Optimizing Portfolio Risk Through Diversification: Application of The Black-Litterman Model

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ABSTRACT

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| **Aim:** The study investigates how the risk reduction strength of different assets and their impact on minimizing portfolio risk. It seeks to recommend an optimal investment strategy using the Black-Litterman model to balance risk and return, helping investors make informed decisions to enhance portfolio stability and financial resilience.**Study design:** We adopt a quantitative approach, by employing the Black-Litterman model to analyze portfolio risk reduction. Monthly financial data from 2018 to 2022 is used to evaluate the impact of asset allocation on risk minimization, focusing on assessing various asset combinations to determine the most effective diversification strategy.**Place and Duration of Study:** The study took place at the Department of Mathematical Sciences, Adekunle Ajasin University, Akungba Akoko, Nigeria, where we explored data from Yahoo finance of Gold, Oil and Gas which span from 2018 to 2022.**Methodology:** Data from Yahoo Finance (2018-2022) covering Gold, Oil, and Natural Gas was analyzed. The Black-Litterman model was used to compute portfolio risk. The Augmented Dickey Fuller test verified stationary conditions of time series data before the model implementation. Mean-variance optimization techniques determined asset allocation. Various portfolios were compared to identify those with the lowest risk levels.**Results:** Gold exhibited the highest risk reduction strength (8.7%), followed by oil (8.37%) and natural gas (0.47%). Portfolios containing gold had significantly lower risk levels. The benchmark portfolio had 0.0038 risk, while portfolios excluding gold had higher risks, confirming gold’s effectiveness in minimizing overall portfolio risk.**Conclusion:** The study confirms that diversification alone does not guarantee risk minimization unless optimal asset selection is applied. Portfolios with high-risk reduction assets like gold significantly lower overall risk. Investors should prioritize assets with strong risk reduction capabilities to enhance portfolio stability, particularly during economic downturns or financial crises. |

***Keywords:*** *Portfolio, Diversification, Black Litterman, Investment, Asset, Risk, Return*

1. INTRODUCTION

Diversification can be termed as investing in several assets so as to minimize risk(s) or maximize return(s) in a portfolio. Investors use this as opportunity to develop from their small firm into several other market products [1]. The study of diversification has gained the attention of many scholars, researchers, investors, and business experts, because it is a vital area of study in business. In the case of researchers, they have studied the antecedents of diversification and its performance in economic growth [2]. While, investors explore the benefits derived from diversification by investing in several securities based on the recommendation of scholars of financial management. The advantage of investing in a diverse portfolio of securities was clearly demonstrated in a more recent study [4].

Moreover, Diversification is an approach by which firm multiply from its main business into other product market [5]. Study reveals that corporate management strongly involved diversification activities and many scholars established this fact. Diversification advances debt capacity, reduce the chances of bankruptcy by introducing new products/markets [6] and improves asset placement and productivity. A diversified firm can move funds from a cash surplus unit to a deficit unit without taxes or transaction costs. Diversified firms pool unsystematic risk and reduce the variability of operating cash flow enjoy comparative benefit in hiring because key employees may have a higher sense of job security [7]. The Black-Litterman model, introduced by [3], builds upon two fundamental theories of modern portfolio theory: the Capital Asset Pricing Model (CAPM) and Harry Markowitz's Mean-Variance Optimization (MPT). In this research, the Black-Litterman model (BLM) is employed to assess the risk and return of a portfolio. BLM determines the optimal asset allocation by integrating investor views with prior market information. It offers a structured quantitative framework for incorporating investor perspectives and merging them with a prior distribution to generate a refined combined distribution. The study utilizes monthly data on Gold, Copper, and Oil sourced from Yahoo Finance DataStream. To ensure stationarity, the Augmented Dickey-Fuller (ADF) test is applied, transforming non-stationary time series data into stationary data at the first difference. The primary objective of this paper is to examine the risk reduction potential of each asset and their respective impacts on minimizing portfolio risk. The structure of this paper is as follows: Section 2 presents a literature review, Section 3 outlines the methodology, Section 4 discusses data analysis and results, and Section 5 concludes the study.

**2. LITERATURE REVIEW**

Modern Portfolio Theory (MPT) is a financial framework designed to optimize a portfolio by maximizing expected returns while minimizing risk. The theory was first introduced by Harry Markowitz in 1952, whose groundbreaking work laid the foundation for many advancements in the field of finance. He formulated the portfolio selection problem as an optimization of the mean-variance tradeoff, emphasizing that the risk faced by investors is best measured at the portfolio level rather than at the individual asset level. Markowitz observed that a stock’s risk should not be assessed solely by its variance but also by its covariance with other assets in the portfolio. He further highlighted that an optimal portfolio should ideally consist of assets that are perfectly negatively correlated, though in practice, many assets tend to be positively correlated. This key insight led to the development of the diversification principle, which remains central to modern investment strategies [3]. A crucial contribution of Markowitz’s model was his analysis of how diversification is influenced by the number of securities in a portfolio and their covariance relationships [8]. Empirical studies have further built on this idea, using data on sectoral employment and value-added contributions to demonstrate that economic growth follows a pattern of diversification across different stages, while sectoral concentration exhibits a U-shaped relationship with per capita income [9].

Mean-variance (MV) optimization remains the most effective portfolio optimization theory; however, its practical implementation is challenging due to the extreme sensitivity of asset weights to input estimates, which are difficult to obtain accurately. Additionally, MV does not allow investors to incorporate their views on relative asset performance or express confidence in their expected returns. To address these limitations, Black and Litterman enhanced the original MV model by integrating Markowitz’s mean-variance optimization with the Capital Asset Pricing Model (CAPM) [3]. First introduced in 1990 and further refined a year later, their model introduced a strategic asset allocation framework that incorporates investor views on a global scale. Unlike CAPM, which assumes that expected returns are always in equilibrium, the Black-Litterman model accounts for deviations from the mean, recognizing that market imbalances naturally work to restore equilibrium. As a result, investors can achieve better returns by blending their market insights with equilibrium information [3]. Another crucial aspect of the Black-Litterman framework is its emphasis on risk-taking in alignment with investor views, which should only be undertaken when supported by strong evidence [10].

The Black-Litterman Model (BLM) employs a Bayesian approach to integrate investor views on the expected returns of one or more assets with the market equilibrium vector of expected returns, resulting in a new, blended estimate of expected returns. This updated return vector leads to an intuitive portfolio with reasonable asset weights [11, 17], producing more stable results compared to classical mean-variance optimization. Researchers have explored asset distribution and model simplification to enhance its applicability. For instance, [11] extended the model by applying conditional distribution theory directly to the return vector, modifying both the return vector and covariance matrix. While the mean vector of returns remained consistent with Black-Litterman’s original framework, the conditional covariance matrix was newly derived, reducing the sensitivity of mean-variance optimization to investor volatility estimates. Additionally, [12] introduced a novel method incorporating quantitative views in the form of linear inequalities into mean-variance portfolio optimization, assessing risk-adjusted measures (expected alpha) based on qualitative views that could be adjusted with confidence levels. On the other hand, [13] criticized previous BLM studies and introduced behavioral finance into the discussion, arguing that investors with a home bias tend to have lower confidence in foreign assets compared to domestic ones. This bias results in portfolio weights that remain closer to benchmark weights when compared to the allocation of domestic assets.

[14] proposed an alternative approach to determining portfolio weights by linking them to the eigenvalues of the prior covariance matrix. In contrast, [15] criticized the Alternative Reference Model, arguing that the only valid prior estimate should come from a statistical model rather than a combination of opinion and fact. Their critique was solely based on the Alternative Reference Model, which they deemed irrelevant to the canonical Reference Model, focusing instead on the fundamental statistical properties of time series. Meanwhile, [16] highlighted that investor-generated views could stem from various sources, including fundamental analysis, quantitative models, or even blind belief, emphasizing the diverse nature of inputs that influence portfolio construction.

**3. METHODOLOGY**

Here, we adopted the Mean-Variance Optimization and Black Litternan model. Consider the minimization constraint:



 (1)

where  and  is the covariance matrix with dimension  with entries given as. Together with the constraints

 (2)

 (3)

Defining the Lagrange multiplier for the constraints

 (4)

 (5)

Now, we differentiate equation (5) with respect to  gives the following first order conditions (FOC)

 (6)

 (7)

 (8)

 (9)

 (10)

Fixed points for this FOC equations can be obtained with regular methods or stiffly stable methods, see [18], [19] and [20].

The minimum variance portfolio is obtained from equations (9) and (10), and the generalized n-assets case is obtained as

 (11)

 (12)

where  covariance matrix, *ER* is ,  are scalars, are weights of assets,  are variances, and  equation (10) arbitrarily set  to any fixed value, we have 

linear equation and  unknowns, the  and . Solving the linear system of equations can be done to obtain the optimal weights for each points on the minimum variance portfolio. The estimated expected returns , standard deviations and covariances  are obtained. The optimal weight  are then substituted into  and  to give one point on the efficient frontier.

A portfolio of *n* assets is denoted by a vector  with. Let the returns of an asset be denoted by  and expected return of asset *i* be. Then the expected return vector is, *(i=1, 2,…,n)*. The covariance matrix is denoted by. The covariance of assets *i* and *j* is given as . The return  of portfolio is estimated by

 (13)

 (14)

 (15)

 (16)

The variance of return of the portfolio can be computed as:

 (17)

 (18)

 (19)

 (20)

 (21)

The expected return of equilibrium portfolio as:

 (22)

where,  is the expected return of market equilibrium, is the risk aversion,

 The equation below is known as the Black Litterman model and represents the expected return vectors that is produced from a Bayesian mixing of the implied equilibrium. Excess return vector  and the vector of investor views *Q*

 (23)

**4. DATA**

The sample data, consisting of monthly prices for Gold, Natural Gas, and Oil, was obtained from Yahoo Finance DataStream, covering the period from January 2018 to September 2022. Since the raw data was non-stationary, it was transformed into a stationary series through first differencing.

**5. RESULT AND DISCUSSION**

The asset allocation results derived from the Black-Litterman model are utilized to estimate portfolio risk and asset contributions. As previously stated, this study aims to assess the risk reduction capacity of individual assets and their impact on minimizing overall portfolio risk. The analysis focuses on three assets: Gold, Oil, and Natural Gas. Table 1 presents their respective risk reduction strengths, with Gold exhibiting the highest strength at 8.7%, followed by Oil at 8.37%, and Natural Gas at 0.47%. This indicates that Gold contributes the most to risk reduction, while Natural Gas has the least effect. The second objective of this study is to evaluate how these risk reduction strengths influence portfolio risk minimization. Table 2 categorizes portfolios based on their asset composition and associated risk levels. The benchmark portfolio, which includes all three assets, has a risk value of 0.0038. The assets are then allocated into three separate portfolios: Portfolio 1 (Gold and Oil) with a risk of 0.0085, Portfolio 2 (Gold and Natural Gas) with a risk of 0.0875, and Portfolio 3 (Oil and Natural Gas) with a risk of 0.0908. The results clearly indicate that Portfolio 1 has the lowest risk, as it comprises the two assets with the highest risk reduction strengths. Conversely, Portfolio 3, which lacks Gold—the asset with the highest risk reduction capacity—exhibits the highest risk. Table 1 summarizes the key finding that including assets with higher risk reduction strength helps minimize overall portfolio risk. However, Table 2 presents risk values (0.0038, 0.0085) that require clarification regarding their measurement type, as they could represent standard deviation, variance, or an uncertainty factor. A more detailed specification of computational procedures, along with the formulas used, would enhance readers’ understanding of the methodology applied in this study.

**Table 1:** **Assets’ Risk Reduction strength**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Gold | Oil | Natural gas |
| Risk reduction strength | 0.0870 | 0.0837 | 0.0047 |

**Table 2:** **Assets’ portfolios with corresponding risk**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Benchmark Portfolio | Portfolio1 | Portfolio2 | Portfolio3 |
|  | Gold | Gold | Gold | Oil |
|  | Oil | Oil | Oil | Natural gas |
|  | Natural gas |  |  |  |
| Portfolio risk | 0.0038 | 0.0085 | 0.0875 | 0.0908 |

 **6. CONCLUSION**

This paper proposes a method for minimizing portfolio risk by analyzing the risk reduction capacity of individual assets and their impact on overall portfolio risk. While diversification is a key strategy for risk minimization, its effectiveness depends on proper implementation. In this study, the black-litterman model (blm) was employed for asset allocation, as it is currently the most robust model in finance for this purpose. The blm results were used to estimate the risk associated with both individual assets and portfolios, leading to the conclusion that portfolio 1 is the optimal choice for rational investors. The study reveals that portfolio 1 has the lowest risk, primarily due to the presence of gold and the absence of natural gas. To effectively minimize portfolio risk, investors should first assess the risk reduction strength of each asset, then compute portfolio risk to identify the optimal investment option. Additionally, investors should prioritize assets with high risk reduction capacity, such as gold, while avoiding those with weaker risk mitigation properties, like natural gas. Based on these findings, this study affirms that gold plays a crucial role in risk reduction, making it a valuable hedge and safe haven during financial crises

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