*Original Research Article*

INVESTMENT APPROACH FOR OPTIMIZING RISK

Optimizing Portfolio Risk through Diversification: Application of the Black-Litterman Model

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ABSTRACT

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| **Aim:** The study aims to investigate the risk reduction strength of different assets and their impact on minimizing portfolio risk. It seeks to recommend an optimal investment strategy using the Black-Litterman model to balance risk and return, helping investors make informed decisions to enhance portfolio stability and financial resilience.  ***This research study mainly investigates the risk reduction strength of unique assets, analyses their impact on decreasing portfolio risk, and suggests an optimal investment strategy using the Black-Litterman model. This model combines prior market information with investor views to balance risk & return, enabling informed decision-making to boost portfolio stability and financial stability.***  **Study design:** We adopt a quantitative approach, by employing the Black-Litterman model to analyze portfolio risk reduction. Monthly financial data from 2018 to 2022 is used to evaluate the impact of asset allocation on risk minimization, focusing on assessing various asset combinations to determine the most effective diversification strategy.  ***Investors incorporated their personal opinions into the Black-Litterman model to improve asset allocation while also minimizing portfolio risk.***  **Place and Duration of Study:** The study took place at the Department of Mathematical Sciences, Adekunle Ajasin University, Akungba Akoko, Nigeria, where we explored data from Yahoo finance of Gold, Oil and Gas which span from 2018 to 2022.  **Methodology:** Data from Yahoo Finance (2018-2022) covering Gold, Oil, and Natural Gas was analyzed. The Black-Litterman model was used to compute portfolio risk. Augmented Dickey-Fuller (ADF) test ensured stationarity. Mean-variance optimization techniques determined asset allocation. Various portfolios were compared to identify those with the lowest risk levels.  ***The Augmented Dickey-Fuller test verified stationary conditions of time series data before the model implementation.***  **Results:** Gold exhibited the highest risk reduction strength (8.7%), followed by oil (8.37%) and natural gas (0.47%). Portfolios containing gold had significantly lower risk levels. The benchmark portfolio had 0.0038 risk, while portfolios excluding gold had higher risks, confirming gold’s effectiveness in minimizing overall portfolio risk.  ***All portfolios that included gold exhibited reduced risk exposure compared to ones without gold as the benchmark standard reached 0.0038 but the exclusion of gold increased risk to 0.0908 thus confirming gold's remarkable role as a risk reducer.***  **Conclusion:** The study confirms that diversification alone does not guarantee risk minimization unless optimal asset selection is applied. Portfolios with high-risk reduction assets like gold significantly lower overall risk. Investors should prioritize assets with strong risk reduction capabilities to enhance portfolio stability, particularly during economic downturns or financial crises.  ***During times of economic turmoil investors should focus on gold alongside other high-risk reduction assets in order to build a more resilient portfolio for risk reduction*.** |

*Keywords: Portfolio, Diversification, Black Litterman, Investment, Asset, Risk, Return*

1. INTRODUCTION

Diversification is investing in many assets in order to minimize risk or maximize return in the portfolio. It is an opportunity by which investors develop from his small firm into other market products [1]. Study on diversification has caught the attention of many management scholars and is one of the vital areas of study in business. Among others, researchers have studied the antecedents of diversification and the financial performance [2]. Investors indeed would explore the benefit of diversification by investing on 10 to 15 securities as suggested by scholars of financial management. The benefit of investing in a large number of securities was visibly established in a more recent study [4].

Moreover, Diversification is an approach by which firm multiply from its main business into other product market [5]. Study reveals that corporate management strongly involved diversification activities and many scholars established this fact. Diversification advances debt capacity, reduce the chances of bankruptcy by introducing new products/markets [6] and improves asset placement and productivity. A diversified firm can move funds from a cash surplus unit to a deficit unit without taxes or transaction costs. Diversified firms pool unsystematic risk and reduce the variability of operating cash flow enjoy comparative benefit in hiring because key employees may have a higher sense of job security [7]. Black Litterman model was developed by [3]. It builds on the knowledge of two main theories of modern portfolio theory, the capital asset pricing model (CAPM) and Harry Markowitz mean-variance optimization theory (MPT). Black Litterman model (BLM) is used in this research work to evaluate the risk and return of portfolio. BLM is a model that determines optimal asset allocation in a portfolio, it provides a clear way to specify investor’s views with prior information, BLM gives a quantitative framework for specifying the investor’s views, and a clear way to combine those investor’s views with an intuitive prior to arrive at a new combined distribution. The sample data is explored from monthly data of Gold, Copper and Oil from Yahoo finance DataStream. Augmented Dickey Fuller (ADF) test is adopted to transform non-stationary time series data to stationary time series data at first difference. The aim of this paper is to investigate the reduction strength of each asset and the effects of each asset in minimizing risk of portfolio. The remaining parts of this paper are organized as follow: section two reviews the literature, section three presents the methodology. Data analysis and results are considered in section four while section five concludes the paper.

**2. LITERATURE REVIEW**

Modern Portfolio theory is a finance theory that attempts to minimize risk of the portfolio and maximize portfolio expected return. Harry Markowitz (1952) was the first to discover the theory of modern portfolio. His discovery was filled with insights and ideas that anticipated many of the subsequent growth in the field. He originated a portfolio problem as a choice of the mean variance portfolio of assets. He observed that risk encountering by investors was portfolio risk which would lead to a basic and important point that the risk of a stock should not only be estimated just by the variance of the stock but also by the covariance. Moreover, he also mentioned that the best (optimal) portfolio should consist of assets that are perfectly negatively correlated. He noted that there are many perfectly positively correlated assets in circulation. This observation gives rise to the theory of diversification [3]. The most important aspect of Markowitz model was his description of the impact on portfolio diversification by the number of securities within a portfolio and their covariance relationships [8]. They used data on sectoral level of employment and value added to generate new and robust evidence that economic growth through stages of diversification and that sectoral concentration follows a U-shaped pattern in relation to per capital income [9].

It is observed that mean-variance (MV) optimization is still the best theory of portfolio optimization but it is difficult to implement in practice. The asset weights are extremely sensitive to inputs and the inputs are difficult to achieve. Furthermore through MV it is not possible for investors to express their opinions on relative asset performance and their confidence in their selected expected asset returns Black and Litterman improved on the original MV model by combining mean-variance optimization of Markowitz and CAPM [3]. The original model was first developed in 1990 and a year later they elaborate on the strategic asset allocation that is embedded with investor’s views in a global sense. The model does not consider the assumption that expected returns are always at equilibrium with CAPM. Rather as expected returns deviate from the mean, imbalances in the markets will attempt to drive them back. Therefore, it is observed that investors would make more returns by combining their views about returns with the information in the equilibrium [3] Moreover, additional vital feature of the BL structure is that investors should be willing to take risk according to their views and this should be done when they have strong evidence to support their views [10].

BLM uses Bayesian approach to syndicate the views from the investor with respect to the expected returns of one or more assets with the market equilibrium vector of expected returns to provide a new, mixed estimate of expected returns. The new vector of returns results to intuitive portfolio gives a reasonable portfolio weight [11]. Hence, the model produces better stable result than classical mean-variance optimization. Some researchers have tried to study the asset distribution and model simplification. [11] made effort to extend the model by demystifying it; they applied the conditional distribution theory straight to the return vector. The authors amended both the return vector and the covariance matrix. The outcome is the mean vector returns that are the same to those of Black litterman, whereas the conditional covariance matrix is new. It also minimizes the sensitivity of the mean variance optimization to an investor’s volatility estimates. [12] introduced new method for quantitative views, taking the form of linear inequalities attached to MV portfolio optimization. The authors evaluated the risk-adjusted measure (expected alpha) conditioned on qualitative views that in turn can be combined with a degree of confidence. [13] criticized all the previous studies on BLM and included behavioral finance in her study. She explained that an investor with home bias would have less or no confidence in the views about foreign assets than domestic assets. This would make the weights of assets closer to the benchmark weights when compared with the weights of the domestic assets.

[14] introduced another different method to measure the weight of the weights to the eigenvalues from the prior covariance matrix. [15] criticized the Alternative Reference Model. Actually, it is a combination of opinion and fact that the only valid prior estimate is from statistical model. The authors arguments only based on Alternative reference model which is not relevant to the canonical Reference Model. The focus of their article is on basic statistical properties of time series. [16] stressed that the generated views may ooze from fundamental analysis, quantitative models or blind belief.

**3. METHODOLOGY**

The methodologies adopted are Mean-Variance Optimization and Black Litternan model. Consider the following minimization constraint:



 (1)

where  and  is the  covariance matrix . The constraints are

 (2)

 (3)

For short selling, we use Lagrange multiplier for the constraints

 (4)

 (5)

Differentiating Equation (5) with respect to  gives the following first order conditions (FOC)

 (6)

 (7)

 (8)

 (9)

 (10)

For minimum variance portfolio

From equations (9) and (10), the generalized n-assets case can be written as

 (11)

 (12)

where  covariance matrix, *ER* is ,  are scalars, are weights of assets,  are variances, and  equation (10) arbitrarily set  to any fixed value, we have 

linear equation and  unknowns, the  and . These linear equations are easily solved to give the optimal weights for one point on the minimum variance portfolio. We estimate expected returns , standard deviations and covariances  . Having obtained the optimal weight  these substituted in and  to give one point on the efficient frontier.

A portfolio of *n* assets is denoted by a vector  with. Let the returns of an asset be denoted by  and expected return of asset *i* be. Then the expected return vector is, *(i=1, 2…,n)*. The covariance matrix is denoted by. The covariance of assets *i* and *j* is given as . The return  of portfolio is estimated by

 (13)

 (14)

 (15)

 (16)

The variance of return of the portfolio can be computed as:

 (17)

 (18)

 (19)

 (20)

 (21)

The expected return of equilibrium portfolio as:

 (22)

where,  is the expected return of market equilibrium, is the risk aversion,

The equation below is known as the Black Litterman model and represents the expected return vectors that is produced from a Bayesian mixing of the implied equilibrium. excess return vector  and the vector of investor views *Q*

 (23)

**4. DATA**

The sample data was explored from monthly data of Gold, Natural Gas and Oil from Yahoo finance DataStream. The data spans from January, 2018 to September, 2022. The actual data for this study are non-stationary. The non-stationary data was transformed to stationary by first differencing.

**5. RESULT AND DISCUSSION**

The assets allocations results divulge from Black Litterman model is used for estimation of portfolios’ risk and assets. As stated above that the aim of this paper is to investigate assets’ reduction strength and their effects in minimizing portfolio risk. Three assets are used in this study; Gold, Oil and Natural gas. Table 1 presents risk reduction strength of the three assets; gold possesses 8.7% strength; oil contains 8.37% strength and Natural gas has 0.47% strength. This shows that gold possesses higher risk reduction strength followed by oil. The second aim of this paper is to investigate the effects of these assets strengths in minimizing risk of portfolio. Table 2 presents portfolios with their assets and corresponding risk; Benchmark portfolio contains all the three assets used in the paper with 0.0038 risk, hence these assets are partitioned into three portfolios. Portfolio 1 contains gold and oil with 0.0085 risk, portfolio 2 comprises of gold and natural gas with 0.0875 risk while portfolio 3 consists of oil and natural gas with 0.0908 risk. It is shown vividly that portfolio 1divulges minimum risk compared with other portfolios, reason is that the two assets in the portfolio possessed higher risk reduction strength. Portfolio 3 with higher risk proved the absence of gold with highest risk reduction strength. Figure 1, clearly shows the summary of the study that if asset with higher risk reduction strength is included in a portfolio, it minimizes the portfolio’s risk.

**Table 1:** **Assets’ Risk Reduction strength**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Gold | Oil | Natural gas |
| Risk reduction strength | 0.0870 | 0.0837 | 0.0047 |

**Table 2:** **Assets’ portfolios with corresponding risk**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Benchmark Portfolio | Portfolio1 | Portfolio2 | Portfolio3 |
|  | Gold | Gold | Gold | Oil |
|  | Oil | Oil | Oil | Natural gas |
|  | Natural gas |  |  |  |
| Portfolio risk | 0.0038 | 0.0085 | 0.0875 | 0.0908 |

Table 2 presents risk values (0.0038, 0.0085) that require clarification regarding their measurement type because they could represent standard deviation, variance, or uncertainty factor. Wider specification of computational procedures together with presented formulas will help readers understand the methods used.

**6. CONCLUSION**

This paper proposed a method to minimize risk of portfolio. In view of this, investigation of risk reduction strength of each asset was carried out and the effects of each asset in minimizing risk of portfolio. Diversification is a strategic approach for minimizing risk of portfolio but if not done according to optimal method, it may not fulfill its purpose. In this study BLM was used for assets allocations being the best assets allocation model in finance at present. The results of BLM were used to estimate both risk exhibits by portfolios and assets. Hence, it is observed that portfolio1 is the best portfolio to invest for rational investors. Moreover, it is observed that portfolio1 has the lowest risk; this is as a result of the presence of gold and absence of natural gas in the portfolio. According to this study, in order to minimize risk of portfolios, there is need for investors to; first estimate the risk of each asset to know the strength of risk reduction of all the assets, second to compute risk of portfolios in order to identify the one with minimum risk to decide on the best portfolio. Moreover, investors should endeavour to add high risk reduction assets like gold to portfolios and get rid of less risk reduction asset like natural gas in order to minimize portfolio risk. In view of this fact, we wish to state that gold is of great strength in risk reduction that can serve as hedge and safe haven during financial crisis.

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