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| Journal Name: | **Asian Research Journal of Mathematics** |
| Manuscript Number: | **Ms\_ARJOM\_134080** |
| Title of the Manuscript: | **Anticenter-Symmetric Bialgebras** |
| Type of the Article | **Research paper** |

PART 1: Comments

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|  | **Reviewer’s comment**  **Artificial Intelligence (AI) generated or assisted review comments are strictly prohibited during peer review.** | **Author’s Feedback** *(Please correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)* |
| **Please write a few sentences regarding the importance of this manuscript for the scientific community. A minimum of 3-4 sentences may be required for this part.** | The paper under review explains a bialgebraic framework to define a new class of Yang— Baxter equations for anticenter-symmetric algebras where solutions these equations are available by constructive methods. In this regard, the Manin triples of anticenter-symmetric algebras is applied to equip each anticenter-symmetric algebra with a bialgebra structure which leads to an anticenter-symmetric modification of Yang—Baxter equations. Solutions of this new class of Yang—Baxter equations are studied on the basis of $\mathcal{O}$-operators and pre-anticenter-symmetric algebras. |  |
| **Is the title of the article suitable?**  **(If not please suggest an alternative title)** | **Yes** |  |

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| **Is the abstract of the article comprehensive? Do you suggest the addition (or deletion) of some points in this section? Please write your suggestions here.** | **Yes** |  |
| **Is the manuscript scientifically, correct? Please write here.** | The paper under review is an interesting research task which contains new results. The presented mathematical details are correct but the paper requires some modifications.  **Comments to Author:**  In my opinion, the paper under review **can be considered for publication after performing the following corrections or changes**.   1. Page 3, the proof for Proposition 2.4 should be added with appropriate references. 2. Page 4, an appropriate reference for Proposition 2.5 should be added to the text. 3. Page 4, the last paragraph “Then ... is called matched pair ...” should be rewritten as a Definition with an appropriate reference. 4. Page 6, appropriate references for Definition 3.1 and Definition 3.2 should be added to the text. 5. Page 12, an appropriate reference for Definition 5.1 should be added to the text. 6. Page 13, an appropriate reference for Definition 5.5 should be added to the text.   Thanks to the interconnection between Yang—Baxter equations and Rota—Baxter Lie bialgebras, the results of this research has potential to be applied for the study of integrable systems on anticenter-symmetric algebras and also, formulation of a new deformation theory for these bialgebras. In this regard, **I recommend adding** some new paragraphs together with appropriate references in terms of the following remarks.   1. Since Manin triples of Rota—Baxter Lie algebras can be characterized in terms of bialgebras, I **recommend adding** a paragraph in Introduction or Conclusion to address this note and then explain or clarify its interconnection with modified Yang—Baxter equations generated by the bialgebraic framework of this paper. In regard to this note, it is useful if the author could add the following work   , which provide the mathematical background of this idea, to the reference list.   * + C. Bai, L. Guo, G. Liu, T. Ma, Rota—Baxter Lie bialgebras, classical Yang—Baxter equations and special L-dendriform bialgebras, Algebr Represent Theor, Vol. **27**, 1347 – 1372, 2024.   https://doi.org/10.1007/s10468-024-10261-1 |  |

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|  | 1. Since the Rota-Baxter condition addresses a modified version of Yang—Baxter equations, I **recommend adding** a paragraph in Introduction or Conclusion to address the possibility of studying integrable systems at the level of anticenter-symmetric algebras. In regard to this note, it is useful if the author could add the following works, which provide the mathematical background of this idea, to the reference list.    * L. Guo, An introduction to Rota—Baxter algebra, Surveys of modern mathematics, Vol. **4**, International Press, Somerville, MA; Higher Education Press, Beijing, *2012.* https://archive.intlpress.com/site/pub/files/preview/bookpubs/00000391.pdf 2. Deformation theory of (Lie) algebras via Rota—Baxter maps, Nijenhuis relations and their applications have been vastly studied. I **recommend adding** a paragraph in Introduction or Conclusion to address the possibility of applying those deformation techniques to anticenter-symmetric algebras and their bialgebraic version. In regard to this note, it is useful if the author could add the following works, which provide the mathematical background of this idea, to the reference list.    * I. Dorfman, Dirac structures and integrability of nonlinear evolution equations. Nonlinear Science: Theory and Applications. *John Wiley & Sons, Ltd., Chichester,* 1993.   <http://lib.ysu.am/disciplines_bk/f7739dfd9bc2b90c9030fd443cf22e92.pdf>   * + K. Ebrahimi-Fard, On the associative Nijenhuis relation. *Electron. J. Combin.* **11**   (2004), no. 1, Research Paper 38, 13 pp https://doi.org/10.37236/1791   * + A. Shojaei-Fard, Application of deformed Lie algebras to non-perturbative quantum field theory, J. Indian Math. Soc. (N.S.), Vol. **84**, no. 1-2, 109—129, 2017. https://doi.org/10.18311/jims/2017/5839   + H. Lang, Y. Sheng, Factorizable Lie bialgebras, quadratic Rota—Baxter Lie algebras and Rota—Baxter Lie bialgebras, Commun. Math. Phys., Vol. **397**, 763 – 791, 2023. https://doi.org/10.1007/s00220-022-04501-y   + Gurevich, D., Rubtsov, V. (1992), Yang--Baxter equation and deformation of associative and Lie algebras. In: Kulish, P.P. (eds) Quantum Groups. Lecture Notes in Mathematics, Vol **1510**. Springer, Berlin, Heidelberg. https://doi.org/10.1007/BFb0101177   + T. Ma, L. Long, Nijenhuis operators and associative D-bialgebras, J. Algeb., Vol.   **639**, 150—186, 2024.  https://doi.org/10.1016/j.jalgebra.2023.09.047 |  |

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| **Are the references sufficient and recent? If you**  **have suggestions of additional references, please mention them in the review form.** | Some new references should be added to the paper. They are addressed in the comments to the author. |  |
| **Is the language/English quality of the article suitable for scholarly communications?** | **Yes** |  |
| **Optional/General** comments | The paper under review is an interesting research task which contains new results, however, adding some new paragraphs together with appropriate references is **recommended** to author to improve the mathematical impact of this paper. |  |

PART 2:

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|  | **Reviewer’s comment** | **Author’s comment** *(if agreed with the reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)* |
| **Are there ethical issues in this manuscript?** | *(If yes, Kindly please write down the ethical issues here in detail)*  No |  |

**Reviewer details:**

**Ali Shojaei-Fard, Iran**