Hybrid Poisson-Gaussian Stochastic Modeling for Simulating Ethereum Price Dynamics

Abstract

This study presents a novel framework for modeling and simulating the price dynamics of Ethereum through the utilization of Poisson and Gaussian processes. The method utilizes the distinctive traits of bitcoin price dynamics, which frequently experience abrupt surges and persistent variations. We establish a state-space model that incorporates a Poisson process to account for substantial price fluctuations and a Gaussian process to represent the continuous price dynamics. The model facilitates the derivation of essential statistical parameters, such as the lambda rate of significant changes, mean returns, and volatility. We substantiate our methodology by simulating future price trajectories derived from historical data, illustrating its efficacy in encapsulating the intrinsic volatility and patterns of Ethereum prices. The findings demonstrate that our approach can provide significant insight for traders and analysts in making informed judgments in the volatile cryptocurrency market.

Keywords: Ethereum, Poisson process, Gaussian process, price dynamics, statistical modeling, and simulation.

1 Introduction

In recent years, there has been a notable increase in interest in cryptocurrencies, with Ethereum becoming one of the most prominent entities in this unpredictable industry. Ethereum, as a decentralized platform facilitating smart contracts and decentralized applications (DApps), experiences price fluctuations driven by various factors, including market sentiment, regulatory changes, and technological progress (2). Comprehending these dynamics is essential for investors, traders, and policymakers alike (3).

Modeling price dynamics in financial markets has conventionally depended on stochastic processes, especially those derived from Brownian motion. Nonetheless, the distinctive features of cryptocurrency markets—such as abrupt price fluctuations and heavy-tailed distributions—frequently contravene the assumptions foundational to these models (4). Thus, there is an increasing demand for advanced statistical approaches capable of precisely elucidating the complexity of price behavior in cryptocurrencies (5).

This research seeks to tackle these problems by formulating a statistical model that integrates Poisson and Gaussian processes to examine Ethereum price movements. The Poisson process is utilized to model substantial price swings or leaps, whereas the Gaussian process represents the constant variations in pricing. This dual methodology facilitates a more thorough comprehension of Ethereum's price dynamics over time.

The objectives of this study are as follows:

1. To present a full overview of Ethereum's relevance in the cryptocurrency industry and the importance of appropriately estimating its price movements.
2. To highlight the barriers involved in forecasting bitcoin valuations and underscore the necessity for sophisticated statistical techniques.
3. To formulate and validate a statistical model employing Poisson and Gaussian processes that accurately replicates Ethereum price movements.

The rest of this paper is structured as follows: Section 2 reviews existing research. Section 3 explains the theory behind the Poisson and Gaussian processes in our model. Section 4 describes how we collected and prepared the data, fitted the model, and simulated price paths. In Section 5, we discuss the results of our analysis, and Section 6 covers what our findings mean. Finally, Section 7 wraps up with a summary of what we contributed and ideas for future research.

2 Literature Review

The modeling and forecasting of cryptocurrency price dynamics have garnered considerable attention in the past decade owing to the spectacular growth and volatility of digital assets like as Ethereum. In contrast to conventional financial markets, cryptocurrencies display distinct characteristics including significant volatility, large tails, return clustering, and sudden fluctuations, requiring advanced and hybrid modeling techniques. Among the prevalent stochastic tools, **Poisson** and **Gaussian processes** have become essential for elucidating the dual characteristics of continuous and discrete market behaviors.

2.1 Gaussian Processes in Price Modeling

Gaussian processes (GPs) are well-established in financial modeling, especially for representing the continuous and diffusive characteristics of asset price fluctuations. Gaussian models are based on the classical Geometric Brownian Motion (GBM) framework, which forms the foundation of the Black-Scholes model (14). General practitioners presume normally distributed returns and continuous-time stochastic processes, rendering them appropriate for modeling the overarching trend and daily variations of asset values.

The presumption of regularly distributed returns and stable volatility frequently proves inadequate in the realm of bitcoin markets. Research, including (15), indicates that cryptocurrencies, such as Ethereum, display non-Gaussian characteristics, such as fat tails and volatility clustering; hence, questioning the adequacy of solely Gaussian models. Notwithstanding these constraints, Gaussian processes continue to be advantageous, particularly when used with non-linear kernels in Bayesian machine learning for adaptable regression modeling (16).

2.2 Poisson Processes and Jump Dynamics

Researchers have integrated Poisson jump processes into financial models to account for sudden and discontinuous fluctuations in asset prices, frequently instigated by exogenous shocks like news events, regulatory actions, or substantial trades. Poisson processes characterize the occurrence and consequences of infrequent or distinct events and have been widely applied in jump-diffusion models, as demonstrated in the research by (17) and subsequently expanded by (18). These models combine continuous Brownian motion with a discrete Poisson process to represent both typical market behavior and abrupt fluctuations.

In the realm of Ethereum, (19) and (20) employed jump-diffusion and compound Poisson models to encapsulate the discontinuous price dynamics characteristic of cryptocurrency marketplaces. Their findings endorse the utilization of hybrid models that integrate the Gaussian foundation with Poisson jumps to more precisely represent price fluctuations. This is especially pertinent for Ethereum because of its susceptibility to protocol enhancements, decentralized application (DApp) engagement, and fluctuations in speculative demand.

2.3 Hybrid Models and Simulation Approaches

Due to the insufficiency of either Gaussian or Poisson models, hybrid models that integrate both have gained popularity. These models replicate Ethereum price dynamics by include both the steady trend (Gaussian) and abrupt fluctuations (Poisson). For instance, (21) employed GARCH-Jump models to simultaneously capture volatility and jump dynamics. Their findings demonstrated that incorporating jump intensity markedly enhances forecasting precision for cryptocurrencies.

Monte Carlo simulations are frequently utilized to examine the behavior of hybrid stochastic models under diverse settings. Ametrano et al. (22) underscored that a genuine modeling of cryptocurrency price dynamics must incorporate heteroskedasticity, jump components, and mean-reverting behavior, particularly over brief time intervals. Moreover, (23) examined the amalgamation of machine learning with stochastic simulations, demonstrating that these models surpass conventional static approaches in the unpredictable cryptocurrency environment.

2.4 Ethereum-Specific Modeling Considerations

Ethereum possesses distinct attributes in contrast to other cryptocurrencies like Bitcoin. It facilitates smart contracts, decentralized finance (DeFi) apps, and network-based usage indicators, including gas fees and transaction throughput, all of which affect its pricing. Research by (24) and (25) indicates that Ethereum has elevated informational inefficiency and heightened responsiveness to network activity, hence supporting the need for dynamic models capable of adapting to structural changes and abrupt disturbances.

Recent studies by (26) utilized non-linear time series models to analyze Ethereum price and volatility data, uncovering substantial dependencies on both external and internal cryptocurrency market components. This corresponds effectively with the application of compound stochastic processes that facilitate the modeling of such structural complexities.

3 Theoretical Framework

This section outlines the theoretical foundations that support our methodology for modeling Ethereum price movements through Poisson and Gaussian processes. These processes are essential for capturing both the discrete transitions and continuous variations inherent in cryptocurrency markets (6).

3.1 Poisson Process

The Poisson process (7) is a stochastic model that defines the occurrence of random events across time, defined by its rate parameter λ, which signifies the average number of occurrences occurring within a specified interval. The process demonstrates essential characteristics like independence, where the count of events in non-overlapping intervals is independent; stationarity, signifying that the likelihood of events relies exclusively on the duration of the interval; and memorylessness, which denotes that forthcoming events are unaffected by prior occurrences. The mathematical expression for the chance of observing k occurrences within a time interval t is as follows:

$$P(N(t) = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \tag{3.1}$$

where $N(t)$ represents the quantity of events occurring inside the time interval t. In financial environments, substantial price fluctuations can frequently be represented by a Poisson process. This study employs this paradigm to identify abrupt fluctuations in Ethereum prices, allowing us to measure both the frequency and intensity of these notable variations.

3.2 Gaussian Process

A Gaussian process comprises a set of random variables, where every finite subset adheres to a joint Gaussian distribution, defined by a mean function $m(x)$ and a covariance function $k(x, x')$. This framework facilitates the modeling of continuous functions, rendering it especially effective for representing smooth price fluctuations in financial data. The mathematical relationship between the mean and covariance can be articulated as:

$$f(x) \sim \mathcal{GP}(m(x), k(x, x')) \tag{3.2}$$

where $f(x)$ is the modeled function. Our study use Gaussian processes to model the continuous variations in Ethereum prices, accurately reflecting underlying patterns and volatility while accommodating the noise intrinsic to financial data (8; 9).

The integration of Poisson and Gaussian processes offers a robust framework for simulating Ethereum price movements. We employ the Poisson process to represent discrete price fluctuations resulting from market events, whereas the Gaussian process delineates the continuous progression of prices over time. This dual methodology improves our capacity to assess and forecast price fluctuations in a highly volatile market.

4 Methodology

This section outlines the methods utilized in this study to examine Ethereum price movements through Poisson and Gaussian processes. The procedure encompasses data acquisition, preprocessing, model testing and simulation of price trajectories.

4.1 Data Collection

The dataset utilized for this analysis comprises daily closing prices of Ethereum, obtained from Kaggle (1). The data ranges from January 1, 2016, to January 11, 2021, encompassing over 1,800 observations. Each entry includes the date and the corresponding closing price, facilitating a thorough examination of price fluctuations over time (10).

4.2 Data Preprocessing

Before analysis, we implemented multiple preprocessing measures to guarantee the validity of data and suitability for modeling. Missing values were resolved via forward-filling to ensure continuity (5). Daily returns were computed utilizing both percentage change and logarithmic approaches:

$$\text{Returns} = \frac{\text{Price}_t - \text{Price}_{t-1}}{\text{Price}_{t-1}} \tag{4.1}$$

$$\text{Log Returns} = \ln\left(\frac{\text{Price}_t}{\text{Price}_{t-1}}\right) \tag{4.2}$$

Substantial changes were identified as any return beyond an absolute threshold of 5%, and these were annotated in the dataset for subsequent analysis. Furthermore, we calculated the total number of significant changes and the interarrival intervals between these occurrences to analyze their frequency (11).

4.3 Model Fitting

The subsequent stage entailed fitting both the Poisson and Gaussian processes to the preprocessed data.

Fitting the Poisson Process: The rate parameter (λ) for the Poisson process was determined from the number of significant changes seen across the entire length of the dataset. This entailed computation:

$$\lambda = \frac{\text{Number of Events}}{\text{Total Days}} \tag{4.3}$$

This parameter allows us to model the frequency of significant price jumps.

Fitting the Gaussian Process: A rolling window approach was used to estimate the mean and standard deviation of daily returns over a specified period (default set to 30 days). We derived the annualized volatility using the fitted parameters.

$$\text{Annualized Volatility} = \sigma \times \sqrt{252} \tag{4.4}$$

where σ is the standard deviation of daily returns.

4.4 Simulation of Price Paths

To simulate future price paths, we combined both processes:

1. Generating Daily Returns: Daily returns were generated using a normal distribution characterized by the fitted mean (μ) and standard deviation (σ) obtained from the Gaussian process fitting.

2. Incorporating Poisson Jumps: We determined the number of jumps on each simulated day using a Poisson random variable with a rate parameter λ. The sizes of these jumps were drawn from a normal distribution with a specified mean and standard deviation.

3. Calculating Price Paths: The price paths were computed iteratively by applying the generated returns (both continuous and jump) to simulate future prices over a specified number of days (thus, 252 trading days) (12).

This thorough methodology enables us to accurately estimate Ethereum's price movements, encompassing both discrete jumps and continuous oscillations characteristic of cryptocurrency marketplaces.

5 Results

This section presents the results derived from fitting Poisson and Gaussian processes to the Ethereum price data, along with the consequences of simulating future price trajectories.

5.1 Model Parameters

The fitted parameters from both the Poisson and Gaussian processes provide insight into the dynamics of Ethereum prices, as summarized in Table 1.

Parameter	Value
Poisson Process Results	
Lambda (events per day)	$\lambda = 0.2308$
Average days between significant events	4.33
Gaussian Process Results	
Mean daily return	$\mu = 0.0041$
Daily volatility	$\sigma = 0.0544$
Annualized volatility	$\sigma_{\text{annualized}} = 0.8631$

Table 1: Model parameters for the Poisson and Gaussian processes applied to Ethereum price dynamics.

These parameters suggest that substantial price fluctuations transpire roughly every 4.33 days, with an average return of approximately 0.41% per day, illustrating the pronounced volatility inherent in the cryptocurrency market.

5.2 Simulation Outcomes

Utilizing the tested models, we projected future price trajectories for Ethereum over a designated timeframe of 252 trading days. The simulation entailed producing daily returns utilizing the Gaussian process and integrating discontinuous jumps represented by the Poisson process.

The simulated price trajectories demonstrated considerable volatility, indicative of both ongoing oscillations and abrupt shifts characteristic of cryptocurrency markets. For example, some simulated trajectories exhibited swift price escalations followed by corrections, reflecting the actual patterns found in Ethereum's historical price data.

5.3 Visualizations

To facilitate understanding of our findings, we provide graphical representations of the results:

- Analysis of Ethereum Price Dynamics:

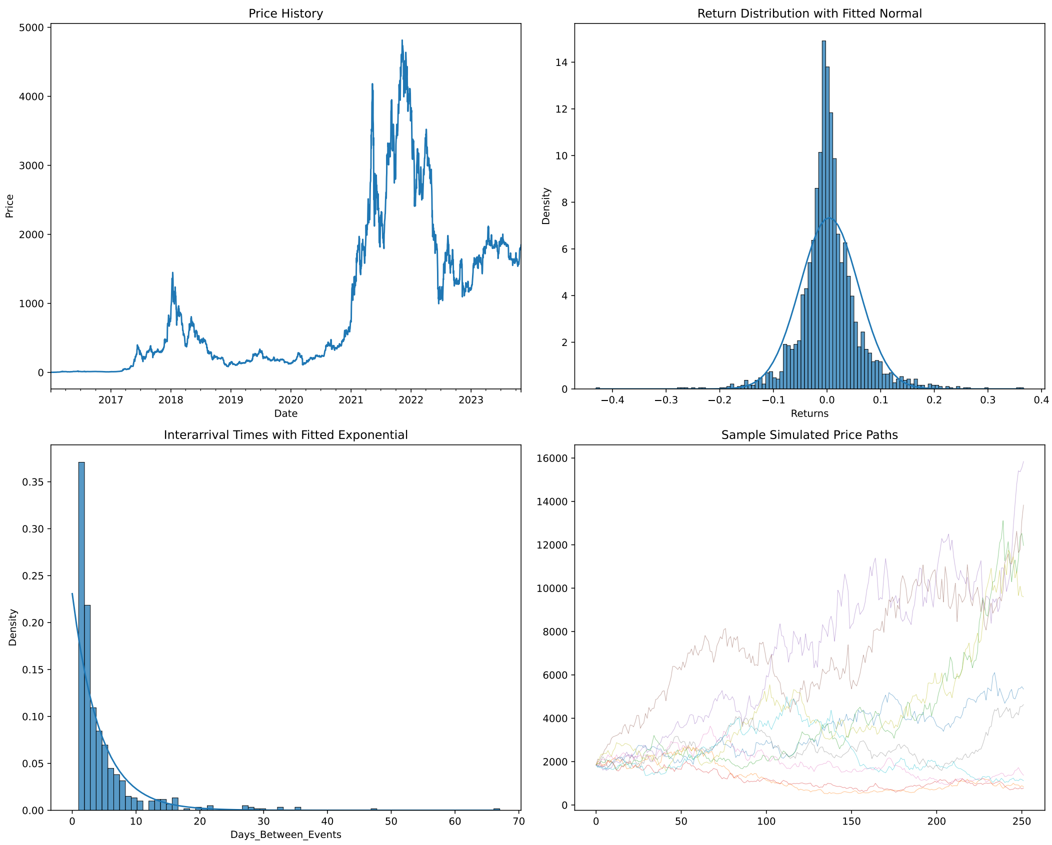

Figure 1: Analysis of Ethereum Price Dynamics

Figure 1 effectively demonstrates the suitability of integrating Gaussian and Poisson processes to explain Ethereum price patterns. The Ethereum price history from 2017 to 2023 demonstrates significant volatility and prominent price surges, particularly in 2021, suggesting speculative bubbles and market corrections. A histogram showing returns, along with a normal distribution, has a shape that is more peaked and has longer tails, which means that big price changes happen more often than a typical bell curve would suggest. This observation underscores the constraints of normality assumptions in accurately reflecting actual market behavior. The graph showing the time between significant price changes closely matches an exponential distribution, supporting the use of a Poisson process to model sudden market events. The simulated price trajectories produced by the hybrid model exhibit both continuous trends and sudden fluctuations, which are comparable to actual Ethereum price dynamics. Together, these graphs support the idea that a Poisson-Gaussian hybrid model is a better and more effective tool for predicting, managing risks, and modeling different situations in cryptocurrency markets.

6 Discussion

This section interprets the conclusions derived from our modeling and simulation of Ethereum price dynamics, compares our methodology with current models, and addresses the limits identified throughout the study.

6.1 Interpretation of Results

The estimated parameters from both the Poisson and Gaussian processes yield significant insights into Ethereum's price dynamics. The predicted lambda rate of 0.2308 signifies that substantial price fluctuations transpire approximately every 4.33 days, underscoring the volatility inherent to cryptocurrencies. The average daily return of 0.0041 indicates a slight rising trend; however, the annualized volatility of 0.8631 reflects the substantial risk involved in trading Ethereum.

The simulation results indicated that our model accurately represents both continuous variations and abrupt changes in pricing. The simulated trajectories closely mirror actual price fluctuations, affirming our selection of Poisson and Gaussian processes as appropriate for simulating Ethereum's behavior. This dual approach facilitates a more sophisticated comprehension of market dynamics, especially during times of increased volatility.

6.2 Comparison with Existing Models

Conventional financial models frequently utilize Brownian motion (13) to characterize price dynamics, presupposing normally distributed returns and continuous trajectories. This assumption neglects the substantial fluctuations and heavy-tailed distributions evident in Bitcoin markets. By integrating Poisson processes to explain abrupt price movements with Gaussian processes for continuous variations, our methodology offers a more precise depiction of Ethereum's price dynamics.

Our method is more adaptable than current models that mainly use Brownian motion or other standard random processes, allowing it to better capture the unique features of bitcoin price changes. This advantage is especially pertinent in the realm of algorithmic trading tactics that necessitate resilient models to guide decision-making in volatile markets.

6.3 Limitations

Although our modeling approach has strengths, we must acknowledge several limitations:

- Data Constraints: The analysis depends on past pricing data from a singular source, potentially introducing biases or mistakes. Various exchanges may demonstrate distinct price behaviors owing to variances in market depth and liquidity.

- Assumptions: Our model presupposes that the parameters are invariant over time, which may not be valid in swiftly evolving market conditions. Future studies may investigate time-varying parameters to improve model precision.

- Model Complexity: Although our dual-process methodology captures critical dynamics, it concurrently elevates computational complexity. The necessity for comprehensive simulations may restrict feasible implementations in real-time trading contexts.

In conclusion, although our study effectively presents a unique framework for simulating Ethereum price dynamics through Poisson and Gaussian processes, additional research is required to overcome these limitations and enhance the model's usefulness in financial markets.

7 Conclusion

This study presents a statistical framework for estimating the price dynamics of Ethereum through Poisson and Gaussian processes. Our analysis reveals that substantial price fluctuations transpire at an average frequency of roughly 0.2308 events per day, accompanied by a mean daily return of 0.0041 and an annualized volatility of 0.8631, underscoring the elevated risk associated with trading this cryptocurrency. The dual-process approach effectively includes both sudden changes and steady movements, providing a better understanding than traditional models based on Brownian motion. The ramifications of this research transcend Ethereum, since the framework can be modified for other cryptocurrencies and financial instruments exhibiting the same tendencies. Subsequent research may improve model precision by integrating time-varying factors and utilizing the methodology on high-frequency trading data. Our research enhances the literature on cryptocurrency price modeling by presenting a strong framework that tackles the distinct issues of erratic markets.

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