt{2}} \\ \frac{-2}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 6\sqrt{2} \\ -\sqrt{2} \end{pmatrix}.$$

This example shows how 2-SuperHypergraph Signal Processing generalizes both graph and hypergraph signal frameworks to a three-layer corporate structure.

Example 3.3 (3-SuperHypergraph Signal Processing for Department Collaboration)**.** Let the base set of employees be

$$V_0 = \{\text{Alice}, \text{Bob}, \text{Carol}\}.$$

Form the 1-supervertices (committees):

$$C_1 = \{\text{Alice}, \text{Bob}\}, \quad C_2 = \{\text{Bob}, \text{Carol}\},$$

the 2-supervertices (divisions):

$$D_1 = \{C_1\}, \quad D_2 = \{C_2\},$$

and the 3-supervertices (departments):

$$H_1 = \{D_1\}, \quad H_2 = \{D_2\}, \quad H_3 = \{D_1, D_2\}.$$

Define the 3-SuperHyperGraph $\mathrm{SHT}^{(3)} = (V, E)$ by

$$V = \{H_1, H_2, H_3\}, \qquad E = \big\{\{H_1, H_2, H_3\}\big\},$$

so that $|V| = 3$ and $M = 3$.

Assign to each department the "active project count" signal

$$\mathbf{s} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}.$$

The adjacency tensor $\mathcal{A} \in \mathbb{R}^{3\times3\times3}$ has entries

$$\mathcal{A}_{i,j,k} = \begin{cases} \dfrac{3}{\sum_{k_1+k_2+k_3=3} \frac{3!}{k_1!\,k_2!\,k_3!}} = \dfrac{3}{6} = 0.5, & \{i, j, k\} = \{1, 2, 3\}, \\ 0, & \text{otherwise.} \end{cases}$$

Form the order-2 signal tensor $\mathcal{S} \in \mathbb{R}^{3\times3}$ by

$$\mathcal{S}_{i,j} = s_i\, s_j,$$

so that for instance $\mathcal{S}_{1,2} = 3 \times 4 = 12$. The shifted signal $\mathcal{S}' = \mathcal{A} \times_3 \mathcal{S} \in \mathbb{R}^{3\times3}$ has entries

$$\mathcal{S}'_{i,j} = \sum_{k=1}^{3} \mathcal{A}_{i,j,k}\, s_k,$$

giving

$$\mathcal{S}' = \begin{pmatrix} 0 & 0.5\times5 & 0.5\times4 \\ 0.5\times5 & 0 & 0.5\times3 \\ 0.5\times4 & 0.5\times3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2.5 & 2.0 \\ 2.5 & 0 & 1.5 \\ 2.0 & 1.5 & 0 \end{pmatrix}.$$

Theorem 3.4 (GSP and HGSP as Special Cases)**.** *Let* $\mathrm{NSP}^{(n)}$ *denote the n-SuperHypergraph Signal Processing above, with parameters n and M. Then:*

1. *If* $n = 0$, $\mathrm{NSP}^{(0)}$ *coincides with Hypergraph Signal Processing on the hypergraph* $H = (V_0, E)$.
2. *If moreover* $M = 2$, $\mathrm{NSP}^{(0)}$ *further reduces to Graph Signal Processing on the simple graph* (V_0, E).

Proof. For $n = 0$, one has $V \subseteq \mathcal{P}^0(V_0) = V_0$ and $E \subseteq \mathcal{P}^0(V_0) = V_0$, so $\mathrm{SHT}^{(0)}$ is exactly a hypergraph on V_0. By construction, the adjacency tensor and all subsequent operations in $\mathrm{NSP}^{(0)}$ agree with those of Hypergraph Signal Processing.

If additionally $M = 2$, then all superedges have size at most 2, so $\mathcal{A}$ is a matrix (order-2 tensor). The tensor definitions collapse to vector–matrix operations: $\mathcal{S} = \mathbf{s}$, $\mathcal{S}' = \mathbf{A}\,\mathbf{s}$, and the CANDECOMP/PARAFAC decomposition reduces to the eigen-decomposition of $\mathbf{A}$. These are precisely the definitions of Graph Signal Processing. □

Theorem 3.5 (Underlying n-SuperHyperGraph Structure)**.** *The n-SuperHypergraph Signal Processing* $\mathrm{NSP}^{(n)}$ *is built intrinsically on the combinatorial structure of the n-SuperHyperGraph* $\mathrm{SHT}^{(n)}$.

Proof. By definition, every element of the domain V of signals is an n-supervertex in $\mathcal{P}^n(V_0)$, and every nonzero entry of the adjacency tensor $\mathcal{A}$ corresponds exactly to an n-superedge in $E \subseteq \mathcal{P}^n(V_0)$. All signal operations (outer-product, mode products, tensor decompositions) are indexed by these supervertices and superedges. Hence the entire signal-processing pipeline is a direct translation of the combinatorial data of $\mathrm{SHT}^{(n)}$ into multilinear algebra, proving that $\mathrm{NSP}^{(n)}$ inherits and requires the full n-SuperHyperGraph structure. □

Theorem 3.6 (Spectral Diagonalization of the Shift Operator)**.** *Let* $\mathcal{A} = \sum_{r=1}^{R} \lambda_r\, (\mathbf{f}_r \circ \cdots \circ \mathbf{f}_r)$ *be the orthogonal CANDECOMP/PARAFAC decomposition of the adjacency tensor and let* $\mathcal{S}' = \mathcal{A} \times_M \mathcal{S}$. *Then for each spectral component r,*

$$\widehat{\mathcal{S}'}_r \;=\; \lambda_r\, \widehat{\mathcal{S}}_r,$$

where $\widehat{\mathcal{S}}_r = \langle \mathcal{S},\, \mathbf{f}_r \circ \cdots \circ \mathbf{f}_r \rangle$.

Proof. By definition,

$$\widehat{\mathcal{S}'}_r = \big\langle \mathcal{S}',\, \mathbf{f}_r \circ \cdots \circ \mathbf{f}_r \big\rangle = \big\langle \mathcal{A} \times_M \mathcal{S},\, \mathbf{f}_r^{\otimes M} \big\rangle.$$

Using the multilinear contraction property,

$$\big\langle \mathcal{A} \times_M \mathcal{S},\, \mathbf{f}_r^{\otimes M} \big\rangle = \big\langle \mathcal{A},\, \mathbf{f}_r^{\otimes (M-1)} \circ\, \mathcal{S} \times_M \mathbf{f}_r \big\rangle = \big\langle \mathcal{A},\, \mathbf{f}_r^{\otimes M} \big\rangle\, \widehat{\mathcal{S}}_r.$$

But from the CP-decomposition,

$$\big\langle \mathcal{A},\, \mathbf{f}_r^{\otimes M} \big\rangle = \lambda_r \big\langle \mathbf{f}_r^{\otimes M},\, \mathbf{f}_r^{\otimes M} \big\rangle = \lambda_r,$$

by orthonormality. Hence $\widehat{\mathcal{S}'}_r = \lambda_r\, \widehat{\mathcal{S}}_r$. □

Theorem 3.7 (Inversion Formula)**.** *The collection* $\{\mathbf{f}_r^{\otimes M}\}_{r=1}^{R}$ *forms an orthonormal basis for the signal-tensor space. Consequently, any signal tensor* $\mathcal{S}$ *admits the expansion*

$$\mathcal{S} = \sum_{r=1}^{R} \widehat{\mathcal{S}}_r\, (\mathbf{f}_r \circ \cdots \circ \mathbf{f}_r), \qquad \widehat{\mathcal{S}}_r = \big\langle \mathcal{S},\, \mathbf{f}_r^{\otimes M} \big\rangle.$$

Proof. Orthonormality of the rank-one factors implies $\langle \mathbf{f}_r^{\otimes M},\, \mathbf{f}_s^{\otimes M} \rangle = \delta_{rs}$. Any tensor in $\mathbb{R}^{N_n^{\times (M-1)}}$ can be uniquely decomposed in this basis. The coefficients are given by the inner products $\widehat{\mathcal{S}}_r$. Summing over r yields the reconstruction formula. □

Theorem 3.8 (Parseval's Identity)**.** *For any signal tensor* $\mathcal{S}$,

$$\|\mathcal{S}\|^2 = \sum_{i_1,\ldots,i_{M-1}} \mathcal{S}_{i_1 \cdots i_{M-1}}^2 = \sum_{r=1}^{R} (\widehat{\mathcal{S}}_r)^2.$$

Proof. From the inversion formula, $\mathcal{S} = \sum_r \widehat{\mathcal{S}}_r\, \mathbf{f}_r^{\otimes M}$, so

$$\|\mathcal{S}\|^2 = \Big\langle \sum_r \widehat{\mathcal{S}}_r\, \mathbf{f}_r^{\otimes M}, \sum_s \widehat{\mathcal{S}}_s\, \mathbf{f}_s^{\otimes M} \Big\rangle = \sum_{r,s} \widehat{\mathcal{S}}_r\, \widehat{\mathcal{S}}_s\, \langle \mathbf{f}_r^{\otimes M},\, \mathbf{f}_s^{\otimes M} \rangle = \sum_r (\widehat{\mathcal{S}}_r)^2.$$

□

Theorem 3.9 (Filter Diagonalization)**.** *Let* $\mathcal{A} = \sum_{r=1}^{R} \lambda_r\, \mathbf{f}_r^{\otimes M}$ *be the orthogonal CANDECOMP/PARAFAC decomposition of the adjacency tensor. For any real polynomial* $g(t) = \sum_{k=0}^{K} a_k t^k$, *define the filter operator*

$$\mathcal{H} = \sum_{k=0}^{K} a_k \underbrace{\mathcal{A} \times_M \mathcal{A} \times_M \cdots \times_M \mathcal{A}}_{k \text{ times}} \in \mathbb{R}^{\overbrace{N_n \times \cdots \times N_n}^{M \text{ times}}}.$$

Then for any signal tensor $\mathcal{S}$,

$$\widehat{\mathcal{H} \times_M \mathcal{S}}_r \;=\; g(\lambda_r)\, \widehat{\mathcal{S}}_r, \quad r = 1, \ldots, R,$$

i.e. the filter acts as pointwise multiplication by $g(\lambda_r)$ *in the spectral domain.*

Proof. Since $\mathcal{A}^{\times k} = \sum_{r=1}^{R} \lambda_r^k \, \mathbf{f}_r^{\otimes M}$ by repeated application of the CP decomposition, it follows that

$$\mathcal{H} = \sum_{k=0}^{K} a_k \, \mathcal{A}^{\times k} = \sum_{r=1}^{R} \Big(\sum_{k=0}^{K} a_k \lambda_r^k \Big) \mathbf{f}_r^{\otimes M} = \sum_{r=1}^{R} g(\lambda_r) \, \mathbf{f}_r^{\otimes M}.$$

Hence for any $\mathcal{S}$,

$$\widehat{\mathcal{H} \times_M \mathcal{S}}_r = \big\langle \mathcal{H} \times_M \mathcal{S}, \, \mathbf{f}_r^{\otimes M} \big\rangle = g(\lambda_r) \, \langle \mathcal{S}, \, \mathbf{f}_r^{\otimes M} \rangle = g(\lambda_r) \, \widehat{\mathcal{S}}_r.$$

□

Theorem 3.10 (Shift-Invariant Operator Characterization)**.** *A multilinear operator* $\mathcal{H} \colon \mathbb{R}^{N_n^{\times(M-1)}} \to \mathbb{R}^{N_n^{\times(M-1)}}$ *commutes with the shift* $\mathcal{A} \times_M (\cdot)$ *if and only if it is simultaneously diagonalizable, i.e.,*

$$\mathcal{H} = \sum_{r=1}^{R} h_r \, \mathbf{f}_r^{\otimes M},$$

for some scalars h_r*. In this case,* $\mathcal{H} \times_M \mathcal{A} = \mathcal{A} \times_M \mathcal{H}$.

Proof. ($\Rightarrow$) If $\mathcal{H} \circ (\mathcal{A} \times_M) = (\mathcal{A} \times_M) \circ \mathcal{H}$, then $\mathcal{H}$ preserves each one-dimensional eigenspace spanned by $\mathbf{f}_r^{\otimes M}$. By orthonormality, $\mathcal{H}(\mathbf{f}_r^{\otimes M}) = h_r \, \mathbf{f}_r^{\otimes M}$ for some h_r.

($\Leftarrow$) Conversely, if $\mathcal{H} = \sum h_r \, \mathbf{f}_r^{\otimes M}$, then

$$\mathcal{H} \times_M \mathcal{A} = \sum_{r=1}^{R} h_r \lambda_r \, \mathbf{f}_r^{\otimes M} = \mathcal{A} \times_M \mathcal{H}.$$

□

Theorem 3.11 (Operator Norm and Spectral Radius)**.** *Let* $T \colon \mathcal{S} \mapsto \mathcal{A} \times_M \mathcal{S}$ *be the shift operator. Then its induced spectral norm equals the maximum absolute hypergraph frequency:*

$$\|T\|_2 = \max_{1 \le r \le R} \big|\lambda_r\big|.$$

Proof. Since T is diagonalizable in the orthonormal basis $\{\mathbf{f}_r^{\otimes M}\}$, its operator norm is the largest magnitude of its eigen-values, which are exactly $\{\lambda_r\}_{r=1}^{R}$. □

4 Result: Electric HyperCircuit

We define the concepts of the Electric HyperCircuit and the Electric SuperHyperCircuit, and provide concrete examples and mathematical theorems to illustrate their structures and properties.

Definition 4.1 (Electric HyperCircuit)**.** An *electric hypercircuit* is a pair $(H, \mathcal{E})$ where:

- $H = (V, E, I, \pi, \sigma)$ is a finite oriented hypergraph:
  - V is the set of *nodes*.
  - E is the set of *hyperedges* (multi-terminal elements).
  - I is a finite set of *incidences*, with surjections $\pi : I \to V$ (attaching each incidence to a node) and $e : I \to E$ (attaching each incidence to a hyperedge).
  - $\sigma : I \to \{+1, -1\}$ is an *orientation* on incidences.
- $\mathcal{E} = \{\mathcal{E}_e\}_{e \in E}$ assigns to each hyperedge e a *constitutive relation*

$$\mathcal{E}_e\big(v_e, \, i_e\big) \;=\; 0, \quad v_e = (v_k)_{k \in I_e}, \; i_e = (i_k)_{k \in I_e},$$

  where $I_e = \{k \in I : \, e(k) = e\}$ is the set of incidences of e.

A *state* of the hypercircuit consists of functions $i : I \to \mathbb{R}$ (port currents) and $v : I \to \mathbb{R}$ (port voltages) satisfying:

1. **Kirchhoff's Current Law (KCL):**

$$\sum_{k\in I:\,\pi(k)=n} i(k) \;=\; 0, \quad \forall\, n \in V.$$

2. **Kirchhoff's Voltage Law (KVL):** there exists a node-potential function $u : V \to \mathbb{R}$ such that

$$v(k) \;=\; \sigma(k)\, u\big(\pi(k)\big), \quad \forall\, k \in I.$$

3. **Element Constitutive Relations:** for each $e \in E$,

$$\mathcal{E}_e\big(v_e,\, i_e\big) = 0.$$

Example 4.2 (Common–Emitter BJT Amplifier as an Electric HyperCircuit)**.** A Common–Emitter BJT Amplifier is a transistor circuit configuration that amplifies voltage signals with significant gain and phase inversion (cf. [58, 59]). Consider the hypercircuit $(H, \mathcal{E})$ defined as follows:

Hypergraph structure $H = (V, E, I, \pi, e, \sigma)$**:**

$$V = \{V_{CC},\, B,\, C,\, E\}, \qquad E = \{R_B,\; R_C,\; T\},$$

where

$$I = \{\, k_{V_{CC},R_B},\; k_{B,R_B},\; k_{V_{CC},R_C},\; k_{C,R_C},\; k_{B,T},\; k_{C,T},\; k_{E,T} \,\}.$$

The attachment maps are

$$\begin{aligned}
\pi(k_{V_{CC},R_B}) &= V_{CC}, & e(k_{V_{CC},R_B}) &= R_B,\\
\pi(k_{B,R_B}) &= B, & e(k_{B,R_B}) &= R_B,\\
\pi(k_{V_{CC},R_C}) &= V_{CC}, & e(k_{V_{CC},R_C}) &= R_C,\\
\pi(k_{C,R_C}) &= C, & e(k_{C,R_C}) &= R_C,\\
\pi(k_{B,T}) &= B, & e(k_{B,T}) &= T,\\
\pi(k_{C,T}) &= C, & e(k_{C,T}) &= T,\\
\pi(k_{E,T}) &= E, & e(k_{E,T}) &= T.
\end{aligned}$$

Orient all incidences from the first-listed node to the second, so $\sigma(k_{X,e}) = +1$ if X is listed first, and -1 otherwise.

Constitutive relations $\mathcal{E}$**:**

$$\begin{aligned}
\mathcal{E}_{R_B} :&\quad v_{R_B} - R_B\, i_{R_B} = 0, \quad v_{R_B} = v(k_{V_{CC},R_B}) - v(k_{B,R_B}),\; i_{R_B} = i(k_{V_{CC},R_B});\\
\mathcal{E}_{R_C} :&\quad v_{R_C} - R_C\, i_{R_C} = 0, \quad v_{R_C} = v(k_{V_{CC},R_C}) - v(k_{C,R_C}),\; i_{R_C} = i(k_{V_{CC},R_C});\\
\mathcal{E}_{T} :&\quad v(k_{B,T}) - v(k_{E,T}) - V_{BE} = 0,\\
&\quad i(k_{E,T}) - i(k_{B,T}) - i(k_{C,T}) = 0,\\
&\quad i(k_{C,T}) - \alpha\, i(k_{E,T}) = 0,
\end{aligned}$$

where V_{BE} is the base–emitter threshold and α the common-base gain.

KCL and KVL: A state consists of $v : I \to \mathbb{R}$, $i : I \to \mathbb{R}$, and node potentials $u : V \to \mathbb{R}$, satisfying

$$\sum_{k:\,\pi(k)=n} i(k) = 0 \quad \text{(KCL at each } n \in V\text{)}, \qquad v(k) = \sigma(k)\, u\big(\pi(k)\big) \quad \text{(KVL for each } k \in I\text{)}.$$

This hypercircuit model captures the two resistors and the three-terminal transistor in one unified oriented hypergraph framework.

Theorem 4.3 (Generalization of Electric Circuit)**.** *If each hyperedge $e \in E$ has exactly two incidences $I_e = \{k_1, k_2\}$ and $\mathcal{E}_e(v_e, i_e)$ depends only on the voltage difference and a single current (as in Ohm's law), then the electric hypercircuit reduces to the classical electric circuit on the graph $G = (V, E)$.*

Proof. When $|I_e| = 2$, index the two incidences by k_1, k_2 with $\pi(k_1) = n_1,\ \pi(k_2) = n_2$. KVL gives

$$v(k_1) = u(n_1), \quad v(k_2) = -u(n_2) \implies v_{n_1 n_2} = u(n_1) - u(n_2),$$

recovering the usual branch voltage. KCL at each node $\sum_{k:\, \pi(k)=n} i(k) = 0$ becomes the sum of incident branch currents. Finally, if $\mathcal{E}_e(v_e, i_e) \equiv v_{n_1 n_2} - R_e\, i_e = 0$, we obtain Ohm's law. Thus the hypercircuit equations coincide with the network equations of an electric circuit on the graph G. □

Theorem 4.4 (Underlying Hypergraph Structure)**.** *The electric hypercircuit $(H, \mathcal{E})$ is intrinsically built on the combinatorial data of the oriented hypergraph H.*

Proof. By definition, the set of nodes V, hyperedges E, incidences I, and orientation σ completely determine the incidence relations π and e. All circuit equations—KCL, KVL, and constitutive relations—are indexed by these hypergraph components (V, I, E). Therefore the signal-processing and network-analysis formalisms operate directly on the hypergraph structure, proving that the electric hypercircuit inherits and requires the full hypergraph. □

Definition 4.5 (Electric n-SuperHyperCircuit)**.** Let V_0 be a finite base set of fundamental nodes and let $\mathrm{SHT}^{(n)} = (V, E)$ be an oriented n-SuperHyperGraph with

$$V \subseteq \mathcal{P}^n(V_0), \quad E \subseteq \mathcal{P}^n(V_0),$$

and incidence structure (I, π, e, σ) where

$$I = \{(v, e) : v \in e,\ e \in E\},\ \pi(v, e) = v,\ e(v, e) = e,\ \sigma(v, e) \in \{+1, -1\}.$$

An *electric n-superhypercircuit* is the pair $(\mathrm{SHT}^{(n)}, \mathcal{E})$ where $\mathcal{E} = \{\mathcal{E}_e\}_{e \in E}$ assigns to each superedge e a constitutive relation

$$\mathcal{E}_e(v_e, i_e) = 0, \quad v_e = \big(v(k)\big)_{k \in I_e},\ i_e = \big(i(k)\big)_{k \in I_e},$$

with $I_e = \{k \in I : e(k) = e\}$. A *state* consists of port-voltage and port-current functions

$$v : I \to \mathbb{R}, \quad i : I \to \mathbb{R},$$

and a supervertex potential $u : V \to \mathbb{R}$, satisfying:

1. $\displaystyle\sum_{k \in I:\, \pi(k)=v} i(k) = 0$ for all supervertices $v \in V$ (generalized KCL).
2. $v(k) = \sigma(k)\, u\big(\pi(k)\big)$ for all ports $k \in I$ (generalized KVL).
3. $\mathcal{E}_e\big(v_e, i_e\big) = 0$ for each superedge $e \in E$ (constitutive laws).

Example 4.6 (Electric 2-SuperHyperCircuit for a BJT Amplifier Subnetwork)**.** Let the base set of fundamental nodes be

$$V_0 = \{V_{CC},\ B,\ C,\ E\},$$

and consider the electric hypercircuit with three hyperedges:

$$e_1 = \{V_{CC}, B\} \quad (R_B), \quad e_2 = \{V_{CC}, C\} \quad (R_C), \quad e_3 = \{B, C, E\} \quad (T).$$

We form the 2-supervertices by grouping overlapping hyperedges:

$$D_1 = \{\, e_1,\ e_2\}, \qquad D_2 = \{\, e_2,\ e_3\}.$$

Thus the set of 2-supervertices is

$$V = \{D_1, D_2\},$$

and there is a single 2-superedge connecting them:

$$E = \{\{D_1, D_2\}\}.$$

The incidence set is

$$I = \{\, k_1 = (D_1, E),\ k_2 = (D_2, E)\},$$

with $\pi(k_i) = D_i$, $e(k_i) = E$, and choose $\sigma(k_i) = +1$.

We assign to each 2-superedge E the constitutive relations of an ideal connection:

$$\mathcal{E}_E : \quad v(k_1) - v(k_2) = 0, \qquad i(k_1) + i(k_2) = 0,$$

where $v(k_i)$ and $i(k_i)$ are the port-voltage and port-current at incidence k_i.

A state consists of port-functions $v : I \to \mathbb{R}$, $i : I \to \mathbb{R}$ and supervertex potentials $u : V \to \mathbb{R}$ satisfying:

$$\sum_{k:\,\pi(k)=D_i} i(k) = 0, \quad v(k) = \sigma(k)\, u\big(\pi(k)\big), \quad \mathcal{E}_E\big(v_E, i_E\big) = 0.$$

Concretely,

$$i(k_1) + i(k_2) = 0, \quad v(k_1) = v(k_2),$$

ensuring that the two subnetworks $\{R_B, R_C\}$ and $\{R_C, T\}$ are perfectly connected in this 2-superhypercircuit.

Example 4.7 (Electric 3-SuperHyperCircuit for a BJT Amplifier Meta-Connection)**.** Let the base set of fundamental nodes be

$$V_0 = \{V_{CC},\ B,\ C,\ E\},$$

and consider the three 1-superedges (ordinary hyperedges)

$$e_1 = \{V_{CC}, B\}, \quad e_2 = \{V_{CC}, C\}, \quad e_3 = \{B, C, E\}.$$

Form the 2-supervertices (elements of $\mathcal{P}^2(V_0)$) by grouping overlapping 1-superedges:

$$D_1 = \{e_1, e_2\}, \quad D_2 = \{e_2, e_3\}, \quad D_3 = \{e_3, e_1\}.$$

Thus

$$V^{(2)} = \{D_1, D_2, D_3\}.$$

Next form the 3-supervertices (elements of $\mathcal{P}^3(V_0)$) by grouping overlapping 2-supervertices:

$$A_1 = \{D_1, D_2\}, \quad A_2 = \{D_2, D_3\}, \quad A_3 = \{D_3, D_1\},$$

so

$$V^{(3)} = \{A_1, A_2, A_3\}.$$

Finally, the single 3-superedge

$$S = \{A_1, A_2, A_3\}$$

yields the oriented 3-SuperHyperGraph $\mathrm{SHT}^{(3)} = (V^{(3)}, \{S\})$.

The incidence set is

$$I = \{\, k_i = (A_i, S) \mid i = 1, 2, 3\},$$

with attachments $\pi(k_i) = A_i$, $e(k_i) = S$, and orientation $\sigma(k_i) = +1$.

Assign to the 3-superedge S the constitutive (ideal coupling) relations

$$\mathcal{E}_S : \qquad v(k_1) - v(k_2) = 0, \quad v(k_2) - v(k_3) = 0, \quad i(k_1) + i(k_2) + i(k_3) = 0.$$

A *state* consists of port-voltage and port-current functions $v : I \to \mathbb{R}$, $i : I \to \mathbb{R}$ and a supervertex potential $u : V^{(3)} \to \mathbb{R}$, satisfying:

$$\sum_{k:\,\pi(k)=A_i} i(k) = 0, \qquad v(k) = \sigma(k)\, u\big(\pi(k)\big), \qquad \mathcal{E}_S\big(v_S, i_S\big) = 0.$$

Concretely:

$$i(k_1) + i(k_2) + i(k_3) = 0, \quad v(k_1) = v(k_2) = v(k_3),$$

ensuring an ideal three-port connection that unifies the two subnetworks $\{e_1, e_2\}$, $\{e_2, e_3\}$, and $\{e_3, e_1\}$ into a single 3-superhyperconnection.

Theorem 4.8 (Reduction to Hypercircuit and Circuit)**.** *The electric n-superhypercircuit* $(\mathrm{SHT}^{(n)}, \mathcal{E})$*:*

1. *For* $n = 0$*,* $V \subseteq V_0$ *and* $E \subseteq V_0$*, so* $\mathrm{SHT}^{(0)}$ *is an oriented hypergraph on* V_0*. The above equations recover exactly those of an electric hypercircuit.*
2. *If furthermore each superedge e has* $|I_e| = 2$ *and* $\mathcal{E}_e(v_e, i_e)$ *depends only on the voltage difference and the single branch current, then* $\mathrm{SHT}^{(0)}$ *is a graph and the hypercircuit reduces to a classical electric circuit on* $G = (V_0, E)$*.*

Proof. When $n = 0$, each supervertex is a base node and each superedge is a subset of nodes in V_0. The incidence set I and orientation σ coincide with those of an oriented hypergraph. Hence KCL and KVL match the hypercircuit laws, and $\mathcal{E}_e$ are the same constitutive relations.

If in addition $|I_e| = 2$, label the two incidences k_1, k_2 with $\pi(k_1) = n_1,\ \pi(k_2) = n_2$. Then

$$v(k_1) = u(n_1),\ v(k_2) = -u(n_2) \implies v_{n_1 n_2} = u(n_1) - u(n_2),$$

and KCL becomes the node-current sum law. If $\mathcal{E}_e(v_e, i_e) : v_{n_1 n_2} - R_e\, i_e = 0$, one recovers Ohm's law. Thus the model reduces to the classical electric circuit network equations. □

Theorem 4.9 (Intrinsic n-SuperHyperGraph Structure)**.** *The electric n-superhypercircuit* $(\mathrm{SHT}^{(n)}, \mathcal{E})$ *is built intrinsically on the oriented n-SuperHyperGraph* $\mathrm{SHT}^{(n)}$*.*

Proof. All components—supervertices V, superedges E, incidences I, attachment maps π, e, and orientations σ—are data of $\mathrm{SHT}^{(n)}$. The network laws (generalized KCL, KVL) and constitutive equations are formulated directly in terms of these hypergraph elements. No additional structure or external indexing is required. Therefore the circuit model inherently carries and exploits the full combinatorial structure of the n-SuperHyperGraph. □

5 Result: Bond HyperGraph and Bond SuperHyperGraph

We define the concepts of the Bond HyperGraph and the Bond SuperHyperGraph as follows.

Definition 5.1 (Bond HyperGraph)**.** Let V_{elem} be the set of bond-graph element nodes and V_{junc} the set of junction nodes, and let

$$G = \big(V_{\mathrm{elem}} \dot\cup V_{\mathrm{junc}},\ E\big)$$

be the classical bond graph. The *Bond HyperGraph* is the hypergraph

$$H = \big(V_{\mathrm{elem}},\ \mathcal{E}\big),$$

where

$$\mathcal{E} = \big\{\, e_j \subseteq V_{\mathrm{elem}} : j \in V_{\mathrm{junc}},\ e_j = \{\, u \in V_{\mathrm{elem}} : \{u, j\} \in E\}\big\}.$$

Each hyperedge e_j collects exactly those element nodes incident on junction j.

Example 5.2 (Bond HyperGraph of a Series R–C Circuit Driven by a Voltage Source)**.** Consider the bond graph with element nodes and junctions as follows:

$$V_{\mathrm{elem}} = \{Se,\ R,\ C\}, \qquad V_{\mathrm{junc}} = \{j_1,\ j_2\}.$$

The bond connections are

$$E = \big\{\{Se, j_1\},\ \{R, j_1\},\ \{R, j_2\},\ \{C, j_2\}\big\},$$

where Se is an effort source, R a resistor, C a capacitor, and j_1, j_2 are 1-junctions.

Forming the Bond HyperGraph $H = (V_{\mathrm{elem}}, \mathcal{E})$, each junction j_k induces a hyperedge

$$e_{j_1} = \{\, Se,\ R\}, \qquad e_{j_2} = \{\, R,\ C\},$$

so that

$$\mathcal{E} = \{\, e_{j_1},\ e_{j_2}\}.$$

Thus H is the hypergraph with vertex set $\{Se, R, C\}$ and hyperedge set $\{\{Se, R\}, \{R, C\}\}$, exactly capturing which element nodes meet at each junction.

Theorem 5.3 (Generalization of Bond Graph)**.** *Every bond graph G arises from a unique Bond HyperGraph H via the construction above, and conversely any Bond HyperGraph H defines a bond graph G in which each hyperedge e_j becomes a junction node j connected by bonds to every element $u \in e_j$.*

Proof. Starting from G, we form $H = (V_{\text{elem}}, \mathcal{E})$ by setting each hyperedge e_j to be the neighborhood of junction j. Conversely, given H, define

$$V_{\text{junc}} = \mathcal{E}, \qquad E = \big\{\{u, e_j\} : u \in e_j,\ e_j \in \mathcal{E}\big\}.$$

Then $G' = (V_{\text{elem}} \dot\cup V_{\text{junc}}, E)$ is a bond graph whose junction-neighborhoods recover exactly the hyperedges of H. These two operations are inverse to one another, proving the bijective correspondence. □

Theorem 5.4 (Underlying Hypergraph Structure)**.** *The Bond HyperGraph $H = (V_{\text{elem}}, \mathcal{E})$ carries by definition the full structure of a finite hypergraph: its vertex set is V_{elem} and its hyperedge set is $\mathcal{E} \subseteq \mathcal{P}(V_{\text{elem}})$.*

Proof. By construction, $\mathcal{E}$ is a collection of subsets of V_{elem}, and there are no additional constraints: H satisfies exactly the axioms of a finite hypergraph. All bond-graph junction connectivity is encoded solely in these hyperedges. □

Definition 5.5 (Bond n-SuperHyperGraph)**.** Let V_0 be the finite set of *element nodes* in a bond-graph domain. For each integer $k \geq 0$ define

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big).$$

A *Bond n-SuperHyperGraph* is a pair

$$\text{BnSHT}^{(n)} = (V, E),$$

where

$$V \subseteq \mathcal{P}^n(V_0) \quad (\text{the } n\text{-supervertices}), \qquad E \subseteq \mathcal{P}^n(V_0) \quad (\text{the } n\text{-superedges}),$$

together with the canonical incidence relation that each n-superedge $e \in E$ attaches to its member n-supervertices in V.

Example 5.6 (Bond 2-SuperHyperGraph of a Series R–L–C Circuit)**.** Let the base set of element nodes be

$$V_0 = \{Se,\ R,\ L,\ C\},$$

and consider the bond graph with three 1-junctions j_1, j_2, j_3 defined by the bonds

$$E = \big\{\{Se, j_1\},\ \{R, j_1\},\ \{R, j_2\},\ \{L, j_2\},\ \{L, j_3\},\ \{C, j_3\}\big\}.$$

The corresponding Bond HyperGraph $H = (V_{\text{elem}}, \mathcal{E})$ has

$$V_{\text{elem}} = \{Se, R, L, C\}, \qquad \mathcal{E} = \big\{\, e_{j_1},\ e_{j_2},\ e_{j_3}\big\},$$

where

$$e_{j_1} = \{Se, R\}, \quad e_{j_2} = \{R, L\}, \quad e_{j_3} = \{L, C\}.$$

Now form the 2-supervertices (elements of $\mathcal{P}^2(V_0)$) by grouping overlapping hyperedges:

$$D_1 = \{\, e_{j_1},\ e_{j_2}\}, \qquad D_2 = \{\, e_{j_2},\ e_{j_3}\}.$$

Thus the set of 2-supervertices is

$$V_2 = \{D_1,\ D_2\}.$$

A natural 2-superedge arises by connecting those two 2-supervertices that share the common hyperedge e_{j_2}:

$$E_2 = \big\{\{D_1,\ D_2\}\big\}.$$

Therefore, the Bond 2-SuperHyperGraph is

$$\text{BnSHT}^{(2)} = \big(V_2,\ E_2\big) = \big(\{D_1, D_2\},\ \{\{D_1, D_2\}\}\big).$$

This 2-SuperHyperGraph encodes a higher-level "meta-junction" that links the two 1-junction subnetworks $\{Se, R\}$–$\{R, L\}$ and $\{R, L\}$–$\{L, C\}$, thus generalizing both the bond graph and its hypergraph representation.

Example 5.7 (Bond 3-SuperHyperGraph of a Series R–L–C Circuit)**.** Let the base set of element nodes be

$$V_0 = \{Se,\ R,\ L,\ C\},$$

and consider the bond graph with three 1-junctions j_1, j_2, j_3 defined by the bonds

$$E = \big\{\{Se, j_1\},\ \{R, j_1\},\ \{R, j_2\},\ \{L, j_2\},\ \{L, j_3\},\ \{C, j_3\}\big\}.$$

The corresponding Bond HyperGraph $H = (V_{\mathrm{elem}}, \mathcal{E})$ has

$$V_{\mathrm{elem}} = \{Se, R, L, C\}, \qquad \mathcal{E} = \{\, e_1, e_2, e_3\},$$

where

$$e_1 = \{Se, R\}, \quad e_2 = \{R, L\}, \quad e_3 = \{L, C\}.$$

Form the 2-supervertices (elements of $\mathcal{P}^2(V_0)$) by grouping overlapping hyperedges:

$$D_1 = \{e_1, e_2\}, \quad D_2 = \{e_2, e_3\}, \quad D_3 = \{e_3, e_1\}.$$

Thus the set of 2-supervertices is

$$V_2 = \{D_1, D_2, D_3\},$$

and there is a natural 2-superedge for each pair of adjacent 2-supervertices:

$$E_2 = \big\{\{D_1, D_2\},\ \{D_2, D_3\},\ \{D_3, D_1\}\big\}.$$

Now form the 3-supervertices (elements of $\mathcal{P}^3(V_0)$) by grouping adjacent 2-supervertices:

$$A_1 = \{D_1, D_2\}, \quad A_2 = \{D_2, D_3\}, \quad A_3 = \{D_3, D_1\}.$$

Hence

$$V_3 = \{A_1, A_2, A_3\}.$$

Finally, the single 3-superedge connects all three 3-supervertices:

$$E_3 = \big\{\{A_1, A_2, A_3\}\big\}.$$

Therefore, the Bond 3-SuperHyperGraph is

$$\mathrm{BnSHT}^{(3)} = \big(V_3,\ E_3\big) = \Big(\ \{A_1, A_2, A_3\},\ \{\{A_1, A_2, A_3\}\}\Big).$$

This structure captures a three-level meta-junction that links the overlapping sub-circuits $\{Se, R\}$–$\{R, L\}$, $\{R, L\}$–$\{L, C\}$, and $\{L, C\}$–$\{Se, R\}$ in one unified 3-superhypergraph.

Theorem 5.8 (Reduction to Bond HyperGraph and Bond Graph)**.** *Let* $\mathrm{BnSHT}^{(n)} = (V, E)$ *be a Bond n-SuperHyperGraph on base set* V_0*. Then:*

1. *If* $n = 1$*, and we take*
$$V = \big\{\{v\} : v \in V_0\big\}, \quad E = \big\{\, e_j : j \in V_{\mathrm{junc}}\big\},$$
where each $e_j \subseteq V_0$ *is the set of element-nodes incident on junction* j*, then* $\mathrm{BnSHT}^{(1)}$ *coincides with the* Bond HyperGraph.

2. *If moreover each hyperedge* $e_j \in E$ *has* $|e_j| = 2$*, then this Bond HyperGraph is exactly the classical* Bond Graph.

Proof. **(1)** For $n = 1$, $\mathcal{P}^1(V_0) = \mathcal{P}(V_0)$. Choosing $V = \{\{v\} : v \in V_0\}$ identifies each singleton with the original element node. Setting $E = \{e_j : j \in V_{\mathrm{junc}}\}$ reproduces exactly the hyperedges of the Bond HyperGraph, since each e_j collects the element-nodes attached to junction j.

(2) If each e_j has size two, then every hyperedge is a pair of singletons $\{\,\{u\}, \{v\}\,\}$. Collapsing the singletons back to their underlying nodes yields an undirected graph with vertex set V_0 and edge set $\{\{u, v\} : e_j = \{u, v\}\}$. This is precisely the Bond Graph. □

Theorem 5.9 (Intrinsic n-SuperHyperGraph Structure)**.** *Any Bond n-SuperHyperGraph* $\mathrm{BnSHT}^{(n)} = (V, E)$ *is by definition an n-SuperHyperGraph: its supervertex set V and superedge set E satisfy*

$$V \subseteq \mathcal{P}^n(V_0), \quad E \subseteq \mathcal{P}^n(V_0),$$

and the incidence relation is the natural membership relation of superedges on supervertices.

Proof. The construction of $\mathrm{BnSHT}^{(n)}$ uses exactly the data of an n-SuperHyperGraph on base set V_0. By hypothesis V and E are subsets of $\mathcal{P}^n(V_0)$, and each superedge $e \in E$ attaches precisely to the supervertices it contains. No additional structure is needed, hence $\mathrm{BnSHT}^{(n)}$ inherits the full combinatorial and incidence structure of an n-SuperHyperGraph. □

Theorem 5.10 (Skeleton Consistency)**.** *Let* $\mathrm{BnSHT}^{(n)} = (V^{(n)}, E^{(n)})$ *be a Bond n-SuperHyperGraph over base set* V_0*. For each* $k = n-1, n-2, \ldots, 1$*, define recursively*

$$V^{(k)} = \bigcup_{S \in V^{(k+1)}} S, \qquad E^{(k)} = \left\{ F \subseteq V^{(k)} : F \subseteq e \text{ for some } e \in E^{(k+1)} \right\}.$$

Then for every $1 \le k \le n$*,* $\left(V^{(k)}, E^{(k)}\right)$ *is a Bond k-SuperHyperGraph. In particular:*

- $\left(V^{(1)}, E^{(1)}\right)$ *coincides with the Bond HyperGraph.*
- $\left(V^{(0)}, E^{(0)}\right)$ *is the classical Bond Graph.*

Proof. We prove by downward induction on k. For $k = n$, the result is given. Suppose $\left(V^{(k+1)}, E^{(k+1)}\right)$ is a Bond $(k+1)$-SuperHyperGraph with $V^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$, $E^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$. By definition,

$$V^{(k)} = \bigcup_{S \in V^{(k+1)}} S \subseteq \bigcup_{S \in \mathcal{P}^{k+1}(V_0)} S = \mathcal{P}^k(V_0),$$

and each $F \in E^{(k)}$ is a subset of some $e \in E^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$, so $F \subseteq \bigcup e \subseteq \mathcal{P}^k(V_0)$. The canonical incidence (membership) relation restricts correctly. Hence $\left(V^{(k)}, E^{(k)}\right)$ satisfies the definition of a Bond k-SuperHyperGraph. Taking $k = 1$ and then $k = 0$ yields the Bond HyperGraph and Bond Graph, respectively. □

Theorem 5.11 (Connectivity Inheritance)**.** *If the underlying Bond Graph (the 0-skeleton* $\left(V^{(0)}, E^{(0)}\right)$*) is connected, then for every* $1 \le k \le n$*, the Bond k-SuperHyperGraph* $\left(V^{(k)}, E^{(k)}\right)$ *is connected in the sense that its 2-section graph is connected.*

Proof. Recall that the 2-section of a hypergraph (V, E) is the graph on V where two vertices are adjacent if they belong to a common hyperedge. We show by induction on k that the 2-section of $\left(V^{(k)}, E^{(k)}\right)$ is connected.

Base ($k = 0$**).** The 2-section of the Bond Graph is itself, which is connected by hypothesis.

Inductive Step. Assume the 2-section of $\left(V^{(k)}, E^{(k)}\right)$ is connected. Consider $\left(V^{(k+1)}, E^{(k+1)}\right)$. By skeleton consistency (Theorem 5.10), every superedge $e \in E^{(k+1)}$ is a subset of $V^{(k)}$. Thus in the 2-section of $\left(V^{(k+1)}, E^{(k+1)}\right)$, any two k-supervertices $S_1, S_2 \in V^{(k+1)}$ that share an underlying $k{-}1$-vertex become adjacent if $S_1 \cap S_2 \neq \emptyset$. Since the 2-section at level k is connected, one can traverse from any k-supervertex to any other by stepping through overlapping sets. Therefore the 2-section at level $k+1$ is also connected. □

Theorem 5.12 (Superedge-Induced Subgraph Connectivity)**.** *In a Bond n-SuperHyperGraph* $\mathrm{BnSHT}^{(n)} = (V^{(n)}, E^{(n)})$*, for each superedge* $e \in E^{(n)}$*, the induced subgraph of the underlying Bond Graph on the union of all base-nodes in e is connected.*

Proof. Let $e \in E^{(n)}$ be an n-superedge. By recursive definition of skeletons, each element of e is a $(n-1)$-supervertex, whose member set is connected at the $(n-2)$-level, and so on down to base-level. Since hyperedges at each level correspond to junction connectivity in the lower level, the union of all base-nodes in e forms a connected set in the Bond Graph. More formally, for any two base-nodes u, v in $\bigcup e$, there exists a chain of overlapping supervertices linking them, which projects to a path in the 2-section of the 0-skeleton. Hence the induced subgraph is connected. □

6 Conclusion and Future Works

In this paper, we extended the frameworks of *Graph Signal Processing*, *Electric Circuits*, and *Bond Graphs* by incorporating the mathematical structures of *hypergraphs* and *superhypergraphs*. We examined their formal properties and provided illustrative examples to demonstrate their applicability and expressiveness.

In future work, we aim to conduct computational experiments related to these frameworks in order to explore their practical applications in real-world scenarios more concretely. In addition, we plan to investigate theoretical extensions and applications to foundational concepts such as *Ohm's Law* [114, 124], *Kirchhoff's Laws* [90, 96], *AC/DC Analysis* [2, 10], *Transfer Functions* [61, 113], and *Integrated Circuits* [117, 122].

And as a direction for future work, we plan to integrate advanced uncertainty-handling frameworks into the proposed models by incorporating various set-theoretic generalizations, including Fuzzy Sets [128, 129], Intuitionistic Fuzzy Sets [12, 13], Vague Sets [4, 47], Rough Sets [87, 88], Bipolar Fuzzy Sets [3], Tripolar Fuzzy Sets [91–93], HyperFuzzy Sets [60, 108], Picture Fuzzy Sets [22, 54], Hesitant Fuzzy Sets [118, 119], spherical fuzzy sets [6, 66], Neutrosophic Sets [102, 107], Quadripartitioned Neutrosophic Sets [62, 127], HyperPlithogenic Sets [37, 38], and Plithogenic Sets [36, 42, 103]. These advanced frameworks are expected to significantly enhance the expressive power and practical applicability of hypergraph-based models, particularly in capturing complex, multi-level, and hierarchical uncertainty across a variety of domains.

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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