Hypergraph and Superhypergraph Approaches in Electronics: A Hierarchical Framework for Modeling Power-Grid Hypernetworks and Superhypernetworks

Abstract

Graphs are fundamental tools for modeling pairwise relationships in complex systems. However, many real-world networks—such as energy infrastructures—require more expressive frameworks to capture multi-node and hierarchical interactions. A *hypergraph* extends the classical graph by allowing edges, called *hyperedges*, to connect multiple vertices simultaneously. Building upon this, a *superhypergraph* introduces recursively nested powerset structures, enabling the modeling of layered and self-referential relationships.

In the context of network science, these extensions give rise to *hypernetworks* and *superhypernetworks*, which generalize classical networks to capture higher-order connectivity patterns. In this study, we present formal mathematical definitions for two such advanced structures tailored to the energy domain: the *Power-Grid HyperNetwork* and the *Power-Grid SuperHyperNetwork*. These models offer a powerful new lens for representing and analyzing the complexity of modern power systems.

Keywords: Superhypergraph, Hypergraph, Power-Grid Networks, HyperNetworks, SuperHyperNetworks

1 Introduction

1.1 HyperGraph Theory and SuperHyperGraph Theory

Graph theory is a branch of mathematics concerned with the study of networks, where nodes (called vertices) are connected by links (called edges) [21,22,24]. Graphs have been extensively investigated and applied across numerous disciplines, including social science [54,58,61], artificial intelligence [13,62], graph neural networks (GNNs) [35,36,44], and general network analysis [18,43].

Mathematical structures can often be extended to hyperstructures and superhyperstructures by employing the power set and n-th iterated powerset constructions (cf. [74,76]). These extended frameworks are particularly effective for representing hierarchical and multi-layered systems in both theoretical and applied settings.

When applied to graph theory, these extensions yield two well-known generalizations: the *hypergraph* and the *superhypergraph*. A hypergraph allows each edge—known as a *hyperedge*—to connect more than two vertices simultaneously, enabling the representation of complex many-to-many relationships [11,14,52]. A superhypergraph builds upon this by incorporating recursively nested powerset structures, allowing for hierarchical and self-referential interactions among groups of hyperedges [28,72,73]. Graphs are commonly used to represent networks, and in this context, *hypernetworks* and *superhypernetworks* emerge as the network counterparts of hypergraphs and superhypergraphs, respectively.

1.2 Power-Grid Network and Power-Grid HyperNetwork

Electronics is the branch of science and engineering that studies and applies the flow and control of electric current in circuits [83,85,86]. Electrical power systems are responsible for the generation, transmission, and distribution of electricity through a network of interconnected components, ensuring consistent and reliable energy delivery to residential, commercial, and industrial users [23,26,50]. These systems are closely related to energy systems [25,51] and infrastructure networks [49,64].

Graph theory has been extensively applied in the analysis and modeling of energy systems and infrastructure networks [90,91]. A *power-grid network* provides a structural representation of electrical power systems, in which nodes correspond to generation stations, substations, or consumers, and edges represent the physical transmission lines that connect them [5,17,84]. This framework effectively captures the topological and operational characteristics essential for maintaining the stability and efficiency of electricity distribution.

It is also worth noting that related concepts such as *Smart Grid Systems* have been developed in the context of modernizing and optimizing power-grid infrastructures [7,53,63].

1.3 Our Contribution

In this study, we propose formal mathematical definitions for two higher-order generalizations of the power-grid network: the *Power-Grid HyperNetwork* and the *Power-Grid SuperHyperNetwork*. We investigate their structural properties and provide illustrative examples that highlight their expressive capabilities. These models offer a novel theoretical framework for representing complex, multi-way, and hierarchical interactions within power systems. While this work remains theoretical in scope, we hope it will stimulate further research into practical applications in real-world energy infrastructure and smart grid systems.

2 Preliminaries and Definitions

This section provides an overview of the fundamental concepts and definitions essential for the discussions in this paper.

2.1 Classical Structure, Hyperstructure, and n-Superhyperstructure

A *Classical Structure* represents a general mathematical concept, while a *Hyperstructure* can be defined using the power set, and an *n-Superhyperstructure* can be defined using the n-th powerset [75]. Intuitively, the n-th powerset is a repeated application of the powerset operation. Relevant definitions and simple examples are provided below.

Definition 2.1 (Base Set)**.** A *base set* S is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of S.

Definition 2.2 (Powerset)**.** [27, 66] The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 2.3 (n-th Powerset)**.** (cf. [27, 75])

The n-th powerset of a set H, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the powerset of H with the empty set removed.

Example 2.4 (Hierarchical Bus Clustering via n-th Powerset)**.** Let the base set of buses in a simple power grid be

$$H = \{b_1, b_2, b_3\},$$

where each b_i denotes a bus node. Then the first powerset is

$$P_1(H) = \big\{ \{b_1\}, \{b_2\}, \{b_3\}, \{b_1, b_2\}, \{b_1, b_3\}, \{b_2, b_3\}, \{b_1, b_2, b_3\} \big\}.$$

We interpret each element of $P_1(H)$ as a subnetwork of connected buses (for example, a single transmission corridor or loop).

Next, the second powerset is

$$P_2(H) = P\big(P_1(H)\big),$$

whose elements are sets of subnetworks. For instance,

$$U_1 = \big\{\{b_1, b_2\}, \{b_2, b_3\}\big\}, \quad U_2 = \big\{\{b_1\}, \{b_1, b_3\}\big\} \in P_2(H).$$

Here, U_1 represents a cluster of two parallel transmission paths, and U_2 groups together a single-line segment with a two-bus loop.

Continuing to the third level, the third powerset is

$$P_3(H) = P\big(P_2(H)\big),$$

which clusters collections of bus-subnetwork clusters. For example,

$$W = \{\, U_1,\, U_2 \} \;\in\; P_3(H)$$

models the joint interaction of the two distinct subnetwork clusters, such as simultaneous contingencies on both clusters.

This n-th powerset construction provides a systematic way to move from individual buses ($P_0(H) = H$) to subnetworks ($P_1(H)$), to clusters of subnetworks ($P_2(H)$), and beyond, enabling multi-level analysis of hierarchical dependencies and contingency scenarios in power-grid systems.

Definition 2.5 (Classical Structure)**.** (cf. [69,75]) A *Classical Structure* is a mathematical framework defined on a non-empty set H, equipped with one or more *Classical Operations* that satisfy specified *Classical Axioms*. Specifically:

A *Classical Operation* is a function of the form:

$$\#_0 : H^m \to H,$$

where $m \geq 1$ is a positive integer, and H^m denotes the m-fold Cartesian product of H. Common examples include addition and multiplication in algebraic structures such as groups, rings, and fields.

Definition 2.6 (Hyperoperation)**.** (cf. [81,82]) A *hyperoperation* is a generalization of a binary operation where the result of combining two elements is a set, not a single element. Formally, for a set S, a hyperoperation $\circ$ is defined as:

$$\circ : S \times S \to \mathcal{P}(S),$$

where $\mathcal{P}(S)$ is the powerset of S.

Definition 2.7 (Hyperstructure)**.** (cf. [27,69,75]) A *Hyperstructure* extends the notion of a Classical Structure by operating on the powerset of a base set. Formally, it is defined as:

$$\mathcal{H} = (\mathcal{P}(S), \circ),$$

where S is the base set, $\mathcal{P}(S)$ is the powerset of S, and $\circ$ is an operation defined on subsets of $\mathcal{P}(S)$. Hyperstructures allow for generalized operations that can apply to collections of elements rather than single elements.

Definition 2.8 (SuperHyperOperations)**.** (cf. [75]) Let H be a non-empty set, and let $\mathcal{P}(H)$ denote the powerset of H. The n-th powerset $\mathcal{P}^n(H)$ is defined recursively as follows:

$$\mathcal{P}^0(H) = H, \quad \mathcal{P}^{k+1}(H) = \mathcal{P}(\mathcal{P}^k(H)), \quad \text{for } k \geq 0.$$

A *SuperHyperOperation* of order (m, n) is an m-ary operation:

$$\circ^{(m,n)} : H^m \to \mathcal{P}_*^n(H),$$

where $\mathcal{P}_*^n(H)$ represents the n-th powerset of H, either excluding or including the empty set, depending on the type of operation:

- If the codomain is $\mathcal{P}_*^n(H)$ excluding the empty set, it is called a *classical-type* (m, n)*-SuperHyperOperation*.
- If the codomain is $\mathcal{P}^n(H)$ including the empty set, it is called a *Neutrosophic* (m, n)*-SuperHyperOperation*.

These SuperHyperOperations are higher-order generalizations of hyperoperations, capturing multi-level complexity through the construction of n-th powersets.

Definition 2.9 (n-Superhyperstructure)**.** (cf. [75]) An *n-Superhyperstructure* further generalizes a Hyperstructure by incorporating the n-th powerset of a base set. It is formally described as:

$$\mathcal{SH}_n = (\mathcal{P}_n(S), \circ),$$

where S is the base set, $\mathcal{P}_n(S)$ is the n-th powerset of S, and $\circ$ represents an operation defined on elements of $\mathcal{P}_n(S)$. This iterative framework allows for increasingly hierarchical and complex representations of relationships within the base set.

Example 2.10 (SuperHyperOperations and n-Superhyperstructure in a Simple Power-Grid)**.** Let the base set of buses be

$$S = \{b_1, b_2, b_3\}.$$

We construct the first two iterated powersets:

$$\mathcal{P}^1(S) = \{\{b_1\}, \{b_2\}, \{b_3\}, \{b_1, b_2\}, \{b_1, b_3\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}\},$$

$$\mathcal{P}^2(S) = \mathcal{P}(\mathcal{P}^1(S)),$$

whose elements are sets of subnetworks of S.

1. SuperHyperOperation of order $(2, 1)$

Define the binary SuperHyperOperation

$$\circ^{(2,1)} : S \times S \longrightarrow \mathcal{P}^1(S)$$

by letting $\circ^{(2,1)}(b_i, b_j)$ be the set of buses on the unique minimal path between b_i and b_j. Concretely:

$$\circ^{(2,1)}(b_1, b_2) = \{b_1, b_2\}, \quad \circ^{(2,1)}(b_2, b_3) = \{b_2, b_3\}, \quad \circ^{(2,1)}(b_1, b_3) = \{b_1, b_2, b_3\}.$$

Then the pair

$$\mathcal{SH}_1 = (\mathcal{P}^1(S), \circ^{(2,1)})$$

is a *Hyperstructure* on $\mathcal{P}^1(S)$.

2. SuperHyperOperation of order $(2, 2)$

Next, define the binary SuperHyperOperation

$$\circ^{(2,2)} : S \times S \longrightarrow \mathcal{P}^2(S)$$

by mapping two buses to the set of all one-hop subnetworks on their connecting path. For example:

$$\circ^{(2,2)}(b_1, b_3) = \{\{b_1, b_2\}, \{b_2, b_3\}\} \in \mathcal{P}^2(S),$$

while

$$\circ^{(2,2)}(b_1, b_2) = \{\{b_1, b_2\}\}, \quad \circ^{(2,2)}(b_2, b_3) = \{\{b_2, b_3\}\}.$$

Then

$$\mathcal{SH}_2 = (\mathcal{P}^2(S), \circ^{(2,2)})$$

is a *2-Superhyperstructure*, capturing two levels of hierarchical clustering.

Interpretation:

- $\circ^{(2,1)}$ yields subnetworks (paths) between pairs of buses.
- $\circ^{(2,2)}$ yields clusters of these subnetworks, representing all one-edge segments along a path.
- $\mathcal{SH}_1$ and $\mathcal{SH}_2$ demonstrate how power-grid elements can be modeled with increasing levels of hierarchical complexity using SuperHyperOperations and n-Superhyperstructures.

2.2 SuperHyperGraph

In classical graph theory, a hypergraph extends the idea of a conventional graph by permitting edges—called hyperedges—to join more than two vertices. This broader framework enables the modeling of more intricate relationships between elements, thereby enhancing its utility in various fields [11, 38, 39].

A *SuperHyperGraph* is an advanced extension of the hypergraph concept, integrating recursive powerset structures into the classical model. This concept has been recently introduced and extensively studied in the literature [1, 15, 34, 73].

Definition 2.11 (Hypergraph)**.** [11, 14] A *hypergraph* $H = (V(H), E(H))$ consists of:

- A nonempty set $V(H)$ of vertices.
- A set $E(H)$ of hyperedges, where each hyperedge is a nonempty subset of $V(H)$, thereby allowing connections among multiple vertices.

Unlike standard graphs, hypergraphs are well-suited to represent higher-order relationships. In this paper, we restrict ourselves to the case where both $V(H)$ and $E(H)$ are finite.

Definition 2.12 (n-SuperHyperGraph)**.** [72, 73]

Let V_0 be a finite base set of vertices. For each integer $k \geq 0$, define the iterative powerset by

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{k+1}(V_0) = \mathcal{P}(\mathcal{P}^k(V_0)),$$

where $\mathcal{P}(\cdot)$ denotes the usual powerset operation. An *n-SuperHyperGraph* is then a pair

$$\mathrm{SHT}^{(n)} = (V, E),$$

with

$$V \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}^n(V_0).$$

Each element of V is called an *n-supervertex* and each element of E an *n-superedge*.

Example 2.13 (2-SuperHyperGraph of Regional Power-Grid Clusters)**.** Let the base set of substation nodes in a regional power grid be

$$V_0 = \{ s_A,\ s_B,\ s_C,\ s_D \},$$

where each s_X denotes a geographic substation. The first iterated powerset is

$$\mathcal{P}^1(V_0) = \big\{\{s_A\}, \{s_B\}, \{s_C\}, \{s_D\}, \{s_A, s_B\}, \{s_B, s_C\}, \{s_C, s_D\}, \dots\big\},$$

each element representing a connected subnetwork of one or two substations.

The second powerset is

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big),$$

whose elements are sets of subnetworks. We select two 2-supervertices:

$$U_1 = \big\{\{s_A, s_B\},\ \{s_B, s_C\}\big\}, \quad U_2 = \big\{\{s_C, s_D\},\ \{s_A, s_D\}\big\}.$$

Here U_1 clusters the north–central corridor, and U_2 clusters the south–west ring.

We then form the 2-SuperHyperGraph

$$\mathrm{SHT}^{(2)} = \big(V,\ E\big),$$

with

$$V = \{ U_1,\ U_2 \} \ \subseteq\ \mathcal{P}^2(V_0), \quad E = \big\{\{U_1\},\ \{U_2\},\ \{U_1, U_2\}\big\} \ \subseteq\ \mathcal{P}^2(V_0).$$

Interpretation:

- Each 1-supervertex (element of $\mathcal{P}^1(V_0)$) is a physical subnetwork segment.
- Each 2-supervertex U_i groups two such segments into a higher-level corridor or ring.
- A 2-superedge $\{U_1, U_2\}$ links both clusters, representing interdependencies between the north–central and south–west corridors under contingency scenarios.
- This construction captures two hierarchical levels of power-grid connectivity, enabling analysis of both local subnetworks and their interactions at a regional scale.

2.3 Hypernetwork and n-SuperHypernetwork

Network theory has been explored and applied across a wide range of disciplines [10,12,92]. In this subsection, we present the formal definitions of the *Hypernetwork* and the *n-SuperHypernetwork*, which generalize classical network structures to capture higher-order and hierarchical relationships.

Definition 2.14 (Network)**.** A *network* (or *graph*) is an ordered triple

$$N = (V,\ E,\ w)$$

where

- V is a nonempty finite set of *vertices* (or *nodes*);
- $E \subseteq \{\{u, v\} \mid u, v \in V,\ u \neq v\}$ is the set of *undirected edges*, each joining two distinct vertices;
- $w\colon E \longrightarrow \mathbb{R}_{\geq 0}$ is a *weight function* assigning a nonnegative real weight to each edge (omitted if unweighted).

If edges are *directed*, one instead writes

$$N = (V,\ A,\ w), \quad A \subseteq V \times V,$$

and each $(u, v) \in A$ is an *arc* from u to v. In either case, one may also include an optional *vertex-labeling* $\ell_V : V \longrightarrow L_V$ to record vertex types.

Definition 2.15 (Hypernetwork)**.** (cf. [4,16,40]) A *hypernetwork* is an ordered triple

$$H = (V,\ \mathcal{E},\ w)$$

where

- V is a nonempty finite set of *nodes*;
- $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ is the set of *hyperedges*, each hyperedge $e \in \mathcal{E}$ being a nonempty subset of nodes (allowing multi-node interactions);
- $w\colon \mathcal{E} \longrightarrow \mathbb{R}_{\geq 0}$ is a *weight or attribute function* on hyperedges (omitted if unweighted).

A *directed hypernetwork* may be defined by replacing $\mathcal{E} \subseteq \mathcal{P}(V)$ with a set of *ordered* tuples of nodes or by equipping each $e \in \mathcal{E}$ with a head-tail partition. One can further add a *node-labeling* $\ell_V : V \longrightarrow L_V$ and a *hyperedge-labeling* $\ell_{\mathcal{E}} : \mathcal{E} \longrightarrow L_{\mathcal{E}}$ to record types or properties.

Example 2.16 (Massive MIMO Antenna Hypernetwork)**.** MIMO antenna systems use multiple transmit and receive antennas to improve communication capacity, reliability, and spectral efficiency in wireless networks (cf. [42,46,47]). Consider a massive MIMO base station equipped with eight antennas:

$$V = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}.$$

We form hyperedges corresponding to different beamforming subarrays, with weights given by their measured array gains (in dBi):

$$\mathcal{E} = \{e_1 = \{A_1, A_2, A_3, A_4\},\ e_2 = \{A_5, A_6, A_7, A_8\},\ e_3 = \{A_2, A_4, A_6, A_8\}\},$$

$$w(e_1) = 15, \quad w(e_2) = 17, \quad w(e_3) = 14.$$

Then the *Antenna Hypernetwork* is

$$H_{\text{MIMO}} = (V,\ \mathcal{E},\ w).$$

Interpretation:

- Nodes A_i represent individual antenna elements.
- Hyperedges e_h represent multi-antenna subarrays used to form beams.
- Weight $w(e_h)$ is the measured beamforming gain of subarray e_h.

Definition 2.17 (n-SuperHypernetwork)**.** [29] Let V_0 be a finite base set of *nodes*. Define the n-th iterated powerset recursively by

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big) \quad (k \geq 0).$$

An *n-superhypernetwork* is a tuple

$$\mathcal{N}^{(n)} \;=\; \big(V,\; \mathcal{E},\; w\big)$$

where

- $V \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-supernodes*;
- $\mathcal{E} \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-superedges*, each superedge $e \in \mathcal{E}$ being a nonempty subset of V;
- $w : \mathcal{E} \to \mathbb{R}_{\geq 0}$ is an optional *weight function* assigning a nonnegative real weight (or confidence) to each superedge.

In other words, both vertices and hyperedges of the network are drawn from the n-th powerset of the base node set, capturing up to n levels of hierarchical grouping.

Example 2.18 (2-SuperHypernetwork of Antenna Subarray Clusters)**.** Starting from the same base antennas $V_0 = \{A_1, \ldots, A_8\}$, their first powerset yields all possible subarrays; we focus on the three hyperedges from Example 2.16. The second powerset clusters these subarrays:

$$\mathcal{P}^1(V_0) \supseteq \{\, e_1, e_2, e_3\}, \quad \mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}^1(V_0)).$$

Select two 2-supernodes:

$$U_1 = \{e_1, e_2\}, \qquad U_2 = \{e_2, e_3\}.$$

These lie in $V^{(2)} \subseteq \mathcal{P}^2(V_0)$. We then form the 2-SuperHypernetwork

$$\mathcal{N}^{(2)} = \big(V^{(2)},\, \mathcal{E}^{(2)},\, w^{(2)}\big),$$

with

$$V^{(2)} = \{U_1, U_2\}, \quad \mathcal{E}^{(2)} = \big\{\{U_1\}, \{U_2\}, \{U_1, U_2\}\big\}.$$

The weight function aggregates subarray gains:

$$w^{(2)}(\{U_1\}) = \frac{w(e_1) + w(e_2)}{2} = \frac{15 + 17}{2} = 16,$$

$$w^{(2)}(\{U_2\}) = \frac{w(e_2) + w(e_3)}{2} = \frac{17 + 14}{2} = 15.5,$$

$$w^{(2)}(\{U_1, U_2\}) = \frac{w(e_1) + 2\,w(e_2) + w(e_3)}{4} = \frac{15 + 2 \cdot 17 + 14}{4} = 15.75.$$

Interpretation:

- Each 1-supernode e_h is an antenna subarray.
- Each 2-supernode U_k clusters two subarrays, representing composite beam patterns.
- Weight $w^{(2)}(U_k)$ is the average array gain of the clustered subarrays.
- The 2-superedge $\{U_1, U_2\}$ models interactions between different beamforming clusters.

2.4 Power-grid Network

A *power-grid network* models the structure of electrical power systems, where nodes represent generation stations, substations, or consumers, and edges represent physical power transmission lines.

Definition 2.19 (Power-Grid Network)**.** A *power-grid network* is a quadruple

$$\mathcal{G} = (V, E, Y, S)$$

where:

- $V = \{1, 2, \ldots, N\}$ is the set of *buses* (nodes).
- $E \subseteq \{\{i, j\} : i \neq j\}$ is the set of *branches* (transmission lines).
- $Y = [Y_{ij}] \in \mathbb{C}^{N\times N}$ is the *bus admittance matrix*, defined by

$$Y_{ij} = \begin{cases} -y_{ij}, & \{i, j\} \in E, \\ \sum_{k:\{i,k\}\in E} y_{ik} + y_i^{\text{sh}}, & i = j, \\ 0, & \text{otherwise}, \end{cases}$$

  where each branch $\{i, j\}$ has series admittance $y_{ij} = g_{ij} + jb_{ij}$ and y_i^{sh} is the shunt admittance at bus i.
- $S = (S_1, \ldots, S_N) \in \mathbb{C}^N$ is the vector of *net complex power injections*, $S_i = P_i + jQ_i$.

The voltage phasor vector $V = (V_1, \ldots, V_N)^\top \in \mathbb{C}^N$ and injection satisfy

$$I = YV, \qquad S_i = V_i I_i^* \quad (i = 1, \ldots, N),$$

which yield the *AC power-flow equations*:

$$P_i = \sum_{j=1}^{N} |V_i||V_j|\big(G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)\big),$$
$$Q_i = \sum_{j=1}^{N} |V_i||V_j|\big(G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)\big),$$

where $Y_{ij} = G_{ij} + jB_{ij}$ and $V_i = |V_i|e^{j\theta_i}$.

Example 2.20 (Three-Bus DC Approximation)**.** Consider a simple three-bus network under the DC power-flow approximation:

$$V_0 = \{1, 2, 3\}, \quad E = \big\{\{1, 2\}, \{2, 3\}, \{1, 3\}\big\},$$

with susceptances $b_{12} = b_{23} = b_{13} = 10$ pu, and power injections

$$P_1 = +1.0, \quad P_2 = -0.4, \quad P_3 = -0.6 \quad \text{(pu)},$$

assuming $|V_i| = 1$ pu and ignoring losses. Let bus 1 be the slack bus with $\theta_1 = 0$. The DC flow equations are

$$P_i = \sum_{j:\{i,j\}\in E} b_{ij}\,(\theta_i - \theta_j), \quad i = 2, 3.$$

Hence

$$\begin{cases} -0.4 = 10(\theta_2 - 0) + 10(\theta_2 - \theta_3), \\ -0.6 = 10(\theta_3 - 0) + 10(\theta_3 - \theta_2). \end{cases}$$

Solving gives $\theta_2 \approx -0.0467$ rad and $\theta_3 \approx -0.0533$ rad. The branch power flows (from i to j) are

$$P_{12} = 10(\theta_1 - \theta_2) \approx 0.467, \quad P_{23} = 10(\theta_2 - \theta_3) \approx 0.067, \quad P_{13} = 10(\theta_1 - \theta_3) \approx 0.533 \quad \text{(pu)}.$$

This three-bus example illustrates the use of graph structure, susceptance matrix, and power-flow equations to determine voltage angles and line flows.

3 Result: Power-grid Hypernetwork

In this subsection, we present the formal definitions of the Power-Grid *Hypernetwork* and the *Power-Grid n-SuperHypernetwork*.

Definition 3.1 (Power-Grid HyperNetwork)**.** Let $V = \{1, 2, \ldots, N\}$ be the set of *buses*, and let

$$\mathcal{E} = \{e_1, e_2, \ldots, e_M\} \subseteq \mathcal{P}(V)$$

be a collection of *hyperedges*, each $e_h \subseteq V$ representing a network element connecting one or more buses (e.g. a two-terminal line, a three-winding transformer, or a multi-bus substation). Define the *incidence set*

$$I = \{(v, e) \in V \times \mathcal{E} : v \in e\},$$

with attachment maps $\pi(v, e) = v$ and $e(v, e) = e$. Assign to each hyperedge e_h a *device admittance* $y_h \in \mathbb{C}$ (or a shunt admittance vector). Then the *Power-Grid HyperNetwork* is the tuple

$$\mathcal{H} = (V, \mathcal{E}, I, \pi, e, \{y_h\}_{h=1}^{M}).$$

Example 3.2 (Three-Bus HyperNetwork with a Three-Winding Transformer)**.** Three-winding transformers have three sets of windings, enabling power transfer between three voltage levels or systems with improved flexibility (cf. [56, 65, 68]). Let the set of buses be

$$V = \{1, 2, 3\}.$$

We model two transmission lines and one three-winding transformer as hyperedges:

$$\mathcal{E} = \left\{ e_{12},\ e_{23},\ e_{123} \right\} \ \subseteq\ \mathcal{P}(V),$$

where

$$e_{12} = \{1, 2\}, \quad e_{23} = \{2, 3\}, \quad e_{123} = \{1, 2, 3\}.$$

Device admittances:

- Line e_{12} has admittance $y_{12} = 0.01 - j\,0.05$ (pu).
- Line e_{23} has admittance $y_{23} = 0.015 - j\,0.045$ (pu).
- Three-winding transformer e_{123} is represented by star-equivalent admittances to a fictitious neutral:

  $$y_{1T} = 0.02 - j\,0.10, \quad y_{2T} = 0.018 - j\,0.09, \quad y_{3T} = 0.016 - j\,0.08 \quad \text{(pu)}.$$

  We collect these into the vector $y_{123} = (y_{1T},\ y_{2T},\ y_{3T}) \in \mathbb{C}^3$.

Incidence set and attachment maps:

$$I = \big\{(1, e_{12}),\ (2, e_{12}),\ (2, e_{23}),\ (3, e_{23}),\ (1, e_{123}),\ (2, e_{123}),\ (3, e_{123})\big\},$$

with $\pi(v, e) = v$ and $\epsilon(v, e) = e$.

Power-Grid HyperNetwork:

$$\mathcal{H} = \big(V,\ \mathcal{E},\ I,\ \pi,\ \epsilon,\ \{y_{12}, y_{23}, y_{123}\}\big).$$

This hypernetwork captures:

- Two-terminal devices as 2-bus hyperedges e_{12}, e_{23}.
- One multi-terminal device as the 3-bus hyperedge e_{123} with vector admittance y_{123}.

Restricting $\mathcal{H}$ to the two-bus hyperedges $\{e_{12}, e_{23}\}$ recovers the standard three-bus power-grid network $G = (V, \{e_{12}, e_{23}\})$.

Theorem 3.3 (HyperNetwork Structure)**.** *The Power-Grid HyperNetwork $\mathcal{H}$ is a finite hypernetwork: its vertex set V and hyperedge set $\mathcal{E}$ satisfy*

$$V \subseteq V, \quad \mathcal{E} \subseteq \mathcal{P}(V),$$

and the incidence relation I is exactly the membership relation of vertices in hyperedges.

Proof. By construction, each $e_h \in \mathcal{E}$ is a subset of V. The incidence set I pairs each bus $v \in V$ with exactly those hyperedges e_h that contain it. These definitions coincide with the axioms of a hypernetwork (or hypergraph with incidence), proving that $\mathcal{H}$ indeed carries the full hypernetwork structure. □

Theorem 3.4 (Reduction to Power-Grid Network)**.** *If every hyperedge $e_h \in \mathcal{E}$ has cardinality $|e_h| = 2$, then $\mathcal{H}$ reduces to a classical Power-Grid Network $\mathcal{G} = (V, E, Y, S)$ by letting*

$$E = \big\{\{i, j\} : e_h = \{i, j\}\big\}, \qquad Y_{ij} = -y_h \text{ for } e_h = \{i, j\},$$

and setting shunt injections accordingly. Conversely, any Power-Grid Network can be represented as the special case of $\mathcal{H}$ with only two-bus hyperedges.

Proof. When $|e_h| = 2$, write $e_h = \{i, j\}$. The two-terminal device admittance y_h becomes the off-diagonal bus admittance entry $Y_{ij} = -y_h$, and the diagonal entries Y_{ii}, Y_{jj} are formed by summing adjacent admittances plus shunts. All AC power-flow equations and injection definitions coincide with those of the classical $\mathcal{G} = (V, E, Y, S)$. Conversely, each branch in $\mathcal{G}$ is a two-bus hyperedge in $\mathcal{H}$. Thus $\mathcal{H}$ generalizes $\mathcal{G}$. □

Theorem 3.5 (Bus Admittance Matrix from HyperNetwork)**.** *Let $\mathcal{H} = (V, \mathcal{E}, I, \pi, \epsilon, \{y_h\})$ be a Power-Grid HyperNetwork with $N = |V|$ buses and $M = |\mathcal{E}|$ hyperedges, each carrying a complex admittance y_h. Define the incidence matrix $H \in \{0, 1\}^{N \times M}$ by*

$$H_{ih} = \begin{cases} 1, & \text{if bus } i \in e_h, \\ 0, & \text{otherwise.} \end{cases}$$

Then the nodal admittance matrix $Y \in \mathbb{C}^{N \times N}$ is given by

$$Y \;=\; H \operatorname{diag}(y_1, \dots, y_M)\, H^T.$$

Proof. By nodal analysis, each hyperedge e_h contributes admittance y_h equally between every pair of buses in e_h and adds y_h to each diagonal entry for buses in e_h. Algebraically, letting $h_h \in \{0, 1\}^N$ be column h of H, the contribution to Y from e_h is

$$y_h\, h_h\, h_h^T,$$

so summing over all hyperedges yields

$$Y \;=\; \sum_{h=1}^{M} y_h\, h_h\, h_h^T \;=\; H \operatorname{diag}(y_1, \dots, y_M)\, H^T.$$

□

Theorem 3.6 (Positive Semidefiniteness and Connectivity)**.** *Let Y be as in Theorem 3.5. Then:*

1. *Y is Hermitian and positive semidefinite.*
2. $\operatorname{rank}(Y) = N - c$, *where c is the number of connected components of the hypernetwork (viewed as a bipartite graph on $V \cup \mathcal{E}$).*

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Proof. 1. Since $\operatorname{diag}(y_1, \ldots, y_M)$ is Hermitian for real or complex admittances and H is real, $Y = H \operatorname{diag}(y) H^T$ is Hermitian. For any $x \in \mathbb{C}^N$,

$$x^* Y x = x^* H \operatorname{diag}(y) H^T x = \sum_{h=1}^{M} y_h \left|h_h^T x\right|^2 \geq 0$$

whenever $\Re(y_h) \geq 0$, proving positive semidefiniteness.

2. The nullspace of Y is $\{x : Yx = 0\} = \{x : h_h^T x = 0 \ \forall h\}$. Hence x must be constant on each connected component of the incidence bipartite graph. Therefore $\dim \ker(Y) = c$, and by the rank–nullity theorem, $\operatorname{rank}(Y) = N - \dim \ker(Y) = N - c$.

□

Theorem 3.7 (Equivalent Two-Terminal Reduction)**.** *Any Power-Grid HyperNetwork $\mathcal{H}$ can be transformed to an equivalent classical network $\mathcal{G} = (V, E, Y')$ by replacing each hyperedge e_h of size $|e_h| = k$ with a complete subgraph on the same buses, assigning each branch admittance*

$$y'_{ij} = \frac{y_h}{k-1}, \quad \textit{for all distinct } i, j \in e_h.$$

The resulting nodal admittance matrix Y' coincides with Y from Theorem 3.5.

Proof. In the complete subgraph representation, each bus $i \in e_h$ connects to each other bus $j \in e_h$ with admittance $y_h/(k-1)$. The total off-diagonal entry contributed by e_h to Y'_{ij} is $-y_h/(k-1)$, and summing contributions from all pairs within e_h yields the same net off-diagonal term as in $Y = H \operatorname{diag}(y) H^T$. Diagonal entries also match because each bus i accumulates $\sum_{j \neq i} y_h/(k-1) = y_h$. Thus $Y' = Y$. □

Definition 3.8 (Power-Grid n-Superhypernetwork)**.** Let $V_0 = \{1, 2, \ldots, N\}$ be the set of *buses*. For each $k \geq 0$, define the iterated powerset

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\big(\mathcal{P}^k(V_0)\big).$$

A *Power-Grid n-Superhypernetwork* is a tuple

$$\mathcal{N}^{(n)} = \big(V^{(n)}, \mathcal{E}^{(n)}, w\big),$$

where

- $V^{(n)} \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-supernodes*;
- $\mathcal{E}^{(n)} \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-superedges*, each $e \in \mathcal{E}^{(n)}$ a nonempty subset of $V^{(n)}$;
- $w : \mathcal{E}^{(n)} \to \mathbb{C}$ assigns to each superedge e a *generalized admittance* y_e.

Intuitively, each n-superedge models a device or subnetwork connecting multiple $(n-1)$-supernodes, capturing up to n hierarchical levels of interconnection.

Example 3.9 (2-SuperHyperNetwork of a Four-Bus Power System)**.** Let the base set of buses be

$$V_0 = \{1, 2, 3, 4\},$$

and suppose the following line admittances (in pu):

$$y_{12} = 0.01 - j0.05, \quad y_{23} = 0.015 - j0.045, \quad y_{34} = 0.012 - j0.048, \quad y_{14} = 0.02 - j0.10.$$

Iterated powersets:

$$\mathcal{P}^1(V_0) = \big\{\{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,4\}, \ldots\big\},$$

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big).$$

2-supernodes (elements of $V^{(2)} \subseteq \mathcal{P}^2(V_0)$):

$$U_1 = \{\{1,2\}, \{2,3\}\}, \qquad U_2 = \{\{3,4\}, \{1,4\}\}.$$

Here, U_1 clusters the north-east line corridor and U_2 clusters the south-west loop.

2-superedges (elements of $\mathcal{E}^{(2)}$):

$$\mathcal{E}^{(2)} = \{\{U_1\}, \{U_2\}, \{U_1, U_2\}\}.$$

Weight function $w : \mathcal{E}^{(2)} \to \mathbb{C}$:

$$w(\{U_1\}) = y_{12} + y_{23} = (0.01 - j0.05) + (0.015 - j0.045) = 0.025 - j0.095,$$

$$w(\{U_2\}) = y_{34} + y_{14} = (0.012 - j0.048) + (0.02 - j0.10) = 0.032 - j0.148,$$

$$w(\{U_1, U_2\}) = (y_{12} + y_{23}) + (y_{34} + y_{14}) = (0.025 - j0.095) + (0.032 - j0.148) = 0.057 - j0.243.$$

Power-Grid 2-SuperHyperNetwork:

$$\mathcal{N}^{(2)} = (V^{(2)}, \mathcal{E}^{(2)}, w), \quad V^{(2)} = \{U_1, U_2\}.$$

Interpretation:

- Each 1-supernode $\{i, j\}$ corresponds to a physical transmission line between buses i and j.
- Each 2-supernode U_k groups two line segments into a higher-level corridor or loop.
- The weight $w(\{U_k\})$ aggregates admittances of the underlying lines, modeling the equivalent admittance of that corridor.
- The 2-superedge $\{U_1, U_2\}$ captures interactions between the north-east corridor and the south-west loop, representing a multi-level cluster in the grid topology.

Example 3.10 (2-SuperHyperNetwork of a Five-Bus Ring System)**.** Consider a simple five-bus ring network with buses

$$V_0 = \{1, 2, 3, 4, 5\}.$$

The physical transmission lines and their per-unit admittances are:

$$y_{12} = 0.020 - j\,0.100, \quad y_{23} = 0.025 - j\,0.125, \quad y_{34} = 0.018 - j\,0.090,$$

$$y_{45} = 0.015 - j\,0.085, \quad y_{51} = 0.022 - j\,0.110.$$

First powerset (1-supernodes):

$$\mathcal{P}^1(V_0) = \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{5,1\}, \dots\}.$$

Second powerset (2-supernodes):

$$\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}^1(V_0)).$$

Select two 2-supernodes:

$$U_1 = \{\{1,2\}, \{2,3\}, \{3,4\}\}, \qquad U_2 = \{\{3,4\}, \{4,5\}, \{5,1\}\}.$$

Here, U_1 clusters the north-east corridor (buses 1–4) and U_2 clusters the south-west loop (buses 3–1).

2-superedges:

$$\mathcal{E}^{(2)} = \{\{U_1\}, \{U_2\}, \{U_1, U_2\}\}.$$

Weight function $w : \mathcal{E}^{(2)} \to \mathbb{C}$**:**

$$w(\{U_1\}) = y_{12} + y_{23} + y_{34} = (0.020 + 0.025 + 0.018) - j\,(0.100 + 0.125 + 0.090) = 0.063 - j\,0.315,$$

$$w(\{U_2\}) = y_{34} + y_{45} + y_{51} = 0.055 - j\,0.285,$$

$$w(\{U_1, U_2\}) = \sum_{(i,j)\in\{12,23,34,45,51\}} y_{ij} = 0.100 - j\,0.510.$$

Resulting 2-SuperHyperNetwork:

$$\mathcal{N}^{(2)} = (V^{(2)}, \mathcal{E}^{(2)}, w), \quad V^{(2)} = \{U_1, U_2\}.$$

Interpretation:

- Each 1-supernode $\{i, j\}$ corresponds to a single transmission line between buses i and j.
- Each 2-supernode U_k groups contiguous line segments into a higher-level corridor or loop.
- The weight $w(\{U_k\})$ is the aggregate admittance of that corridor.
- The hyperedge $\{U_1, U_2\}$ represents the interaction between the two corridors, capturing multi-scale connectivity in the ring topology.

Theorem 3.11 (Intrinsic n-Superhypernetwork Structure)**.** *The Power-Grid n-Superhypernetwork $\mathcal{N}^{(n)}$ is an n-superhypernetwork on base set V_0: its node set $V^{(n)}$ and hyperedge set $\mathcal{E}^{(n)}$ satisfy*

$$V^{(n)} \subseteq \mathcal{P}^n(V_0), \qquad \mathcal{E}^{(n)} \subseteq \mathcal{P}^n(V_0),$$

and the natural membership relation provides the incidence structure.

Proof. By definition $V^{(n)} \subseteq \mathcal{P}^n(V_0)$ and $\mathcal{E}^{(n)} \subseteq \mathcal{P}^n(V_0)$. Each superedge $e \in \mathcal{E}^{(n)}$ is by construction a nonempty subset of $V^{(n)}$. The incidence relation $I = \{(v, e) : v \in V^{(n)}, e \in \mathcal{E}^{(n)}, v \in e\}$ then exactly matches the membership relation of an n-superhypernetwork. Hence $\mathcal{N}^{(n)}$ satisfies all axioms of an n-superhypernetwork. □

Theorem 3.12 (Reduction to Power-Grid HyperNetwork)**.** *If $n = 1$, then $\mathcal{N}^{(1)} = (V^{(1)}, \mathcal{E}^{(1)}, w)$ coincides with the Power-Grid HyperNetwork defined by*

$$V^{(1)} = V_0, \qquad \mathcal{E}^{(1)} \subseteq \mathcal{P}(V_0),$$

where each hyperedge e connects one or more buses and carries weight $w(e)$ = device admittance. Conversely, any Power-Grid HyperNetwork is obtained as $\mathcal{N}^{(1)}$ for appropriate choice of $\mathcal{E}^{(1)}$ and w.

Proof. For $n = 1$, $\mathcal{P}^1(V_0) = \mathcal{P}(V_0)$. Setting $V^{(1)} = V_0$ and choosing $\mathcal{E}^{(1)} \subseteq \mathcal{P}(V_0)$ as the collection of device-connection sets yields precisely the Power-Grid HyperNetwork structure. The weight function w assigns to each hyperedge its corresponding admittance or transformer ratio. Conversely, any hypernetwork on buses with hyperedges representing multi-terminal devices matches this form for $\mathcal{N}^{(1)}$. Thus $\mathcal{N}^{(1)}$ and the Power-Grid HyperNetwork are equivalent. □

Theorem 3.13 (Skeleton Consistency)**.** *Let $\mathcal{N}^{(n)} = (V^{(n)}, \mathcal{E}^{(n)}, w)$ be a Power-Grid n-Superhypernetwork on base buses V_0. Define recursively for $k = n-1, n-2, \ldots, 0$:*

$$V^{(k)} = \bigcup_{S\in V^{(k+1)}} S, \qquad \mathcal{E}^{(k)} = \{F \subseteq V^{(k)} : F \subseteq e \text{ for some } e \in \mathcal{E}^{(k+1)}\}.$$

Then for each $0 \le k \le n$, the tuple $\mathcal{N}^{(k)} = (V^{(k)}, \mathcal{E}^{(k)}, w|_{\mathcal{E}^{(k)}})$ is itself a Power-Grid k-Superhypernetwork. In particular:

- $\mathcal{N}^{(1)}$ *coincides with the Power-Grid HyperNetwork.*
- $\mathcal{N}^{(0)}$ *reduces to the classical Power-Grid Network.*

Proof. We prove by downward induction on k. For $k = n$, $\mathcal{N}^{(n)}$ is given. Assume $\mathcal{N}^{(k+1)} = (V^{(k+1)}, \mathcal{E}^{(k+1)}, w)$ satisfies $V^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$, $\mathcal{E}^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$. Then

$$V^{(k)} = \bigcup_{S \in V^{(k+1)}} S \subseteq \mathcal{P}^k(V_0),$$

and each $F \in \mathcal{E}^{(k)}$ sits inside some $e \in \mathcal{E}^{(k+1)} \subseteq \mathcal{P}^{k+1}(V_0)$, so $F \subseteq \mathcal{P}^k(V_0)$. The restriction of w to $\mathcal{E}^{(k)}$ endows $\mathcal{N}^{(k)}$ with the same weight structure. Thus $\mathcal{N}^{(k)}$ is a valid Power-Grid k-Superhypernetwork. For $k = 1$ and $k = 0$, this recovers the hypernetwork and the classical network, respectively. □

Theorem 3.14 (Connectivity Inheritance)**.** *If the underlying Power-Grid Network $\mathcal{N}^{(0)}$ is connected, then for every $1 \leq k \leq n$, the 2-section graph of $\mathcal{N}^{(k)}$ is connected.*

Proof. The 2-section of a hypernetwork $(V^{(k)}, \mathcal{E}^{(k)})$ has vertices $V^{(k)}$ with an edge between two supernodes S, T whenever $S \cap T \neq \emptyset$. For $k = 0$, this is the original network, which is connected by hypothesis. Assume the 2-section at level $k - 1$ is connected. At level k, any two k-supernodes $S, T \in V^{(k)}$ that share a $(k - 1)$-supernode become adjacent in the 2-section. Since the $(k - 1)$ 2-section is connected, there is a sequence of overlaps linking S to T. Hence the 2-section at level k remains connected. By induction, all skeletons inherit connectivity. □

Theorem 3.15 (Subedge-Induced Subnetwork Connectivity)**.** *In $\mathcal{N}^{(n)} = (V^{(n)}, \mathcal{E}^{(n)}, w)$, each n-superedge $e \in \mathcal{E}^{(n)}$ induces a connected subnetwork in the classical Power-Grid Network on the union of its constituent base buses.*

Proof. Let $e \in \mathcal{E}^{(n)}$. By Skeleton Consistency (Theorem 3.13), e corresponds at level $n - 1$ to a collection of $(n - 1)$-supernodes whose union forms a connected 2-section at level $n - 1$. Recursively descending through levels, this ensures that the union of base-level buses in e is connected in the 0-skeleton network. Thus the induced classical subnetwork on these buses is connected. □

4 Conclusion and Future Works

In this study, we introduced formal mathematical definitions for two higher-order generalizations of power-grid systems: the *Power-Grid HyperNetwork* and the *Power-Grid SuperHyperNetwork*. These structures extend classical power-grid network models by capturing multi-node interactions and hierarchical relationships, which are critical for analyzing complex and interconnected energy infrastructures.

As future work, we aim to further enhance these models by incorporating advanced uncertainty-handling frameworks. Specifically, we plan to investigate fuzzy and neutrosophic extensions of power-grid networks using: Fuzzy Sets [88, 89], Intuitionistic Fuzzy Sets [8, 9], Vague Sets [3, 37], Rough Sets [59, 60], Soft Sets [55, 57], Bipolar Fuzzy Sets [2, 67], HyperFuzzy Sets [45, 78], Picture Fuzzy Sets [19, 41], Hesitant Fuzzy Sets [79,80], Neutrosophic Sets [70,77], Quadripartitioned Neutrosophic Sets [48,87], Pentapartitioned Neutrosophic Sets [6,20], HyperPlithogenic Sets [30–32], and Plithogenic Sets [28,33,71].

Such extensions are expected to significantly increase the expressiveness and practical relevance of hypernetwork-based models by enabling robust representation of ambiguity, variability, and multi-level uncertainty within power-grid systems.

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Conflicts of Interest

The authors confirm that there are no conflicts of interest related to the research or its publication.

Disclaimer

This work presents theoretical concepts that have not yet undergone practical testing or validation. Future researchers are encouraged to apply and assess these ideas in empirical contexts. While every effort has been made to ensure accuracy and appropriate referencing, unintentional errors or omissions may still exist. Readers are advised to verify referenced materials on their own. The views and conclusions expressed here are the authors' own and do not necessarily reflect those of their affiliated organizations.

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